Experimental Study about the Applicability of Traffic-induced Vibration for Bridge Monitoring

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Abstract—**Traffic-induced vibration is bridge vibration that passing vehicles excite. To estimate the modal parameters of the monitored bridge, SVD (Singular Value Decomposition) method and FDD (Frequency Domain Decomposition) method assume different load conditions, but both of them are not usually satisfied by traffic loads. In this study, the result differences of these two methods are experimentally investigated. To compare them, MAC (Modal Accuracy Criteria) value is applied. This study reveals that the tendency of MAC value changes due to bridge damage. This means that there is a probability of bridge damage detection method using SVD and FDD.**

Index Terms—**FDD, traffic-induced vibration, SHM, SVD**

I. Introduction

BRIDGE inspection usually indicates visual check done by veteran engineers. They can evaluate structural health of the monitored bridge by checking its appearance visually and sometimes by testing sound of hammering. Because of that, inspectors are required to have sufficient knowledge and experience. Because the inspection accuracy usually depends on the skill of the engineers, the training skilled engineers is always a time-consuming and important issue of the society.

However, Japan is facing a serious aging problem at the moment. It can be predicted that many skilled technicians will retire and that the recruiting will be more difficult. Therefore, the Japanese government is now interested in the application of updated technologies to this field. For example, it can be examined to fill this shortage of human resources with AI. This means that it is necessary to develop technology to substitute professional intuition with data processing.[1]

To objectively evaluate bridge damages, vibration-based SHM has been intensively developed.[2], [3], [4] The vibration includes information of the bridge as a mechanical system. Therefore, it is said that the bridge state can be estimated from vibration data. Usually, the vibration data used in SHM is free damping oscillation, because its frequency directly indicates the eigen frequency of the system. The disadvantage of using the free vibration is that the amplitude is small. On the other hand, the vibration excited by the running vehicle has a large amplitude. The disadvantage of using traffic-induced vibration is that the main part of the obtained vibration is transient response. This means that analyzing the traffic-induced vibrations is a higher level technology than the free vibrations.The popular kind of damage that often occurs on a

bridge is local. The local abnormalities usually affect higher modal parameters, of which amplitude is very small. To find the very small changes in the high order mode component, it is necessary to install expensive sensors on each bridge, or to analyze the traffic-induced vibrations.

In this study, the analysis method is examined to extract the modal parameters of the bridge system from the traffic-induced vibrations. As the examined methods, two methods that have different assumptions are adopted: SVD (singular value decomposition) method and FDD (frequency domain decomposition) method[5], [6]. To verify the applicability of them, experimental models of vehicle and bridge are made. The bridge model is made with paper so that its eigen frequency becomes similar to the real one. While the vehicle model is running over the bridge model, the traffic-induced vibrations are measured. The vehicle weight and the bridge state are varied for parametric study. The model of vehicle runs over the bridge repeatedly, in each case. The bridge damages are modeled by removing a member.

II. Theory Basis

A. Analysis: Mode Shape as Damage Detection Indicator

A bridge is a structure as a dynamic system composed of several materials and members. When it resists the loads, its response depends on position. If only one sensor was installed on the bridge, some information would be overlooked. In the case of multipoint measurement, the overlooked information is reduced and the possibility of damage detection increases. Because of that, the multipoint measurement is adopted in this experiment. As a damage detection indicator that can be obtained from the multipoint measurements, the mode shape of bridge is said to be effective.

The Mode shape of the bridge is the eigen vector of the production of the inverse mass matrix and the stiffness matrix. This means that the mode shape indicates the distribution of the flexural rigidity. It is known that the amplitude of mode shape at/around the damage point tends to become larger than before. If it is possible to accurately estimate it from vibration data, the bridge damage can be identified.

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Letting $\mathbf{M}$ and $\mathbf{K}$ be the mass and stiffness matrices of the bridge, respectively, the equation of motion of the bridge can be written in the follow:

$$\mathbf{M}\ddot{\boldsymbol{y}}(t) + \mathbf{K}\boldsymbol{y}(t) = \boldsymbol{f}(t) \tag{1}$$

where $\boldsymbol{y}(t)$ and $\boldsymbol{f}(t)$ are the vibration outputs and the load inputs of the bridge system, respectively. The operator $\ddot{()}$ indicates the second time differential. After vehicles passed the bridge, the input $\boldsymbol{f}(t)$ becomes the zero vector. In this case, the vibration of the bridge becomes free. If the k-th order eigen frequency of the bridge is ω_k, the output can be assumed as the follow:

$$\boldsymbol{y}(t) = \boldsymbol{a}_k \sin(\omega_k t) \tag{2}$$

where $\boldsymbol{a}_k$ is the k-th order mode shape. For simplicity, the initial phase is ignored, but essentially it does not matter. By substituting Eq. (2) and $\boldsymbol{f}(\mathrm{t}) = \mathbf{0}$ into Eq.(1), we obtain

$$[\mathbf{M}^{-1}\mathbf{K}]\{\boldsymbol{a}_k(t)\} = \omega_k^2\{\boldsymbol{a}_k(t)\}. \tag{3}$$

From this equation, we see that the mode shape $\boldsymbol{a}_k(t)$ is the eigen vector of the matrix $\mathbf{M}^{-1}\mathbf{K}$. Eq.(3) can be rewritten in the following equation.

$$\mathbf{M}^{-1}\mathbf{K} = \mathbf{A}\boldsymbol{\Sigma}\mathbf{A}^{\mathrm{T}} \tag{4}$$

where $\mathbf{A}$ is the modal matrix, of which the k-th row is corresponding to the k-th order mode shape vector $\boldsymbol{a}_k(t)$, and $\boldsymbol{\Sigma}$ is the diagonal matrix, of which k-th element is ω_k^2. The output response $\boldsymbol{y}(t)$ can be expressed as the follow:

$$\boldsymbol{y}(t) = \mathbf{A}\boldsymbol{q}(t) \tag{5}$$

where $\boldsymbol{q}(t)$ is the basis coordinate vector. Data processing becomes a problem to find the mode shape matrix $\mathbf{A}$ from the obtained data $\boldsymbol{y}(t)$.

B. *Mode Shape Estimation: SVD method*

If the measured data is written in the form of data matrix $\mathbf{Y} \in \mathrm{R}^{n\times N}$, it is

$$\mathbf{Y} = [\boldsymbol{y}(t_0), \boldsymbol{y}(t_1), \cdots, \boldsymbol{y}(t_N)]. \tag{6}$$

The problem of finding $\mathbf{A}$ can be taken as the decomposition of $\mathbf{Y}$ into $\mathbf{A}$ and $\mathbf{Q}$, as shown below:

$$\mathbf{Y} = \mathbf{AQ} \tag{7}$$

where

$$\mathbf{Q} = [\boldsymbol{q}(t_0), \boldsymbol{q}(t_1), \cdots, \boldsymbol{q}(t_N)]. \tag{8}$$

To estimate $\mathbf{A}$, SVD (Singular Value Decomposition) can be applied to the data, if it can be assumed that $\boldsymbol{q}(t)$ shows the non-correlativity. The non-correlativity is a similar to the orthogonality. An arbitrary orthogonal data matrix $\mathbf{X} \in \mathrm{R}^{n\times N}$ ($n < N$) satisfies

$$\mathbf{X}\mathbf{X}^{\mathrm{T}} = \mathbf{I} \tag{9}$$

where $\mathbf{I} \in \mathrm{R}^{n\times n}$ is a unit matrix. Similarly, an arbitrary non-correlative data matrix $\mathbf{X}$ satisfies

$$\mathbf{X}\mathbf{X}^{\mathrm{T}} = \boldsymbol{\Sigma} \tag{10}$$

where $\boldsymbol{\Sigma} \in \mathrm{R}^{n\times n}$ is a diagonal matrix. SVD method is a kind of orthogonal decomposition. By SVD, An arbitrary matrix $\mathbf{Y} \in \mathrm{R}^{n\times N}$ can be expressed uniquely as

$$\mathbf{Y} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}} \tag{11}$$

where $\mathbf{U} \in \mathrm{R}^{n\times n}$ is an orthogonal matrix, $\boldsymbol{\Sigma} \in \mathrm{R}^{n\times n}$ is a diagonal matrix and $\mathbf{V} \in \mathrm{R}^{N\times n}$ is an orthogonal matrix. Now, $\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}$ is a non-correlative data matrix, as shown below:

$$(\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}})(\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}})^{\mathrm{T}} = \boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}\mathbf{V}\boldsymbol{\Sigma}^{\mathrm{T}} = \boldsymbol{\Sigma}\mathbf{I}\boldsymbol{\Sigma}^{\mathrm{T}} = \boldsymbol{\Sigma}\boldsymbol{\Sigma}^{\mathrm{T}} = \boldsymbol{\Sigma}^2. \tag{12}$$

Therefore, $\mathbf{U}$ and $\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}$ can be taken as the estimated matrices of $\mathbf{A}$ and $\mathbf{Q}$, respectively.

For accurate estimation of $\mathbf{A}$, the non-correlative condition of $\mathbf{Q}$ should be satisfied. It is known that this is satisfied by free vibrations, vibrations of which input is white Gauss noise, and steady-state vibrations. However, it is clearly expected that the traffic-induced vibration does not always satisfy this condition. This reduces the estimation accuracy based on the SVD method.

C. *Mode Shape Estimation: FDD method*

Letting $\overline{\boldsymbol{y}}(\omega)$ be the Fourier transform of $\boldsymbol{y}(t)$, Eq. (5) can be rewritten in

$$\overline{\boldsymbol{y}}(\omega) = \mathbf{A}\overline{\boldsymbol{q}}(\omega) \tag{13}$$

where $\overline{\boldsymbol{q}}(\omega)$ is the Fourier transform of $\boldsymbol{q}(t)$, and ω is the angular frequency. The cross-power spectra $\overline{\boldsymbol{y}}(\omega)\overline{\boldsymbol{y}}^{\mathrm{T}}(\omega)$ is

$$\overline{\boldsymbol{y}}(\omega)\overline{\boldsymbol{y}}^{\mathrm{T}}(\omega) = \mathbf{A}\overline{\boldsymbol{q}}(\omega)\overline{\boldsymbol{q}}^{\mathrm{T}}(\omega)\mathbf{A}^{\mathrm{T}}. \tag{14}$$

Assuming that $\overline{\boldsymbol{q}}(\omega)\overline{\boldsymbol{q}}^{\mathrm{T}}(\omega)$ is always a diagonal matrix, the cross-power spectra $\overline{\boldsymbol{y}}(\omega)\overline{\boldsymbol{y}}^{\mathrm{T}}(\omega)$ can be decomposed by SVD method. The SVD of $\overline{\boldsymbol{y}}(\omega)\overline{\boldsymbol{y}}^{\mathrm{T}}(\omega)$ is

$$\overline{\boldsymbol{y}}(\omega)\overline{\boldsymbol{y}}^{\mathrm{T}}(\omega) = \mathbf{U}(\omega)\,\boldsymbol{\Sigma}(\omega)\,\mathbf{U}^{\mathrm{T}}(\omega) \tag{15}$$

where $\mathbf{U}(\omega)$ and $\boldsymbol{\Sigma}(\omega)$ are the estimated mode shape and cross-power spectra of the basis coordinates, respectively. The first column vector of $\mathbf{U}(\omega)$ becomes the estimated mode shape $\boldsymbol{\phi}(\omega)$, which is the most predominant mode shape at that frequency. This process estimating mode shapes is called FDD (Frequency Domain Decomposition). The first element of $\boldsymbol{\Sigma}(\omega)$ is a singular value spectrum.

D. *Data Processing: MAC*

To compare the estimated results, MAC (Modal Accuracy Criteria) is adopted. If the SVD and FDD results are $\boldsymbol{\phi}_i$ and $\boldsymbol{\phi}(\omega)$, respectively, the MAC value can be defined as

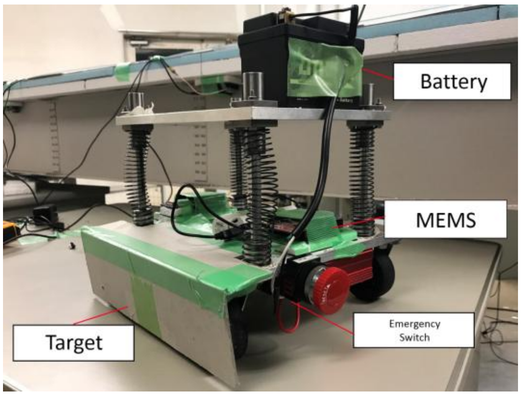
$$m_i(\omega) = \frac{|\boldsymbol{\phi}_i \cdot \boldsymbol{\phi}(\omega)|^2}{(\boldsymbol{\phi}_i \cdot \boldsymbol{\phi}_i)\big(\boldsymbol{\phi}(\omega) \cdot \boldsymbol{\phi}(\omega)\big)} \tag{16}$$

where $(\cdot)$ denotes the inner product. If this value is close to 1, these two vectors are similar to each other.

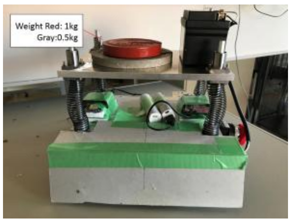
SVD and FDD have different assumptions for estimating the mode shapes. The value of $m_i(\omega)$ is idealy 1, while the difference between the assumptions affects the results. If the effect of the gap depends on the bridge damage, we can find the changes in the tendency of $m_i(\omega)$ after damage.

III. EXPERIMENTAL SETUP

To verify the possibility of damage detection based on the tendency of MAC values $m_i(\omega)$, a laboratory experiment using a vehicle-bridge interaction model is carried out. The vehicle model is a self-propelled car with an electric motor and springs. The springs are modeling vehicular suspensions. **Figure 1** shows the vehicle model. Its basic parameters are also shown in **Table 1**. The sprung-mass is varied by adding the weights.



(a) The overview of the model



(b) The additional weight

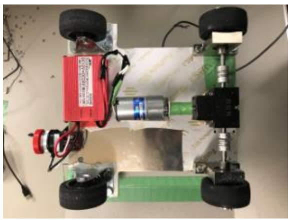

(c) The bottom view and the electric motor

Figure 1 The photos of the vehicle model: (a) To detect the vehicle position by two laser sensors at the entry and exit of the bridge, a cover target is installed. To check the dynamic response of the vehicle, two MEMS sensors are put on the unsprung-mass at both of the front and rear axles. The sprung- and unsprung-mass plates are connected by 4 shafts with springs. To make the vibration of the sprung-mass be smooth, the ball-bearing systems are fixed on it at the shafts.

Table 1 The parameters of the vehicle model: The sprung-mass is varied by adding several weights. The speed is fixed.

Dimension			Performance		
Length	Width	Height	Speed	Weight (Additional)	Natural Frequency
270 mm	280 mm	200 mm	1.34 m/s	6.92 kg (0.5, 1.0, 1.5, 2.0kg)	2.55 Hz

The bridge model is made of cardboards and steel bolts. Its dimension is designed to have the same natural frequencies as the real steel bridges. The details is also similar. **Figure 2** is the diagram of the members.

The bridge damage can be modeled by removing a brace or a cross-girder. **Figure 3** is the photos of the bridge model, and the parameters of the girders are also shown in **Table 2**.

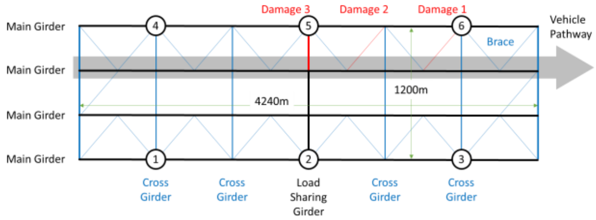


Figure 2 The top view of bridge model: The girders, stiffeners, gusset plates and braces are made of cardboards. They are assembled with glues modeling weld, and are connected with bolts. The Styrofoam deck is on the paper girder bridge. 6 MEMS Sensors are installed on the deck.


(a) The structure of paper bridge


(b) The overview


(c) The detail

Figure 3 The photos of the bridge model: (a) This model simulates a steel girder bridge. (b) There is a Styrofoam deck on the paper bridge model. On the deck, the road profile is introduced by putting an unevenness made by the same cardboard.

Table 2 The parameters of the bridge model:

Common	Span Length	4240 mm
	Cardboard Thickness	1 mm
	Intervals between Main Girders	400 mm
Main Girder	Web Height	260 mm
	Flange Width	70 mm
	Stiffeners Intervals	250 mm
Deck	Styrofoam Thickness	52 mm
Performance	Design Natural Frequency	5.30 Hz

In this experiment, we set several run scenarios shown in **Table 3**. The vehicle model runs 50 times for each case. After damage cases, the bridge model is repaired and the vehicle runs to check whether the bridge condition is intact or not. In the case of Damage 1, the brace located from 9L/12 to 10L/12 under the pathway is removed. It is a model of edge damage. In Damage 2, the removed brace is closer to the mid-span, located from 7L/12 to 8L/12. In Damage 3, the load-sharing girder at mid-span under the pathway is removed as a serious damage model.

Table 3 The parameters of the bridge model: 4 bridge conditions and 5 vehicle models are examined in this experiment.

No.	Bridge	Vehicle	Count
1	Intact	(0) 6.92 kg	50
2		(1) 7.42 kg	50
3		(2) 7.92 kg	50
4		(3) 8.42 kg	50
5		(4) 8.92 kg	50
6	Damage 1	(0) 6.92 kg	50
7		(1) 7.42 kg	50
8		(2) 7.92 kg	50
9		(3) 8.42 kg	50
10		(4) 8.92 kg	50
11	Intact	(4) 8.92 kg	50
12	Damage 2	(0) 6.92 kg	50
13		(1) 7.42 kg	50
14		(2) 7.92 kg	50
15		(3) 8.42 kg	50
16		(4) 8.92 kg	50
17	Intact	(4) 8.92 kg	50
18	Damage 3	(0) 6.92 kg	50
19		(1) 7.42 kg	50
20		(2) 7.92 kg	50
21		(3) 8.42 kg	50
22		(4) 8.92 kg	50
23	Intact	(4) 8.92 kg	50

IV. Results and Discussions

A. Results of SVD and FDD methods

Figure 4 shows the results of SVD method applied to the vibrations measured by 6 MEMS sensors on the paper bridge, while the 8.92kg vehicle is running over it. In each case, while the vehicle model runs in 50 times, the obtained results are always same under the same bridge condition. It is cleared that the traffic-induced vibration has high repeatability.

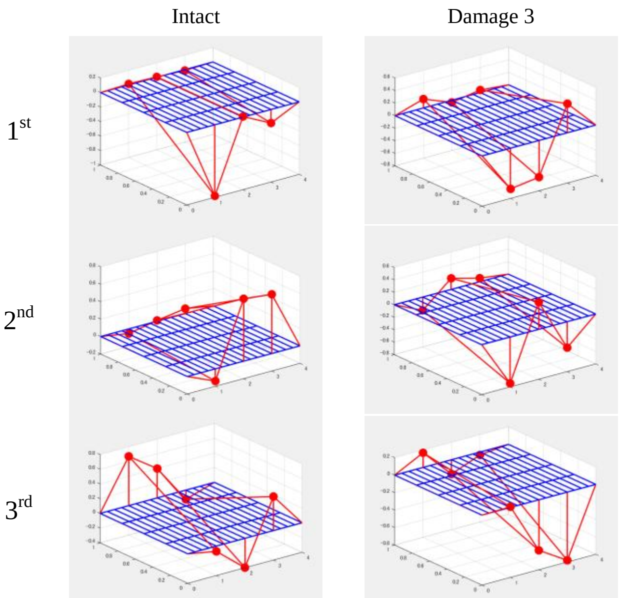


Figure 4 The examples of the estimated mode shapes, in the cases of "Intact" and "Damage 3": The front side is the vehicle pathway.

According to **Figure 4**, the amplitude of the mode shape at the vehicle pathway side is larger than the opposite side. The difference between these two cases, "Intact" and "Damage 3", at least, is clear. If the conditions of vehicle and bridge are same, the same mode shapes are obtained repeatedly. However, the estimation accuracy seems to be low. The reason of the low accuracy can be the error caused by the problem that traffic-induced vibrations did not satisfy the SVD's assumption well. It is known well that we often face such a low accuracy problem at the estimation of the mode shapes. Because of this problem, it is still difficult to identify or detect a damage of a monitored structure.

The obtained mode shapes in 50 runs are very similar to

Many researchers and engineers facing this problem often improve the accuracy by replacing sensors with expensive ones, increasing its number, adopting a different estimation method, and/or updating the method algorithm. The problem of this implement is that we get the different results in each improvement. On the other hand, the concept of the proposed method is to accept this error and to implement this problem by comparing two methods.

B. Comparison of Results by MAC

Figure 5 shows the singular value spectra of FDD method in the case of using 8.92kg vehicle. The spectra of "Damage 1" and "Damage 2" show the same tendencies with that of "Intact", while the spectra amplitude of "Damage 3" becomes larger. The peak frequencies do not change after the damage. Many previous studies have shown that it is efficient to check changes of the natural frequencies due to bridge's serious damages. However, the moderate damages do not change the frequency and it is difficult to detect differences.

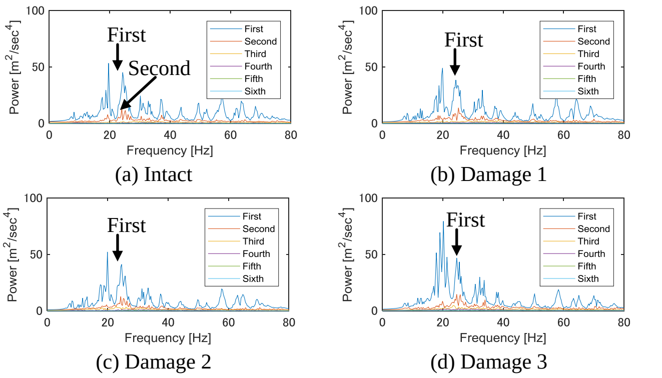
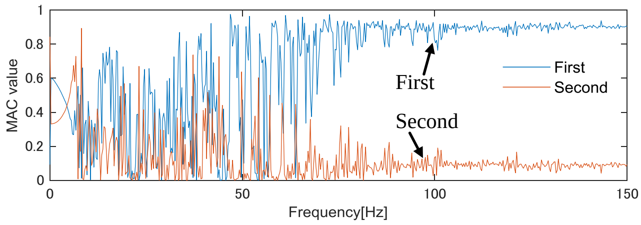


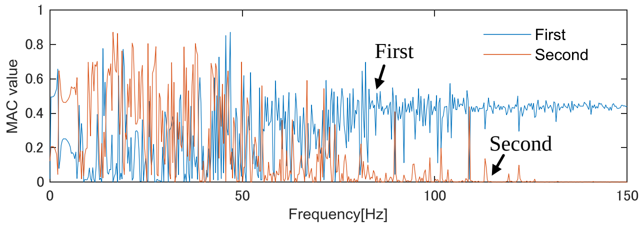
Figure 5 The examples of the Singular Value Spectra:

FDD method can also estimate the mode shapes. The MAC value $m_i(\omega)$ is shown in **Figure 6**. It is confirmed that the same MAC values are obtained repeatedly, if the conditions of the vehicle and bridge models are the same.

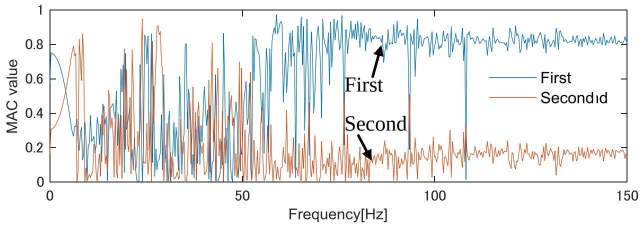
In the range from 0 Hz to 75 Hz, the MAC values of each case are strongly fluctuated so that it is difficult to know the tendency. This means that the 1st and 2nd mode shapes estimated by SVD and low-frequency mode shapes estimated by FDD are not similar. From **Figure 6** (a), for the case of "Intact", in the range from 0 to 75 Hz, FDD method estimates different mode shapes from SVD. In the higher frequency than 75 Hz, the MAC value becomes closer to 1. If there is no damage on the bridge, the higher frequency mode shapes estimated by FDD is same with thh 1st predominant mode shapes estimated by SVD. From **Figure 6** (b), for the case of "Damage 1", in which a brace around the exit edge is removed, the adopted two methods gives different result in any frequency range. There is a possibility that bridge damage affect the satisfaction of the assumptions of the estimation method. **Figure 6** (c) shows MAC for the case of "Damage 2", in which a brace around the mid-span is removed. After the MAC values become stable, several vertical peaks can be confirmed in the cases of brace damage, as shown in **Figure 6** (b) and (c). **Figure 6** (d) indicates the MAC result of the case of "Damage 3", in which the cross girder is removed. This damage can be taken as more serious than other two damage cases. The MAC value of 1st mode shapes estimated by SVD is less than 2nd one, even after it becomes stable. The order of SVD estimation is determined in order of strongest predominant. Thus, FDD method usually estimates a mode shape similar to the SVD's 1st mode shape. This inversion phenomenon between 1st and 2nd mode shapes may indicate the damage.



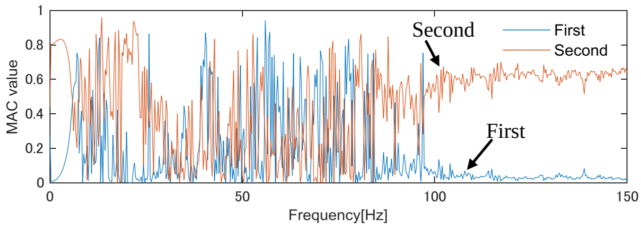
(a) Intact



(b) Damage 1



(c) Damage 2



(d) Damage 3

Figure 6 The example of MAC of SVD and FDD results: The blue curve indicates the MAC of 1st mode shape estimated by SVD and that by FDD. The red one does the MAC of 2nd mode shape estimated by SVD and that of FDD. The vehicle weight is 8.92kg. If the MAC value is less than 0.9, two compared mode shapes are not so similar. In all cases, less than 75 Hz, FDD method gives the different mode shapes from SVD.

C. Discussion about the results

From this experimental study, it is possible that the data processing method using MAC between SVD and FDD can be applied to the traffic-induced vibrations to detect a bridge damage. A general demerit of using traffic-induced vibration is low accuracy. The proposed method does not depend on the accuracy of each method, by focusing on the difference of its assumption. The assumptions of SVD and FDD methods are affected by bridge damage and it is feasible to detect the changes of accuracy by the method using MAC.

The low accuracy, however, could not be resulted only from the assumption problem, in which the traffic-induced vibrations do not satisfy the assumptions of SVD and/or FDD, but also from the less number of sensors, and from the other factors.

To put this method into practical use, it is necessary to check the effect of vehicle parameters: speed, axles number, weight, frequency. They will be future works.

V. Conclusions

In this study, an experiment based on the vehicle-bridge interaction model is carried out, to examine a data processing method comparing SVD and FDD methods. In the laboratory experiment, a paper bridge is used as the bridge model. From the examination, the followings are clarified:

1) SVD method shows the low estimation accuracy for the bridge mode shapes, by using traffic-induced vibrations.
2) FDD method estimates different mode shapes in the range of low frequency.
3) MAC value between SVD and FDD is always fluctuated in low accuracy, while it becomes stable in higher frequency.
4) If there are several vertical peaks in the MAC curve at the stable frequencies, it is possible that there is a damage.
5) If a serious damage occurs on the bridge, an inversion of the order of MAC value can be observed.

These results are just obtained in the limited conditions. The accuracy may be improved by using more sensors. The MEMS sensors are very cheap and labor-saving so it is very feasible to increase the number of sensors. On the other hand, in this experimental study, the variation of vehicle model is limited. In the actual bridge, the bridge is generated by many kinds of vehicles. The dynamic response must be depending on the dynamic characteristic of vehicles. The tendency of MAC curve in the cases of varied vehicle models should be investigated. They are future issues to make it be a feasible bridge monitoring technology.

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