Degradation Data Analysis Using Wiener Process and MCMC Approach

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Abstract—**Traditional reliability assessment methods are based on lifetime data. However, the lifetime data of high reliability product are difficult to obtain even by the accelerated life test. In this paper, a very effective method is presented to assess the reliability via the degradation data of product, where the degradation path of product is characterized by mixed effect wiener process model. Considering that the mixed effect degradation model is very complicated, the Bayesian Markov Chain Monte Carlo (MCMC) method is used to obtain the unknown parameters and the corresponding reliability assessment is carried out. At last, a numerical example about laser data is given to demonstrate that degradation data can provide more information about the product than lifetime data and pseudo lifetime data.**

Index Terms—**Degradation data, Wiener process, Bayesian inference, MCMC**

I. INTRODUCTION

TRADITIONAL reliability assessment methods are focused on the use of lifetime data. For the highly reliable product, it is difficult to obtain sufficient lifetime data. Compared with the lifetime data, the degradation data can provide more life informative. Degradation, such as wear, erosion and fatigue, is very common for most mechanical products. In addition, degradation can be described by a continuous performance process in terms of time. Considering that the stochastic process model can flexibly describe the failure generating mechanisms and the operating environment characteristics, many authors have used the different stochastic processes (i.e. Markov chain, Gamma processes, and Wiener processes et al.) to model degradation data, such as Singpurwalla (1995), Cox (1999), and Aalen (2001), et al. Among those stochastic process models, Wiener process has been widely studied, such as Tseng et al. (2003), Lee and Tang (2007), Park and Padgett (2006), et al.

A well-adopted form for the Wiener process $\{X(t), t\geq 0\}$ can be expressed as

$$X(t) = \mu t + \sigma B(t) \quad (1)$$

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where $B(t)$ is the standard Brownian motion, μ is the drift degradation rate, σ is the diffusion coefficient. The wiener process has independent and normally distributed increments, i.e., $\Delta X(t) = X(t+\Delta t) - X(t)$ is independent of $X(t)$, and $\Delta X(t) \sim N(\mu\Delta t, \sigma^2\Delta t)$.

Note that the above Wiener degradation process models do not take into account the differences between individuals. In fact, the differences between individuals can not be ignored, because each item possibly experiences different sources of variations during its operation. For degradation model to be realistic, the random effects should be incorporated into the process to represent the heterogeneity. Recently, Peng and Tseng (2009), Wang (2010) and Si et al. (2012) considered the random effect Wiener process, and the reliability assessment of the performance degradation product are obtained. However, they only used the MLE method to obtain the estimation of unknown parameters, and they only utilized the current degradation data without considering the prior information about the unknown parameters.

In this paper, Wiener process model with mixed effect is proposed to characterize the performance degradation path of product, and Bayesian inference method is used to obtain the estimation of the parameters. Considering the complexity of mixed effect degradation model, the estimations of unknown parameter are obtained by the Bayesian Markov Chain Monte Carlo (MCMC) method, and goodness of fit measures is given. The results show that MCMC method is better than the MLE method, and the uncertainty is smaller than the MLE method.

II. DEGRADATION MODEL BASED ON WIENER PROCESS

Assume that the degradation path of a product is governed by Equation (1), and ξ is predefined threshold. Given the threshold value ξ , the product's lifetime T can be defined as

$$T = \inf\{t \geq 0 \mid X(0) = 0, X(t) \geq \xi\} \quad (2)$$

It is well known that the lifetime T follows an inverse Gaussian distribution with probability density function (PDF) as

$$f_T(t \mid \mu, \sigma) = \frac{\xi}{\sqrt{2\pi\sigma^2 t^3}} \exp\left(-\frac{(\xi - \mu t)^2}{2\sigma^2 t}\right) \quad (3)$$

Then, the reliability at time t can be expressed as

$$R(t \mid \mu, \sigma) = \Phi\left(-\frac{\mu t - \xi}{\sigma\sqrt{t}}\right) - \exp\left(\frac{2\mu\xi}{\sigma^2}\right)\Phi\left(-\frac{\mu t + \xi}{\sigma\sqrt{t}}\right) \quad (4)$$

To capture the difference between individual, we assume that μ and $B(t)$ are independent, and assume that μ follows $N(\eta, \sigma_\eta^2)$. Then we can get the mixed effect model as following

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$$\begin{cases} X(t) = \mu t + \sigma B(t) \\ \mu \sim N(\beta, \sigma_\beta^2) \end{cases} \tag{5}$$

where the parameter μ is a random effect representing between item variation, and σ is a fixed effect that is common to all items.

Based on the Equation (5), when the drift parameter μ is a random variable, the PDF of the lifetime T can be reconstructed by the total law of probability as follow

$$f_T(t \mid \sigma) = \int_{-\infty}^{+\infty} f_T(t \mid \mu, \sigma) \frac{1}{\sigma_\eta} \varphi\left(\frac{\mu - \eta}{\sigma_\eta}\right) d\mu$$

$$= \sqrt{\frac{\xi^2}{2\pi(\sigma^2 + \sigma_\eta^2 t)t^3}} \exp\left(-\frac{(\xi - \eta t)^2}{2(\sigma^2 t + \sigma_\eta^2 t^2)}\right) \tag{6}$$

where $\varphi(\cdot)$ is the probability density function of the standard normal distribution.

Then, the reliability at time t can be expressed as

$$R(t) = \Phi\left(-\frac{\eta t - \xi}{\sqrt{\sigma_\eta^2 t^2 + \sigma^2 t}}\right) - \exp\left(\frac{2\eta\sigma^2\xi + 2\sigma_\eta^2\xi^2}{\sigma^4}\right) \times \Phi\left(-\frac{2\sigma_\eta^2 \xi t + \sigma^2(\eta t + \xi)}{\sigma^2\sqrt{\sigma_\eta^2 t^2 + \sigma^2 t}}\right) \tag{7}$$

where $\Phi(\cdot)$ is the distribution function of the standard normal distribution.

III. BAYESIAB INFERENCE AND MCMC APPROACH

The Bayesian inference is a method of estimating the unknown parameters of a given distribution by combining the previous knowledge of these parameters with the new information contained in the observed data. The previous information of these parameters is reflected by the prior distribution, and the new information is incorporated through the likelihood function, then the posterior distribution is obtained about the unknown parameters. The likelihood function, prior and posterior distributions are described in the following sections.

A. Likelihood functions of the unknown parameters

Let $X_i(t_{ij})$ denote degradation measurements of product i at time t_{ij}, for i = 1,2,…,N, j = 1,2, … , M. In general, the degradation data can be expressed as the following form

$$X = \begin{pmatrix} X_1(t_{11}) & X_1(t_{12}) & \cdots & X_1(t_{1M}) \\ X_2(t_{21}) & X_2(t_{22}) & \cdots & X_2(t_{2M}) \\ \vdots & \vdots & \ddots & \vdots \\ X_N(t_{N1}) & X_N(t_{N2}) & \cdots & X_N(t_{NM}) \end{pmatrix} \tag{8}$$

Set

$$\Delta X_i(t_{ij}) = X_i(t_{ij}) - X_i(t_{i(j-1)}),\ t_{i0} = 0,\ X_i(t_{i0}) = 0 \tag{9}$$

According to the independent increment property of the Wiener process, $\Delta X_i(t_{ij})$ has the following distribution

$$\Delta X_i(t_{ij}) \sim N(\mu_i \Delta t_{ij}, \sigma^2 \Delta t_{ij}) \tag{10}$$

where $\Delta t_{ij} = t_{ij} - t_{i(j-1)}$, μ_i is the drift degradation rate of the product i. Therefore, the conditional PDF of $\Delta X_i(t_{ij})$ is

$$g(\Delta X_i(t_{ij}) \mid \mu_i) = \frac{1}{\sqrt{2\pi\sigma^2\Delta t_{ij}}} \exp\left(-\frac{(\Delta X_i(t_{ij}) - \mu_i \Delta t_{ij})^2}{2\sigma^2 \Delta t_{ij}}\right) \tag{11}$$

When the uncertainty of μ_i is taken into account, the PDF of $\Delta X_i(t_{ij})$ is given by

$$g(\Delta X_i(t_{ij})) = \frac{1}{\sqrt{2\pi\Delta t_{ij}(\sigma_\beta^2 \Delta t_{ij} + \sigma^2)}} \exp\left(-\frac{(\Delta X_i(t_{ij}) - \mu_\beta \Delta t_{ij})^2}{2\Delta t_{ij}(\sigma_\beta^2 \Delta t_{ij} + \sigma^2)}\right) \tag{12}$$

and the cumulative distribution function (CDF) of $\Delta X_i(t_{ij})$ is

$$G(\Delta X_i(t_{ij})) = \Phi\left(\frac{\Delta X_i(t_{ij}) - \mu_\beta \Delta t_{ij}}{\sqrt{2\pi\Delta t_{ij}(\sigma_\beta^2 \Delta t_{ij} + \sigma^2)}}\right) \tag{13}$$

where $\Phi(\cdot)$ is the distribution function of the standard normal distribution.

Then, the log-likelihood function of unknown parameters $\mu_\beta, \sigma_\beta^2$ and σ_B^2 can be given as

$$L(\mu_\beta, \sigma_\beta, \sigma_B \mid X) = \prod_{i=1}^{N}\prod_{j=1}^{M} \frac{1}{\sqrt{2\pi\Delta t_{ij}(\sigma_\beta^2 \Delta t_{ij} + \sigma^2)}} \exp\left(-\frac{(\Delta X_i(t_{ij}) - \mu_\beta \Delta t_{ij})^2}{2\Delta t_{ij}(\sigma_\beta^2 \Delta t_{ij} + \sigma^2)}\right) \tag{14}$$

B. Prior distribution

For simplicity, we assume that the unknown parameter μ_β has normal prior distribution. Considering that the unknown parameters σ_β^2 and σ_B^2 are positive quantities, a natural choice for the prior of each parameter has gamma prior, then

$$\begin{aligned} \pi_1(\mu_\beta) &\propto \frac{1}{\theta_\beta} \exp\left(-\frac{(\mu_\beta - \theta_1)^2}{2\theta_\beta^2}\right) \\ \pi_2(\sigma_\beta^2) &\propto (\sigma_\beta^2)^{-(\gamma_2+1)} \exp(-\theta_2 / \sigma_\beta^2) \\ \pi_3(\sigma_B^2) &\propto (\sigma_B^2)^{-(\gamma_3+1)} \exp(-\theta_3 / \sigma_B^2) \end{aligned} \tag{15}$$

where θ_β, θ_1, γ_2, θ_2, γ_3 and θ_3 are chosen to reflect prior knowledge about unknown parameters $\mu_\beta, \sigma_\beta^2$ and σ_B^2. Note that when $\gamma_i = \theta_i = 0$, $i = 2,3$, it is corresponding to the case of non-informative priors.

C. Posterior distribution

Let $\eta = (\mu_\beta, \sigma_\beta^2, \sigma_B^2)$ denote the unknown parameters and X denote the degradation data. The joint posterior distribution $\pi(\eta \mid X)$ is obtained by combining the joint prior distribution of η with the likelihood $L(X \mid \eta)$ according to Bayes' theorem

$$\pi(\eta \mid X) = \frac{\pi(\eta)L(X \mid \eta)}{\int \pi(\eta)L(X \mid \eta)d\eta} \propto \pi(\eta)L(X \mid \eta) \qquad (16)$$

where

$$L(\eta \mid X) = \prod_{i=1}^{N}\prod_{j=1}^{M} \frac{1}{\sqrt{2\pi\Delta t_{ij}(\sigma_\beta^2 \Delta t_{ij} + \sigma^2)}} \exp\left(-\frac{(\Delta X_i(t_{ij}) - \mu_\beta \Delta t_{ij})^2}{2\Delta t_{ij}(\sigma_\beta^2 \Delta t_{ij} + \sigma^2)}\right)$$

$, \pi(\eta) = \pi_1(\mu_\beta)\pi_2(\sigma_\beta^2)\pi_3(\sigma_B^2)$

Considering that the joint posterior distribution is very complicated, the MCMC simulation techniques implemented in this study to numerically evaluate the posterior distributions of the parameters.

D. MCMC approach

MCMC approach is a simulation technique when the analytical posterior distribution is difficult to be computed. A Markov chain is generated by sampling the current point based on the previous one. MCMC method works successfully in Bayesian computing. By using MCMC method, it is possible to generate samples from the posterior distribution and to use these samples to estimate the desired features of the posterior distribution. The MCMC techniques, including the Metropolis–Hastings (M–H) algorithm [18, 19] and the Gibbs sampler [20, 21] have become very popular methods for generating a sample from a complicated model in recent years.

The Gibbs sampler is a special case of MCMC algorithm. It generates a sequence of samples from the full conditional probability distributions of two or more random variables. Gibbs sampling requires decomposing the joint posterior distributions into full conditional distributions for each parameter in the model and then sampling from them. From Equation (16), we know that

$$\pi(\eta \mid X) \propto L(\mu_\beta, \sigma_\beta, \sigma_B \mid X)\pi_1(\mu_\beta \mid \gamma_1, \theta_1)\pi_2(\sigma_\beta^2 \mid \gamma_2, \theta_2)\pi_3(\sigma_B^2 \mid \gamma_3, \theta_3) \quad (17)$$

Based on the Equation (17), the posterior inference for parameters can be obtained, but it is not easy to get the detailed results. Therefore, the MCMC method with the Gibbs sampler to carry out Bayesian inference is used for the model parameters. Let $(-j)$ denote some vector without the jth component. Then the full conditional can be written as

$$\pi(\mu_\beta \mid \mu_\beta^{(-j)}, \sigma_\beta, \sigma_B) \propto L(\mu_\beta, \sigma_\beta, \sigma_B \mid X)\frac{1}{\theta_\beta}\exp\left(-\frac{(\mu_\beta - \theta_1)^2}{2\theta_\beta^2}\right)$$

$$\pi(\sigma_\beta \mid \mu_\beta, \sigma_\beta^{(-j)}, \sigma_B) \propto L(\mu_\beta, \sigma_\beta, \sigma_B \mid X)(\sigma_\beta^2)^{-(\gamma_2+1)}\exp(-\theta_2 / \sigma_\beta^2) \quad (18)$$

$$\pi(\sigma_B \mid \mu_\beta, \sigma_\beta, \sigma_B^{(-j)}) \propto L(\mu_\beta, \sigma_\beta, \sigma_B \mid X)(\sigma_B^2)^{-(\gamma_3+1)}\exp(-\theta_3 / \sigma_B^2)$$

We used the Bayesian software package OpenBUGS [23] to carry out the Gibbs sampling, after which we estimated the model parameters.

IV. Numerical Example

In this section, a numerical example about laser data[1] is used to demonstrate the validity of the proposed method and results. The performance characteristic of a laser device represents its operating current. When the operating current reaches at a predefined threshold level, this device is considered to be failed. Table I shows the plot of operating current over time for 15 tested units. The measured frequency of its current is 250 hours, and the experiment is terminated at 4000 hours. For example, the degradation response for the 10th unit, whose degradation is the fastest and observed every 250 hours from 0 to 4,000 hours, is 0.00, 0.41, 1.49, 2.38, 3.00, 3.84, 4.50, 5.25, 6.26, 7.05, 7.80, 8.32, 8.93, 9.55, 10.45, 11.28, 12. 21. The failure threshold ξ is 10. That is, although a laser is working at 10, it is still perceived as being failed.

From the Table I, the degradation curves of the lasers are approximate linear and there are obvious different degradation path of all test units, therefore, we use the mixed effect Wiener process with linear drift model to fit the degradation data. Pseudo lifetimes can be obtained by fitting lines to each degradation curve and calculating the times when the fitted lines reach the failure threshold (Meeker and Escobar, 1998; Tseng, Hamada, and Chiao, 1995).If the degradation path is described by the Wiener process, the pseudo lifetimes are follow the inverse Gaussian distribution.

TABLE I
The laser data

	Operating current						
	0	250	500		3500	3750	4000
1	0	0.47	0.93	…	9.49	9.87	10.94
2	0	0.71	1.22	…	8.42	8.91	9.28
3	0	0.71	1.17	…	6.02	6.45	6.88
4	0	0.36	0.62	…	5.61	5.95	6.14
5	0	0.27	0.61	…	6.32	7.10	7.59
6	0	0.36	1.39	…	9.95	10.4	11.0
7	0	0.36	0.92	…	5.57	6.1	7.17
8	0	0.46	1.07	…	5.46	5.81	6.24
9	0	0.51	0.93	…	6.84	7.20	7.88
10	0	0.41	1.49	…	10.45	11.28	12.21
11	0	0.44	1.00	…	6.54	6.96	7.42
12	0	0.39	1.80	…	6.99	7.37	7.88
13	0	0.30	0.74	…	7.39	7.85	8.09
14	0	0.44	0.70	…	6.14	6.51	6.88
15	0	0.51	0.83	…	5.84	6.16	6.62

A. Parameters estimation and data analysis

Firstly, we use the mixed effects model to fit the degradation data. Based on the MCMC method, we can get the Bayesian estimation of the unknown parameters. Table II presents posterior estimator summaries for μ_β, σ_β, σ_B based on the 50,000 samples, including the mean and standard deviation, as well as the 0.025, 0.050, 0.500, 0.950, 0.975 quantiles.

To test the goodness-of-fit, we firstly obtain each unit's pseudo failure time which is the time of degradation path to threshold ξ =10. The empirical CDF and the CDF obtained

TABLE II
Posterior estimator summaries based on degradation data

Parameter	Mean	SD	Quantiles				
			0.025	0.050	0.500	0.950	0.975
μ_β	0.002052	0.0001087	0.001839	0.001874	0.002052	0.002232	0.002265
σ_β	0.000378	0.0002266	0.000379	0.000375	0.000378	0.000382	0.000383
σ_B	0.1123	0.0005520	0.01019	0.01034	0.01121	0.01216	0.01238
$R(4500)$	0.6542	0.09394	0.4580	0.4902	0.6597	0.8006	0.8233

from the estimated inverse Gaussian distribution are simultaneously displayed in Fig. 1. From the Figure 1, we can find that the estimated failure time distribution based on the Wiener process agrees well with the empirical distribution.

Then, we can use the posteriors estimation results to inference about the reliability function $R(t)$ at each time t. For example, from the posterior estimator summaries, the reliability at 4500 hours is 0.6542, its median value is 0.6597 and its 95% credible interval is (0.4580, 0.8233), respectively.

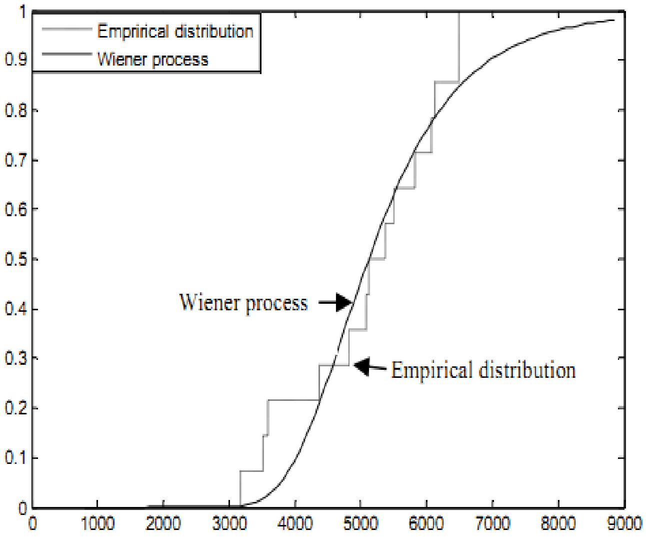

Fig.1 The empirical CDF and the CDF of the laser data.

B. *Comparison with the MLE method*

We use the laser data to compare our methods with the work of Peng and Tseng[12], in which the MLE is used to obtain the unknown parameters. For comparison, we summarize the corresponding estimation results of the parameters in the Table III.

From the Table III, we can find that our estimation results are slightly differences from the results in Ref [12]. Furthermore, we obtain the PDFs of the lifetime T at the different estimation method, as shown in Fig. 2, and the corresponding reliability curves are shown in Fig.3.

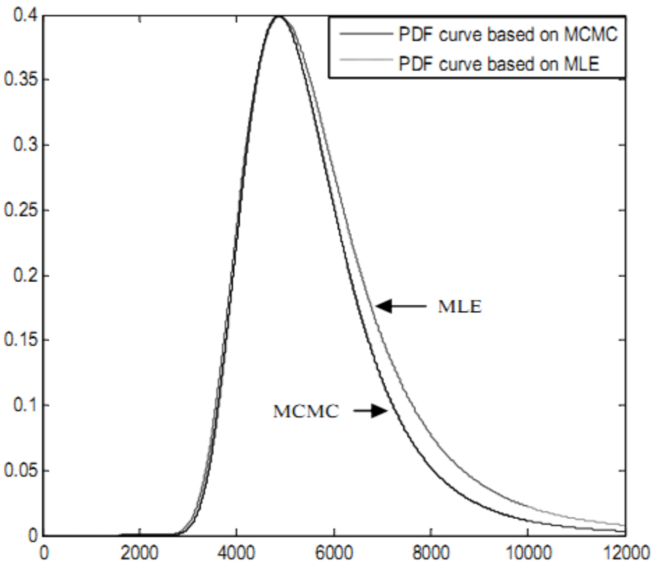

Fig.2 The PDF curves of the lifetime via different methods

From Fig. 2, we can see that the PDFs of the estimated lifetime for MCMC method and the MLE method have a little

TABLE III
THE ESTIMATION RESULTS VIA DIFFERENT METHODS

	μ_β	σ_β	σ_B
MCMC	0.002052	0.0003783	0.01123
MLE	0.002037	0.0004215	0.01012

difference, and the estimated PDF under the MLE method of the lifetime covers a wider range, that is to say its uncertainty is larger than the MCMC method.

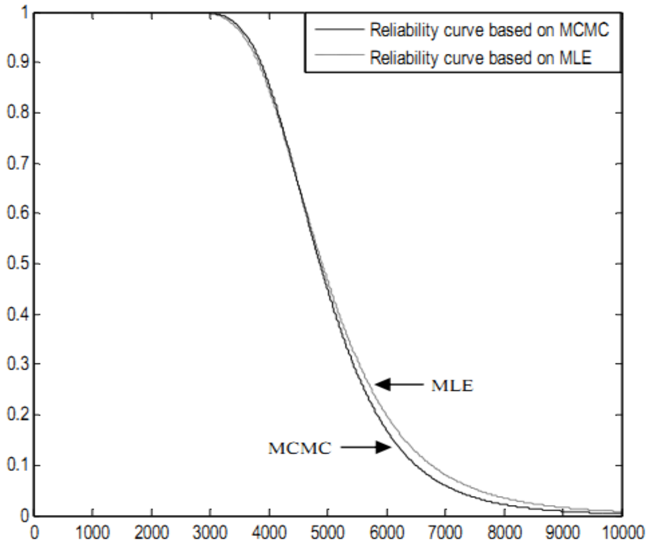

Fig.3 The reliability curves of the lifetime via different methods

C. *Comparison with the lifetime data*

In this section, by using the laser data, we compare the degradation data with the lifetime data to confirm that degradation data can provide more information about the product. When the threshold $\xi=10$, we can find that there are three units (names unit 1, unit 6 and unit 10) failed and the lifetime data consist of three interval censored observations (3750, 4000), (3500, 3750) and (3250, 3500), respectively. Note that the degradation is described by the mixed effect Wiener process, and the lifetime distribution is given by the

TABLE VI
POSTERIOR ESTIMATOR SUMMARIES BASED ON LIFETIME DATA

			Quantiles				
Parameter	Mean	SD	0.025	0.050	0.500	0.950	0.975
μ_β	0.00207	0.000184	0.001702	0.00176	0.00207	0.00237	0.00241
σ_β	0.000378	0.000242	0.000374	0.00038	0.00038	0.00038	0.00038
σ_B	0.01772	0.00174	0.01715	0.01718	0.01738	0.01957	0.02105
$R(4500)$	0.6196	0.1411	0.3395	0.3762	0.6318	0.8396	0.8705

Equation (6). A Bayesian analysis method using the same priors for μ_β, σ_β and σ_B as above, the posterior summaries presented in Table IV as follow:

Comparing the Table IV and the Table II, we can find that the 95% credible intervals of the unknown parameters under the lifetime data are wider than under the degradation data. For example, the 95% credible intervals for $R(4500)$ is now (0.3395, 0.8705). This illustrates the increased uncertainty as compared with those obtained from degradation data.

D. Comparison with the pseudo lifetime data

In this section, we compare the degradation data and the pseudo lifetime data to verify that degradation data generally can provide more information. When the threshold ξ =10, we can find that twelve units (names units 2, 3, 4, 5, 7, 8, 9, 11, 12, 13, 14, 15) do not reach the failure threshold. From Ref [1, 16], the pseudo lifetime can be obtained by the failure threshold ξ and the rate of degradation intensity $\mu_{\beta i}$. Note that the pseudo lifetime distribution is given by the Equation (6). Similarly, a Bayesian analysis method using the same priors for μ_β, σ_β and σ_B as above, the posterior summaries presented in Table V as follow:

TABLE V
POSTERIOR ESTIMATOR SUMMARIES BASED ON PSEUDO LIFETIME DATA

Parameter	Mean	SD	Quantiles				
			0.025	0.050	0.500	0.950	0.975
μ_β	0.002004	0.000139	0.001726	0.001778	0.002009	0.002234	0.002272
σ_β	0.000378	0.000002	0.000374	0.000375	0.000378	0.000382	0.000383
σ_B	0.0267	0.009887	0.01701	0.01717	0.0249	0.04478	0.04963
R(4500)	0.6506	0.0907	0.44665	0.4983	0.6537	0.7954	0.8233

Comparing the Table V and the Table II, we can find that the 95% credible intervals of the unknown parameters under the pseudo lifetime data are wider than under the degradation data. For example, the 95% credible intervals for R(4500) is now (0.44665, 0.8233). This also illustrates the increased uncertainty under the pseudo lifetime data.

V. CONCLUSION

In this paper, we have shown that the degradation data can be modeled by a Wiener process model with mixed effects, and we illustrate the advantages to assess reliability via degradation data. By using the Bayesian MCMC approach, the unknown parameters of the complicated degradation model can be obtained and the corresponding reliability assessment is carried out. At last, a numerical example about laser data is given to demonstrate that degradation data can provide more information about the product than lifetime data and pseudo lifetime data.

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