Asymptotic Stabilization of Nonholonomic Mobile Robots With Spatial Constraint

Yanling Shang, Hongsheng Li and Xiulan Wen

***Abstract*—This paper investigates the problem of global stabilization of nonholonomic mobile robots with spatial constraint. A nonlinear mapping is first introduced to transform the state-constrained system into a new unconstrained one. Then, by employing the backstepping technique and switching control strategy, a state feedback controller is successfully constructed to guarantee that the states of closed-loop system are asymptotically regulated to zero without violation of the constraint. Finally, simulation results provided to illustrate the validity of the proposed approach.**

***Index Terms*—nonholonomic mobile robots, spatial constraint, backstepping, asymptotic stabilization.**

I. Introduction

THE nonholonomic systems have received considerable attention have received considerable attention during the last decades because their widespread applications in modelling many practical systems, such as mobile robots, car-like vehicle, under-actuated satellites and so on [1-4]. Nonholonomic mobile robots have good flexibility, since they could realize autonomous movement in the case of nobody involving. However, due to the limitations imposed by Brockett's condition[6], this class of nonlinear systems cannot be stabilized by stationary continuous state-feedback, although it is controllable. There are currently several effective control methodologies that overcome the topological obstruction. The idea of using time-varying smooth controllers was first proposed in [6], in order to stabilize a mobile robot. For driftless systems in chained form, several novel approaches have been proposed for the design of periodic, smooth, or continuous stabilizing controllers [7, 8]. Most of the time-varying control scheme suffer from a slow convergence rate and oscillation. However, it has been observed that a discontinuous feedback control scheme usually results in a fast convergence rate. An elegant approach to constructing discontinuous feedback controller was developed in [9]. The drawback is that there is a restriction on the initial conditions of the controlled system. This limitation has been overcome by a switching state or output control scheme [10]. Subsequently, [11-19] further developed the discontinuous feedback control strategy based on different control targets, respectively. However, the effect of the constraint is not addressed in the above-mentioned results.

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As a matter that the constraints which can represent not only physical limitations but also performance requirements are common in practical systems. Violation of the constraints may cause performance degradation or system damage. In recent years, driven by practical needs and theoretical challenges, the control design for constrained nonlinear systems has become an important research topic [20-22]. However, less attention has been paid to the space-constrained nonholonomic mobile robots.

This paper addresses the asymptotic stabilization by state feedback for nonholonomic mobile robots with spatial constraint. The contributions can be highlighted as follows. (i) The stabilization problem of nonholonomic systems with spatial constraint is studied for the first time. (ii) A nonlinear mapping is introduced, under which the constrained interval is mapped to the whole real number field, and then the conventional control technique can be directly applied without considering the range of initial values. (iii) With the help of switching strategy eliminating the phenomenon of uncontrollability of $u_0 = 0$, a systematic state feedback control design procedure by using the backstepping technique is proposed such that the states of closed-loop system are regulated to zero asymptotically while the state constraints are not violated.

The rest of this paper is organized as follows. In Section II, the problem formulation and preliminaries are given. Section III presents the input-state-scaling transformation the backstepping design procedure, the switching control strategy and the main result. Section IV gives simulation results to illustrate the theoretical finding of this paper. Finally, concluding remarks are proposed in Section V.

II. Problem Formulation

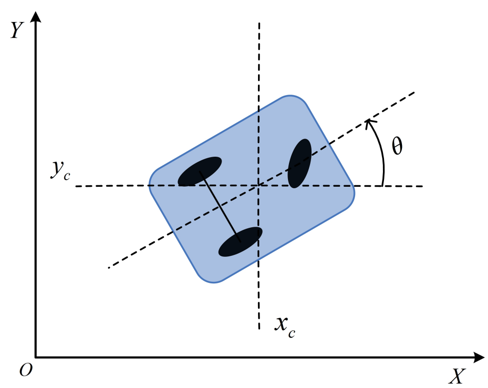

Fig. 1. The planar graph of a mobile robot.

Consider a tricycle-type mobile robot shown in Fig. 1. The

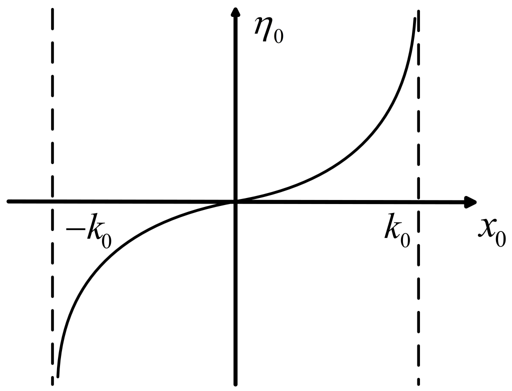

Fig. 2. Schematic illustration of the nonlinear mapping $\mathscr{H}_0$.

kinematic equations of this robot are represented by

$$\begin{aligned} \dot{x}_c &= v\cos\theta \\ \dot{y}_c &= v\sin\theta \\ \dot{\theta} &= \omega \end{aligned} \quad (1)$$

where (x_c, y_c) denotes the position of the center of mass of the robot, θ is the heading angle of the robot, v is the forward velocity while ω is the angular velocity of the robot.

Introducing the following change of coordinates

$$\begin{aligned} &x_0 = x_c, \quad x_1 = y_c, \quad x_2 = \tan\theta, \\ &u_0 = v\cos\theta, \quad u_1 = w\sec^2\theta, \end{aligned} \quad (2)$$

system (1) is transformed into the chained form as

$$\begin{aligned} \dot{x}_0 &= u_0 \\ \dot{x}_1 &= u_0 x_2 \\ \dot{x}_2 &= u_1 \end{aligned} \quad (3)$$

Note that the state (x_0, x_1) can be see as the displacement from the parking position. As we all know, when the robots initial position is far away from the parking position, it usually can move directly to the parking position. The robots body angle can be aligned without difficulties and no more maneuvers are needed. However, when the robot's initial position is close to the parking position, it might not be feasible to get to the parking position while aligning the robots body angle at the same time. Therefore it is very necessary to develop finite-time control techniques for state constrained nonholonomic systems for giving this difficulty a straightforward solution.

Due to physical limitations, in this paper we assume that the states x_0 and x_1 are constrained in the compact sets

$$\Omega_{x_i} = \{-k_i < x_i < k_i\}, \quad i = 0, 1 \quad (4)$$

where k_i's are positive constants.

The objective of this paper is to design a state feedback controller such that the states of closed-loop system are globally asymptotically regulated to zero without violation of the constraint.

III. Nonlinear mapping

To prevent the states x_0 and x_1 from violating the constraints, we define a nonlinear mapping that will be used to develop the control design and the main results.

Define a one-to-one nonlinear mapping $\mathscr{H} : (x_0, x) \to (\eta_0, \eta)$ as follows:

$$\begin{aligned} \eta_0 &= \mathscr{H}_0(x_0) = ln\Big(\frac{k_0 + x_0}{k_0 - x_0}\Big) \\ \eta_1 &= \mathscr{H}_1(x_1) = ln\Big(\frac{k_1 + x_1}{k_1 - x_1}\Big) \\ \eta_2 &= \mathscr{H}_2(x_2) = x_2 \end{aligned} \quad (5)$$

where $\mathscr{H}_0$ is shown in Fig. 2. It is clear that function $\mathscr{H}_0$ is a continuous elementary function. From (5), we get

$$x_0 = \mathscr{H}_0^{-1} = k_0\Big(1 - \frac{2}{e^{\eta_0} + 1}\Big) \quad (6)$$

then the derivative of x_0 is given by

$$\dot{x}_0 = \frac{2k_0 e^{\eta_0}}{(e^{\eta_0} + 1)^2}\dot{\eta}_0 \quad (7)$$

Substituting (7) into the first equation of (3), we have

$$\dot{\eta}_0 = \frac{1}{2k_0}(e^{\eta_0} + e^{-\eta_0} + 2)u_0 \quad (8)$$

Similarly, we can obtain

$$\dot{\eta}_1 = \frac{1}{2k_1}(e^{\eta_1} + e^{-\eta_1} + 2)x_2 \quad (9)$$

By noting that $\dot{\eta}_2 = \dot{x}_2$, we can rewrite the system (3) as

$$\begin{aligned} \dot{\eta}_0 &= d_0(\eta_0)u_0 \\ \dot{\eta}_1 &= d_1(\eta_1)u_0\eta_2 \\ \dot{\eta}_2 &= u_1 \end{aligned} \quad (10)$$

where

$$\begin{aligned} d_0(\eta_0) &= \frac{1}{2k_0}(e^{\eta_0} + e^{-\eta_0} + 2) \\ d_1(\eta_1) &= \frac{1}{2k_1}(e^{\eta_1} + e^{-\eta_1} + 2) \end{aligned} \quad (11)$$

Remark 1. On the basis of the nonlinear mapping $\mathscr{H}$, we see that the state η_0 (η_1) is an unconstrained variable defined in the whole real number field R. Moreover, x_0 (x_1) remains the constraint interval $|x_0| < k_0$ ($|x_1| < k_1$) regardless of the number of η_0 (η_1). That is, the control design for the unconstrained transformed system (10) is equivalent to the control design for the constrained initial system (3).

IV. Robust controller design

In this section, we focus on designing robust controller for system (10) provided that $\eta_0(t_0) \neq 0$, while the case where the initial condition $\eta_0(t_0) = 0$ will be treated in Section V. The inherently triangular structure of system (10) suggests that we should design the control inputs u_0 and u_1 in two separate stages.

A. Design u_0 for η_0-subsystem

For η_0-subsystem, we take the following control law

$$u_0 = -\frac{1}{d_0}\lambda_0\eta_0 \quad (12)$$

where λ_0 is a positive design parameter. Under the control law (12), the following lemma can be established.

Lemma 1. For any initial $t_0 \geq 0$ and any initial condition $\eta_0(t_0) \in R/\{0\}$, the corresponding solution $\eta_0(t)$ exists and satisfies $\lim_{t\to\infty}\eta_0(t) = 0$. Furthermore, the control u_0 given by (12) also exists; does not cross zero and satisfies $\lim_{t\to\infty} u_0(t) = 0$.

Proof. Choosing the Lyapunov function $V_0 = \eta_0^2/2$, a simple computation gives

$$\dot{V}_0 \le -\lambda_0\eta_0^2 \le 0 \tag{13}$$

which implies

$$|\eta_0(t)| \le |\eta_0(t_0)|e^{-\lambda_0(t-t_0)} \tag{14}$$

Consequently, η_0 is globally exponentially convergent and does not cross zero for all $t \in (t_0, \infty)$ provided that $\eta_0(t_0) \ne 0$. Furthermore, from (12), we can conclude that the u_0 exists, does not cross zero for all $t \in (t_0, \infty)$ independent of the η-subsystem and satisfies $\lim_{t\to\infty} u_0(t) = 0$.

B. Input-state-scaling transformation

From the above analysis, we can see the η_0-state in (10) can be globally exponentially regulated to zero via u_0 in (12) as $t \to \infty$. However, it is troublesome in controlling the η-subsystem via the control input u_1 because, in the limit (i.e. $u_0 = 0$), the η-subsystem is uncontrollable. To avoid the phenomenon, the following discontinuous input-state-scaling transformation is employed.

$$z_i = \frac{\eta_i}{u_0^{n-i}}, \quad i = 1, 2 \tag{15}$$

under which, the η-subsystem is transformed into

$$\begin{aligned} \dot{z}_1 &= \tilde{d}_1(\eta_0, z_1)z_2 + \tilde{f}_1(t, \eta_0, z_1, u_0) \\ \dot{z}_2 &= u_1 \end{aligned} \tag{16}$$

where

$$\begin{aligned} \tilde{d}_1(\eta_0, z_1) &= d_1(\eta_1) \\ \tilde{f}_1(t, \eta_0, \bar{z}_i, u_0) &= -\frac{(n-i)z_i}{u_0}\frac{\partial u_0}{\partial \eta_0} d_0 u_0 \end{aligned} \tag{17}$$

The following lemma gives the estimation of nonlinear function $\bar{f}_i$.

Lemma 2. There exist nonnegative smooth functions $\tilde{\varphi}_i$ such that

$$|\tilde{f}_1(t, \eta_0, \bar{z}_i, u_0)| \le \tilde{\varphi}_1(\eta_0, \bar{z}_i)|z_1| \tag{18}$$

Proof. The estimation can be easily obtained from, (12), (17) and the transformation (15). The detailed proof is omitted here.

C. Backstepping Design for u_1

In this subsection, we shall construct a state feedback controller u_1 by backstepping technique.

Step 1. We first consider the z_1-subsystem of (16) and take z_2 as the control input. Construct the first Lyapunov function candidate $V_1 = z_1^2/2$ whose time derivative satisfies

$$\dot{V}_1 = \tilde{d}_1 z_1 z_2 + z_1\tilde{f}_1 \le \tilde{d}_1 z_1 z_2 + z^2\tilde{\varphi}_1 \tag{19}$$

By choosing the first virtual controller z_2^* for z_2 as

$$z_2^* = -\frac{1}{\tilde{d}_1}(2 + \bar{\varphi}_1)z_1 := -\alpha_1(\eta_0, z_1)z_1 \tag{20}$$

where $\alpha_1 > 0$ is a smooth function, we have

$$\dot{V}_1 \le -2z_1^2 + \tilde{d}_1 z_1(z_2 - z_2^*) \tag{21}$$

Step 2. Let $\xi_2 = z_2 - z_2^*$ and consider the Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}\xi_2^2 \tag{22}$$

Clearly

$$\begin{aligned} \dot{V}_2 \le{}& -2z_1^2 + \tilde{d}_1 z_1(z_2 - z_2^*) + \xi_2 u_1 \\ & -\xi_2\frac{\partial z_2^*}{\partial z_1}(\tilde{d}_1 z_2 + \tilde{f}_2) - \xi_2\frac{\partial z_2^*}{\partial \eta_0} d_0 u_0 \end{aligned} \tag{23}$$

Now we estimate each term on the right-hand side of (23). To begin with, we have

$$\tilde{d}_1 z_1(z_2 - z_2^*) \le \frac{1}{3}z_1^2 + l_{21}\xi_2^2 \tag{24}$$

where $l_{21} > 0$ is a constant.

Then, according to (18), it follows that

$$-\xi_2\frac{\partial z_2^*}{\partial z_1}(\tilde{d}_1 z_2 + \tilde{f}_1) \le \frac{1}{3}z_1^2 + l_{22}\xi_2^2 \tag{25}$$

for a smooth function $l_{22} \ge 0$.

Since z_k^* is a smooth function and satisfies

$$z_2^*(\eta_0, 0) = 0, \quad \frac{\partial z_2^*}{\partial \eta_0}(\eta_0, 0) = 0 \tag{26}$$

Based on the completion of squares, it is deduced that there is a smooth function $l_{23} \ge 0$ such that

$$-\xi_2\frac{\partial z_2^*}{\partial \eta_0} d_0 \le \frac{1}{3}z_1^2 + l_{23}\xi_2^2 \tag{27}$$

Substituting (24)–(25) and (27) into (23) yields

$$\dot{V}_2 \le -z_1^2 + \xi_2 u_1 + \xi_2(l_{21} + l_{22} + l_{23}) \tag{28}$$

Now, it easy to see that the smooth actual controller

$$\begin{aligned} u_1 &= -(1 + l_{21} + l_{22} + l_{23})\xi_2 \\ &:= -\alpha_2(\eta_0, z_1, \xi_2)\xi_2 \end{aligned} \tag{29}$$

renders

$$\dot{V}_2 \le -(z_1^2 + \xi_2^2) \tag{30}$$

We have thus far completed the controller design procedure for $\eta_0(t_0) \ne 0$. Without loss of generality, we can assume that $t_0 = 0$.

V. Switching control design and main results

In the preceding section, we have given controller design for $\eta_0(0) \ne 0$. Now, we discuss how to select the control laws u_0 and u_1 when $\eta_0(0) = 0$. In the absence of disturbances, the most commonly used control strategy is using constant control $u_0 = u_0^* \ne 0$ in time interval $[0, t_s)$. However, for system (10), the choice of constant feedbacks may lead to the solution of the η_0-subsystem blow up before the given switching time t_s. In order to prevent this finite escape phenomenon from happening, we give the switching control strategy for control input u_0 by the use of state measurement of the η_0-subsystem in (10) instead of frequently-used time measurement.

When $\eta_0(0) = 0$, we choose u_0 as follow:

$$u_0 = u_0^*, \ \ u_0^* > 0$$

At $\eta_0(0) = 0$, we know that $\dot{\eta}_0(0) = d_0(0)u_0(0) = d_0(0)u_0^* > 0$. Thus for a small positive constant δ, there exists a small neighborhood Ω of $\eta_0(0) = 0$ such that

$|d_0\eta_0| \le \delta$. Suppose that η_0^* satisfies $|\eta_0^*| = \delta$. In Ω, η_0 is increasing until $|\eta_0| > \delta$.

Now, we define the switching control law u_0 as

$$u_0 = u_0^*, \quad u_0^* > 0, \quad |\eta_0| \le |\eta_0^*| < \delta \tag{31}$$

During the time period satisfying $|\eta_0| \le |\eta_0^*|$, using u_0 defined in (31) and new $u_1 = u_1^*(\eta_0, z)$ obtained by the similar control design method as (29), it is concluded that the η-state of (10) cannot blow up for $|\eta_0| \le |\eta_0^*|$. At this time, $\eta_0(t_s)$ is not zero ($|\eta_0(t_s)| > |\eta_0^*|$), then, we switch to the control inputs u_0 and u_1 into (12) and (29), respectively. Thus, the following results are obtained.

Lemma 3. If the proposed control design procedure together with the above switching control strategy is applied to system (10), then, for any initial conditions in the state space $(\eta_0, \eta)^T \in R^3$, system (10) is globally asymptotic-regulated at origin.

Proof. According to the above analysis, it suffices to prove the statement in the case where $\eta_0(0) \neq 0$.

Since we have already proven that $\lim_{t\to\infty} \eta_0(t) = 0$ in Lemma 1, we just need to show that $\lim_{t\to\infty} \eta(t) = 0$. In this case, noting that V_n is positive definite and radially unbounded, by (22) and (30), we get $\lim_{t\to\infty} z(t) = 0$. Furthermore, from the input-state-scaling transformation (15), we conclude that $\lim_{t\to\infty} \eta(t) = 0$. Thereby, the proof of Lemma 3 is completed.

With the help of Lemma 3, we are ready to state the main results of this paper.

Theorem 1. If the proposed control design procedure together with the above switching control strategy is applied to system (3), then, for any initial conditions $(x_0(0), x(0)) \in \Theta = \{(x_0, x)^T \in R^3| - k_i < x_i < k_i, i = 0, 1\}$, the following properties hold.

(i) The states x_0 and x_1 stay in the compact sets $\Omega_{x_i} = \{-k_i < x_i < k_i\}, i = 0, 1$, that is, the state constraints are not violated.

(ii) All the states of closed-loop system are asymptotically regulated to zero.

Proof. From Lemma 3, we can easily see that the states $\eta_i(t)$, $i = 0, 1, 2$ are bounded, and $\lim_{t\to\infty} \eta_i(t) = 0$. The bounded states $\eta_i(t)$, $i = 0, 1$ together with the nonlinear mapping (5) lead to

$$|x_0(t)| = k_0\left|1 - \frac{2}{e^{\eta_0(t)} + 1}\right| < k_0 \tag{32}$$

and

$$|x_1(t)| = k_1\left|1 - \frac{2}{e^{\eta_1(t)} + 1}\right| < k_1 \tag{33}$$

that is, the states x_i will remain in the sets Ω_{x_i}, $i = 0, 1$ and never violate the constraints. Furthermore, $\lim_{t\to\infty} \eta_i(t) = 0$, $i = 0, 1, \cdots, n$ and (5) imply that $\lim_{t\to\infty} x_2(t) = 0$ and

$$\begin{aligned}\lim_{t\to\infty} x_0 &= \lim_{t\to\infty} k_0\left(1 - \frac{2}{e^{\eta_0(t)} + 1}\right) \\ &= k_0\left(1 - \frac{2}{e^{\lim_{t\to\infty}\eta_0(t)} + 1}\right) = 0\end{aligned} \tag{34}$$

$$\begin{aligned}\lim_{t\to\infty} x_1 &= \lim_{t\to\infty} k_1\left(1 - \frac{2}{e^{\eta_1(t)} + 1}\right) \\ &= k_1\left(1 - \frac{2}{e^{\lim_{t\to\infty}\eta_1(t)} + 1}\right) = 0\end{aligned} \tag{35}$$

Thus, the proof is completed.

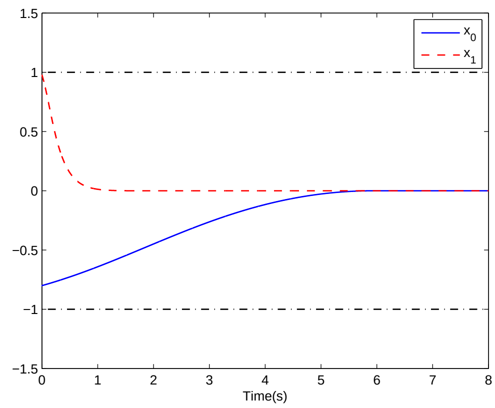

Fig. 3. The state trajectories of the robot.

VI. Simulation Results

In this section, we illustrate the effectiveness of the proposed approach with the boundedness of $k_i = 1$, i.e., $|x_0| < 1$ and $|x_1| < 1$.

If $\eta_0(0) = 0$, controls u_0 and u_1 are set as in Section 4 in interval $[0, t_s)$, such that $\eta_0(t_s) \neq 0$, then we can adopt the controls developed below. Therefore, without loss of generality, we assume that $\eta_0(0) \neq 0$. For the η_0-subsystem, we can choose the control law $u_0 = -2\lambda_0\eta_0/(e^{\eta_0} + e^{-\eta_0} + 2)$. By introducing the input-state-scaling transformation $z_1 = \eta_1/u_0^2$, $z_2 = \eta_2/u_0$, $z_3 = x_3$, the η-subsystem of (**??**) is transformed into

$$\begin{aligned}\dot{z}_1 &= \frac{1}{2}(e^{z_1 u_0} + e^{-z_1 u_0} + 2)z_2 - \frac{\dot{u}_0}{u_0}z_1 \\ \dot{z}_2 &= u_1\end{aligned} \tag{36}$$

According to the design procedure shown in Section 3, we can explicitly construct a state feedback controller

$$u_1 = -b_2(z_2 + b_1 z_1) \tag{37}$$

with appropriate nonnegative smooth functions b_1 and b_2, to globally asymptotically stabilize z-subsystem (36). In the simulation, by choosing initial value $(x_0(0), x_1(0), x_2(0)) = (0.8, -0.98, -2)$ and the gains for the control laws as $k_0 = 3$, $\lambda_0 = 0.3$, $b_1 = 4/(e^{z_1} + e^{-z_1} + 2)$ and $b_2 = 1.5$, Fig. 3 is obtained to exhibit the responses of the closed-loop system. From the figure, it can be seen that all the closed-loop system states are asymptotically regulated to zero and the output constraint is never violated, which accords with the main results established in Theorem 1 and also demonstrates the effectiveness of the control method proposed in this paper.

VII. Conclusion

This paper has studied the problem of asymptotic stabilization by state feedback for nonholonomic mobile robots with spatial constraint. Based on the nonlinear mapping, and by using backstepping technique, a constructive design procedure for state feedback control is given. Together with a novel switching control strategy, the designed controller can guarantee that the closed-loop system states are asymptotically regulated to zero while the constraint is not violated. In this direction, there are still remaining problems to be investigated. For example, an interesting research problem is how to design an output feedback stabilizing controller for the constrained nonholonomic mobile robots studied in the paper.

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