

# Pricing and Service Decisions in a Supply Chain with Fairness Reference

Shengju Sang

**Abstract**—This paper considers the pricing and retail service level decisions between one manufacturer and one retailer in a two stage supply chain. Three supply chain models including centralized decision-making without fairness preference, decentralized decision-making without fairness preference and decentralized decision-making with fairness preference are proposed, and their optimal pricing and retail service level strategies are also derived. Finally, the results of the proposed models are analyzed via a numerical example. It is found that, the service level and the utility of supply chain system in the centralized supply are higher than those in the decentralized supply chain when the manufacturer is fairness neutral, the wholesale price in the decentralized supply with fairness preference is higher than that without fairness concern, and the manufacturer's fairness preference is not only harmful to the utilities of the retailer and supply chain system, but also harmful to herself utility.

**Index Terms**—supply chain, pricing, retail service level, fairness reference

## I. INTRODUCTION

IN recent years, the retail service of the retailer is crucial for winning the market share in a supply chain. For instance, the retailer can stimulate market demand by advertising the features of the products, providing return service and shopping assistant, etc. Since the retail service also incurs significant investment, it is vital for the retailer and the manufacturer to make the optimal pricing and service decisions in the channel.

Over the past decades, many researchers have shown interest in pricing and service decisions in a supply chain. Iyer [1] analyzed the problem how the manufacturer coordinated two competing retailers in a supply chain with pricing and service competition. Tsay and Agrawal [2] showed that the manufacturer and two competing retailers could achieve coordination with price and service competition only under very limiting conditions. Xiao and Yang [3–4] took the risk of the retailers into consideration and developed the price and service competition models under demand uncertainty. Wu [5] studied the pricing and service decisions problem with one retailer and two manufacturers where one manufacturer produced the new product and the other produced the remanufactured product.

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Han et al. [6] analyzed both price competition and service competition problem between two competing retailers and one common manufacturer with the aid of the Stackelberg game. Giri and Sarker [7] studied the same problem where the demand was random but influenced by the prices and service levels of both two competing retailers. In recent years, Jena and Sarmah [8] investigated the price and service co-opetition strategies with two competing remanufacturing firms and one common retailer with uncertain demand. Sang [9] examined the pricing, service level and selling effort decisions in a decentralized supply chain where the retailer and the manufacturer pursued three different power structures. Some studies, such as Yan and Pei [10] showed that retail services not only alleviated the competition and conflict of the dual-channel, but also improve the supply chain performance in a competitive market. Dan et al. [11] studied the optimal decisions on prices and retail services in a decentralized dual-channel and a centralized supply chain. Wang and Zhao [12] applied a game-theoretical approach to study the pricing and service decisions in a dual-channel supply chain with manufacturer's direct channel service and retail service. Zhan et al. [13] developed a pricing and service competition model in O2O supply chain where the service levels in online and offline channel were different. Some researches also studied the pricing and service decisions in an uncertain environment, where the parameters of the market were considered as fuzzy variables and uncertain variables. For instance, Samadi et al. [14] considered a pricing, marketing and service planning inventory model using geometric programming in a fuzzy environment where the demand was a function of selling price, marketing and service quality. Zhao et al. [15] studied the pricing and service decisions problem with two competitive manufacturers and one common retailer where the parameters of the market demand were considered as triangular fuzzy variables. Zhao and Wang [16] used a game-theoretical approach to study the pricing and service decisions with two competing retailers and one manufacturer in a fuzzy uncertain environment. Sang [17] studied the pricing and retail service decisions in an uncertain environment where the costs and parameters of the demand were regarded as uncertain variables. Sang [18] also analyzed the pricing and service level decisions by a game-theoretical approach in an uncertain environment where the manufacturer offered the service level.

Our work is also related to previous research on fairness concerns in a supply chain. The existing research on behavioral showed that fairness strongly affected the decisions of the supply chain members in practice. Cui et al.

[19] introduced the fairness concerns of the members into supply chain and showed that the wholesale price contract could coordinate this supply chain when supply chain members were concerned about fairness in a linear demand model. Caliskan-Demirag et al. [20] extended the work of Cui et al. [19] to a nonlinear demand function and showed that the supply chain member could achieve coordination when only the retailer was fairness concerned. Yang et al. [21] investigated the cooperative advertising problem in a distribution channel with fairness concerns and analyzed the impact of the fairness concerns of retailer. Du et al. [22] studied the newsboy problem in a dyadic supply chain where both the retailer and the supplier were concerned with fairness. Ho et al. [23] studied the supply chain contract with one supplier and two retailers in which the retailer exhibited peer-induced fairness concerns and distributional fairness. Wu and Niederhoff [24] studied the effect of fairness concerns on supply chain performance in the two-party newsvendor setting. Zhang and Ma [25] considered the pricing decisions with a fair caring retailer in a dual-channel supply chain.

To our knowledge, no one has studied the pricing and service level decisions with fairness concerns. Therefore, in this paper, we consider a supply chain composing of one manufacturer and one retailer. The manufacturer dominates the channel and is fairness sensitive. However, the retailer, who is disadvantaged in transactions, is fairness neutral. We mainly studied three supply chain models including centralized decision-making without fairness preference, decentralized decision-making without fairness preference and decentralized decision-making with fairness preference of the manufacturer. We try to find how the fairness preference of the manufacturer affects the decisions of the manufacturer and the retailer.

The remainder of this paper is organized as follows. In Section II, the notations and problem related to this paper are described. Sections III, IV and V separately develop the centralized decision-making without fairness preference, the decentralized decision-making without fairness preference and the decentralized decision-making with fairness preference. The numerical example is shown in Section VI, and conclusions are given in the last section.

## II. PROBLEM DESCRIPTIONS

We consider a two stage supply chain consisting of one manufacturer, who sells her product through one retailer. The market demand faced by the retailer is sensitive to two factors: retail price and retail service level. The market demand function is decreasing in retail price  $p$  and increasing in retail service level  $s$ , and can be described as follows

$$D = \alpha - \beta p + \gamma s \quad (1)$$

where the parameter  $\alpha$  denotes the market base of the product, the parameter  $\beta$  is price elastic coefficient in restraining the market demand, and the parameter  $\gamma$  is price elastic coefficient in stimulating the market demand.

We assume that the cost of achieving retail service level requires fixed investment, which is give by  $\frac{1}{2}\theta s^2$ , where the parameter  $\theta$  is the service cost coefficient of the retailer.

Further, let  $w$  denote the wholesale price per unit charged to the retailer by the manufacturer,  $c_m$  the manufacturer's cost of producing her product, and  $c_r$  the retailer's cost of operating his product.

According to the problem descriptions, the profits of the manufacturer, the retailer and the supply chain system can be expressed as follows

$$\Pi_M = (w - c_m)(\alpha - \beta p + \gamma s) \quad (2)$$

$$\Pi_R = (p - w - c_r)(\alpha - \beta p + \gamma s) - \frac{1}{2}\theta s^2 \quad (3)$$

$$\Pi_{SC} = (p - c_m - c_r)(\alpha - \beta p + \gamma s) - \frac{1}{2}\theta s^2 \quad (4)$$

We assume that the retailer is fairness neutral and his utility  $U_R$  equals his profit  $\Pi_R$ , that is

$$U_R = \Pi_R = (p - w - c_r)(\alpha - \beta p + \gamma s) - \frac{1}{2}\theta s^2 \quad (5)$$

The manufacturer is fairness sensitive and her utility  $U_M$  is given as follows

$$U_M = \Pi_M + \lambda(\Pi_M - \bar{\Pi}_M) \quad (6)$$

where  $\lambda$  is the manufacturer's fairness concern parameter and  $\lambda \geq 0$ ,  $\bar{\Pi}_M$  denotes the manufacturer's Nash bargaining fairness reference.

Let  $\bar{\Pi}_R$  be the retailer's Nash bargaining fairness reference. Since the fairness references come from the Nash bargaining solution for the fair distribution of the supply chain system's profit between the manufacturer and the retailer, it is clear that

$$\bar{\Pi}_M + \bar{\Pi}_R = \Pi_{SC}$$

The supply chain system's utility is

$$U_{SC} = U_M + U_R \quad (7)$$

Nash bargaining solution is derived by maximizing the Nash product  $U_M \times U_R$  as the following model

$$\begin{cases} \max \varphi(\Pi_M, \Pi_R) = U_M \times U_R \\ \text{s. t. } \Pi_M + \Pi_R = \Pi_{SC} \\ \Pi_M, \Pi_R > 0 \end{cases} \quad (8)$$

Substituting  $U_M$  in (6),  $U_R = \Pi_R$ , and  $\Pi_M + \Pi_R = \Pi_{SC}$  into (8), we can have

$$\varphi(\Pi_M, \Pi_{SC}) = [\Pi_M + \lambda(\Pi_M - \bar{\Pi}_M)](\Pi_{SC} - \Pi_M)$$

The first-order and second-order derivatives of  $\varphi(\Pi_M, \Pi_{SC})$  with respect to  $\Pi_M$  are

$$\frac{\partial \varphi(\Pi_M, \Pi_{SC})}{\partial \Pi_M} = -2(1 + \lambda)\Pi_M + \lambda\bar{\Pi}_M + (1 + \lambda)\Pi_{SC}$$

$$\frac{\partial^2 \varphi(\Pi_M, \Pi_{SC})}{\partial \Pi_M^2} = -2(1 + \lambda) < 0$$

Hence,  $\varphi(\Pi_M, \Pi_{SC})$  is strictly concave with  $\Pi_M$ .

Additionally, we have  $\Pi_M = \bar{\Pi}_M$  at the equilibrium. Thus, let  $\frac{\partial \varphi(\Pi_M, \Pi_{SC})}{\partial \Pi_M} = 0$ , we can derive the Nash bargaining

solution as follows

$$\bar{\Pi}_M = \frac{1+\lambda}{2+\lambda} \Pi_{SC}$$

Then, the utility of the manufacturer is

$$U_M = (1+\lambda)\Pi_M - \frac{\lambda(1+\lambda)}{2+\lambda} \Pi_{SC} \quad (9)$$

### III. CENTRALIZED DECISION-MAKING WITHOUT FAIRNESS PREFERENCE

In the centralized supply chain, we assume both the manufacturer and the retailer are fairness-neutral, and they cooperate to determine the optimal retail price and service level to maximize the utility of the supply chain system. Thus, the profit of the supply chain can be considered as his utility, that is

$$U_{SC} = \Pi_{SC} = (p - c_m - c_r)(\alpha - \beta p + \gamma s) - \frac{1}{2}\theta s^2 \quad (10)$$

**Theorem 1.** If  $2\beta\theta - \gamma^2 > 0$ , then the optimal solutions of the supply chain system are

$$p^* = \frac{[\alpha - \beta(c_m + c_r)]\theta}{2\beta\theta - \gamma^2} + c_m + c_r$$

$$s^* = \frac{[\alpha - \beta(c_m + c_r)]\gamma}{2\beta\theta - \gamma^2}$$

**Proof.** Referring to (10), we can get the first order derivatives of  $U_{SC}$  with respect to  $p$  and  $s$  as follows

$$\frac{\partial U_{SC}}{\partial p} = -2\beta p + \gamma s + \alpha + \beta(c_m + c_r) \quad (11)$$

$$\frac{\partial U_{SC}}{\partial s} = -\theta s + \gamma p - \gamma(c_m + c_r) \quad (12)$$

Then, the second order derivatives of  $U_{SC}$  with respect to  $p$  and  $s$  can be shown as

$$\frac{\partial^2 U_{SC}}{\partial p^2} = -2\beta, \quad \frac{\partial^2 U_{SC}}{\partial p \partial s} = \gamma,$$

$$\frac{\partial^2 U_{SC}}{\partial s^2} = -\theta, \quad \frac{\partial^2 U_{SC}}{\partial s \partial p} = \gamma$$

Thus, the Hessian matrix of  $U_{SC}$  can be obtained as follows

$$H = \begin{vmatrix} \frac{\partial^2 U_{SC}}{\partial p^2} & \frac{\partial^2 U_{SC}}{\partial p \partial s} \\ \frac{\partial^2 U_{SC}}{\partial s \partial p} & \frac{\partial^2 U_{SC}}{\partial s^2} \end{vmatrix} = \begin{vmatrix} -2\beta & \gamma \\ \gamma & -\theta \end{vmatrix}$$

Note that the Hessian matrix is negative definite, since  $\beta$  and  $\gamma$  are positive variables, and  $2\beta\theta - \gamma^2 > 0$ . Consequently,  $U_{SC}$  is strictly jointly concave in  $p$  and  $s$ .

Setting (11) and (12) to zero, the first order conditions can be shown as

$$-2\beta p + \gamma s + \alpha + \beta(c_m + c_r) = 0 \quad (13)$$

$$-\theta s + \gamma p - \gamma(c_m + c_r) = 0 \quad (14)$$

Solving (13) and (14), we have

$$p^* = \frac{[\alpha - \beta(c_m + c_r)]\theta}{2\beta\theta - \gamma^2} + c_m + c_r \quad (15)$$

$$s^* = \frac{[\alpha - \beta(c_m + c_r)]\gamma}{2\beta\theta - \gamma^2} \quad (16)$$

The proof of Theorem 1 is completed.

Substituting  $p^*$  and  $w^*$  into (10), we derive the optimal profit of the supply chain system  $U_{SC}^*$  in the centralized supply chain as follows

$$U_{SC}^* = \frac{[\alpha - \beta(c_m + c_r)]^2 \theta}{2(2\beta\theta - \gamma^2)} \quad (17)$$

### IV. DECENTRALIZED DECISION-MAKING WITHOUT FAIRNESS PREFERENCE

In the decentralized supply chain, the manufacturer and the retailer make independent decisions to maximize their individual utilities. In this case, the manufacturer act as the Stackelberg leader, and the manufacturer and the retailer are supposed to be fairness-neutral. Thus, the profits of the manufacturer and the retailer equal to their utilities. Then, the utilities of the manufacturer and the retailer can be expressed as follows

$$U_M = (w - c_m)(\alpha - \beta p + \gamma s) \quad (18)$$

$$U_R = (p - w - c_r)(\alpha - \beta p + \gamma s) - \frac{1}{2}\theta s^2 \quad (19)$$

In the Stackelberg game, the manufacturer first sets the wholesale price  $w$ , and then the retailer decides the retail price and service level after observing the wholesale price.

We first derive the optimal decisions of the retailer.

**Theorem 2.** If  $2\beta\theta - \gamma^2 > 0$ , then the optimal reaction functions of the retailer with respect to the wholesale price  $w$  are

$$p^{**}(w) = \frac{[\alpha - \beta(w + c_r)]\theta}{2\beta\theta - \gamma^2} + w + c_r$$

$$s^{**}(w) = \frac{[\alpha - \beta(w + c_r)]\gamma}{2\beta\theta - \gamma^2}$$

**Proof.** Referring to (19), we can get the first order derivatives of  $U_R$  with respect to  $p$  and  $s$  as follows

$$\frac{\partial U_R}{\partial p} = -2\beta p + \gamma s + \alpha + \beta(w + c_r) \quad (20)$$

$$\frac{\partial U_R}{\partial s} = -\theta s + \gamma p - \gamma(w + c_r) \quad (21)$$

Then, the second order derivatives of  $U_R$  with respect to  $p$  and  $s$  can be shown as

$$\frac{\partial^2 U_R}{\partial p^2} = -2\beta, \quad \frac{\partial^2 U_R}{\partial p \partial s} = \gamma,$$

$$\frac{\partial^2 U_R}{\partial s^2} = -\theta, \quad \frac{\partial^2 U_R}{\partial s \partial p} = \gamma$$

Thus, the Hessian matrix of  $U_R$  can be obtained as follows

$$H = \begin{vmatrix} \frac{\partial^2 U_R}{\partial p^2} & \frac{\partial^2 U_R}{\partial p \partial s} \\ \frac{\partial^2 U_R}{\partial s \partial p} & \frac{\partial^2 U_R}{\partial s^2} \end{vmatrix} = \begin{vmatrix} -2\beta & \gamma \\ \gamma & -\theta \end{vmatrix}$$

Note that the Hessian matrix is negative definite, since  $\beta$  and  $\gamma$  are positive variables, and  $2\beta\theta - \gamma^2 > 0$ . Consequently,  $U_R$  is strictly jointly concave in  $p$  and  $s$ .

Setting (20) and (21) to zero, the first order conditions can be shown as

$$-2\beta p + \gamma s + \alpha + \beta(w + c_r) = 0 \quad (22)$$

$$-\theta s + \gamma p - \gamma(w + c_r) = 0 \quad (23)$$

Solving (22) and (23), we have

$$p^{**}(w) = \frac{[\alpha - \beta(w + c_r)]\theta}{2\beta\theta - \gamma^2} + w + c_r \quad (24)$$

$$s^{**}(w) = \frac{[\alpha - \beta(w + c_r)]\gamma}{2\beta\theta - \gamma^2} \quad (25)$$

The proof of Theorem 2 is completed.

After knowing the reaction functions of the retailer, the manufacturer would use them to maximize her utility by choosing the optimal wholesale price.

**Theorem 3.** If  $2\beta\theta - \gamma^2 > 0$ , then the optimal solutions of the manufacturer and the retailer in the decentralized supply chain are

$$w^{**} = \frac{\alpha - \beta(c_m + c_r)}{2\beta} + c_m$$

$$p^{**} = \frac{(3\beta\theta - \gamma^2)[\alpha - \beta(c_m + c_r)]\theta}{2\beta(2\beta\theta - \gamma^2)} + c_m + c_r$$

$$s^{**} = \frac{[\alpha - \beta(c_m + c_r)]\gamma}{2(2\beta\theta - \gamma^2)}$$

**Proof.** Substituting  $p^{**}(w)$  and  $s^{**}(w)$  into (18), we can get the utility of the manufacturer  $U_M$  as follows

$$U_M = \frac{\beta\theta}{2\beta\theta - \gamma^2} (w - c_m)(\alpha - \beta w - \beta c_r) \quad (26)$$

From (26), we can get the first and second order derivatives of  $U_M$  with respect to  $w$  as follows

$$\frac{\partial U_M}{\partial w} = \frac{\beta\theta}{2\beta\theta - \gamma^2} (-2\beta w + \alpha + \beta c_m - \beta c_r) \quad (27)$$

$$\frac{\partial^2 U_M}{\partial w^2} = -\frac{2\beta^2\theta}{2\beta\theta - \gamma^2}$$

Since  $\beta$  and  $\theta$  are positive variables, and  $2\beta\theta - \gamma^2 > 0$ , then  $\frac{\partial^2 U_M}{\partial w^2} < 0$ . Consequently,  $U_M$  is strictly concave in  $w$ .

Setting (27) to zero, the first order condition can be shown as

$$\frac{\beta\theta}{2\beta\theta - \gamma^2} (-2\beta w + \alpha + \beta c_m - \beta c_r) = 0 \quad (28)$$

Solving (28), we have

$$w^{**} = \frac{\alpha - \beta(c_m + c_r)}{2\beta} + c_m \quad (29)$$

Substituting  $w^{**}$  into (24) and (25), we can obtain the optimal retail price and service level of the retailer as follows

$$p^{**} = \frac{(3\beta\theta - \gamma^2)[\alpha - \beta(c_m + c_r)]\theta}{2\beta(2\beta\theta - \gamma^2)} + c_m + c_r \quad (30)$$

$$s^{**} = \frac{[\alpha - \beta(c_m + c_r)]\gamma}{2(2\beta\theta - \gamma^2)} \quad (31)$$

The proof of Theorem 3 is completed.

Substituting  $w^{**}$ ,  $p^{**}$  and  $s^{**}$  into (18) and (19), we can derive the optimal profits of the manufacturer, the retailer and the supply chain system in the decentralized supply chain as follows

$$U_M^{**} = \frac{[\alpha - \beta(c_m + c_r)]^2 \theta}{4(2\beta\theta - \gamma^2)} \quad (31)$$

$$U_R^{**} = \frac{[\alpha - \beta(c_m + c_r)]^2 \theta}{8(2\beta\theta - \gamma^2)} \quad (32)$$

$$U_{SC}^{**} = U_M^{**} + U_R^{**} = \frac{3[\alpha - \beta(c_m + c_r)]^2 \theta}{8(2\beta\theta - \gamma^2)} \quad (33)$$

**Theorem 4.** In comparison of the optimal solutions obtained from the centralized and decentralized decision-making supply chain, we have

$$U_{SC}^* = \frac{4}{3} U_{SC}^{**},$$

$$s^* = 2s^{**},$$

$$\text{If } \frac{1}{2} < \frac{\beta\theta}{\gamma^2} < 1, \text{ then } p^* > p^{**};$$

$$\text{If } \frac{\beta\theta}{\gamma^2} = 1, \text{ then } p^* = p^{**};$$

$$\text{If } \frac{\beta\theta}{\gamma^2} > 1, \text{ then } p^* < p^{**}$$

**Proof.** By observing the optimal utility of the supply chain system and retail service level in the centralized and decentralized supply chain, we have

$$\frac{U_{SC}^*}{U_{SC}^{**}} = \frac{4}{3}, \text{ and } \frac{s^*}{s^{**}} = 2$$

On the basis of the optimal retail prices  $p^*$  and  $p^{**}$ , we obtain

$$p^* - p^{**} = -\frac{[\alpha - \beta(c_m + c_r)](\beta\theta - \gamma^2)}{2(2\beta\theta - \gamma^2)}$$

$$\text{If } \frac{1}{2} < \frac{\beta\theta}{\gamma^2} < 1, \text{ then we have } p^* - p^{**} > 0. \text{ If } \frac{\beta\theta}{\gamma^2} = 1, \text{ then}$$

$$\text{we have } p^* - p^{**} = 0. \text{ If } \frac{\beta\theta}{\gamma^2} > 1, \text{ then we have } p^* - p^{**} < 0.$$

The proof of Theorem 4 is completed.

V. DECENTRALIZED DECISION-MAKING WITH FAIRNESS PREFERENCE

In this section, we analyze the decisions of the manufacturer and the retailer in the decentralized supply chain considering the manufacturer's fairness preference.

The utility of the manufacturer is

$$U_M = (1 + \lambda)\Pi_M - \frac{\lambda(1 + \lambda)}{2 + \lambda}\Pi_{SC}$$

$$= (1 + \lambda)(w - c_m)(\alpha - \beta p + \gamma s)$$

$$- \frac{(1 + \lambda)\lambda}{2 + \lambda} \left[ (p - c_m - c_r)(\alpha - \beta p + \gamma s) - \frac{1}{2}\theta s^2 \right] \quad (34)$$

**Theorem 5.** If  $2\beta\theta - \gamma^2 > 0$ , then the optimal solutions of the manufacturer and the retailer in the decentralized supply chain with fairness preference are

$$w^{***} = \frac{(2 + \lambda)[\alpha - \beta(c_m + c_r)]}{(4 + \lambda)\beta} + c_m$$

$$p^{***} = \frac{[2\beta\theta + (2 + \lambda)(2\beta\theta - \gamma^2)][\alpha - \beta(c_m + c_r)]}{(4 + \lambda)(2\beta\theta - \gamma^2)\beta}$$

$$+ c_m + c_r$$

$$s^{***} = \frac{2[\alpha - \beta(c_m + c_r)]\gamma}{(4 + \lambda)(2\beta\theta - \gamma^2)}$$

**Proof.** In this case, since the retailer is fairness neutral, then from Theorem 2, we can obtain the optimal reaction functions of the retailer with respect to the wholesale price  $w$  as follows

$$p^{***}(w) = p^{**}(w) = \frac{[\alpha - \beta(w + c_r)]\theta}{2\beta\theta - \gamma^2} + w + c_r \quad (35)$$

$$s^{***}(w) = s^{**}(w) = \frac{[\alpha - \beta(w + c_r)]\gamma}{2\beta\theta - \gamma^2} \quad (36)$$

Substituting  $p^{***}(w)$  and  $s^{***}(w)$  into (34), we can derive the profit of the manufacturer as follows

$$U_M = \frac{(1 + \lambda)\beta\theta}{2\beta\theta - \gamma^2}(w - c_m)(\alpha - \beta w - \beta c_r)$$

$$- \frac{(1 + \lambda)\lambda\theta(\alpha - \beta w - \beta c_r)(\alpha + \beta w - 2\beta c_m - \beta c_r)}{2(2 + \lambda)(2\beta\theta - \gamma^2)} \quad (37)$$

From (37), we can get the first and second order derivatives of  $U_M$  with respect to  $w$  as follows

$$\frac{\partial U_M}{\partial w} = \frac{(1 + \lambda)\beta\theta}{2\beta\theta - \gamma^2}(-2\beta w + \alpha + \beta c_m - \beta c_r)$$

$$- \frac{(1 + \lambda)\lambda\beta^2\theta}{(2 + \lambda)(2\beta\theta - \gamma^2)}(-w + c_m) \quad (38)$$

$$\frac{\partial^2 U_M}{\partial w^2} = - \frac{(\lambda^2 + 7\lambda + 4)\beta^2\theta}{(2 + \lambda)(2\beta\theta - \gamma^2)} \quad (39)$$

Since  $\beta, \theta$  and  $\lambda$  are positive variables, and  $2\beta\theta - \gamma^2 > 0$ ,

then  $\frac{\partial^2 U_M}{\partial w^2} < 0$ . Consequently,  $U_M$  is strictly concave in  $w$ .

Setting (38) to zero, the first order condition can be shown as

$$\frac{(1 + \lambda)\beta\theta}{2\beta\theta - \gamma^2}(-2\beta w + \alpha + \beta c_m - \beta c_r)$$

$$- \frac{(1 + \lambda)\lambda\beta^2\theta}{(2 + \lambda)(2\beta\theta - \gamma^2)}(-w + c_m) = 0 \quad (40)$$

Solving (40), we have

$$w^{***} = \frac{(2 + \lambda)[\alpha - \beta(c_m + c_r)]}{(4 + \lambda)\beta} + c_m \quad (41)$$

Substituting  $w^{***}$  into (35) and (36), we can obtain the optimal retail price and service level of the retailer as follows

$$p^{***} = \frac{[2\beta\theta + (2 + \lambda)(2\beta\theta - \gamma^2)][\alpha - \beta(c_m + c_r)]}{(4 + \lambda)(2\beta\theta - \gamma^2)\beta}$$

$$+ c_m + c_r \quad (42)$$

$$s^{***} = \frac{2[\alpha - \beta(c_m + c_r)]\gamma}{(4 + \lambda)(2\beta\theta - \gamma^2)} \quad (43)$$

The proof of Theorem 5 is completed.

Substituting  $w^{***}, p^{***}$  and  $s^{***}$  into (34) and (19), we can derive the optimal profits of the manufacturer, the retailer and the supply chain system in the decentralized supply chain with fairness preference as follows

$$U_M^{***} = \frac{2(1 + \lambda)[\alpha - \beta(c_m + c_r)]^2\theta}{(4 + \lambda)(2 + \lambda)(2\beta\theta - \gamma^2)} \quad (44)$$

$$U_R^{***} = \frac{2[\alpha - \beta(c_m + c_r)]^2\theta}{(4 + \lambda)^2(2\beta\theta - \gamma^2)} \quad (45)$$

$$U_{SC}^{***} = U_M^{***} + U_R^{***}$$

$$= \frac{2(\lambda^2 + 6\lambda + 6)[\alpha - \beta(c_m + c_r)]^2\theta}{(4 + \lambda)^2(2 + \lambda)(2\beta\theta - \gamma^2)} \quad (46)$$

**Theorem 6.** In comparison of the optimal solutions obtained from the centralized supply chain without fairness preference and with fairness preference, we have

$$w^{***} > w^{**},$$

$$s^{***} < s^{**},$$

$$\text{If } \frac{1}{2} < \frac{\beta\theta}{\gamma^2} < 1, \text{ then } p^{***} > p^{**};$$

$$\text{If } \frac{\beta\theta}{\gamma^2} = 1, \text{ then } p^{***} = p^{**};$$

$$\text{If } \frac{\beta\theta}{\gamma^2} > 1, \text{ then } p^{***} < p^{**}$$

**Proof.** It is easy to verify that

$$w^{***} - w^{**} = \frac{\lambda[\alpha - \beta(c_m + c_r)]}{2(4 + \lambda)\beta} > 0$$

$$s^{***} - s^{**} = -\frac{\lambda[\alpha - \beta(c_m + c_r)]\gamma}{2(4 + \lambda)(2\beta\theta - \gamma^2)} < 0$$

$$p^{***} - p^{**} = -\frac{\lambda[\alpha - \beta(c_m + c_r)](\beta\theta - \gamma^2)}{2(4 + \lambda)\beta(2\beta\theta - \gamma^2)}$$

If  $\frac{1}{2} < \frac{\beta\theta}{\gamma^2} < 1$ , then we have  $p^{***} - p^{**} > 0$ . If  $\frac{\beta\theta}{\gamma^2} = 1$ , then we have  $p^{***} - p^{**} = 0$ . If  $\frac{\beta\theta}{\gamma^2} > 1$ , then we have  $p^{***} - p^{**} < 0$ .

The proof of Theorem 6 is completed.

### VI. NUMERICAL EXAMPLE

In this section, we tend to further elucidate the proposed three supply chain models with a numerical example. We will analyze that the effective of the service investment parameter  $\theta$  and the manufacturer's fairness reference parameter  $\lambda$  on the optimal solutions. The other parameter values are  $\alpha=100, \beta=5, \gamma=5, c_m=5$  and  $c_r=1$ .

The optimal solutions with different of the service cost coefficient  $\lambda$  in the centralized supply chain (CSC) and decentralized supply chain (DSC) without fairness preference are listed in Table I.

TABLE I  
OPTIMAL SOLUTIONS WITH DIFFERENT  $\theta$

	$\theta$	$p$	$s$	$w$	$U_M$	$U_R$	$U_{SC}$
CSC	3.00	48.00	70.00	—	—	—	1470.0
	4.00	24.67	23.33	—	—	—	653.33
	5.00	20.00	14.00	—	—	—	490.00
	6.00	18.00	10.00	—	—	—	420.00
	7.00	16.89	7.78	—	—	—	381.11
DSC	3.00	34.00	35.00	12.00	735.00	367.50	1102.5
	4.00	22.33	11.67	12.00	326.67	163.33	490.00
	5.00	20.00	7.00	12.00	245.00	122.50	367.50
	6.00	19.00	5.00	12.00	210.00	105.00	315.00
	7.00	18.44	3.86	12.00	190.56	95.28	285.83

The optimal solutions with different of the manufacturer's fairness reference parameter  $\lambda$  in the decentralized supply chain with fairness preference are listed in Table II.

TABLE II  
OPTIMAL SOLUTIONS WITH DIFFERENT  $\lambda$

$\lambda$	$p$	$s$	$w$	$U_M$	$U_R$	$U_{SC}$
0	34.00	35.00	12.00	735.00	367.50	1102.50
1.00	31.20	28.00	13.40	784.00	235.20	1019.20
2.00	29.33	23.33	14.33	735.00	163.33	898.33
3.00	28.00	20.00	15.00	672.00	120.00	792.00
4.00	27.00	17.50	15.55	612.50	91.88	704.38
5.00	26.22	15.56	15.89	560.00	72.59	632.59

From Tables I and II, we can obtain the results as follows

- 1) The service level and the utility of the supply chain system decrease as the service investment parameter  $\theta$  increases in both centralized and decentralized supply chain without consideration for the fairness of the

manufacturer, which means that the service investment parameter is a negative factor for both manufacturer's and retailer's utilities. The service level and the utility of supply chain system in the centralized supply are higher than those in the decentralized supply chain when the manufacturer is fairness neural.

- 2) The retail price in the centralized supply chain is higher than that in the decentralized supply chain when  $\theta < 5.00$ , the retail prices are the same in the centralized supply chain and the decentralized supply chain when  $\theta = 5.00$ , and the retail price in the centralized supply chain is lower than that in the decentralized supply when  $\theta > 5.00$ .
- 3) The first row in Table II shows the results at  $\lambda = 0$ , which are just the solutions without consideration for the fairness of the manufacturer. The retail price, the service level and the profits of the manufacturer and the retailer with fairness preference are lower than those without fairness sensitivity. The wholesale price in the decentralized supply chain with fairness preference is higher than that without fairness concern, which means that the manufacturer's fairness preference can positively affect the manufacturer's wholesale price.
- 4) The manufacturer's fairness preference is not only harmful to the utilities of the retailer and supply chain system, but also harmful to herself utility, which is because they can gain fewer utilities.

### VII. CONCLUSION

In this paper, we explore the retail price and service level decisions models in a two stage supply chain. By using utility theoretical method, we examine the manufacturer's and the retailer's optimal strategies under three different supply chain models, by which we obtain how the manufacturer and the retailer make their own decisions about the wholesale price, retail price and retail service level.

Based on the discussions above, we can obtain three findings. Firstly, the service investment parameter is a negative factor for both manufacturer's and retailer's utilities. Secondly, the retail price in the centralized supply chain may be higher or lower than that in the decentralized supply chain depending on the value of service investment parameter. Thirdly, the manufacturer's fairness preference is not only harmful to the utilities of the retailer and supply chain system, but also harmful to herself utility.

The limitation in this study is that we only consider one retailer and one manufacturer in a two stage supply. Future research can be done for the situations including two or more competing supply chain members or in a multi-stage supply chain.

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