# Research on Improved Algorithm of Frequency Estimation Based on Complex Modulation

Xinyu Dao, Min Gao, and Zhifei Ke

*Abstract*—**Achieving precision frequency estimation plays an important role in radar signal processing, especially on the aspects of target recognition, tracking and location. Applying Fast Fourier Transformation to analyze the spectrum is convenient whereas fence effect and spectrum leakage cannot be easily denied and the error is too much to satisfy practical engineering requirement. Zoom FFT algorithm, a complex modulation algorithm, provides advantages in spectrum analysis and frequency estimation while the magnification time is limited. A method utilizing Newton Interpolation is proposed to improve the precision of frequency estimation, which the requirement of magnification time is unessential. The corresponding frequency correction equations are derived and two scenarios are illustrated to depict the performance of the proposed method. The simulation results verified that the method had an awesome effect on frequency estimation and the error of frequency measurement decreased an average by 30% compared to Zoom FFT algorithm. The real ranging system also indicated that the proposed method could improve ranging precision.**

*Index Terms*—**Frequency estimation, Frequency correction, Complex modulation, Newton interpolation**

## I. Introduction

SPECTRUM analysis is one of the most basic and frequently-used methods in modern signal processing technology, which is wildly applied to mechanical engineering, radar system, instrument and apparatus and other aspects of production practice. Fast Fourier Transformation (FFT), a classical spectrum analysis approach, tends to deal with dynamic signal [1]. However, the discrete spectrum obtained by FFT algorithm may cause serious errors in measurement of frequency, amplitude and phase. It is obviously unable to meet the requirement of practical engineering. For instance, in radio frequency modulation ranging system, ranging is carried out through the beat frequency, taking advantages of the transmitted and received signal from the target. Achieving accurate estimation of beat frequency is the basis of precise ranging [2].

In order to overcome the shortcoming of the FFT algorithm and make frequency estimation more accurate, spectral correction becomes extremely necessary. Some researchers have been dedicating to study relative theory and have borne some fruits [3-12]. Zoom Fast Fourier Transformation (ZFFT), one of the correction methods, has been used in practical engineering because its principal and physical concept is explicit. The technology only concerns certain useful information of the whole spectrum and the resolution could be improved greatly without increasing data of FFT [13, 14]. Nevertheless, there are still two flaws in ZFFT algorithm. One is that it is impossible to magnify the spectrum indefinitely due to the limitation of storage space and another is that the existence of the fence effect has an impact on the accuracy of frequency estimation.

In this paper, we intend to explore an improved method to avoid the affection of the fence effect. Firstly, the frequency spectrum is amplified by the ZFFT algorithm. The amplified spectrum is still discrete. Then, based on that, by reconstructing the original signal using the Newton interpolation, the discrete spectrum is turned into a definite continuous spectrum and the desired frequency could be ascertained. The requirement of magnification time is not rigorous in the approach, in other words, the higher accuracy of frequency estimation could be obtained with less computational cost. The structure of this paper is arranged as follows: Firstly, the principal of ZFFT algorithm is presented briefly. Then the method based on Newton Interpolation to correct frequency is derived. Next, the performance of ZFFT and the proposed algorithm is discussed in detail. Finally, numerical simulation experiments and real distance measurement are provided to demonstrate the superior performance of the proposed algorithm on frequency estimation.

## II. Principle of Complex Modulation with ZFFT Algorithm

The kernel of the ZFFT algorithm is a complex modulation of the input signal. The modulated signal will experience low pass filtered, resampled, FFT, and frequency component modulated [15]. The principal of ZFFT algorithm is presented in Fig.1.

Assume the input signal is $x(n)$, after complex modulation, we get:

$$y(n) = x(n) * \exp(-j2\pi n f_0 / f_s) \qquad (1)$$

Where, $f_0$ and $f_s$ represent the center frequency of the input signal and the sample rate, respectively. The corresponding frequency spectrum relation is:

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$$Y(e^{j\omega}) = X(j(\omega + 2\pi f_0)) \quad (2)$$

The function of the low pass filter is to prevent aliasing. Assume the magnification time is $M$. The cutoff frequency of the low pass filter is $f_s/2M$. The frequency band becomes narrow after filtering, therefore, resampling the signal with the lower sample frequency $f_s/M$, we could get the signal $y_M(m)$, the corresponding spectrum is:

$$Y_M(e^{j\omega}) = \frac{1}{M} Y(e^{-j\frac{\omega}{M}}) \quad (3)$$

Then, after FFT, we get:

$$Y_M(K) = \frac{1}{M} Y(\frac{K}{M}) \quad (4)$$

Therefore, the original spectrum is expanded $M$ times by means of ZFFT algorithm.

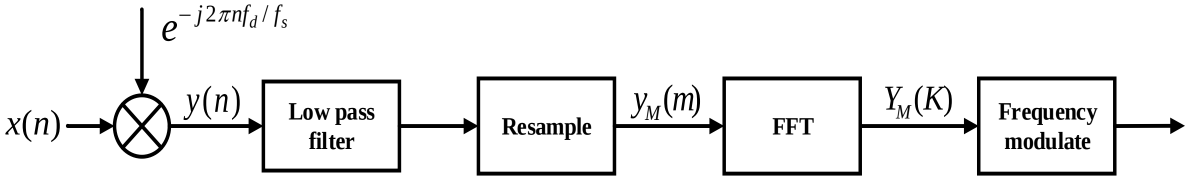



Fig.1 Schematic diagram of ZFFT

## III. Proposed Method for Precision Frequency Estimation

The majority of objects information is usually indicated in the peak of the signal spectrum. Thus, the frequency estimation with respect to the peak position of the signal spectrum is our interests in this literature. Although the accuracy of frequency estimation is improved by means of ZFFT algorithm, there still are some errors due to fence effect. Considering that, the precise estimation for frequency is usually divided into two steps: a coarse estimation and a fine estimation [16]. The Fig.2 shows the frequency spectrum comparison between FFT and ZFFT, while the Fig.3 reveals that there is still deviation $\delta$ between the estimated frequency and the true frequency. The red solid line represents the true frequency while the blue solid line represents the estimated frequency. Hence, ZFFT is regarded as coarse estimation in this paper. Applying Newton interpolation to correct frequency is considered as a fine estimation.

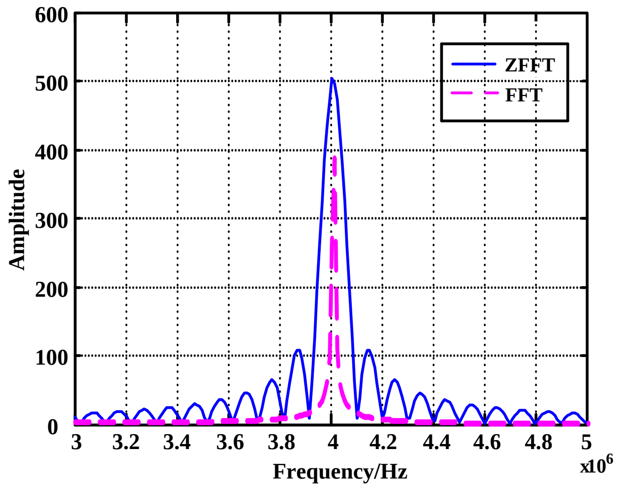



Fig. 2 Spectrum of FFT and ZFFT

Newton interpolation is one of the efficient ways to approximate original function. Suppose $x_0, x_1, \cdots, x_n$ and $y_i = f(x_i)$ $(i = 0, 1, \cdots, n)$ are already known, the approximate function can be described as [17]:

$$\begin{aligned} N_n(x) = & f(x_0) + f[x_0, x_1](x - x_0) \\ & + f[x_0, x_1, x_2](x - x_0)(x - x_1) \\ & + \cdots + f[x_0, x_1, \cdots, x_n](x - x_0) \\ & (x - x_1) \cdots (x - x_{n-1}) \end{aligned} \quad (5)$$

Where, $f[x_0, x_1, \cdots, x_n]$ represents the difference quotient.

$$f[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \quad (6)$$

When $n \geq 2$,

$$\begin{aligned} f[x_0, x_1, \cdots, x_n] &= \frac{f[x_0, \cdots, x_{n-2}, x_n] - f[x_0, \cdots, x_{n-2}, x_{n-1}]}{x_n - x_{n-1}} \\ &= \sum_{i=0}^{n} \frac{f(x_i)}{\omega'(x_i)} \end{aligned} \quad (7)$$

Where, $\omega'(x_i) = \prod_{j=0, j \neq i}^{n} (x_i - x_j)$.

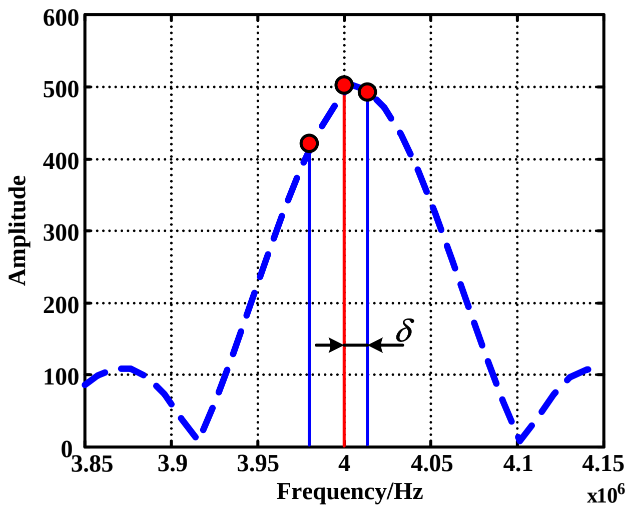



Fig. 3 Partial Spectrum of ZFFT

Theoretically speaking, the more points we select, the better the effect is. However, the complexity and time of computation will increase sharply.

Considering that the frequency spectrum is presented in many discrete points after ZFFT, we set two scenarios selecting several sample points of the spectrum to explore the method of frequency correction. The diagrams of the two scenarios are shown in Fig.4 and Fig.5, respectively.

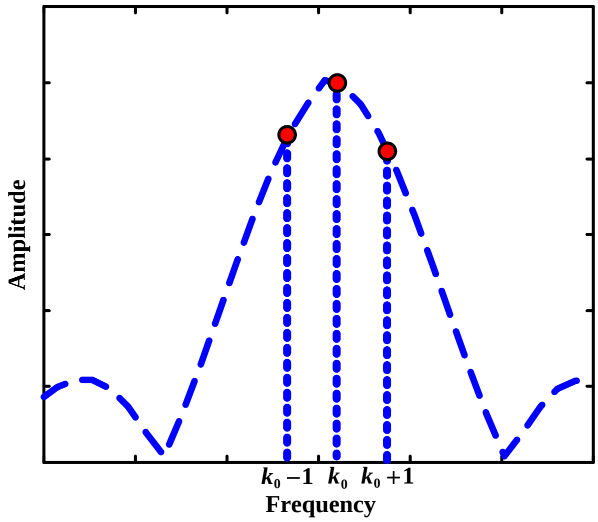



Fig. 4 Schematic diagram of 3 points fitting correction

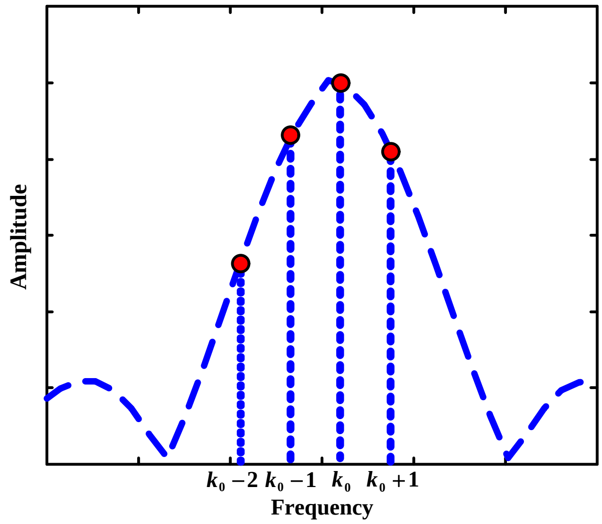



Fig. 5 Schematic diagram of 4 points fitting correction

*A. **Scenario1** 3-Points Fitting Correction (3PFC)*

Assume the peak position and neighboring two sample points position are $k_0, k_0-1, k_0+1$, respectively, and their corresponding amplitude are $Y(k_0), Y(k_0-1), Y(k_0+1)$, respectively. The Newton interpolation function can be shown as follows:

$$\begin{aligned} N_n(k) &= Y(k_0-1) + f[k_0-1, k_0](k-(k_0-1)) \\ &+ f[k_0-1, k_0, k_0+1](k-(k_0-1))(k-k_0) \end{aligned} \tag{8}$$

Where,

$$f[k_0-1, k_0] = Y(k_0) - Y(k_0-1) \tag{9}$$

$$\begin{aligned} f[k_0-1, k_0, k_0+1] &= f[k_0-1, k_0+1] - f[k_0-1, k_0] \\ &= \frac{1}{2}Y(k_0-1) - Y(k_0) + \frac{1}{2}Y(k_0+1) \end{aligned} \tag{10}$$

In order to acquire the frequency with respect to peak magnitude, the extreme value of $N_n(k)$ need to be calculated, then let the extreme value is zero, we get:

$$k_p = -\frac{f[k_0-1, k_0]}{2f[k_0-1, k_0, k_0+1]} + k_0 - \frac{1}{2} \tag{11}$$

Where $k_p$ denotes the estimation position of peak magnitude, thus, the estimation frequency is:

$$\hat{f}_0 = k_p \Delta f \tag{12}$$

Where $\Delta f$ represents the resolution of the spectrum, $\Delta f = f_s / N$, $N$ is the number of samples to be transformed.

*B. **Scenario 2** 4-Points Fitting Correction (4PFC)*

On the basis of the 3-points fitting correction method, only one more point (either ( $k_0-2$ , $Y(k_0-2)$ ) or $(k_0+2, Y(k_0+2))$ ) is added in Scenario2. Here, we choose point $(k_0-2, Y(k_0-2))$ to demonstrate the method. In a similar way of Scenario1, the Newton interpolation function is:

$$\begin{aligned} N_n(k) &= Y(k_0-2) + f[k_0-2, k_0-1](k-(k_0-2)) \\ &+ f[k_0-2, k_0-1, k_0](k-(k_0-2))(k-(k_0-1)) \\ &+ f[k_0-2, k_0-1, k_0, k_0+1](k-(k_0-2)) \cdot \\ &(k-(k_0-1))(k-k_0) \end{aligned} \tag{13}$$

Where,

$$f[k_0-2, k_0-1] = Y(k_0-1) - Y(k_0-2) \tag{14}$$

$$\begin{aligned} f[k_0-2, k_0-1, k_0] &= \frac{1}{2}Y(k_0-2) - Y(k_0-1) \\ &+ \frac{1}{2}Y(k_0) \end{aligned} \tag{15}$$

$$\begin{aligned} f[k_0-2, k_0-1, k_0, k_0+1] &= \frac{1}{6}Y(k_0-2) - Y(k_0) \\ &+ \frac{1}{3}Y(k_0+1) \end{aligned} \tag{16}$$

Let $a = f[k_0-2, k_0-1]$ , $b = f[k_0-2, k_0-1, k_0]$ , $c = f[k_0-2, k_0-1, k_0, k_0+1]$ and calculate the extreme value of $N_n(k)$ , let the extreme value is zero, we can get:

$$k_{p1} = \frac{-(b+3c) + \sqrt{b^2 - 3bc + 3c^2 - 3ac}}{3c} + k_0 \tag{17}$$

and

$$k_{p2} = \frac{-(b+3c) - \sqrt{b^2 - 3bc + 3c^2 - 3ac}}{3c} + k_0 \tag{18}$$

In previous studies, it is clearly that $Y(k_0) > Y(k_0-1) > Y(k_0-2)$ and $Y(k_0) > Y(k_0+1)$ , thus, $b+3c<0$, $c<0$. Then, we can know that $k_{p1} < k_{p2}$ .From the monotonicity of the function $N_n(k)$ , it is monotone increasing when $k \in [k_{p1}, k_{p2}]$ while it is decreasing in other intervals. Hence, $k_{p1}$ and $k_{p2}$ represent the index of minimal value and maximum value respectively.

Considering the practical condition of frequency estimation, on the one hand, the index of peak magnitude of the spectrum is the focus of estimation. On the other hand, we merely need segmental information around the peak magnitude. That means the maximum value is the location to correct. Therefore, the corrected frequency can be described as:

$$\hat{f}_0 = k_{p2} \Delta f \tag{19}$$

Where, $\Delta f = f_s / N$ .

## IV. Verification

In the previous sections, ZFFT and proposed method of frequency correction are introduced. In order to verify the effect of above methods, numerical simulation experiments are conducted. Firstly, we explore the accuracy of the above methods with different frequency varied from 1.6MHz to 6.4MHz in steps of 0.4MHz and calculate the respective errors. Furthermore, we investigate the estimated performance and noise sensitivity. Then, we analyze the influence of different deviation $\delta$ within the margin of error. Finally, the practical ranging testing was conducted.

### *A. Accuracy comparison of different methods*

In the background of a certain radar ranging system, we choose the theoretical center frequency of the input signal varied from 1.6MHz to 6.4MHz in steps of 0.4MHz to conduct tests. The input cosine wave amplitude was set to 1. The number of samples $N$ was set to 1024 and the sampling rate was set to 15MHz to satisfy the Nyquist sampling theorem. The estimated frequency and corresponding errors of experiments are listed in Table I and Table II . The data in the bracket indicates the percentage of error reduction. Compared with ZFFT algorithm, the proposed methods make progress in the aspect of frequency precision. The errors of frequency estimation are generally reduced. It can be found that the errors reduce an average by 30 percent according to calculation. The absolute error curve of diverse estimated frequency is presented in Fig. 6. The absolute error caused by 3PFC and 4PFC is commonly lower than ZFFT. Furthermore, the absolute error of the estimated detection distance corresponding to center frequency in ranging system is shown in Fig. 7. The results imply that the proposed method has better precision.

Table I Frequency estimation results with different algorithm

| $f_0$/Hz | ZFFT/Hz | 3PFC/Hz | 4PFC/Hz |
|---|---|---|---|
| **1600000** | 1611328 | 1608615 | 1608231 |
| **2000000** | 2006835 | 2004114 | 2003733 |
| **2400000** | 2402343 | 2399617 | 2399414 |
| **2800000** | 2812500 | 2809768 | 2809389 |
| **3200000** | 3208007 | 3205271 | 3204894 |
| **3600000** | 3603515 | 3600775 | 3600399 |
| **4000000** | 4013671 | 4010926 | 4010522 |
| **4400000** | 4409179 | 4406430 | 4406057 |
| **4800000** | 4804687 | 4801934 | 4801561 |
| **5200000** | 5203195 | 5197265 | 5197066 |
| **5600000** | 5610351 | 5607586 | 5607217 |
| **6000000** | 6005859 | 6003085 | 6002718 |
| **6400000** | 6401867 | 6398578 | 6398215 |

Table II Frequency estimation errors with different algorithm

| $f_0$/Hz | △ZFFT/Hz | △3PFC/Hz | △4PFC/Hz |
|---|---|---|---|
| **1600000** | 11328 | 8615(23.9%) | 8231(27.3%) |
| **2000000** | 6835 | 4114(39.8%) | 3733(45.3%) |
| **2400000** | 2343 | -383(83.7%) | -586(75.0%) |
| **2800000** | 12500 | 9768(21.9%) | 9389(24.9%) |
| **3200000** | 8007 | 5271(34.2%) | 4894(38.9%) |
| **3600000** | 3515 | 775(77.9%) | 399(88.6%) |
| **4000000** | 13671 | 10926(20.1%) | 10522(23.0%) |
| **4400000** | 9179 | 6430(29.9%) | 6057(34.0%) |
| **4800000** | 4687 | 1934(58.7%) | 1561(66.7%) |
| **5200000** | 3195 | -2735(14.3%) | -2934(8.2%) |
| **5600000** | 10351 | 7586(26.7%) | 7217(30.2%) |
| **6000000** | 5859 | 3085(47%) | 2718(53.6%) |
| **6400000** | 1867 | -1422(23.8%) | -1785(43.9%) |

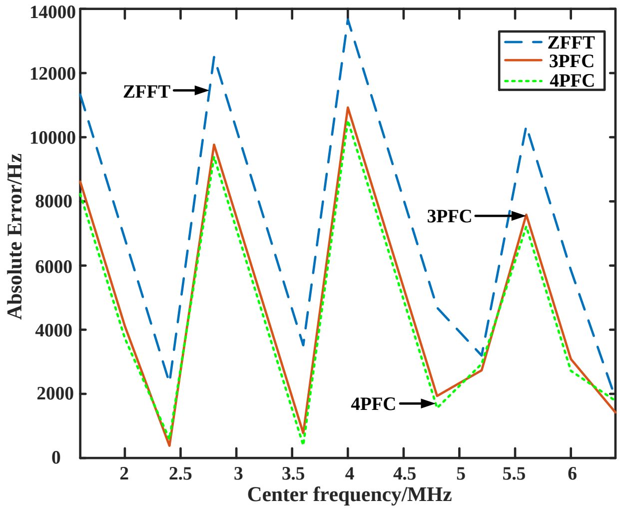



Fig. 6 Absolute error of frequency estimated

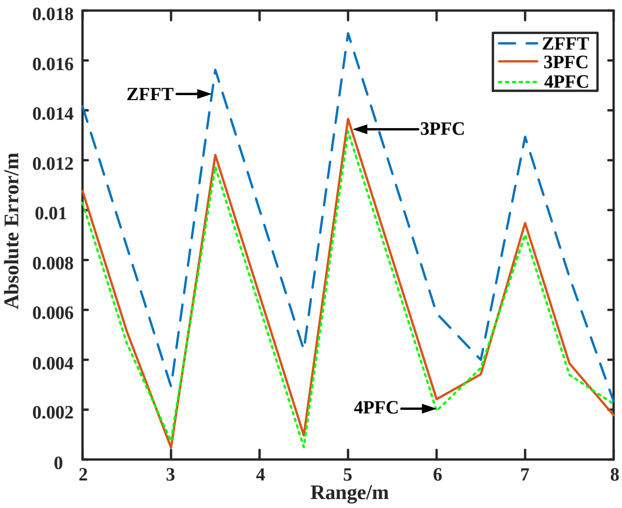



Fig. 7 Absolute error of range estimated

### *B. Influence of noises*

In practical measurement environment, signals are usually affected by noise from kinds of uncontrollable factors. As a result, the accuracy of estimation is probably reduced and the frequency estimation has randomness. Considering that, we add zero-mean Gaussian noise in theoretical input signal to assess sensitivity of the proposed algorithm to white noise.

The noise level is estimated by Signal-Noise Ratio (SNR). Assuming that the SNR is changed from 0dB to 20dB in steps of 1dB, we conduct the Monte-Carlo simulation 1000 times for distinct SNR and observe the estimation performance compared with the Cramer-Rao lower bound (CRLB). The CRLB of the mean squared error is shown in [18]:

$$\sigma_f^2 = \frac{6 f_s^2}{4\pi^2 N(N^2-1)\rho} \tag{20}$$

Where $\rho$ is the SNR, since $N >> 1$, the CRLB of the Root Mean Squared Error (RMSE) can be simplified as:

$$\sigma_f = \sqrt{\frac{3f_s^2}{2\pi^2 N^3 \rho}} \tag{21}$$

The calculating results of Root Mean Squared Error with respect to SNR are plotted in Fig.8. From the Fig. 8, the higher the SNR is, the better the performance of estimation is, which is in agreement with practical experience. The performance of improved algorithms in this paper is obviously more excellent than ZFFT algorithm. With the decrease of SNR, the estimated Root Mean Squared Error increases exponentially, which means ZFFT algorithm is not applicable in the condition of low SNR. However, the 3PFC or 4PFC algorithm reduces the error to a certain extent and approaches the CRLB closely as the SNR increasing.

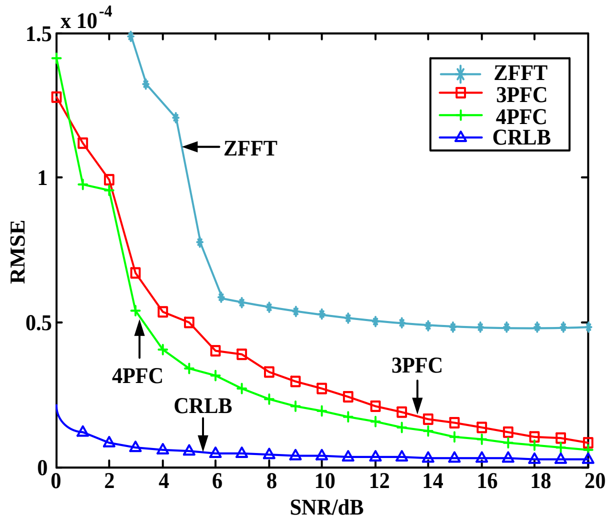



Fig. 8 Comparison of estimation performance with different SNR

*C. Influence of deviation*

In previous depictions, the deviation $\delta$ reflects the index error between the estimated frequency and the true frequency. It is related to the sample frequency. In this subsection, we consider the influence of various deviation in frequency estimation. Fig.9-12 show the RMSE with respect to SNR in the condition of different $\delta$ ( $\delta$ = 0.1, 0.2, 0.3 or 0.4). However, the curves of ZFFT algorithm are not marked in these figures due to its excessive errors.

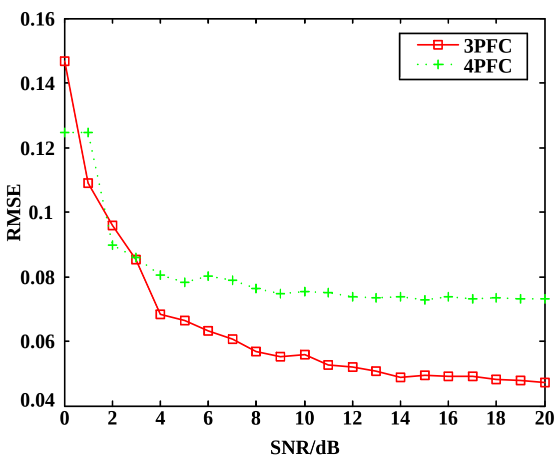



Fig. 9 RMSE of frequency estimation vs. SNR with $\delta = 0.1$

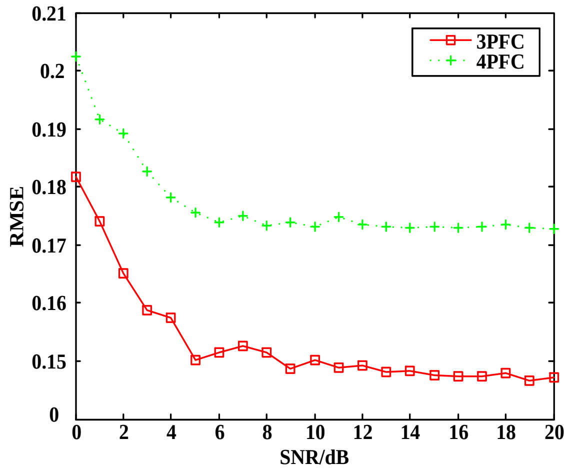



Fig. 10 RMSE of frequency estimation vs. SNR with $\delta = 0.2$

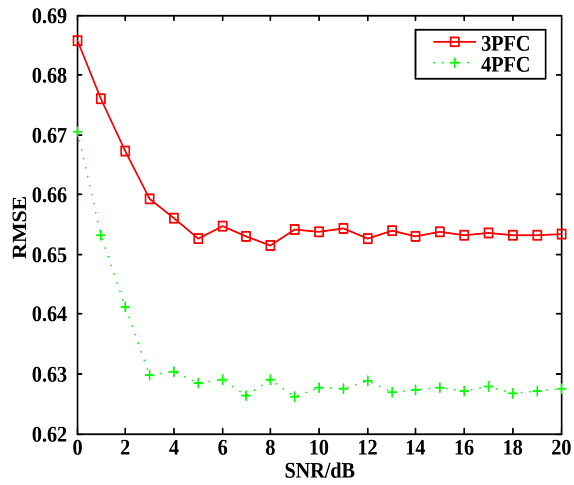



Fig. 11 RMSE of frequency estimation vs. SNR with $\delta = 0.3$

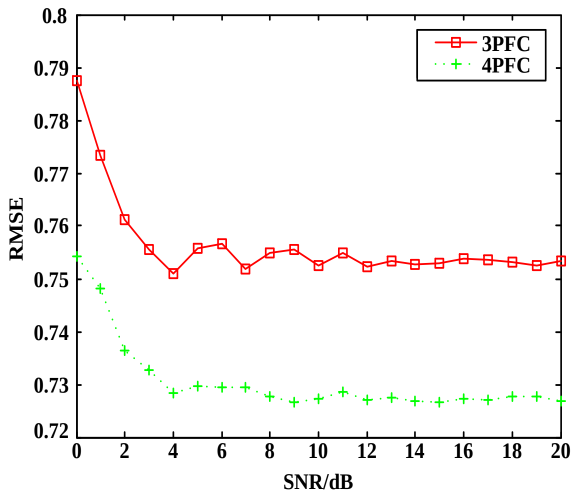



Fig. 12 RMSE of frequency estimation vs. SNR with $\delta = 0.4$

From Fig.9-12, we can see that the noise is the main factor when SNR is below 5dB .With the increase of $\delta$, the root mean squared error increases gradually. The performance of 3PFC has advantages when $\delta$ is small whereas the performance of 4PFC is better when $\delta$ is large.

*D. Experiment Verification*

To verify the application of proposed method in practical test, we designed an experiment based on Frequency Modulated Continuous Wave (FMCW) system. The range measurement diagram is shown in Fig.13. The detection module in the system is illustrated in the Fig.14.

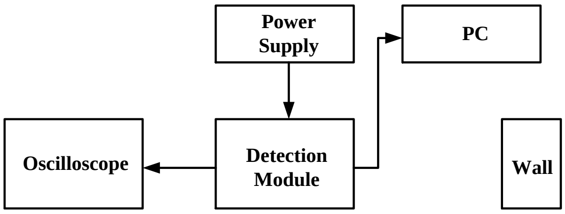



Fig. 13 Diagram of the measurement

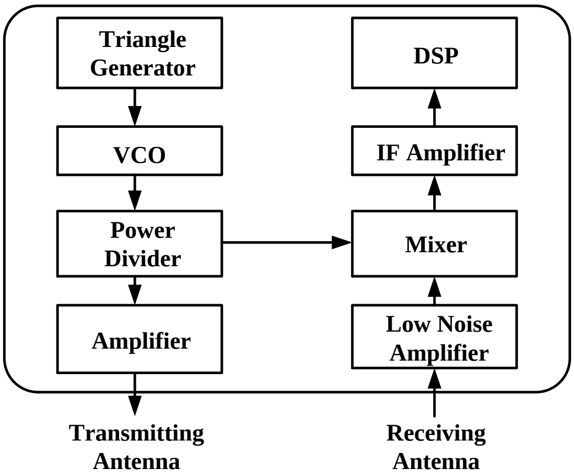



Fig. 14 Structure diagram of the detection module

The carrier frequency is set to be 24.5GHz, frequency deviation is 2MHz. and transmit power is 8mW. The distance between the wall and the detection module is preset to be 2m, 3m, 4m, 5m, 6m, 7m and 8m, respectively. After positioning the detection module, turn on the PC to test three times and observe the range value. The test results of the three methods are presented in Table III~Table V, respectively. The average absolute error of the three group results is shown in Fig. 15. Compared with the simulation results, the practical testing results of the three methods are little worse due to the influence of encapsulating material. However, it can still be found that the proposed methods have better effect on ranging compared with ZFFT. The Fig. 15 indicates that the ranging precision of 3PFC or 4PFC is obviously higher than ZFFT.

In order to illustrate the ranging effect of the three methods better, multiple experiments are carried out at different distances (the number of experiments is set to be 100 times). The root mean square error of each test distance value is calculated as shown in Fig. 16. No matter what the detection distance, the error of 3PFC and 4PFC are obviously lower than ZFFT.

Table III First group results of range measurement

| Range/m | ZFFT/m | 3PFC/m | 4PFC/m |
|---|---|---|---|
| **2** | 2.34 | 1.95 | 2.10 |
| **3** | 2.73 | 3.12 | 2.95 |
| **4** | 3.51 | 3.91 | 4.05 |
| **5** | 4.69 | 5.08 | 5.08 |
| **6** | 5.60 | 6.25 | 6.30 |
| **7** | 6.25 | 6.95 | 7.14 |
| **8** | 8.60 | 7.82 | 8.13 |

Table IV Second group results of range measurement

| Range/m | ZFFT/m | 3PFC/m | 4PFC/m |
|---|---|---|---|
| **2** | 2.52 | 1.90 | 2.05 |
| **3** | 2.83 | 3.02 | 3.08 |
| **4** | 3.51 | 3.94 | 4.09 |
| **5** | 4.59 | 5.08 | 5.06 |
| **6** | 5.45 | 6.17 | 6.16 |
| **7** | 6.29 | 6.95 | 7.16 |
| **8** | 8.60 | 7.88 | 8.15 |

Table V Third group results of range measurement

| Range/m | ZFFT/m | 3PFC/m | 4PFC/m |
|---|---|---|---|
| **2** | 2.51 | 1.92 | 1.95 |
| **3** | 2.78 | 3.08 | 3.06 |
| **4** | 3.51 | 4.06 | 4.05 |
| **5** | 4.54 | 4.92 | 5.06 |
| **6** | 5.48 | 6.15 | 6.18 |
| **7** | 6.26 | 7.05 | 7.13 |
| **8** | 8.58 | 7.90 | 8.11 |

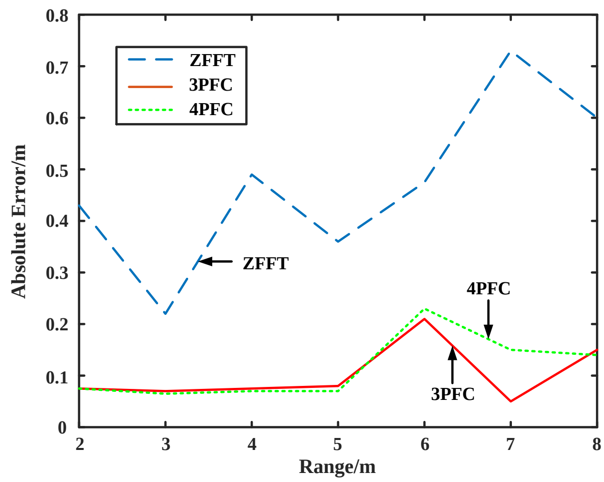



Fig. 15 Absolute error of different range with three methods

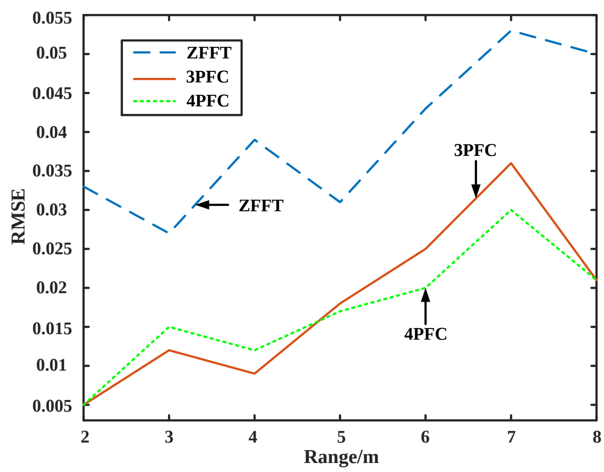



Fig. 16 RMSE of different range with three methods

To further verify the evaluation of proposed methods under dynamic condition, a vehicle testing is designed. Test schematic diagram is depicted in Fig. 17. The detection module is loaded on a vehicle. The corner reflector is used to replace the target. The test range is preset to be 5m. At the same time, taking into account the error in the distance measurement, the indicators are placed respectively at the 4m, 5m and 6m far from the corner reflector. Then, the distance from the vehicle to the corner reflector is divided into four intervals (I1, I2, I3 and I4). An indicator light (as shown in Fig. 18) is connected to the signal output port of the detection module. By observing the indicator light on or off, the ranging accuracy is judged to meet the requirement. When the light flashing in the I2 or I3, the results can be recognized within the margin of error. The car goes into the corner reflector at the speed of 10m/s, 30m/s and 50m/s, respectively. The testing times at each speed are set to be 20. The statistical results of the number of lighting in each interval are calculated. The Fig.19 gives the percentage results of lighting times in each interval.

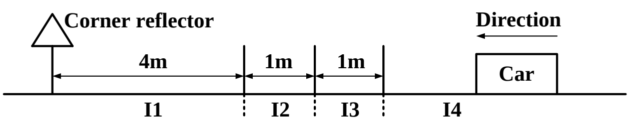



Fig. 17 Schematic diagram of vehicle testing

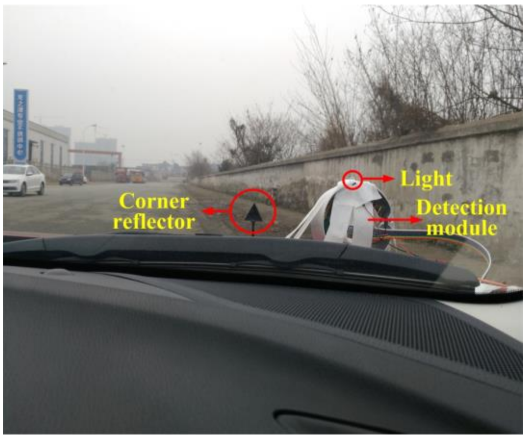
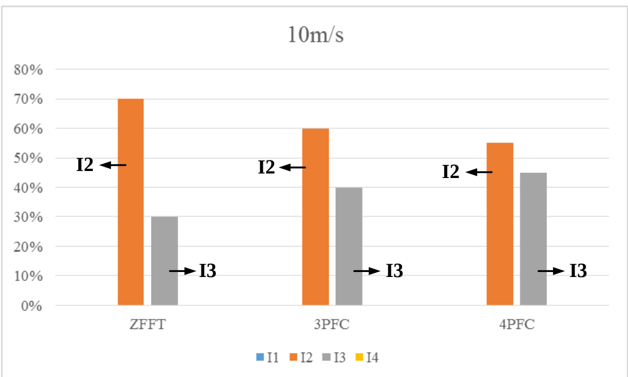
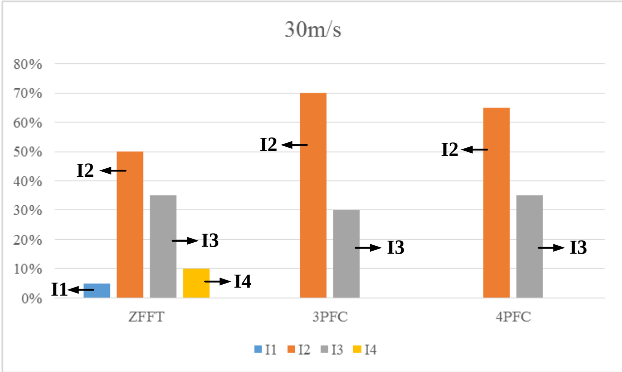



Fig. 18 Scene of vehicle testing

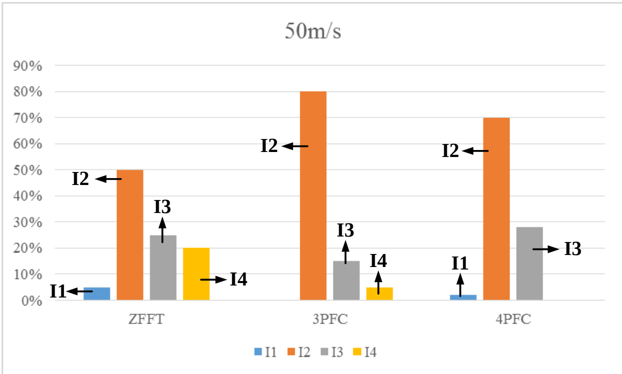
At the speed of 10m/s (Fig.19 (a)), it does not appear much difference in the results of the three methods. All the results meet the error requirement. Nevertheless, as the velocity increasing (Fig.19 (b) and Fig.19 (c)), the error comes to appear in the ZFFT algorithm while the proposed methods still remain the precision.




(a)




(b)




(c)

Fig. 19 Statistic results of vehicle testing with different speed

## V. Conclusion

This paper proposes a frequency correction algorithm based on Newton interpolation, which provides accurate frequency estimation. We theoretically analyze the traditional complex modulation, namely ZFFT algorithm, and derive the corrected expression of the proposed algorithm. The proposed method maintains the complexity of computation at a tolerable level and straightforward to understand. Furthermore, the magnification time of ZFFT algorithm is not required to be large, which saves a lot of storage space in the practical hardware operation and paves the way for the other further research in frequency estimation to satisfy higher requirement. White Gaussian noise is also discussed in experiments, simulation results state that the proposed algorithm has great performance on sensitivity to additive noise compared with ZFFT algorithm. The ranging measurement under practical condition indicates that the proposed method is of great benefit for improving frequency precision.

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