Time-varying Finite Memory Structure Filter to Incorporate Time-Delayed Measurements

Pyung Soo Kim

Abstract— **This paper proposes a finite memory structure (FMS) filter for state estimation in time-varying systems with measurement delay, called the TV-FMS filter. By incorporating time-delayed measurements, the proposed TV-FMS filter is obtained by directly solving a weighted least-square cost function. The proposed TV-FMS filter has inherent properties such as unbiasedness, deadbeat, and robustness. Through extensive computer simulations for three kinds of discrete time-varying noisy signal models with a measurement delay as well as an uncertain model parameter, the proposed TV-FMS filter can be shown to be comparable with the Kalman filter with infinite memory structure (IMS) for a nominal system and better than that for a temporarily uncertain system.**

Index Terms— **Measurement delay, Finite memory structure filter, Infinite memory structure filter, Time-varying system, Uncertain system.**

I. Introduction

TIME delays exist in various practical systems, such as hydraulic processes, chemical systems, temperature processes, and signal transmission systems and are often a primary source of instability and performance degradation. Therefore, to deal with time delays, many control and estimation problems have been researched in the past few decades.

To deal with a *time delay* of measurements in the estimation filtering, several approaches have been researched [1]-[5]. The estimation filters developed in these approaches have the infinite memory structure (IMS) such as Kalman filter of [6][7]. Therefore, these existing approaches may show poor performance and even divergence phenomenon for mismodeling and temporary uncertainties. Recently, therefore, the finite memory structure (FMS) filter has been developed for state estimation in discrete time-invariant systems with *measurement delay* [8][9]. The FMS has been a well-known strategy in the estimation filtering [10]-[14] and thus applied successfully to various engineering problems [15]-[18]. However, since discrete time-varying systems are used quite often for many practical applications such as detection, tracking, and guidance in the aerospace industry, an FMS filter for discrete time-varying systems with measurement delay is necessary.

Therefore, this paper proposes a time-varying FMS filter for state estimation to incorporate time-delayed measurements. The proposed filter is called the TV-FMS filter. The proposed TV-FMS filter is obtained by directly solving a weighted least-square cost function. The proposed TV-FMS filter has inherent properties such as unbiasedness, deadbeat, and robustness. The proposed TV-FMS filter can be shown to be comparable with the Kalman filter with IMS for a nominal system and better than that for a temporarily uncertain system via extensive computer simulations. As simulation models, three kinds of discrete time-varying noisy signal models with a measurement delay as well as an uncertain parameter are considered by a sinusoid signal model, a Van der Pol oscillation signal model, and a discretized DC motor model.

This paper is organized as follows. In Section II, TV-FMS filter is proposed to incorporate time delayed measurements. In Section III, extensive computer simulations are performed. Finally, concluding remarks are presented in Section IV.

II. Time-varying Finite Memory Structure Filter with Incorporating Time Delayed Measurement

The linear discrete time-varying system with measurement delay is considered as follows:

$$\begin{aligned} x_{i+1} &= \Phi_i x_i + G_i w_i, \\ z_i &= H_i x_{i-d} + v_i, \end{aligned} \tag{1}$$

where $x_i \in \Re^n$ is unknown state vector and $z_i \in \Re^q$ is a measured measurement vector. The system noise $w_i \in \Re^p$ and the measurement noise $v_i \in \Re^q$ are zero-mean white Gaussian whose covariances Q_i and R_i are assumed to be positive definite matrix. It is assumed that the delay length d is the positive integer, that is, the system involves delayed measurements.

The main objective of this paper is to estimate the system state x_i at the current time i, given the most recent finite measurements Z_i on the window $[i-M, i]$ as follows

$$Z_i \equiv \begin{bmatrix} z_{i_M} \\ z_{i_M+1} \\ \vdots \\ z_{i-1} \end{bmatrix}, \; W_i \equiv \begin{bmatrix} w_{i_M} \\ w_{i_M+1} \\ \vdots \\ w_{i-1} \end{bmatrix}, \; V_i \equiv \begin{bmatrix} v_{i_M} \\ v_{i_M+1} \\ \vdots \\ v_{i-1} \end{bmatrix}, \tag{2}$$

when there is measurement delay as shown in (1). Note that the delay length d is satisfying $0 \le d < M$.

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On the most recent window $[i-M,i]$, the state x_{i-d} at the time $i-d$ is represented from (1) as follows:

$$x_{i-d} = \Psi_{i,d}^{-1}[x_i - \Theta_i W_i]$$

where

$$\Psi_{i,j} \equiv \Phi_{i-1}\Phi_{i-2}\cdots\Phi_{i-j+1}\Phi_{i-j},$$

$$\Theta_i \equiv \left[\overbrace{0 \quad 0 \quad \cdots \quad 0}^{M-d}\ \overbrace{\Psi_{i,d\text{-}1}G_{i-d} \quad \Psi_{i,d\text{-}2}G_{i-d+1} \quad \cdots \quad G_{i-1}}^{d}\right].$$

Then, the measurement model (1) can be written by

$$z_i = \Xi_i x_i + \Xi_i \Theta_i W_i + v_i, \tag{3}$$

with $\Xi_i = H_i \Psi_{i,d}^{-1}$. Therefore, from (1) and (3), the most recent finite measurements Z_i on the window $[i-M,i]$ can be represented by

$$Z_i = \Gamma_i x_i + \Lambda_i W_i + \Sigma_i \overline{W_i} + V_i, \tag{4}$$

where $\overline{W_i}$ has the same form as (2) for W_i and matrices Γ_i, Λ_i, and Σ_i are as follows:

$$\Gamma_i \equiv \begin{bmatrix} \Xi_{i_M}\Psi_{i,M}^{-1} \\ \Xi_{i_M+1}\Psi_{i,M-1}^{-1} \\ \vdots \\ \Xi_{i-2}\Psi_{i,2}^{-1} \\ \Xi_{i-1}\Psi_{i,1}^{-1} \end{bmatrix},$$

$$\Lambda_i \equiv \begin{bmatrix} \Xi_{i-1}\Psi_{i,1}^{-1}G_{i-1} & \Xi_{i-2}\Psi_{i,2}^{-1}G_{i-2} & \cdots & \Xi_{i_M}\Psi_{i,M}^{-1}G_{i_M} \\ 0 & \Xi_{i-1}\Psi_{i,1}^{-1}G_{i-1} & \cdots & \Xi_{i_M+1}\Psi_{i,M-1}^{-1}G_{i_M+1} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \Xi_{i-1}\Psi_{i,1}^{-1}G_{i-1} \end{bmatrix},$$

$$\Sigma_i \equiv \left[diag\left(\overbrace{\Xi_{i_M}\Theta_{i_M} \quad \Xi_{i_M+1}\Theta_{i_M+1} \quad \cdots \quad \Xi_{i-1}\Theta_{i-1}}^{M}\right)\right].$$

The noise term $\Lambda_i W_i + \Sigma_i \overline{W_i} + V_i$ in (4) is zero-mean white Gaussian with covariance Π_i given by

$$\Pi_i \equiv \Lambda_i\left[diag\left(\overbrace{Q_{i_M} \quad Q_{i_M+1} \quad \cdots \quad Q_{i-1}}^{M}\right)\right]\Lambda_i^T + \Sigma_i\Big[diag$$

$$\left\{\overbrace{diag\left(\overbrace{Q_{i_M} \quad Q_{i_M+1} \quad \cdots \quad Q_{i-1}}^{M}\right) \quad \cdots \quad diag\left(\overbrace{Q_{i_M} \quad Q_{i_M+1} \quad \cdots \quad Q_{i-1}}^{M}\right)}^{M}\right\}$$

$$\Big]\Sigma_i^T + \left[diag\left(\overbrace{R_{i_M} \quad R_{i_M+1} \quad \cdots \quad R_{i-1}}^{M}\right)\right].$$

Now, to get the filtered estimate $\hat{x}_i$, the following weighted least square cost function must be minimized:

$$[Z_i - \Gamma_i x_i]^T \Pi_i^{-1}[Z_i - \Gamma_i x_i]. \tag{5}$$

Taking a derivation of (5) with respect to $\hat{x}_i$ and setting it to zero, the proposed TV-FMS filter $\hat{x}_i$ on the window $[i-M,i]$ is given for the discrete time-varying system with measurement delay as follows:

$$\hat{x}_i = \left(\Gamma_i^T \Pi_i^{-1}\Gamma_i\right)^{-1}\Gamma_i^T \Pi_i^{-1} Z_i. \tag{6}$$

The matrix Γ_i is of full rank since $\{\Phi_i, H_i\}$ is observable for $M \geq n$. In addition, the matrix Π_i is positive definite and thus its inversion exists. Therefore, the matrix $\Gamma_i^T \Pi_i^{-1}\Gamma_i$ is nonsingular and thus its inversion exists.

The proposed time-varying FMS filter has inherent several properties such as unbiasedness, deadbeat, time-invariance, robustness as follows. It is shown in the following theorem that the proposed time-varying FMS filter $\hat{x}_i$ is unbiased for noisy systems and exact for noise-free systems with measurement delay.

Theorem 1. *Assume that $\{\Phi_i, H_i\}$ is observable and $M \geq n$. Then the proposed TV-FMS filter $\hat{x}_i$ on the window $[i-M,i]$ has an unbiasedness property when there are noises and a deadbeat property when there are no noises in the discrete time-varying system (1) with measurement delay.*

Proof. When there are noises on the window $[i-M,i]$ in (1) with measurement delay, $E[Z_i] = \Gamma_i E[x_i]$ since the noise term $\Lambda_i W_i + \Sigma_i \overline{W_i} + V_i$ in (4) is zero-mean. Therefore, the following is true:

$$\begin{aligned} E[\hat{x}_i] &= \left(\Gamma_i^T \Pi_i^{-1}\Gamma_i\right)^{-1}\Gamma_i^T \Pi_i^{-1} E[Z_i] \\ &= \left(\Gamma_i^T \Pi_i^{-1}\Gamma_i\right)^{-1}\Gamma_i^T \Pi_i^{-1} E\left[\Gamma_i x_i + \Lambda_i W_i + \Sigma_i \overline{W_i} + V_i\right] \\ &= \left(\Gamma_i^T \Pi_i^{-1}\Gamma_i\right)^{-1}\Gamma_i^T \Pi_i^{-1}\Gamma_i E[x_i] = E[x_i]. \end{aligned}$$

This completes the proof of the unbiasedness property.

When there are no noises on the window $[i-M,i]$ in (1) with measurement delay as follows:

$$x_{i+1} = \Phi_i x_i, \quad z_i = H_i x_{i-d}, \tag{7}$$

then $Z_i = \Gamma_i x_i$, since the noise term $\Lambda_i W_i + \Sigma_i \overline{W_i} + V_i$ in (4) is removed. Therefore, the following is true:

$$\begin{aligned} \hat{x}_i &= \left(\Gamma_i^T \Pi_i^{-1}\Gamma_i\right)^{-1}\Gamma_i^T \Pi_i^{-1} Z_i \\ &= \left(\Gamma_i^T \Pi_i^{-1}\Gamma_i\right)^{-1}\Gamma_i^T \Pi_i^{-1}\Gamma_i x_i = x_i. \end{aligned}$$

This completes the proof of the deadbeat property. ■

The unbiasedness of the estimate means that its mean value tracks the mean value of the state at every time for noisy systems. The deadbeat of the estimate means that its value tracks exactly the state at every time. The proposed TV-FMS filter $\hat{x}_i$ (6) is proposed assuming that the system (7) has additive system and measurement noises, w_i and v_i as shown in (1). In actual cases, the noises of the signal can exist or not. In either case, the proposed TV-FMS filter is designed with nonzero Q_i and R_i. That is, the deadbeat property in this paper is obtained in the case when the noises do not exist, while the proposed TV-FMS filter is designed with nonzero Q_i and R_i. The deadbeat property indicates finite convergence time and fast tracking ability of the proposed TV-FMS filter. Thus, it can be expected that the proposed TV-FMS filter would be appropriate for fast estimation and detection of signals with unknown times of occurrence, which arise in many problems such as fault detection and diagnosis of various systems, maneuver detection and target tracking of flying objects, etc.

Moreover, according to [10]-[14], it is a general rule of thumb that, the proposed TV-FMS filter might be also robust against mismodeling and temporary uncertainties or due to its finite memory structure.

III. Extensive Computer Simulations for Diverse Dynamic Systems

To illustrate the validity of the proposed TV-FMS filter and to compare with the existing IMS filter, Kalman filter in [3], extensive computer simulations are performed for three kinds of discrete time-varying noisy signal models. Even if a process is represented in state space accurately on a long time scale, it may undergo unpredictable changes, such as jumps in frequency, phase, and velocity. Because these effects typically occur over a short time interval, they are called temporary uncertainties. As representative temporary uncertainties, there are a model uncertainty, an unknown input, and incomplete measurement information, etc. Thus, two kinds of discrete time-varying noisy signal models in computer simulations consider a measurement delay d as well as a uncertain model parameter δ_i.

Firstly, the sinusoid signal model, has received considerable attention in the literature, is considered as follows [11]:

$$\Phi_i = \begin{bmatrix} \cos(\pi/32)+\delta_i & \sin(\pi/32) \\ -\sin(\pi/32) & \cos(\pi/32)+\delta_i \end{bmatrix}, \; G_i = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad H_i = \begin{bmatrix} 1+0.25\delta_i & 0.25\delta_i \end{bmatrix}. \tag{8}$$

The delay length is set by $d = 2$. A temporarily uncertain parameter is taken as $\delta_i = 0.04$ for the interval $100 \le i \le 150$ for the sinusoid signal model (8). System and measurement noise covariances are taken as $Q_i = 0.1^2$ and $R_i = 0.2^2$. For consider diverse situations, three kinds of window lengths are set by $M = 10$, $M = 15$, and $M = 20$.

Secondly, the Van der Pol oscillation signal model for an electronic circuit with vacuum tubes is considered as follows [11]:

$$\Phi_i = \begin{bmatrix} 1+\delta_i & T+\delta_i & T^2/2+\delta_i \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \quad G_i = \begin{bmatrix} 1 \\ 0.3 \\ 0.3 \end{bmatrix}, \; H_i = \begin{bmatrix} 1+2\delta_i & 2\delta_i & 2\delta_i \end{bmatrix}. \tag{9}$$

The delay length is set by $d = 10$. A temporarily uncertain parameter is taken as $\delta_i = 0.005$ for the interval $250 \le i \le 300$ for the Van der Pol oscillation signal model (9). System and measurement noise covariances are taken as $Q_i = 0.1^2$ and $R_i = 0.2^2$. For consider diverse situations, three kinds of window lengths are set by $M = 50$, $M = 60$, and $M = 70$.

Thirdly, the discretized direct current (DC) motor model is considered. The DC motor is the most commonly used motors in the control systems due to their features such cost-efficiency, ease of use, high performance, longevity and quiet operation as follows [18]

$$\Phi_i = \begin{bmatrix} 0.8178+\delta_i & -0.0011 \\ 0.0563 & 0.3678+\delta_i \end{bmatrix}, \quad G_i = \begin{bmatrix} 0.0006 & 0 \\ 0 & 0.0057 \end{bmatrix}, \; H_i = \begin{bmatrix} 1+0.2\delta_i & 0.2\delta_i \end{bmatrix}. \tag{10}$$

The delay length is set by $d = 4$. A temporarily uncertain parameter is taken as $\delta_i = 0.1$ for the interval $150 \le i \le 300$ for the discretized DC motor model (10). System and measurement noise covariances are taken as $Q_i = 0.1^2$ and $R_i = 0.1^2$. For consider diverse situations, three kinds of window lengths are set by $M = 10$, $M = 20$, and $M = 30$.

The proposed TV-FMS filter and the existing IMS filter are compared for the temporarily uncertain system. Figure 1, 2 and 3 show simulations results according to diverse window lengths. In addition, last plots of three figures show simulation results for the certain system without any temporary modeling uncertainty. The proposed TV-FMS filter can be comparable to the existing IMS filter for the certain system. The estimation error of the proposed TV-FMS filter is smaller than that of the existing IMS filter on the interval where modeling uncertainty exist for all cases. In addition, the convergence time of estimation error is much shorter than that of the existing IMS filter after temporary modeling uncertainty disappears. In addition, the proposed TV-FMS filter can be comparable to the existing IMS filter after the effect of temporary modeling uncertainty completely disappears. Therefore, the proposed TV-FMS filter can be more robust than the existing IMS filter when applied to temporarily uncertain systems, although the proposed TV-FMS filter is designed with no consideration for robustness. Moreover, it can be known that the larger window length may yield the longer convergence time of the estimation error, which can degrade the performance of the proposed TV-FMS filter. Therefore, it can be stated that the window length M can be used as an effective design parameter to make the best possible performance of the proposed TV-FMS filter.

IV. Concluding Remarks

In this paper, the TV-FMS filter has been proposed for state estimation in time-varying systems with measurement delay. The proposed TV-FMS filter has been obtained by directly solving a weighted least-square cost function with incorporating time-delayed measurements. It has been shown that the proposed TV-FMS filter has inherent properties such as unbiasedness, deadbeat, and robustness. Computer simulations for three kinds of discrete time-varying noisy signal models, sinusoid signal model, Van der Pol oscillation signal model, discretized DC motor model, have shown that

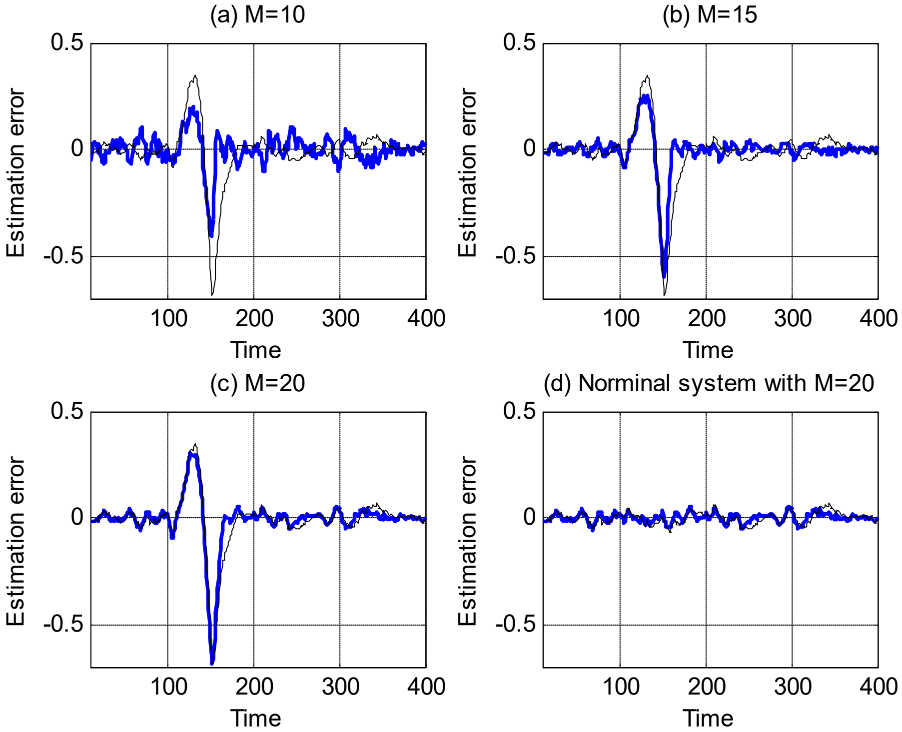

Fig. 1. Estimation errors for sinusoid signal model

the proposed TV-FMS filter can be comparable with the existing IMS filter for a nominal system and better than that for a temporarily uncertain system.

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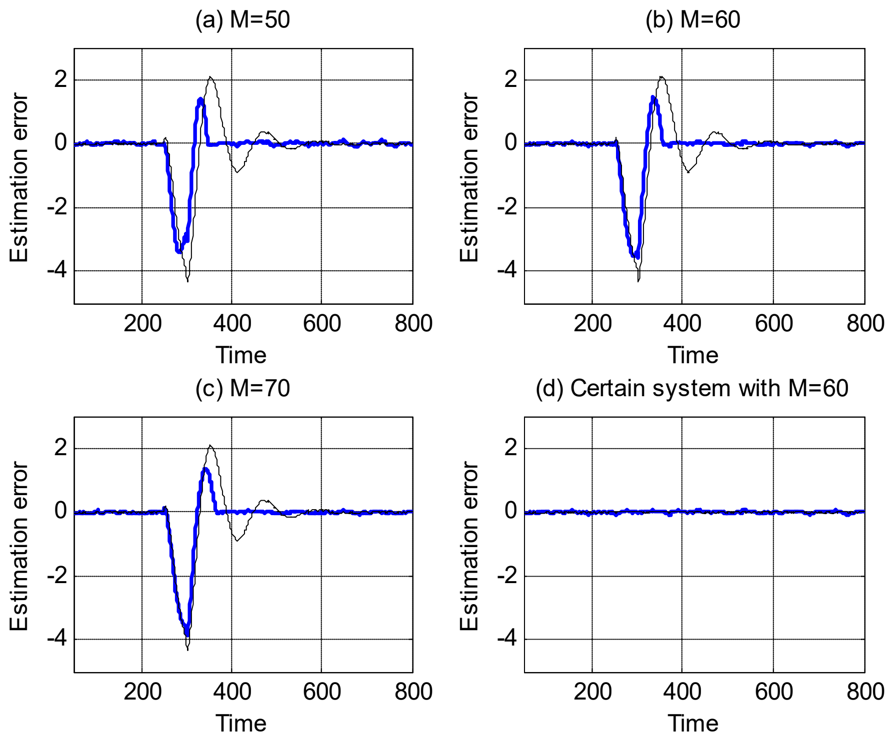

Fig. 2. Estimation errors for Van der Pol oscillation signal model

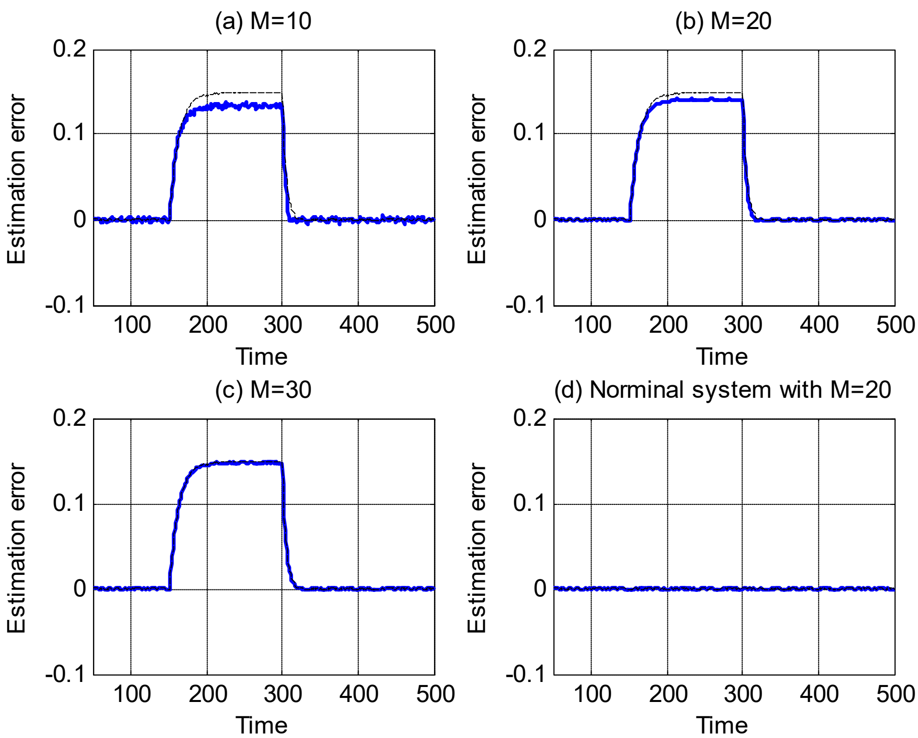

Fig. 3. Estimation errors for DC motor signal model