Sequential Game in Supply Chain Dominated by Manufacturers Considering Selling Effort in a Fuzzy Environment

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Abstract—**In this paper, a two-echelon supply chain that includes one manufacturer and one retailer is considered. In this supply chain, the manufacturer plays the dominant role. To increase sales, the manufacturer needs to engage in selling. This paper assumes that the manufacturer provides this selling effort and bears the cost. To reflect the various factors of uncertainty in a real economy, the market demand function, manufacturing costs, and retail operating costs are considered to be fuzzy variables. The Stackelberg game is adopted to solve the problem between the retailer and the manufacturer. The expected value and the chance constrained models are introduced to solve for optimal decisions. The optimal wholesale price and the marginal profit per unit that are at equilibrium in each model are provided to determine the maximum profit for the retailer and the manufacturer. Finally, a numerical example illustrates the effectiveness of the supply chain game model.**

Key words—**Stackelberg game, Selling effort, Fuzzy supply chain, Chance constrained model, optimal strategy**

I. INTRODUCTION

ADVANCEMENTS in science and technology create challenges for manufacturers because these advancements intensify the competition in a market environment. For manufacturers to succeed in this competitive environment, they must attain customer satisfaction through well-functioning value delivery systems in supply chains. Consequently, heightened research interest exists as regards supply chains and supply chain management. At the same time, a growing number of enterprises are beginning to strengthen the management and coordination of their supply chains to improve their competitive advantage. For example, Procter & Gamble, Hewlett-Packard (HP), and other multinational companies have enhanced their competitiveness in international markets by improving the operation and management of their supply chains. From the perspective of the market system, the core issue of the supply chain is coordinating the relationship among the various members to achieve a win-win situation for all. However, in most cases, each member within a supply chain acts as an independent node that maximizes its own benefits on the basis of its unique power and position within the supply chain.

In 1952, when the German economist Stackelberg studied problems in a market economy, he proposed a leader–follower hierarchical decision-making structure, also known as the Stackelberg problem or the Stackelberg game. The theory holds that a supply chain of independent entities has two types of decision makers that make their own independent decisions. The firm that dominates the supply chain is at a higher decision-making level and the followers are at a subordinate level. After making a decision, the dominant party can predict the followers' reactions and, thus, make the best decision through this process. Then, within the constraints of the dominant player's decision, followers can make their best decisions. The Stackelberg model has a wide range of applications in the fields of relationship coordination among firms, contract design of demand and supply, supply chain distribution channel design, inventory management, and others.

Faced with fierce market competition, enterprises have changed the form of their supply chains. Researchers are increasingly studying competition from the perspective of marketing and the choice of different channel structures.

Reference [1] analyzed the Bertrand competitive behavior of two manufacturers that sell their products through their respective retailers under deterministic demand conditions. By studying the interactions among channel structure decisions, [2] further analyzed the reasons behind the conclusions proposed by [1] and presented the conditions that need to be met. From the perspective of the manufacturer, reference [3] divided customers into two categories—service sensitive and price sensitive—and found that when both types of customers are less sensitive to service, it is more advantageous for manufacturers to construct the network channel. Reference [4] studied different marketing channels in different duopoly markets and found that integrated marketing channels were more price competitive than

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marketing channels of independent middlemen. Reference [5] studied the pricing game and equilibrium under the simple direct channel structure, the simple retail channel structure, and the dual channel structure. They found that the manufacturer's profits under the dual channel structure were always higher than those under the pure retail channel structure. Reference [6] analyzed industrial equilibrium under three structures: vertical integration, the manufacturer Stackelberg game, and the price bargaining structure. Reference [7] considered three situations and a supply chain with only one manufacturer and one retailer. The first situation had only one manufacturer, the second situation had a manufacturer in a manufacturer-led chain, and the third situation had a manufacturer in a retail-dominated chain. The respective strength within the supply chain was analyzed, and the paper concluded that the change in chain dominance had no effect on intra-chain competition.

Coordination is the basis for the stable operation of supply chain management and operation. Supply chain coordination refers to the compatibility of the goals of the members within the chain, such that the supply chain operation is coordinated. Hence, supply chain members that optimize their own interests can ensure that the entire supply chain's interests are maximized. Reference [8] put forward a more practical supply chain coordination mechanism—the Stackelberg game under an uncertain just-in-time (JIT) delivery condition—and considered the interaction form of intra-chain members' decision making to deduce the Stackelberg equilibrium solution. Reference [9] studied the coordination of two competing supply chains in the context of customer service competition, and [10] studied the supply chain competition and coordination problem of a two-channel supply chain using Nash's game theory. The study designed a type of coordination contract mechanism that combines a buyback and a reward-punishment scheme to realize the coordination of a two-channel supply chain. Reference [11] analyzed the optimal order quantity of the members of a dual-channel supply chain under the constraints of a retailer-dominated Stackelberg game and found that the entire system lacked coordination. They proposed a reverse revenue sharing contract and transfer payment combination contract to realize the coordination of the entire two-channel supply chain system. Reference [12] proposed the conditions and assumptions for the overall performance optimization of a supply chain by analyzing several cooperative models. Reference [13] considered product substitution among different channels under the Stackelberg game led by a manufacturer. They studied the inventory coordination problem between traditional and electronic channels by establishing two-party revenue sharing contract models with a coordinated dual channel supply chain.

Product pricing in supply chains has been an intensely studied topic in academic research. Reference [14] considered the duopoly game under linear and nonlinear demand and analyzed the price competition model of a supply chain with two competing retailers. References [15] and [16] expanded reference [14] and analyzed supply chains with two manufacturers and two retailers, respectively. References [17] studied the price competition problem with two manufacturers and one retailer in a fuzzy environment where the costs and parameters of the demand were regarded as fuzzy variables.Reference [18] analyzed the pricing strategy of the supplier and the retailer in different channels and constructed Bertrand and Stackelberg double channel price competition game models that consider the impact of price and service on demand. Reference [19] found that if delivery time is shortened, manufacturers' online prices increase through the establishment of dual channel supply chains. In recent years, the coordination strategy of quantity discounts has been widely studied in manufacturers' and retailers' supply chains. Reference [20] studied three distribution channel models and concluded that a quantity discount is more effective on price sensitivity. Reference [21] constructed a two-stage supply chain containing a manufacturer and a retailer and studied the coordination mechanism of the dual channel of retailer and manufacturer. They concluded that the quantity discount contract can coordinate the retailer's dual channel.

Numerous papers studied channel pricing, inventory management, and competition. However, few related to selling efforts even though a positive relationship exists between selling efforts and competition. Analysis of the status of a supply chain can assist manufacturers in dominating the supply chain, and this status also allows them to engage in greater selling efforts. Therefore, it is more realistic to analyze the supply chain game, including the concept of sales efforts. This paper analyzes the supply chain game dominated by a manufacturer that simultaneously engages in selling.

The paper is organized as follows. In Section II, we briefly described the problem and the notations that will be used in the following sections. In Sections III, we developed the decentralized decision-making system. In Section IV, a numerical example is given to illustrate the solutions for proposed models. Section V summarizes the work.

II. Preliminaries

The fuzzy theory is based on the fuzzy set. The basic concept of the theory assumes the existence of fuzzy phenomena and addresses the concept of fuzzy or uncertain things as its research objective [22]. Fuzzy theory employs $Pos\{A\}$ to describe the probability of event A. $Pos\{A\}$ must have certain properties to ensure its rationality in practice [23].

Suppose Θ is a non-empty set and $P(\Theta)$ is the power set of Θ. Then:

Axiom 1. $P\{\Theta\}=1$.

Axiom 2. $P\{\Phi\}=1$.

Axiom 3. For any set $\{A_i\}$ in $P(\Theta)$, $Pos\{U_i A_i\}=\sup_i Pos\{A_i\}$.

If these three axioms are satisfied, fuzziness is a characteristic with a possibility measure, and the three $(\Theta, P(\Theta), Pos)$ form a possibility space.

Certain definitions and properties as subsequently listed serve as the premise and foundation for the remainder of this paper.

Definition 1. If the fuzzy variable ξ is a function from a possibility space $(\Theta, P(\Theta), Pos)$ to a real line R, then ξ can be said to represent the definition of a fuzzy variable in a possibility space $(\Theta, P(\Theta), Pos)$ [24].

Definition 2. Fuzzy variable ξ is a non-negative (or positive) variable if and only if $Pos\{\xi < 0\} = 0$ (or $Pos\{\xi \le 0\} = 0$) [25].

Proposition 1. Suppose that ξ_i is a mutually independent fuzzy variable, $f_i : R \to R$, $i = 1,2,...m$. Then, $f_1(\xi_1)$, $f_2(\xi_2)$, ..., $f_m(\xi_m)$ are also mutually independent fuzzy variables [24].

Definition 3. Provided that ξ is a fuzzy variable defined in the possibility space $(\Theta, P(\Theta), Pos)$ and $\alpha \in (0,1]$, then:

$\xi_\alpha^L = \inf\{r \mid Pos\{\xi \le r\} \ge \alpha\}$ and $\xi_\alpha^U = \sup\{r \mid Pos\{\xi \ge r\} \ge \alpha\}$ are referred to as the α-pessimistic and α-optimistic values of fuzzy variable ξ.

Here, r is the value of fuzzy variable ξ with possibility α. The α-pessimistic value ξ_α^L is the infimum value of ξ with possibility α, and α-optimistic value ξ_α^U is the supremum value of ξ with possibility α [25].

Example 1.

The triangular fuzzy variable $\xi = (a,b,c)$ has its α-pessimistic value and α-optimistic value:

$$\xi_\alpha^L = a + (b - a)\alpha \text{ and } \xi_\alpha^U = c - (c - b)\alpha.$$

Proposition 2. If there are two mutually independent fuzzy variables expressed as ξ and η, then we can derive the following equations.

(1) For any $\alpha \in (0,1]$, $(\xi + \eta)_\alpha^L = \xi_\alpha^L + \eta_\alpha^L$.

(2) For any $\alpha \in (0,1]$, $(\xi + \eta)_\alpha^U = \xi_\alpha^U + \eta_\alpha^U$.

(3) For any $\alpha \in (0,1]$, $(\xi \cdot \eta)_\alpha^L = \xi_\alpha^L \cdot \eta_\alpha^L$.

(4) For any $\alpha \in (0,1]$, $(\xi \cdot \eta)_\alpha^U = \xi_\alpha^U \cdot \eta_\alpha^U$ [23], [25]–[26].

Definition 4. Let ξ be a fuzzy variable and r_0 be a real number defined from $-\infty$ to ∞. The expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} C_r\{\xi \ge r_0\} dr_0 - \int_{-\infty}^0 C_r\{\xi \le r_0\} dr_0,$$

provided that at least one of the two integrals is finite. In particular, if ξ is a nonnegative fuzzy variable, then

$$E[\xi] = \int_0^{+\infty} C_r\{\xi \ge r_0\} dr_0.$$

Example 2.

The triangular fuzzy variable $\xi = (a,b,c)$ has an expected value

$$E[\xi] = \frac{a + 2b + c}{4}.$$

Proposition 3. References [26]–[27] showed that ξ is a fuzzy variable with limited expectations. Then:

$$E[\xi] = \frac{1}{2}\int_0^1 (\xi_\alpha^L + \xi_\alpha^U) d\alpha.$$

Proposition 4. References [28]–[29] showed that ξ and η are mutually independent fuzzy variables with limited expectations. Then, for any numbers a and b, the formula is as follows:

$$E[a\xi + b\eta] = aE[\xi] + bE[\eta].$$

III. Model Description

In this paper, a two-stage supply chain consisting of a manufacturer and a retailer is examined. The manufacturer sells wholesale goods to the retailer, and then the retailer sells the ordered goods to the customer. To increase sales, the manufacturer engages in selling efforts and bears the costs. The manufacturer and the retailer formulate the optimal wholesale price, the marginal profit per unit, and the selling effort needed to maximize their profits. If the manufacturer is dominant in the supply chain, it becomes the core in the chain. In the Stackelberg model, the dominant player makes the decisions first, the follower then makes decisions according to the dominant player's decisions. Therefore, in the two-stage supply chain Stackelberg game led by a manufacturer, the manufacturer first decides the wholesale price, then the retailer observes the price and decides on the marginal profit per unit. Thus, the manufacturer and the retailer maximize their profits. To construct a two-stage supply chain model in a fuzzy environment, the following basic symbols are used.

Notations:

w The wholesale price per product unit;

c_r The manufacturing cost per product unit;

c_r The retailer's operating cost per product unit;

e The manufacturer's selling effort;

m The retailer's wholesale purchase price and customer sale price differential, referred to as the marginal profit per unit;

Π_m The manufacturer's profit as a function of w and m; and,

Π_r The retailer's profit as a function of w and m.

Provided that the customer demand function is a linear decreasing function in wholesale prices and marginal profit per unit and increasing in the degree of selling effort, the function should be expressed as

$$D = a - bp + ke = a - b(w + m) + ke$$

Here, a and b are two mutually independent non-negative fuzzy variables. a represents the maximum market capacity and b represents the demand to price change rate. Customer demand D is also a fuzzy variable. Because the demand in practice is positive, $Pos\{a - b(w + m) + ke \le 0\} = 0$.

Given these preliminaries, the profit functions of the manufacturer and the retailer can be expressed, respectively, as follows:

$$\Pi_m(w, m) = (w - c_m)D. \quad (1)$$

$$\Pi_r(w, m) = (m - c_r)D. \quad (2)$$

IV. Model for a Two-Stage Supply Chain in a Fuzzy Environment

This paper analyzes a situation in which a manufacturer is dominant in the supply chain. Here, the manufacturer is the key enterprise in the supply chain and the retailer is the follower. Provided that the information between the manufacturer and the retailer is symmetric, according to the Stackelberg game model, the manufacturer makes decisions first.

Furthermore, because this paper considers the manufacturer's selling effort, the manufacturer's decision variable is the wholesale price and the level of selling effort. Hereafter, the retailer formulates the profit of the unit product according to the manufacturer's observed wholesale price, and both the manufacturer and the retailer maximize their profits. Using the previously stated basic assumptions, we can construct the expected value model of the supply chain with the manufacturer in the dominant role.

$$\begin{cases} \max\limits_{w} E[\Pi_m(w,e)] = \max\limits_{w} E\{(w-c_m)(a-b(w+m)+ke)-le^2\} \\ s.t. \\ w-c_m>0 \\ m^* \text{ is the optimal solution of the model at the lower level} \\ \begin{cases} \max\limits_{m} E[(m-c_r)(a-b(w+m)+ke)] \\ s.t. \\ Pos\{a-b(w+m)+ke\le 0\}=0 \end{cases} \\ Pos\{m-c_r\le 0\}=0 \end{cases} \tag{3}$$

Provided that $E[\Pi_r(m)]$ is the retailer's expected profit, the following tenable conclusion is made with respect to the aforementioned two-echelon planning model.

Theorem 1. Suppose that the wholesale price w is constant. Then, if

$$Pos\{a-b\frac{E[a]+wE[b]+eE[k]+E[c_r b]}{2E[b]}\le 0\}=0$$

and

$$Pos\{c_r\ge\frac{E[a]+eE[k]+E[c_r b]-wE[b]}{2E[b]}\}=0$$

then the best reaction function of the retailer to the wholesale price is

$$m^*=\frac{E[a]+eE[k]+E[c_R b]-wE[b]}{2E[b]}. \tag{4}$$

Proposition 5. The best reaction functions of the retailer m^* decrease strictly with w.

Proof.

$$\begin{aligned} &E[\Pi_r(m)] \\ &=\frac{1}{2}\int_0^1\{[(m-c_r)(a-b(w+m)+ke)]_\alpha^{\mathrm{L}}+[(m-c_r)(a-b(w+m)+ke)]_\alpha^{\mathrm{U}}\}d\alpha \\ &=\frac{1}{2}\int_0^1\{[(m-c_r)_\alpha^{\mathrm{L}}(a-b(w+m)+ke)_\alpha^{\mathrm{L}}]+[(w-c_r)_\alpha^{\mathrm{U}}(a-b(w+m)+ke)_\alpha^{\mathrm{U}}]\}d\alpha \\ &=\frac{1}{2}\int_0^1[(m-c_{r\alpha}^{\ \mathrm{U}})(a_\alpha^{\mathrm{L}}-b_\alpha^{\mathrm{U}}(w+m)+k_\alpha^{\mathrm{L}}e)+(m-c_{m\alpha}^{\ \mathrm{L}})(a_\alpha^{\mathrm{U}}-b_\alpha^{\mathrm{L}}(w+m)+k_\alpha^{\mathrm{U}}e)]d\alpha \\ &=\text{-E[b]m}^2+(E[a]+E[bc_r]-wE[b]+eE[k])m+wE[bc_r]-e^2E[l]-\frac{1}{2}\int_0^1(a_\alpha^{\mathrm{L}}c_{r\alpha}^{\ \mathrm{U}}+a_\alpha^{\mathrm{U}}c_{r\alpha}^{\ \mathrm{L}})d\alpha\text{-}\frac{1}{2}\int_0^1 e(k_\alpha^{\mathrm{L}}c_{r\alpha}^{\ \mathrm{U}}+k_\alpha^{\mathrm{U}}c_{r\alpha}^{\ \mathrm{L}})d\alpha \end{aligned} \tag{5}$$

Regarding the previous equations, the first-order and second-order derivatives are

$$\frac{dE[\Pi_r(m)]}{dm}=-2mE[b]+\{E[a]-wE[b]+eE[k]+E[c_r b]\},$$

$$\frac{d^2E[\Pi_r(m)]}{dm^2}=-2E[b]<0.$$

Therefore, $E[\Pi_r(m)]$ is a concave function with the maximum value as follows:

$$m^*(w,e)=-\frac{1}{2}w+\frac{E[a]+eE[k]+E[bc_r]}{2E[b]} \tag{6}$$

Obviously, m^* is a strictly decreasing function related to w and e. Provided that $E[\Pi_m(w,m^*(w))]$ is the retailer's expected profit, the following tenable conclusion is made regarding the aforementioned two-echelon planning model.

Theorem 2.

If

$$Pos\{c_r\ge\frac{E[a]-E[c_m b]+3E[c_r b]}{4E[b]}+\frac{E^2[k]\{E[a]+3E[c_m b]-E[c_r b]\}}{4E[b]\{8E[b]*E[l]-E^2[k]\}}-\frac{1}{2}*\frac{E[k]}{8E[b]*E[l]-E^2[k]}\int_0^1(c_{m\alpha}^{\ U}k_\alpha^L+c_{m\alpha}^{\ L}k_\alpha^U)d\alpha\}=0$$

and

$$Pos\{a-b(\frac{3E[a]+2eE[k]+E[c_M b]+E[c_R b]}{4E[b]}+\frac{E^2[k]\{E[a]+3E[c_M b]-E[c_R b]\}}{4E[b]\{8E[b]*E[l]-E^2[k]\}}-\frac{1}{2}*\frac{E[k]}{8E[b]*E[l]-E^2[k]}\int_0^1(c_{M\alpha}^{\ U}k_\alpha^L+c_{M\alpha}^{\ L}k_\alpha^U)d\alpha\le 0\}=0$$

then the optimal wholesale price, the optimal unit marginal profit, and the optimal degree of selling effort, respectively, are

$$w^*=\frac{E[a]+E[c_m b]-E[c_r b]}{2E[b]}+\frac{E[k]*E[k](E[a]+3E[c_m b]-E[c_r b])}{2E[b]*(8E[b]*E[l]-E[k]*E[k])}-\frac{E[k]*\int_0^1(c_{m\alpha}^{\ U}*k_\alpha^L+c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}$$

$$m^* = \frac{E[a]-E[c_m b]+3E[c_r b]}{4E[b]} + \frac{E[k]*E[k](E[a]+3E[c_m b]-E[c_r b])\text{-}E[k]*2E[b]*\int_0^1 (c_{m\alpha}^{\ U}*k_\alpha^L + c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha}{4E[b]*(8E[b]*E[l]-E[k]*E[k])}$$

$$e^* = \frac{(E[a]+3E[c_m b]-E[c_r b])*E[k]\text{-}2E[b]*\int_0^1 (c_{m\alpha}^{\ U}*k_\alpha^L + c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}$$

Proposition 6.
In $(w^*, e^*, m^*(w^*))$, the manufacturer and retailer achieve their maximum expected profit, which can be represented respectively, as

$$\begin{aligned}
&E[\Pi_m(w^*, e^*, m^*(w^*))] \\
&= (1\text{-}\frac{E[b]}{2})*\{\frac{E[a]+E[bc_m]\text{-}E[\mathrm{bc_r}]}{2E[b]} + \frac{E[k]*E[k]}{2E[b]}*\frac{E[a]+3E[\mathrm{bc_m}]-E[bc_r]}{8E[b]*E[l]-E[k]*E[k]} - \frac{E[k]*\int_0^1 (c_{m\alpha}^{\ U}*k_\alpha^L + c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2 \\
&+E[bc_m]*\frac{E[a]+E[\mathrm{bc_r}]}{2E[b]} + \frac{E[bc_m]*E[k]}{2E[b]}*\{\frac{E[k]*(E[a]+3E[\mathrm{bc_m}]-E[bc_r])\text{-}2E[\mathrm{b}]*\int_0^1 (c_{m\alpha}^{\ U}*k_\alpha^L + c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\} \\
&-E[l]*\{\frac{E[k]*(E[a]+3E[\mathrm{bc_m}]-E[bc_r])\text{-}2E[\mathrm{b}]*\int_0^1 (c_{m\alpha}^{\ U}*k_\alpha^L + c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2\text{-}\frac{1}{2}\int_0^1 e*(c_{m\alpha}^{\ U}*k_\alpha^L + c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha\text{-}\frac{1}{2}\int_0^1 (c_{m\alpha}^{\ U}*a_\alpha^L + c_{m\alpha}^{\ L}*a_\alpha^U)d\alpha
\end{aligned}$$

and

$$\begin{aligned}
&E[\Pi_{\mathrm{r}}(w^*, m^*(w^*))] \\
&= (1\text{-}E[b])*\{\frac{E[a]+3E[\mathrm{bc_r}]-E[bc_m]}{4E[b]} + \frac{E[k]*E[k]}{4E[b]}*\frac{E[a]+3E[\mathrm{bc_m}]-E[bc_r]}{8E[b]*E[l]-E[k]*E[k]}\text{-}\frac{E[k]}{2}*\frac{\int_0^1 (c_{m\alpha}^{\ U}*k_\alpha^L + c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2 \\
&+(\frac{1}{2}\text{-}E[b])*\{(\frac{E[a]+3E[\mathrm{bc_R}]-E[bc_M]}{4E[b]} + \frac{E[k]*E[k]}{4E[b]}*\frac{E[a]+3E[\mathrm{bc_M}]-E[bc_R]}{8E[b]*E[l]-E[k]*E[k]} \\
&\text{-}\frac{E[k]}{2}*\frac{\int_0^1 (c_{m\alpha}^{\ U}*k_\alpha^L + c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]})*(\frac{E[a]+E[bc_m]\text{-}E[\mathrm{bc_r}]}{2E[b]} + \frac{E[k]*E[k]}{2E[b]}*\frac{E[a]+3E[\mathrm{bc_m}]-E[bc_r]}{8E[b]*E[l]-E[k]*E[k]} - \frac{E[k]*\int_0^1 (c_{m\alpha}^{\ U}*k_\alpha^L + c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]})\} \\
&+E[\mathrm{bc_r}]*(\frac{E[a]+E[bc_m]\text{-}E[\mathrm{bc_r}]}{2E[b]} + \frac{E[k]*E[k]}{2E[b]}*\frac{E[a]+3E[\mathrm{bc_m}]-E[bc_r]}{8E[b]*E[l]-E[k]*E[k]} - \frac{E[k]*\int_0^1 (c_{m\alpha}^{\ U}*k_\alpha^L + c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}) \\
&\text{-}E[l]*\{\frac{E[k]*(E[a]+3E[\mathrm{bc_m}]-E[bc_r])}{8E[b]*E[l]-E[k]*E[k]}\text{-}\frac{2E[\mathrm{b}]*\int_0^1 (c_{m\alpha}^{\ U}*k_\alpha^L + c_{m\alpha}^{\ L}*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2\text{-}\frac{1}{2}\int_0^1 e*(c_{r\alpha}^{\ U}*k_\alpha^L + c_{r\alpha}^{\ L}*k_\alpha^U)d\alpha\text{-}\frac{1}{2}\int_0^1 (c_{r\alpha}^{\ U}*a_\alpha^L + c_{r\alpha}^{\ L}*a_\alpha^U)d\alpha
\end{aligned}$$

Proof. The process is the same as for Proposition 5. By substituting m^* in these equations, we obtain

$$\begin{aligned}
&E[\Pi_{\mathrm{m}}(w, e, m^*(w))] \\
&= \frac{1}{2}\int_0^1 \{[(w-c_m)(a-b(w+m)+ke)]_\alpha^{\mathrm{L}} + [(w-c_m)(a-b(w+m)+ke)]_\alpha^{\mathrm{U}}\}d\alpha \\
&= \frac{1}{2}\int_0^1 \{[(w-c_m)_\alpha^{\mathrm{L}}(a-b(w+m)+ke)_\alpha^{\mathrm{L}}] + [(w-c_m)_\alpha^{\mathrm{U}}(a-b(w+m)+ke)_\alpha^{\mathrm{U}}]\}d\alpha \\
&= \frac{1}{2}\int_0^1 [(w-c_{m\alpha}^{\ \mathrm{U}})(a_\alpha^{\mathrm{L}} - b_\alpha^{\mathrm{U}}(w+m) + k_\alpha^{\mathrm{L}}e) + (w-c_{m\alpha}^{\ \mathrm{L}})(a_\alpha^{\mathrm{U}} - b_\alpha^{\mathrm{L}}(w+m) + k_\alpha^{\mathrm{U}}e)]d\alpha \\
&= wE[a] - wmE[\mathrm{b}]\text{-}w^2E[\mathrm{b}] + weE[k] - \frac{1}{2}\int_0^1 e(c_{m\alpha}^{\ U}a_\alpha^L + c_{m\alpha}^{\ \mathrm{L}}a_\alpha^L)d\alpha + mE[c_m b] + wE[c_m b] - \frac{1}{2}\int_0^1 e(c_{m\alpha}^{\ U}k_\alpha^L + c_{m\alpha}^{\ \mathrm{L}}k_\alpha^L)d\alpha \\
&= -\frac{w^2}{2}*E[b] + w*\frac{E[a]+e*E[k]+E[c_m b]-E[c_r b]}{2} + E[c_m b]*\frac{E[a]+e*E[k]+E[c_r b]}{2E[b]} - \frac{1}{2}*\int_0^1 [(a_\alpha^L + e*k_\alpha^L)*c_{m\alpha}^{\ U} + (a_\alpha^U + e*k_\alpha^U)*c_{m\alpha}^{\ L}]d\alpha - e^2*E[l]
\end{aligned}$$

We can calculate the first-order and second-order derivatives of these equations with respect to w and e, respectively, as

$$\frac{dE[\Pi_{\mathrm{m}}(w, e, m^*(w^*))]}{dw} = -w*E[b] + \frac{E[a]+e*E[k]+E[c_m b]-E[c_r b]}{2}$$

$$\frac{d^2E[\Pi_{\mathrm{m}}(w, e, m^*(w^*))]}{dw^2} = -E[b] < 0$$

$$\frac{dE[\Pi_{\mathrm{m}}(w, e, m^*(w^*))]}{de} = -\frac{1}{2}*w*E[k] + \frac{E[c_m b]*E[k]}{2E[b]}$$

$$-\frac{1}{2}*\int_0^1 (c_{m\alpha}^{\ U}k_\alpha^L + c_{m\alpha}^{\ L}k_\alpha^U)d\alpha - 2eE[l]$$

$$\frac{d^2E[\Pi_{\mathrm{m}}(w, e, m^*(w^*))]}{de^2} = -2E[l] < 0$$

$$\frac{\partial^2E[\Pi_m(w, e, m^*(w^*))]}{\partial w \partial e} = \frac{E[k]}{2}$$

$$\frac{\partial^2E[\Pi_m(w, e, m^*(w^*))]}{\partial e \partial w} = \frac{E[k]}{2}.$$

The Hessian matrix of the profit function is as follows:

$$H = \begin{bmatrix} -E[b] & \frac{E[k]}{2} \\ \frac{E[k]}{2} & -2E[l] \end{bmatrix} = 4E[b]E[l] - \frac{E^2[k]}{4}.$$

Therefore, $E[\Pi_m(w,e,m^*(w))]$ is a concave function, which realizes its maximum value in $(w^*,e^*,m^*(w^*))$.

The retailer's maximum profit is:

$$
\begin{aligned}
&E[\Pi_{\mathrm{m}}(w^*,e^*,m^*(w^*))]\\
&=1\text{-}\frac{E[b]}{2}*\{\frac{E[a]+E[bc_m]\text{-}E[\mathrm{bc_r}]}{2E[b]}+\frac{E[k]*E[k]}{2E[b]}*\frac{E[a]+3E[\mathrm{bc_m}]-E[bc_r]}{8E[b]*E[l]-E[k]*E[k]}-\frac{E[k]*\int_0^1(c_{m\alpha}{}^U*k_\alpha^L+c_{m\alpha}{}^L*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2+E[bc_m]*\frac{E[a]+E[\mathrm{bc_r}]}{2E[b]}\\
&+\frac{E[bc_m]*E[k]}{2E[b]}*\{\frac{E[k]*E[a]+3E[\mathrm{bc_m}]-E[bc_r]}{8E[b]*E[l]-E[k]*E[k]}\text{-}\frac{2E[\mathrm{b}]*\int_0^1(c_{m\alpha}{}^U*k_\alpha^L+c_{m\alpha}{}^L*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}-E[l]*\{\frac{E[k]*E[a]+3E[\mathrm{bc_m}]-E[bc_r]}{8E[b]*E[l]-E[k]*E[k]}-\frac{2E[\mathrm{b}]*\int_0^1(c_{m\alpha}{}^U*k_\alpha^L+c_{m\alpha}{}^L*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2\\
&\text{-}\frac{1}{2}\int_0^1 e*(c_{m\alpha}{}^U*k_\alpha^L+c_{m\alpha}{}^L*k_\alpha^U)d\alpha\text{-}\frac{1}{2}\int_0^1(c_{m\alpha}{}^U*a_\alpha^L+c_{m\alpha}{}^L*a_\alpha^U)d\alpha
\end{aligned}
\tag{7}
$$

The manufacturer's maximum profit is:

$$
\begin{aligned}
&E[\Pi_{\mathrm{r}}(m^*,w^*(m^*))]\\
&=(1\text{-}E[b])*\{\frac{E[a]+3E[\mathrm{bc_r}]-E[bc_m]}{4E[b]}+\frac{E[k]*E[k]}{4E[b]}*\frac{E[a]+3E[\mathrm{bc_m}]-E[bc_r]}{8E[b]*E[l]-E[k]*E[k]}\text{-}\frac{E[k]}{2}*\frac{\int_0^1(c_{m\alpha}{}^U*k_\alpha^L+c_{m\alpha}{}^L*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2\\
&+(\frac{1}{2}\text{-}E[b])*\{(\frac{E[a]+3E[\mathrm{bc_R}]-E[bc_M]}{4E[b]}+\frac{E[k]*E[k]}{4E[b]}*\frac{E[a]+3E[\mathrm{bc_M}]-E[bc_R]}{8E[b]*E[l]-E[k]*E[k]}\text{-}\frac{E[k]}{2}*\frac{\int_0^1(c_{m\alpha}{}^U*k_\alpha^L+c_{m\alpha}{}^L*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]})*(\frac{E[a]+E[bc_m]\text{-}E[\mathrm{bc_r}]}{2E[b]}\\
&+\frac{E[k]*E[k]}{2E[b]}*\frac{E[a]+3E[\mathrm{bc_m}]-E[bc_r]}{8E[b]*E[l]-E[k]*E[k]}-\frac{E[k]*\int_0^1(c_{m\alpha}{}^U*k_\alpha^L+c_{m\alpha}{}^L*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]})\}\\
&+E[\mathrm{bc_r}]*(\frac{E[a]+E[bc_m]\text{-}E[\mathrm{bc_r}]}{2E[b]}+\frac{E[k]*E[k]}{2E[b]}*\frac{E[a]+3E[\mathrm{bc_m}]-E[bc_r]}{8E[b]*E[l]-E[k]*E[k]}-\frac{E[k]*\int_0^1(c_{m\alpha}{}^U*k_\alpha^L+c_{m\alpha}{}^L*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]})\\
&-E[l]*\{\frac{E[k]*(E[a]+3E[\mathrm{bc_m}]-E[bc_r])}{8E[b]*E[l]-E[k]*E[k]}-\frac{2E[\mathrm{b}]*\int_0^1(c_{m\alpha}{}^U*k_\alpha^L+c_{m\alpha}{}^L*k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2\text{-}\frac{1}{2}\int_0^1 e*(c_{r\alpha}{}^U*k_\alpha^L+c_{r\alpha}{}^L*k_\alpha^U)d\alpha\text{-}\frac{1}{2}\int_0^1(c_{r\alpha}{}^U*a_\alpha^L+c_{r\alpha}{}^L*a_\alpha^U)d\alpha
\end{aligned}
\tag{8}
$$

Strategy $(w^*,e^*,m^*(w^*))$ is the Stackelberg-Nash equilibrium solution for the supply chain expected value model. In addition, we can also build the $\max i \max$ chance-constrained and $\min i \max$ chance-constrained models.

First, we construct the $\max i \max$ chance-constrained model as follows:

$$
\begin{cases}
\max\limits_{w} \Pi_m\\
s.t.\\
Pos\{(w-c_m)[a-b(w+m^*(w,e))+ke]-le^2 \geq \Pi_m\} \geq \alpha\\
w-c_m>0\\
m^* \text{ is the optimal solution for the lower-level plan}\\
\begin{cases}
\max\limits_{m} \Pi_r\\
s.t.\\
Pos\{(m-c_r)(a-b(w+m)+ke) \geq \Pi_r\} \geq \alpha\\
Pos\{a-b(w+m)+ke \leq 0\}=0\\
Pos\{m-c_r \leq 0\}=0
\end{cases}
\end{cases}
\tag{9}
$$

Where α is the manufacturer's and the retailer's predefined confidence level for all provided available (w,m) strategies and $\max\limits_{m} \Pi_r$ and $\max\limits_{w} \Pi_m$ are the α -optimistic profit values for the manufacturer and retailer, respectively. Therefore, the model represented in (9) is equivalent to the following model:

$$
\begin{cases}
\max\limits_{m}((w-c_m)(a-b(w+m)+ke)-le^2)_\alpha^U\\
s.t.\\
w-c_m>0\\
m^* \text{ is the optimal solution for the lower-level plan}\\
\begin{cases}
\max\limits_{m}((m-c_r)(a-b(w+m)+ke))_\alpha^U\\
s.t.\\
Pos\{a-b(w+m)+ke \leq 0\}=0\\
Pos\{m-c_r \leq 0\}=0
\end{cases}
\end{cases}
\tag{10}
$$

wherein $(\Pi_m(w,m^*(w)))_\alpha^U$ and $(\Pi_r(m))_\alpha^U$ are the α -optimistic
profit values for the manufacturer and retailer, respectively.

Proposition 7.

If $Pos\{c_r \geq \frac{a_\alpha^U-c_{m\alpha}{}^L*b_\alpha^L+k^{2U}_\alpha+3c_{r\alpha}{}^Lb_\alpha^L}{4b_\alpha^L}\}=0$

and

$Pos\{a-b*(\frac{3a_\alpha^U+c_{m\alpha}{}^Lb_\alpha^L+k^{2U}_\alpha+3c_{r\alpha}{}^Lb_\alpha^L}{4b_\alpha^L}-\frac{k^{2U}_\alpha}{2l_\alpha^L}) \leq 0\}=0$

the model represented in (10) has one and only α -optimistic value and the Stackelberg-Nash equilibrium solution is

$$(\frac{a_\alpha^U+c_{m\alpha}{}^L*b_\alpha^L-c_{r\alpha}{}^Lb_\alpha^L}{2b_\alpha^L},\frac{a_\alpha^U-c_{m\alpha}{}^L*b_\alpha^L+k^{2U}_\alpha+3c_{r\alpha}{}^Lb_\alpha^L}{4b_\alpha^L}).$$

Proof. The optimistic value of the retailer's profit is

$$
\begin{aligned}
&\max\limits_{m}(\Pi_r(m))_\alpha^U=((m-c_r)(a-b(w+m)+ke))_\alpha^U\\
&=(m-c_{r\alpha}{}^L)*(a_\alpha^U-b_\alpha^L(w+m)+k_\alpha^Ue)\\
&=-m^2b_\alpha^L+m(a_\alpha^U-wb_\alpha^L+ek_\alpha^U+c_{r\alpha}{}^Lb_\alpha^L)+wc_{r\alpha}{}^Lb_\alpha^L-c_{r\alpha}{}^La_\alpha^U-ec_{r\alpha}{}^Lk_\alpha^U
\end{aligned}
$$

Generally, w and e are exogenous variables for the retailer; therefore, in the previous equation, we can only calculate the first- and second-order derivatives regarding m.

$$\frac{d\max\limits_{m}(\Pi_r(m))_\alpha^U}{dm}=-2mb_\alpha^L+a_\alpha^U-wb_\alpha^L+ek_\alpha^U+c_{r\alpha}{}^Lb_\alpha^L \tag{11}$$

$$\frac{d^2 \max_m (\Pi_r(m))_\alpha^{\mathrm{U}}}{dm^2} = -2b_\alpha^{\mathrm{L}} < 0$$

Therefore, $\max_m (\Pi_r(m))_\alpha^{\mathrm{U}}$ is a concave function and realizes its maximum value in

$$m^* = \frac{a_\alpha^{\mathrm{U}} - wb_\alpha^{\mathrm{L}} + ek_\alpha^U + c_{r\alpha}{}^{\mathrm{L}} b_\alpha^{\mathrm{L}}}{2b_\alpha^{\mathrm{L}}}. \qquad (12)$$

$$\max_w (\prod_m (w,e))_\alpha^U = ((w-c_m)(a-b(w+m)+ke)-le^2)_\alpha^U$$
$$=(w-c_{m\alpha}{}^L)*(a_\alpha^U - b_\alpha^L(w+m)+k_\alpha^U e - l_\alpha^L e^2)$$
$$=-\frac{w^2}{2} b_\alpha^L + w*\frac{a_\alpha^U + ek_\alpha^U - 2e^2 l_\alpha^L + c_{m\alpha}{}^L b_\alpha^L - c_{r\alpha}{}^L b_\alpha^L}{2} + \frac{a_\alpha^U + ek_\alpha^U + c_{r\alpha}{}^L b_\alpha^L}{2} - c_{m\alpha}{}^L a_\alpha^U + c_{m\alpha}{}^L k_\alpha^U e + c_{m\alpha}{}^L l_\alpha^U e^2$$

First, we calculate the first- and second-order derivatives of these equations with regard to w.

$$\frac{d \max_w (\Pi_m(w,e))_\alpha^{\mathrm{U}}}{dw} = -b_\alpha^L w + \frac{a_\alpha^U + ek_\alpha^U - 2e^2 l_\alpha^L + c_{m\alpha}{}^L b_\alpha^L - c_{r\alpha}{}^L b_\alpha^L}{2}$$

$$\frac{d^2 \max_w (\Pi_m(w,e))_\alpha^{\mathrm{U}}}{dw^2} = -b_\alpha^L < 0$$

Second, because the degree of selling effort e is also a decision variable for the manufacturer, the first- and second-order derivatives of these equations with regard to e can be calculated. The result is as follows:

$$\frac{d \max_w (\Pi_m(w,e))_\alpha^{\mathrm{U}}}{de} = 2(c_{m\alpha}{}^L - w) l_\alpha^L e + (w - c_{m\alpha}{}^L) k_\alpha^U,$$

$$\frac{d^2 \max_w (\Pi_m(w,e))_\alpha^{\mathrm{U}}}{de^2} = 2(c_{m\alpha}{}^L - w) l_\alpha^L < 0.$$

Third, we calculate the second partial order derivatives of w and e, respectively.

$$\frac{\partial^2 \max_w (\Pi_m(w,e))_\alpha^{\mathrm{U}}}{\partial w \partial e} = k_\alpha^U - 2el_\partial^L$$

$$\frac{\partial^2 \max_w (\Pi_m(w,e))_\alpha^{\mathrm{U}}}{\partial e \partial w} = k_\alpha^U - 2el_\partial^L$$

$$\frac{\partial^2 \max_w (\Pi_m(w,e))_\alpha^{\mathrm{U}}}{\partial w \partial e} = \frac{k_\alpha^U}{2} - 2el_\partial^L$$

$$\frac{\partial^2 \max_w (\Pi_m(w,e))_\alpha^{\mathrm{U}}}{\partial e \partial w} = k_\alpha^U - 2el_\partial^L$$

We derive the Hessian matrix:

$$H = \begin{bmatrix} -b_\alpha^L & k_\alpha^U - 2el_\partial^L \\ k_\alpha^U - 2el_\partial^L & 2(c_{m\alpha}{}^L - w) l_\alpha^L \end{bmatrix} = 2(c_{m\alpha}{}^L - w) l_\alpha^L (-) b_\alpha^L - (k_\alpha^U - 2el_\partial^L)^2$$

Thus, $\max_w (\Pi_m(w,e))_\alpha^{\mathrm{U}}$ is a strictly decreasing function and the optimal values are

$$e^* = \frac{k_\alpha^U}{2l_\alpha^L},$$

$$w^* = \frac{a_\alpha^U + c_{m\alpha}{}^L * b_\alpha^L - c_{r\alpha}{}^L b_\alpha^L}{2b_\alpha^L}.$$

Substituting w^* and e^* into m^*:

$$m^* = \frac{a_\alpha^U - c_{m\alpha}{}^L * b_\alpha^L + k^{2U}{}_\alpha + 3c_{r\alpha}{}^L b_\alpha^L}{4b_\alpha^L}$$

Subsequently, m^* is a strictly decreasing function regarding w. To derive the optimistic value of the manufacturer's profit, substituting m^* in (12) yields the following.

$$e^* = \frac{(E[a]+3E[c_m b]-E[c_r b])*E[k]-2E[b]*\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U) d\alpha}{8E[b]*E[l]-E[k]*E[k]}$$

Therefore, the retailer's optimistic profit is

$$\max_m (\Pi_r(m))_\alpha^{\mathrm{U}} = \frac{a_\alpha^U - c_{m\alpha}{}^L * b_\alpha^L + k^{2U}{}_\alpha + 3c_{r\alpha}{}^L b_\alpha^L}{16 b_\alpha^L} - \frac{c_{r\alpha}{}^L c_{m\alpha}{}^L b_\alpha^L - c_{r\alpha}{}^L a_\alpha^U}{2} - \frac{c_{r\alpha}{}^L k^{2U}{}_\alpha}{2l_\alpha^L}$$

The manufacturer's optimistic profit is

$$\max_w (\Pi_m(w,e))_\alpha^{\mathrm{U}} = \frac{b_\alpha^L}{2}\frac{(a_\alpha^U + c_{m\alpha}{}^L * b_\alpha^L)^2}{4b_\alpha^L} + \frac{c_{r\alpha}{}^L c_{m\alpha}{}^L b_\alpha^L - c_{m\alpha}{}^L a_\alpha^U}{2} - \frac{k^{2U}{}_\alpha c_{m\alpha}{}^L}{4l_\alpha^L} - \frac{c_{m\alpha}{}^L k^{2U}{}_\alpha}{2}$$

Strategy $(w^*, e^*, m^*(w^*, e^*))$ is the only equilibrium solution of α-optimistic values for the manufacturer and retailer.

In contrast, we can develop a $\min i \max$ chance-constrained programming model for the two-echelon supply chain.

$$\begin{cases} \max_w \min_{\Pi_m} \Pi_m \\ s.t. \\ Pos\{(w-c_m)[a-b(w+m^*(w,e))+ke]-le^2 \le \Pi_m\} \ge \alpha \\ w - c_m > 0 \\ m^* \text{ is the optimal solution for the lower-level plan} \\ \begin{cases} \max_w \min_{\Pi_r} \Pi_r \\ s.t. \\ Pos\{(m-c_r)(a-b(w+m)+ke) \le \Pi_r\} \ge \alpha \\ Pos\{a-b(w+m)+ke \le 0\} = 0 \\ Pos\{m - c_r \le 0\} = 0 \end{cases} \end{cases} \qquad (16)$$

Here, α is the manufacturer's and retailer's predefined confidence level for all provided available (w,m) strategies and $\max_w \min_{\Pi_m} \Pi_m$ and $\max_m \min_{\Pi_r} \Pi_r$ are the α-pessimistic values for the manufacturer and retailer, respectively. Therefore, the model represented in (16) is equivalent to the following model.

$$\begin{cases} \max_w ((w-c_m)(a-b(w+m)+ke)-le^2)_\alpha^L \\ s.t. \\ w - c_m > 0 \\ m^* \text{ is the optimal solution for the lower-level plan} \\ \begin{cases} \max_m ((m-c_r)(a-b(w+m)+ke))_\alpha^L \\ s.t. \end{cases} \end{cases}$$

$$Pos\{a-b(w+m)+ke\le 0\}=0$$
$$Pos\{m-c_r\le 0\}=0$$

Here, $(\Pi_m(w,m^*(w)))_\alpha^L$ and $(\Pi_r(m))_\alpha^L$ are the α -pessimistic values for the retailer and manufacturer, respectively.

Regarding the model represented in (16) and (17), the tenable conclusions are as follows.

Proposition 8.

If $Pos\{c_r \ge \dfrac{a_\alpha^L - c_{m\alpha}{}^U * b_\alpha^U + k^{2L}{}_\alpha + 3c_{r\alpha}{}^U b_\alpha^U}{4b_\alpha^U}\}=0$

and

$$Pos\{a-b*(\frac{3a_\alpha^L + c_{m\alpha}{}^U b_\alpha^U + k^{2L}{}_\alpha + 3c_{r\alpha}{}^U b_\alpha^U}{4b_\alpha^U} - \frac{k^{2L}{}_\alpha}{2l_\alpha^U})\le 0\}=0$$

the model represented in (17) has the one and only α -optimistic value, the Stackelberg-Nash equilibrium solution:

$$(\frac{a_\alpha^L + c_{m\alpha}{}^U * b_\alpha^U - c_{r\alpha}{}^U b_\alpha^U}{2b_\alpha^U}, \frac{a_\alpha^L - c_{m\alpha}{}^U * b_\alpha^U + k^{2L}{}_\alpha + 3c_{r\alpha}{}^U b_\alpha^U}{4b_\alpha^U})$$

Proof. This proof is similar to the proof of Proposition 7.

With respect to the previous analysis, a conclusion for the game equilibrium in the two-echelon supply chain is provided in Table I.

TABLE I

SUMMARY OF FUZZY TWO-ECHELON SUPPLY CHAIN MODEL DOMINATED BY THE MANUFACTURER

Ranking criterion	Optimal unit product profit m^*	Optimal wholesale price w^*	Optimal selling effort degree e^*
Expectation criterion	$\frac{E[a]+3E[\mathrm{bc_R}]-E[bc_M]}{4E[b]} + \frac{E[k]*E[k]}{4E[b]} * \frac{E[a]+3E[\mathrm{bc_M}]-E[bc_R]}{8E[b]*E[l]-E[k]*E[k]} - \frac{E[k]}{2} * \frac{\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}$	$\frac{E[a]+E[bc_M]-E[\mathrm{bc_R}]}{2E[b]} + \frac{E[k]*E[k]}{2E[b]} * \frac{E[a]+3E[\mathrm{bc_M}]-E[bc_R]}{8E[b]*E[l]-E[k]*E[k]} - \frac{E[k]*\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}$	$\frac{E[k]*(E[a]+3E[\mathrm{bc_M}]-E[bc_R])}{8E[b]*E[l]-E[k]*E[k]} - \frac{2E[\mathrm{b}]*\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}$
$\alpha-optimistic$ value criterion	$\frac{a_\alpha^U - c_{M\alpha}{}^L b_\alpha^L + (k_\alpha^U)^2 + 3c_{R\alpha}{}^L b_\alpha^L}{4b_\alpha^L}$	$\frac{a_\alpha^U + c_{M\alpha}{}^L b_\alpha^L - c_{R\alpha}{}^L b_\alpha^L}{2b_\alpha^L}$	$\frac{k_\alpha^U}{2l_\alpha^L}$
$\alpha-pessimistic$ value criterion	$\frac{a_\alpha^L - c_{M\alpha}{}^U b_\alpha^U + (k_\alpha^L)^2 + 3c_{R\alpha}{}^U b_\alpha^U}{4b_\alpha^U}$	$\frac{a_\alpha^L + c_{M\alpha}{}^U b_\alpha^U - c_{R\alpha}{}^U b_\alpha^U}{2b_\alpha^U}$	$\frac{k_\alpha^L}{2l_\alpha^U}$

	Retailer's maximum profit	Manufacturer's maximum profit
Expectation criterion	$(1-E[b])*\{\frac{E[a]+3E[\mathrm{bc_R}]-E[bc_M]}{4E[b]} + \frac{E[k]*E[k]}{4E[b]} * \frac{E[a]+3E[\mathrm{bc_M}]-E[bc_R]}{8E[b]*E[l]-E[k]*E[k]} - \frac{E[k]}{2} * \frac{\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2 + (\frac{1}{2}-E[b])*\{(\frac{E[a]+3E[\mathrm{bc_R}]-E[bc_M]}{4E[b]} + \frac{E[k]*E[k]}{4E[b]} * \frac{E[a]+3E[\mathrm{bc_M}]-E[bc_R]}{8E[b]*E[l]-E[k]*E[k]} - \frac{E[k]}{2} * \frac{\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]})*(\frac{E[a]+E[bc_M]-E[\mathrm{bc_R}]}{2E[b]} + \frac{E[k]*E[k]}{2E[b]} * \frac{E[a]+3E[\mathrm{bc_M}]-E[bc_R]}{8E[b]*E[l]-E[k]*E[k]} - \frac{E[k]*\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]})\} + E[\mathrm{bc_R}]*(\frac{E[a]+E[bc_M]-E[\mathrm{bc_R}]}{2E[b]} + \frac{E[k]*E[k]}{2E[b]} * \frac{E[a]+3E[\mathrm{bc_M}]-E[bc_R]}{8E[b]*E[l]-E[k]*E[k]} - \frac{E[k]*\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}) - \frac{1}{2}\int_0^1 e*(c_{R\alpha}{}^U * k_\alpha^L + c_{R\alpha}{}^L * k_\alpha^U)d\alpha - \frac{1}{2}\int_0^1 (c_{R\alpha}{}^U * a_\alpha^L + c_{R\alpha}{}^L * a_\alpha^U)d\alpha - E[l]*\{\frac{E[k]*(E[a]+3E[\mathrm{bc_M}]-E[bc_R])}{8E[b]*E[l]-E[k]*E[k]} - \frac{2E[\mathrm{b}]*\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2$	$(1-\frac{E[b]}{2})*\{\frac{E[a]+E[bc_M]-E[\mathrm{bc_R}]}{2E[b]} + \frac{E[k]*E[k]}{2E[b]} * \frac{E[a]+3E[\mathrm{bc_M}]-E[bc_R]}{8E[b]*E[l]-E[k]*E[k]} - \frac{E[k]*\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2 + E[bc_M]*\frac{E[a]+E[\mathrm{bc_R}]}{2E[b]} + \frac{E[bc_M]*E[k]}{2E[b]} * \{\frac{E[k]*(E[a]+3E[\mathrm{bc_M}]-E[bc_R])}{8E[b]*E[l]-E[k]*E[k]} - \frac{2E[\mathrm{b}]*\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\} - \frac{1}{2}\int_0^1 e*(c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha - \frac{1}{2}\int_0^1 (c_{m\alpha}{}^U * a_\alpha^L + c_{m\alpha}{}^L * a_\alpha^U)d\alpha - E[l]*\{\frac{E[k]*(E[a]+3E[\mathrm{bc_M}]-E[bc_R])}{8E[b]*E[l]-E[k]*E[k]} - \frac{2E[\mathrm{b}]*\int_0^1 (c_{m\alpha}{}^U * k_\alpha^L + c_{m\alpha}{}^L * k_\alpha^U)d\alpha}{8E[b]*E[l]-E[k]*E[k]}\}^2$
α -optimistic	$\frac{[a_\alpha^U - c_{m\alpha}{}^L b_\alpha^L + (k_\alpha^U)^2 + 3c_{R\alpha}{}^L b_\alpha^L]^2}{16b_\alpha^L} - \frac{c_{R\alpha}{}^L c_{m\alpha}{}^L b_\alpha^L - c_{R\alpha}{}^L a_\alpha^U}{2} - \frac{c_{R\alpha}{}^L (k_\alpha^U)^2}{2l_\alpha^L}$	$\frac{(a_\alpha^U + c_{m\alpha}{}^L b_\alpha^L)^2}{8} + \frac{c_{R\alpha}{}^L c_{m\alpha}{}^L b_\alpha^L - c_{M\alpha}{}^L a_\alpha^U}{2} - \frac{c_{M\alpha}{}^L (k_\alpha^U)^2}{2l_\alpha^L}$
α -pessimistic	$\frac{[a_\alpha^L - c_{m\alpha}{}^U b_\alpha^U + (k_\alpha^L)^2 + 3c_{R\alpha}{}^U b_\alpha^U]^2}{16b_\alpha^U} - \frac{c_{R\alpha}{}^U c_{m\alpha}{}^U b_\alpha^U - c_{R\alpha}{}^U a_\alpha^L}{2} - \frac{c_{R\alpha}{}^U (k_\alpha^L)^2}{2l_\alpha^U}$	$\frac{(a_\alpha^L + c_{m\alpha}{}^U b_\alpha^U)^2}{8} - \frac{c_{M\alpha}{}^U (k_\alpha^L)^2}{2l_\alpha^U} + \frac{c_{R\alpha}{}^U c_{m\alpha}{}^U b_\alpha^U - c_{M\alpha}{}^U a_\alpha^L}{2}$

V. Numerical Experiment

In the previous discussion, the pricing strategy of various retailers in a two-echelon supply chain in which the manufacturer plays the dominant role was solved. A numerical example is provided to illustrate the effectiveness of this game model.

For example, manufacturing costs c_m, operational costs c_r, market capacity a, demand change rate regarding price b, and demand change rate regarding selling effort l are typically evaluated by management decision makers and technical experts. During these evaluations, terms such as "low costs," "big market capacity," and "sensitive demand changing rate" are frequently used to describe the approximate evaluation values. The estimators depend on experience to determine the relationship between fuzzy language variables and the triangle fuzzy value, as shown in Table II.

We assume the following values. The evaluated product market capacity is very large (approximately 5000), the demand change rate with regard to price is very sensitive (approximately 500), the demand change rate with regard to selling effort is very sensitive (approximately 200), the sale costs change rate with regard to selling effort is very sensitive (approximately 100), manufacturing costs are average (approximately 5), and the retailer's operational costs are low (approximately 2). Then, according to the expected value model and fuzzy variable equation, the derived conclusion is as shown in Tables III through V.

TABLE II

RELATIONSHIP BETWEEN LINGUISTIC EXPRESSION AND TRIANGULAR FUZZY VARIABLE

	Language variable	Triangle fuzzy value
Manufacturing costs	Lower (approximately 3)	(2,3,4)
	Medium (approximately 5)	(4,5,6)
	Higher (approximately 7)	(6,7,8)
Operational costs	Lower (approximately 2)	(1,2,3)
	Medium (approximately 4)	(3,4,5)
	Higher (approximately 6)	(5,6,7)
Market capacity	Very big (approximately 5000)	(4900,5000,5100)
	Small (approximately 3000)	(2900,3000,3100)
Demand change rate with regard to price	Very sensitive (approximately 500)	(450,500,550)
	Sensitive (approximately 300)	(280,300,320)
Demand change rate with regard to selling effort	Very sensitive (approximately 200)	(150,200,250)
	Sensitive (approximately 100)	(80,100,120)
Sale cost change rate with regard to selling effort	Very sensitive (approximately 100)	(50,100,150)
	Sensitive (approximately 50)	(30,50,70)

TABLE III

OPTIMAL STRATEGY OF STACKELBERG GAME OF A SUPPLY CHAIN WHEN THE MANUFACTURER PLAYS A DOMINANT ROLE

Ranking criterion	Optimal unit product profit m^*	Optimal wholesale price w^*	Optimal selling effort e^*
Expected value criterion	409.2022656	823.1267535	0.002765957
	Maximum retailer profit	Maximum manufacturer profit	
	1937.670864	4526.388226	

TABLE IV

ANALYSIS OF OPTIMAL PRICING STRATEGY AND SENSITIVITY OF α VARIABLE

α value	Optimistic value criterion			Pessimistic value criterion		
	w^*	m^*	e^*	w^*	m^*	e^*
$\alpha = 1$	418.25	418.25	0.002857143	840.8333333	840.8333333	0.002857143
$\alpha = 0.95$	427.4786017	409.3235656	0.00294964	859.0949153	823.1737705	0.002765957
$\alpha = 0.9$	437.025	400.6846774	0.003043478	877.9896552	806.0870968	0.002676056
$\alpha = 0.85$	446.9059211	392.3196429	0.003138686	897.5508772	789.5460317	0.002587413
$\alpha = 0.8$	457.1392857	384.215625	0.003235294	917.8142857	773.525	0.0025

TABLE V

ANALYSIS OF OPTIMAL PROFITS ACCORDING TO SENSITIVITY OF α VARIABLE

α value	Optimistic value criterion		Pessimistic value criterion	
	Maximum retailer profit	Maximum manufacturer profit	Maximum retailer profit	Maximum manufacturer profit
$\alpha = 1$	549424.1304	3157327.982	549424.1304	3157327.982
$\alpha = 0.95$	563713.0446	3186895.144	535626.9661	3127931.264
$\alpha = 0.9$	578519.206	3216633.179	522297.6994	3098704.561
$\alpha = 0.85$	593869.9009	3246542.517	509413.9923	3069647.443
$\alpha = 0.8$	609794.3646	3276623.589	496954.9029	3040759.483

In Table III, we observe that manufacturers who play a dominant role in the Stackelberg game of a two-echelon supply chain obtain larger profits than retailers. Manufacturers have the power to determine pricing by taking advantage of their dominant roles. Furthermore, manufacturers engage in selling efforts to expand their market capacity to obtain larger profits.

In Tables IV and V, we observe that the optimal strategies and the maximum profits of the Stackelberg game change with the predefined confidence levels of manufacturers and retailers. Under the optimistic value criterion, as the confidence level decreases, the manufacturers' and retailers' optimal wholesale prices, optimal selling effort, and maximum profits gradually increase, but the optimal unit margin profits decrease. Under the pessimistic value criterion, as confidence levels decrease, only the optimal wholesale prices gradually increase when manufacturers' optimal unit margin profits, optimal selling effort, and maximum profits gradually decrease.

VI. CONCLUSION

In this paper, the selling effort as a fuzzy variable is considered to be part of the costs, along with other fuzzy variables, such as market demand, manufacturing costs, and operational costs. We present an expected value model, which is a chance-constrained programming model (the related models of optimistic and pessimistic values) of a two-echelon supply chain with a dominant manufacturer to provide a comprehensive discussion of supply chain games. We use game theory and conduct an analysis of the optimal pricing strategies and maximum profits for both the manufacturer and the retailer in the various models when the manufacturer is dominant. In the Stackelberg game equilibrium, the dominant role yields higher profits for manufacturers, and the optimal pricing strategies are related to the confidence levels predefined by manufacturers and retailers. Thus, one conclusion is that expanding the optimal strategies is related to uncertainty in the market environment.

One limitation of this article is that we only consider one supplier and one retailer in the supply chain. Therefore, one possible extension work is to study the supply chain game with multiple competing retailers or suppliers in a fuzzy decision making environment.

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References

[1] B. T. W. McGuire and R. Staelin, "An industry equilibrium analysis of downstream vertical integration," *Marketing Science*, vol. 2, no. 2, pp. 161–191, Mon. 1983.

[2] K. S. Moorthy, "Strategic decentralization in channels," *Marketing Science*, vol. 7, no. 4, pp. 335–355, 1988.

[3] B. D. Rhee and S. Y. Park, "Online store as a new direct channel and emerging hybrid channel system," *Channels of Distribution*, 2000.

[4] A. T. Coughlan, "Competition and cooperation in marketing channel choice: Theory and application," *Marketing Science*, vol. 4, no. 2, pp. 110–129, 1985.

[5] S. Y. Park and H. T. Keh, "Modelling hybrid distribution channels: A game-theoretic analysis," *Journal of Retailing & Consumer Services*, vol. 10, no. 3, pp. 155–167, 2003.

[6] O. Baron, O. Berman, and D. Wu, "Bargaining within the supply chain and its implication to coordination of supply chains in an industry," *Decision Sciences*, vol. 47, no. 2, pp. 193–218, 2016.

[7] W. Zeng and Y. W. Zhou, "Game analysis of principal and subordinate supply chain competition," *Acta Mathematicae Application Sinica*, vol. 36, no. 5, pp. 769–782, 2013.

[8] Z. Lu, X. Y. Huang, and S. R. Guan, "Research on supply chain coordination and Stackelberg game under uncertain JIT delivery," *Chinese Journal of Management Science*, vol. 11, no. 4, pp. 15–19, 2003.

[9] T. Boyaci and G. Gallego, "Supply chain coordination in a market with customer service competition," *Production & Operations Management*, vol. 13, no. 1, pp. 3–22, 2009.

[10] T. Boyaci, "Competitive stocking and coordination in a multiple-channel distribution system," *IIE Transactions*, vol. 37, no. 5, pp. 407–427, 2005.

[11] Q. Geng and S. Mallik, "Inventory competition and allocation in a multi-channel distribution system," *European Journal of Operational Research*, vol. 182, no. 2, pp. 704–729, 2007.

[12] S. R. Tayur, R. Ganeshan, and M. J. Magazine, *Quantitative models for supply chain management*, City: Kluwer Academic Publishers, 1999.

[13] B. Dan and G. Y. Xu, "Revenue sharing contract of dual channel supply chain coordination under stochastic demand," *Journal of Systems Engineering*, vol. 28, no. 4, pp. 514–521, 2013.

[14] S. C. Choi, "Price competition in a channel structure with a common retailer," *Marketing Science*, vol. 10, no. 4, pp. 271–296, 1991.

[15] E. Lee and R. Staelin, "Vertical strategic interaction: Implications for channel pricing strategy," *Marketing Science*, vol. 16, no. 3, pp. 185–207, 1997.

[16] M. Trivedi, "Distribution channels: An extension of exclusive retailership," *Management Science*, vol. 44, no. 7, pp. 896–909, 1998.

[17] S. Sang, "Price competition of manufacturers in supply chain under a fuzzy decision environment", Fuzzy Optimization and Decision Making, vol.14, no 3, pp. 335 - 363, 2015.

[18] D. Q. Yao and J. J. Liu, "Competitive pricing of mixed retail and e-tail distribution channels," *Omega*, vol. 33, no. 3, pp. 235–247, 2005.

[19] G. Hua, S. Wang, and T. C. E. Cheng, "Price and lead time decisions in dual-channel supply chains," *European Journal of Operational Research*, vol. 205, no. 1, pp. 113–126, 2010.

[20] S. S. Sana, "A production-inventory model of imperfect quality products in a three-layer supply chain," *Decision Support Systems*, vol. 50, no. 2, pp. 539–547, Jan. 2011.

[21] Y. X. Yan and X. Y. Xu, "Research on adaptability and coordination of retailers' dual channel," *Systems Engineering-Theory Methodology Application*, vol. 21, no. 5, pp. 602–608, 2012.

[22] S. Sang, "Coordinating a three stage supply chain with fuzzy demand," *Engineering Letters*, vol. 22, no. 3, pp. 109–117, 2014.

[23] M. Hong, "Stackelberg game in a supply chain led by retailers in a fuzzy environment," *Engineering Letters*, vol. 24, no. 1, pp. 45–51, 2016.

[24] S. Sang, "Pricing and service decisions of supply chain in an uncertain environment," *Engineering Letters*, vol. 24, no. 4, pp. 441–448, 2016.

[25] B. Q. Hu, *Fuzzy Theory Foundation*, Wuhan University Press, 2010.

[26] C. Zhou, R. Zhao, and W. Tang, "Two-echelon supply chain games in a fuzzy environment," *Computers & Industrial Engineering*, vol. 55, no. 2, pp. 390–405, 2008.

[27] S. Nahmias, "Fuzzy variables," *Fuzzy Sets & Systems*, vol. 1, no. 2, pp. 97–110, 1978.

[28] Y. K. Liu and B. Liu, "Expected value operator of random fuzzy variable and random fuzzy expected value models," *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 11, no. 2, pp. 195–215, 2003.

[29] B. Liu, and Y. K. Liu, "Expected value of fuzzy variable and fuzzy expected value models," *IEEE Trans. Fuzzy Systems*, vol. 10, no. 4, pp. 445–450, 2002.