Global Stabilization of Nonholonomic Mobile Robots with Constrained Angular Velocity

Yanling Shang, Ye Yuan and Fangzheng Gao

Abstract—**This paper investigates the problem of global stabilization of nonholonomic mobile robots with constrained angular velocity. By using input-state-scaling technique and backstepping recursive approach, a state feedback control strategy is presented. With the help of a switching control strategy, the designed controller renders that the states of closed-loop system are globally asymptotically regulated to zero. A simulation example is provided to illustrate the effectiveness of the proposed approach.**

Index Terms—**nonholonomic mobile robots , state feedback, constrained angular velocity, backstepping.**

I. Introduction

IN the past decade, nonholonomic systems, which can be modeled with constraints concerning velocity or acceleration as well as coordinates and position angle, have become a hot research topic of the mechanical systems. As a class of typical nonholonomic systems, the mobile robots have caused the extensive concern[1-4]. Nonholonomic mobile robots have good flexibility, since they could realize autonomous movement in the case of nobody involving. However, due to the limitations imposed by Brockett's condition[6], this class of nonlinear systems cannot be stabilized by stationary continuous state-feedback, although it is controllable. There are currently several effective control methodologies that overcome the topological obstruction. The idea of using time-varying smooth controllers was first proposed in [6], in order to stabilize a mobile robot. For driftless systems in chained form, several novel approaches have been proposed for the design of periodic, smooth, or continuous stabilizing controllers [7, 8]. Most of the time-varying control scheme suffer from a slow convergence rate and oscillation. However, it has been observed that a discontinuous feedback control scheme usually results in a fast convergence rate. An elegant approach to constructing discontinuous feedback controller was developed in [9]. The drawback is that there is a restriction on the initial conditions of the controlled system. This limitation has been overcome by a switching state or output control scheme [10]. Subsequently, [11-19] further developed the discontinuous feedback control strategy based on different control targets, respectively. However, the effect of the angular velocity constraint is not addressed in the above-mentioned results.

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Yanling Shang is with School of Software Engineering, Anyang Normal University, Anyang 455000, P. R. China hnnhsyl@126.com

Ye Yuan is with School of Economics, Anyang Normal University, Anyang 455000, P. R. China

Fangzheng Gao is with School of Mathematics and Statistics, Anyang Normal University, Anyang 455000, P. R. China

As a matter that the constraints which can represent not only physical limitations but also performance requirements are common in practical systems. Violation of the constraints may cause performance degradation or system damage. In recent years, driven by practical needs and theoretical challenges, the control design for constrained nonlinear systems has become an important research topic [20-22]. However, less attention has been paid to the constrained nonholonomic systems.

This paper addresses the global stabilization by state feedback for nonholonomic mobile robots with constrained angular velocity. A constructive method in designing global stabilizing controller for such uncertain systems is proposed. The contributions of this paper are listed as follows:(i) by using the nested saturation to handle the technical problem of input saturation, and based on a combined application of the input-state-scaling technique and backstepping recursive approach, a systematic control design procedure is developed for all plants in the considered class, including the ideal chained form system; (ii) the saturated control based switching strategy is adopted to handle the technical problem of uncontrollability at $x_0(0) = 0$, which prevents the finite escape of system and guarantees thatthe states of closed-loop system are globally asymptotically regulated to zero.

The rest of this paper is organized as follows. In Section II, the problem formulation and preliminaries are given. Section III presents the input-state-scaling transformation the backstepping design procedure, the switching control strategy and the main result. Section IV gives simulation results to illustrate the theoretical finding of this paper. Finally, concluding remarks are proposed in Section V.

II. Problem Formulation

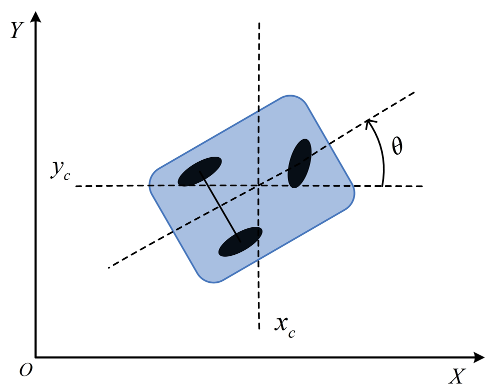


Fig. 1. The planar graph of a mobile robot.

Consider a tricycle-type mobile robot shown in Fig. 1. The kinematic equations of this robot are represented by

$$\begin{aligned} \dot{x}_c &= v\cos\theta \\ \dot{y}_c &= v\sin\theta \\ \dot{\theta} &= \omega \end{aligned} \tag{1}$$

where (x_c, y_c) denotes the position of the center of mass of the robot, θ is the heading angle of the robot, v is the forward velocity while ω is the angular velocity of the robot.

For system (1), by taking the following state and input transformation

$$\begin{aligned} x_0 &= \theta, \\ x_1 &= x_c\sin\theta - y_c\cos\theta \\ x_2 &= x_c\cos\theta + y_c\sin\theta \\ u_0 &= \omega \\ u_1 &= v \end{aligned} \tag{2}$$

one obtains

$$\begin{aligned} \dot{x}_0 &= u_0 \\ \dot{x}_1 &= x_2 u_0 \\ \dot{x}_2 &= u_1 - x_1 u_0 \end{aligned} \tag{3}$$

which belongs to the class of nonholonomic chain systems introduced in [10].

Remark 1. It is evident that, system (3) is a third-order chained form system which has been extensively studied in the literature when the inputs saturation was not taken into consideration. However, in the process of actual movement, the unbounded angular velocity of the robot is impermissible. That is because the overquick rotation will result in robot overturned. Therefore, it is more practical to consider the stabilization problem of the nonholonomic mobile robot subject to saturated angular velocity, that is, consider the stabilization problem of the nonholonomic system (3) with input u_0 saturation.

The objective of this paper is to design a state feedback controller of the form

$$u_0 = u_0(x_0), \quad |u_0(x_0)| \le M, \quad u_1 = u_1(x_0, x) \tag{4}$$

where M is a known bound of u_0, such that the states of closed-loop system are globally asymptotically regulated to zero.

III. Robust controller design

In this section, we proceed to design a robust controller based on backstepping technique. For clarity, the case that $x_0(t_0) \neq 0$ is considered first. Then the case where the initial $x_0(t_0) = 0$ is dealt later. The inherently structure of system (4) suggests that we should design the control inputs u_0 and u_1 in two separate stages.

A. Design u_0 for x_0-subsystem

For x_0-subsystem, we take the following control law

$$u_0(x_0) = -k_0\sigma(x_0) \tag{5}$$

where $k_0 > 0$ is a design constant and

$$\sigma(x_0) = \begin{cases} sign(x_0), & |x_0| > \varepsilon \\ x, & |x_0| \le \varepsilon \end{cases} \tag{6}$$

for a small constant $\varepsilon > 0$ to be determined later.

Remark 2. From (5) and (6), it can clearly be seen that the control law u_0 is bounded by a constant $k_0\varepsilon$, this is, by choosing design parameters k_0 and ε as $k_0\varepsilon < M$, the control law $|u_0(x_0)| \le M$ is guaranteed.

Under (5), the first result of this paper is established, which is crucial for the input-state-scaling transformation in what follows.

Lemma 1. For any initial condition $x_0(t_0) \neq 0$, where $t_0 \ge 0$, the corresponding solution $x_0(t)$ exists and globally asymptotically converges to zero. Furthermore, the control u_0 given by (5) also exists and does not cross zero.

Proof. Taking the Lyapunov function $V_0 = x_0^2/2$, a simple computation gives

$$\begin{aligned} \dot{V}_0 &\le \begin{cases} -k_0|x_0|, & |x_0| > \varepsilon \\ -k_0 x_0^2, & |x_0| \le \varepsilon \end{cases} \\ &\le \begin{cases} -k_0 V_0^{1/2}, & |x_0| > \varepsilon \\ -2k_0 V_0, & |x_0| \le \varepsilon \end{cases} \end{aligned} \tag{7}$$

from which, we can conclude that $x_0(t)$ exists and $x_0(t) \to 0$ as $t \to \infty$.

Next, we will show that $x_0(t)$ does not cross zero. Obviously, it suffices to prove the statement in the case where $|x_0(t)| \le \varepsilon$. In this case, under the control law (5), the x_0-subsystem becomes

$$\dot{x}_0 = -k_0 x_0 \tag{8}$$

Therefore, the solution of x_0- subsystem can be expressed as

$$x_0(t) = x_0(t_0)e^{-k_0(t-t_0)}$$

Consequently, x_0 can be zero only at $t = t_0$, when $x(t_0) = 0$ or $t = \infty$. Since $x_0(t_0) \neq 0$ is assumed, it is concluded that x_0 does not cross zero for all $t \in (t_0, \infty)$ provided that $x_0(t_0) \neq 0$. Furthermore, we can see from (5) that the u_0 exists, does not cross zero for all $t \in (t_0, \infty)$ independent of the x-subsystem and satisfies $\lim_{t\to\infty} u_0(t) = 0$. Thus, the proof of Lemma 1 is completed.

B. Input-state-scaling transformation

From Lemma 1, we can see the x_0-state in (3) can be globally regulated to zero via u_0 in (5) as $t \to \infty$. However, in the limit case, x_0 will converge to the origin, which will cause serious trouble in controlling the x-subsystem via the control input u_1. This difficulty can be well addressed by utilizing the following discontinuous input-state scaling transformation:

$$z_1 = \frac{x_1}{u_0}, \quad z_2 = x_2 \tag{9}$$

Under the new z-coordinates, the x-subsystem is transformed into

$$\begin{aligned} \dot{z}_1 &= z_2 + f_i(x_0, z) \\ \dot{z}_2 &= u_1 + f_n(x_0, z) \end{aligned} \tag{10}$$

where

$$f_i(x_0, z) = \frac{\phi_i(x_0, x, u_0)}{u_0^{n-i}} - (n-i)z_i\frac{\dot{u}_0}{u_0} \tag{11}$$

By transformation (9), we easily obtain the following estimation for nonlinear function f_i.

Lemma 2. For $i = 1, 2$, there are nonnegative smooth functions γ_i such that

$$|f_i(x_0, z)| \le (|z_1| + \cdots + |z_i|)\gamma_i(x_0, z_1, \cdots, z_i) \tag{12}$$

Proof. In view of (9) and (11), we have

$$\begin{aligned}
&|f_i(x_0,z)| \\
&\le \frac{(|x_1|+\cdots+|x_i|)}{|u_0^{n-i}|}\varphi_i(\cdot)+(n-i)|z_i|\Big|\frac{\dot{u}_0}{u_0}\Big| \\
&= \frac{(|z_1u_0^{n-1}|+\cdots+|z_iu_0^{n-i}|)}{|u_0^{n-i}|}\varphi_i(\cdot) \\
&\quad +(n-i)|z_i|\times\begin{cases} |\frac{\dot{u}_0}{u_0}|, & |x_0|>\varepsilon \\ |\frac{\dot{u}_0}{u_0}|, & |x_0|\le\varepsilon \end{cases} \\
&= \frac{(|z_1u_0^{n-1}|+\cdots+|z_iu_0^{n-i}|)}{|u_0^{n-i}|}\varphi_i(\cdot) \\
&\quad +(n-i)|z_i|\times\begin{cases} 0, & |x_0|>\varepsilon \\ k_0, & |x_0|\le\varepsilon \end{cases} \\
&\le (|z_1|+\cdots+|z_i|)\gamma_i(x_0,z_1,\cdots,z_i)
\end{aligned} \tag{13}$$

C. *Backstepping Design for u_1*

In this subsection, the controller u_1 will be recursively constructed by applying backstepping technique to system (10).

Step 1. Begin with z_1-subsystem of (10), where z_2 is regarded as a virtual control. Introducing the transformation

$$e_1 = z_1, \ \ e_2 = z_2 - z_2^* \tag{14}$$

and choosing Lyapunov function

$$V_1 = \frac{1}{2}e_1^2 \tag{15}$$

From (10) and (12), it follows that

$$\dot{V}_1 \le e_1e_2 + e_1z_2^* + e_1^2\gamma_1(x_0,z_1) \tag{16}$$

Obviously, the first virtual controller

$$z_2^* = -e_1(2+\gamma_1(x_0,e_1)) := -e_1\beta_1(x_0,e_1) \tag{17}$$

leads to

$$\dot{V}_1 \le -2e_1^2 + e_1e_2 \tag{18}$$

Step 2. Consider the Lyapunov function

$$V_2 = V_1 + +\frac{1}{2}e_2^2 \tag{19}$$

Clearly

$$\begin{aligned}
\dot{V}_i &\le -2e_1^2 + e_1e_2 \\
&\quad + e_2\Big(z_2 + f_2 - \frac{\partial z_2^*}{\partial x_0}u_0 - \frac{\partial z_2^*}{\partial z_1}(z_1+f_1)\Big)
\end{aligned} \tag{20}$$

Now we estimate each term on the right-hand side of (20). First, it follows (14) that

$$e_1e_2 \le \frac{1}{4}e_1^2 + e_2^2\sigma_1 \tag{21}$$

where σ_1 is a positive constant.

Noting that $z_2^* = -e_1\beta_1$, it implies that z_i^* satisfies

$$z_2^*(x_0,0,0) = 0, \qquad \frac{\partial z_2^*}{\partial x_0}(x_0,0,0) = 0 \tag{22}$$

from (22), (12) and (14), after lengthy but simple calculations based on the completion of squares, there is a smooth nonnegative function σ_{i2} such that

$$\begin{aligned}
&e_2\Big(z_2 + f_2 - \frac{\partial z_2^*}{\partial x_0}u_0 - \frac{\partial z_2^*}{\partial z_1}(z_1+f_1)\Big) \\
&\le \frac{3}{4}e_1^2 + e_2^2\sigma_2(x_0,e_1,e_2)
\end{aligned} \tag{23}$$

Substituting(21) and (23) into (20) gives

$$\dot{V}_2 \le -e_1^2 + e_2u_1 + e_2^2(\sigma_1+\sigma_2) \tag{24}$$

Now, it easy to see that the smooth actual control

$$u_1 = -e_2(1+\sigma_1+\sigma_2) := -e_2\beta_2(x_0,e_1,e_2) \tag{25}$$

renders

$$\dot{V}_2 \le -(e_1^2+e_2^2) \tag{26}$$

which implies $\lim_{t\to\infty} e(t) = 0$. According to the input-state-scaling transformation (9), we conclude that $\lim_{t\to\infty} x(t) = 0$.

The above analysis is summarized into the following theorem:

Theorem 1. For system (3), if control law (5) and the full feedback control law (25) are applied, the globally asymptotic regulation of the closed-loop system is achieved for $x_0(t_0) \neq 0$.

D. *Switching controller and main results*

Without loss of generality, we assume that $t_0 = 0$. When the initial state $x_0(0) \neq 0$, we have given controller (5) and (25) for u_0 and u_1 of system (3). Now, we discuss how to select the control laws u_0 and u_1 when $x_0(0) = 0$. In the absence of disturbances, the most commonly used control strategy is using constant control $u_0 = u_0^* \neq 0$ in time interval $[0,t_s)$. In this paper, we also use this method when $x_0(0) = 0$, with u_0 chosen as

$$u_0 = u_0^* \tag{27}$$

where $0 < u_0^* < M$ is a constant.

Since $x_0(0) = 0$, under (27), the solution of x_0-subsystem can be expressed as

$$x_0(t) = u_0^*t \tag{28}$$

Obviously, we have x_0 does not escape and $x(t_s) \neq 0$, for given any finite $t_s > 0$. Thus, input-state-scaling transformation for the control design can be carried out.

During the time period $[0,t_s)$, using u_0 defined in (27), new control law $u_1 = u_1^*(x_0,x)$ can be obtained by applying the procedure described in Section III-C to the original x-subsystem in (3). Then we can conclude that the x-state of (3) cannot blow up during the time period $[0,t_s)$. Since $x(t_s) \neq 0$ at t_s, we can switch the control input u_0 and u_1 to (5) and (25), respectively.

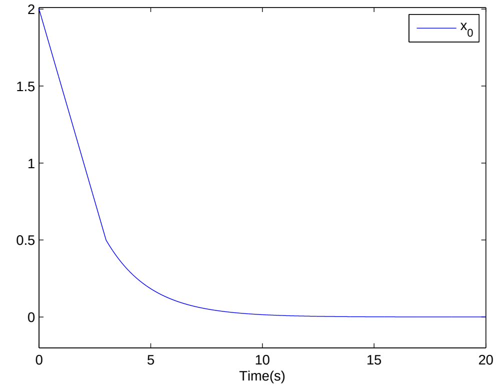
We are now ready to state the main theorem of our paper.

Theorem 2. If the proposed saturated control design procedure together with the above switching control strategy is applied to system (3), then the states of closed-loop system globally asymptotically regulated to zero.

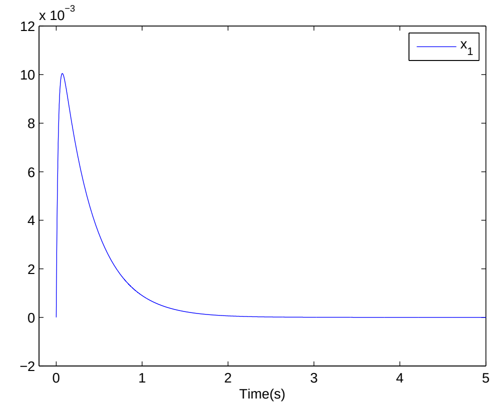
IV. Simulation results

In this section, we illustrate the effectiveness of the proposed approach. with the boundedness of ω being 0.5, i.e., $|u_0| \le 0.5$.

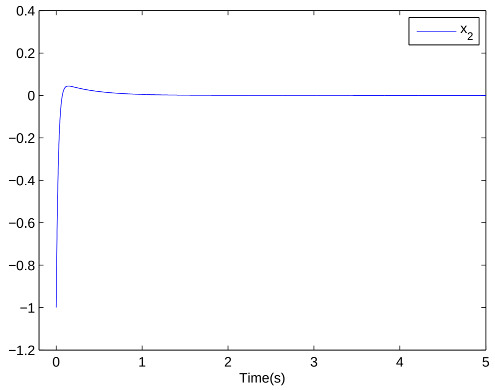
If $x_0(0) = 0$, controls u_0 and u_1 are set as in Section III-D in interval $[0,t_s)$, such that $x(t_s) \neq 0$, then we can adopt the controls developed below. Therefore, without loss of



(a) x_0



(b) x_1



(c) x_2

Fig. 2. System states.

generality, we assume that $x_0(0) \neq 0$. For the x_0-subsystem, we can choose the control law

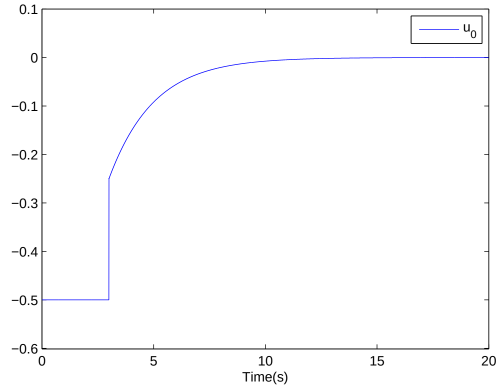
$$u_0(x_0) = \begin{cases} -sign(x_0), & |x_0| > 0.5 \\ -x, & |x_0| \leq 0.5 \end{cases} \tag{29}$$

and introduce the input-state-scaling transformation

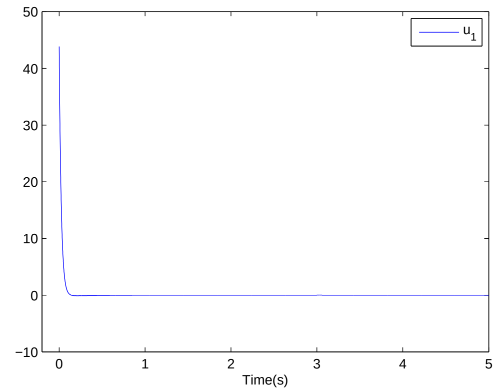
$$z_1 = \frac{x_1}{u_0}, \quad z_2 = x_2 \tag{30}$$

In new z-coordinates, the(x_1, x_2)-subsystem of (3) is rewritten as

$$\begin{aligned} \dot{z}_1 &= z_2 - \frac{\dot{u}_0}{u_0} z_1 \\ \dot{x}_2 &= u_1 - z_1 u_0^2 \end{aligned} \tag{31}$$



(a) u_0



(b) u_1

Fig. 3. Control inputs.

Using (29), it is easy to verify that Lemma 2 holds with $\gamma_1 = \gamma_2 = 1$. By applying the design procedure shown in Section III-C to system (31), we can obtain the following controller

$$u_1 = -\beta_2(z_2 + \beta_1 z_1) \tag{32}$$

where $\beta_1 = 2.1$ and $\beta_1 = 23.45$. When $(x_0(0), x_1(0), x_2(0))$= $(2, -1, 1)$, the simulation results are shown in Figs. 2 and 3, from which, it can be seen that the system states are asymptotically regulated to zero and the amplitude of the control input u_0 is bounded by 0.5.

V. Conclusion

In this paper, the problem of global stabilization of nonholonomic mobile robots with constrained angular velocity. By using input-state-scaling transformation and backstepping technique, a state feedback controller is obtained. Based on switching strategy to eliminate the phenomenon of uncontrollability, the proposed controller can guarantee that the system states globally asymptotically converge to the origin. Simulation results demonstrate the effectiveness of the proposed control design.

References

[1] R. M. Mahamood, "Improving the performance of adaptive PDPID control of two-link flexible robotic manipulator with ILC, " *Engineering Letters*, vol.20, no.3, pp.259–270, 2012.

[2] A. Gorbenko and V. Popov, "Visual landmark selection for mobile robot navigation, " *IAENG International Journal of Computer Science*, vol.40, no.3, pp.134–142, 2013.

[3] Y. L. Shang, D.H. Hou, and F. Z. Gao, “Global output feedback stabilization of nonholonomic chained form systems with communication delay, ” *IAENG International Journal of Applied Mathematics*, vol.46, no.3, pp. 367–371, 2016.

[4] Y. L. Shang, D.H. Hou, and F. Z. Gao, “Finite-time output feedback stabilization for a class of uncertain high order nonholonomic systems, ” *Engineering Letters*, vol.25, no.1, pp. 39–45, 2017.

[5] R. W. Brockett, “Asymptotic stability and feedback stabilization, ” in: R.W. Brockett, R.S. Millman, and H.J. Sussmann (Eds.), *Differentical Geometry Control Theory*, pp. 2961–2963, 1983.

[6] C. Samson, “Velocity and torque feedback control of a nonholonomic cart, ” *Advanced robot control*, Springer Berlin Heidelberg, pp. 125-151, 1991.

[7] C. Samson, “ Control of chained systems application to path following and time-varying point-stabilization of mobile robots, ” *IEEE Transactions on Automatic Control*, vol.40, no.1, pp. 64-77, 1995.

[8] R. T. MCloskey, R. M. Murray, “Exponential stabilization of driftless nonlinear control systems using homogeneous feedback, ” *IEEE Transactions on Automatic Control*, vol.42, no.2, pp. 614-628, 1997.

[9] A. Astolfi, “Discontinuous control of nonholonomic systems, ”*Systems and Control Letters*, vol.27, no.1, pp. 37–45, 1996.

[10] Z. P. Jiang, “Robust exponential regulation of nonholonomic systems with uncertainties, *Automatica*, vol.36, no.2, pp.189–209, 2000.

[11] S. S. Ge, Z. P. Wang, and T. H. Lee, “ Adaptive stabilization of uncertain nonholonomic systems by state and output feedback, *Automatica*, vol.39, no.8, pp.1451–1460, 2003.

[12] Y. Q. Wu, G. L. Ju, and X. Y. Zheng, “Adaptive output feedback control for nonholonomic systems with uncertain chained form, ” *International Journal of Systems Science*, vol.41, no.12, pp. 1537–1547, 2010.

[13] F. Z. Gao, F. S. Yuan, and H. J. Yao, “Robust adaptive control for nonholonomic systems with nonlinear parameterization, ” *Nonlinear Analysis: Real World Applications*, vol.11, no.4, pp. 3242–3250, 2010.

[14] F. Z. Gao, F. S. Yuan, and Y. Q. Wu, “State-feedback stabilisation for stochastic non-holonomic systems with time-varying delays, ” *IET Control Theory and Applications*, vol.6, no.17, pp. 2593–2600, 2012.

[15] F. Z. Gao, F. S. Yuan, H. J. Yao, and X. W. Mu, “Adaptive stabilization of high order nonholonomic systems with strong nonlinear drifts, ” *Applied Mathematical Modelling*, vol.35, no.9, pp.4222–4233, 2011.

[16] F. Z. Gao, Y. L. Shang, and F.S. Yuan, “Robust adaptive finite-time stabilization of nonlinearly parameterized nonholonomic systems, ” *Acta applicandae mathematicae*, vol.123, no.1, pp.157–173, 2013.

[17] F. Z. Gao and F. S. Yuan, “Adaptive finite-time stabilization for a class of uncertain high order nonholonomic systems, ” *ISA Transactions*, vol.54, pp.75–82, 2015.

[18] F. Z. Gao, Y. Q. Wu, and Z. C. Zhang, “Finite-time stabilization of uncertain nonholonomic systems in feedforward-like form by output feedback, ” *ISA Transactions*, vol.59, pp.125–132, 2015.

[19] Y. Q. Wu, F. Z. Gao, and Z. G. Liu, “Finite-time state feedback stabilization of nonholonomic systems with low-order nonlinearities, ” *IET Control Theory and Applications*, vol.9, no.10, pp.1553-1560, 2015.

[20] K. D. Do, “Control of nonlinear systems with output tracking error constraints and its application to magnetic bearings,” *International Journal of Control*, vol.83, no.6, pp. 1199–1216, 2010.

[21] K. P. Tee, B. Ren, and S. S. Ge, “Control of nonlinear systems with time-varying output constraints,” *Automatica*, vol.47, no.11, pp. 2511–2516, 2011.

[22] Y. J. Liu and S. Tong, “Barrier Lyapunov functions-based adaptive control for a class of nonlinear pure-feedback systems with full state constraints,” *Automatica*, vol.64, pp. 70–75, 2016.