Multiple Attribute Decision Making based on Neutrosophic Sets in Venture Capital

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Abstract—**Because of the complexity and uncertainty of objective things and the fuzziness of human thinking, it is possible to express the information in terms of neutrosophic sets in the actual decision making problem, so as to handle the uncertain information. Making decision by using the neutrosophic numbers can minimize the uncertainty of the evaluation data and make the decision goal more objective and reasonable, thus it can make the comprehensive decision better. This paper which based on theory of neutrosophic sets discusses whether decision information is given by single-valued neutrosophic sets or interval neutrosophic sets and obtains the corresponding solutions of decision-making problems.**

Index Terms—**Multiple attribute decision making, Interval neutrosophic set, Single-valued neutrosophic set, Venture investment.**

I. Introduction

VENTURE capital is called VC, which mainly refers to a way of financing for start-ups to provide financial support and acquire shares of enterprise(EN).

Chinese venture investment industry rapidly developed since the middle of the 1990s. Based on the actual situation of the development of venture capital in China, a series of theoretical and empirical research were proposed. Therefore, how to choose investors in venture capital has been a central topic of research. Since the second half of the 20th century, scholars obtained some remarkable achievements in the research and practice of multiple attribute decision-making theory and related methods, including the application of multiple attribute decision-making theory in venture investment and the selection of investment objects. Generally speaking, decision information and decision environment are uncertain, incomplete and inconsistent. For the uncertainty of information, the fuzzy set which has been widely recognized after Zadeh is used to the theory of multiple attribute decision making (MADM). Jiang and Wang [1] investigated an approach to multiple attribute group decision making with interval intuitionistic trapezoidal fuzzy numbers; Wang et al. [2] developed an approach for multi-criteria decision making under dual hesitant fuzzy environment and illustrated an example to show the behavior of the proposed distance measures; Bao et al. [3] aimed at analyzing multi-attribute decision making problems with intuitionistic fuzzy numbers. By establishing the optimal model for the decision matrix in the form of the hesitant fuzzy element, Zhang [4] obtained the decision weight and extended it with interval fuzzy value, and finally applied it to the investment selection problem; Lin et al. [5] studied a signed distance-based approach for the multiple attribute decision making with hesitant fuzzy information.

It seems that the research of the existing uncertain method of multiple attribute decision making mainly focuses on the hesitant fuzzy number, interval fuzzy number, and intuitionistic fuzzy numbers. Uncertainty of attribute leads to the lack of integrity and practicality in the decision-making system. There is still some uncertain information that intuitionistic fuzzy set cannot deal with in real life. Then, Smarandache [6], [7] introduced the concept of neutrosophic set (NS), which includes three parts: truth-membership function, indeterminacy-membership function and falsity-membership function. In order to simplify NS, Wang et al. [8] defined the single-valued neutrosophic set (SVNS) and proved that SVNS has the properties of exchange law, binding law, distribution law and idempotent law. Ye [9] proposed the correlation coefficient and cross entropy of SVNS, and provided the corresponding MADM method. Majumdar and Samanta [10] gave the distance, similarity and entropy of SVNS. For the complexity and uncertainty of objective things, it is difficult to express the truth-membership, indeterminacy-membership and falsity-membership of the evaluation object, and it is appropriate to use the interval number. Wang et al. [11] made a further expansion of SVNS, and proposed interval neutrosophic set (INS), whose truth-membership, indeterminacy-membership and falsity-membership are represented by interval numbers. Then, Broumi and Smarandache [12] gave a generalized distance of INS.

Recently, researchers used many classical MADM methods to solve the multiple attribute decision-making problem of attribute values in single-valued neutrosophic number (SVNN) or interval neutrosophic number (INN), and studied the further application of them. For example, Zhang and Wu [13] proposed a new decision method to solve an MADM problem whose attribute value is SVNN or INN and part of weight information is unknown. Then, Peng et al. [14] used ELECTRE method to classify an MADM problem with the attribute value of SVNN. Biswas et al. [15] used Euclidean distance TOPSIS method to solve an MADM problem with SVNN attribute value. Based on the ELECTRE method, Zhang et al. [16] proposed an MADM problem with INN. Wang and Li [17] added the INS to TODIM method. Wang and Dang [18] improved TODIM method with more feasible. Liu et al. [19] discussed ELECTRE method of INS. Tan et al. [20] applied NS to emergency group decision-making. Song [21] developed the rough set of NS and used this theory to diagnose the fault of the steam turbine. To tackle the latter problem in both color and depth domains, Hu [22] build a robust tracker by utilizing SVNS. A projection-

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based TODIM method with multi-valued neutrosophic sets (MVNSs) for personnel selection was established to consider the risk preference of decision-makers and overcome the defect of the extant fuzzy TODIM methods in [23]. Xu et al. [24] combined NS with TOMID method for MADM. For solving the MADM problems, Yang et al. [25] combined NSs and hesitant fuzzy sets to define the hesitant fuzzy neutrosophic sets. To avoid some impractical operations in certain cases and solve multi-criteria decision-making problems, some new operational laws of simplified neutrosophic numbers based on Einstein operations are defined by Li et al. [26] .

In this paper, we discuss whether the decision-making information given by decision makers is SVNS or INS in venture capital and uses an improved TODIM method to choose the best enterprise for investment. This paper makes some changes on the basis of previous studies to provide some references for future research.

II. Preliminaries of neutrosophic

This section gives a brief overview of concepts and definitions of neutrosophic set (NS), interval neutrosophic set (INS) and single-valued neutrosophic set (SVNS) [27].

A. Neutrosophic set

Definition 1: [6] Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_{A(x)}$, an indeterminacy-membership function $I_{A(x)}$, and a falsity-membership function $F_{A(x)}$, where $T_{A(x)}$, $I_{A(x)}$, $F_{A(x)}$ are the function of finite discrete subset of $[0,1]$. So, $T_{A(x)}$: $X \to [0,1]$, $I_{A(x)}$:$X \to [0,1]$, $F_{A(x)}$:$X \to [0,1]$. A can be expressed by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\},$$

with the condition of $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$.

Definition 2: [6] The complement of a neutrosophic set A is denoted by A^C and is defined as $T_{A^C}(x) = \{1^+\} \ominus T_A(x)$, $I_{A^C}(x) = \{1^+\} \ominus I_A(x)$, $F_{A^C}(x) = \{1^+\} \ominus F_A(x)$ for every x in X.

Definition 3: [6] A neutrosophic set A is contained in the other neutrosophic set B: $A \subseteq B$ if and only if

$\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$,

$\inf I_A(x) \geq \inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$,

$\inf F_A(x) \geq \inf F_B(x)$, and $\sup F_A(x) \geq \sup F_B(x)$ for every $x \in X$.

B. Interval neutrosophic set

Definition 4: [11] Let X be a space of points(objects) with generic elements in X denoted by x. An INSs A in X is characterized by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\},$$

where

$$T_{A(x)} = [\inf T_{A(x)}, \sup T_{A(x)}], I_{A(x)} = [\inf I_{A(x)}, \sup I_{A(x)}],$$

$$F_{A(x)} = [\inf F_{A(x)}, \sup F_{A(x)}]$$

with the condition of

$$0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3.$$

For convenience, an INSs can be expressed to be

$$A = \langle [T^L, T^U], [I^L, I^U], [F^L, F^U]\rangle$$

with $T^L = \inf T_{A(x)}$, $T^U = \sup T_{A(x)}$, $I^L = \inf I_{A(x)}$, $I^U = \sup I_{A(x)}$, $F^L = \inf F_{A(x)}$, $F^U = \sup F_{A(x)}$ and $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$.

Definition 5: [27] The complement of an INS A is denoted by A^C and is defined as $T_{A^C}(x) = F_{A(x)}$, $\inf I_{A^C}(x) = 1 - \sup I_{A(x)}$, $\sup I_{A^C}(x) = 1 - \inf I_{A(x)}$,$F_{A^C}(x) = T_{A(x)}$for any x in X. That is

$$A^C = \langle F_A(x), [1 - \sup I_{A(x)}, 1 - \inf I_{A(x)}], T_A(x)\rangle. \quad (1)$$

Definition 6: [27] There are two INSs A and B in $X = \{x_1, x_2, ..., x_n\}$ are denoted by

$$A = \{\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i) | x_i \in X\rangle\}$$

and

$$B = \{\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i) | x_i \in X\rangle\}.$$

In other words,

$$A = \langle [T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U]\rangle$$

and

$$B = \langle [T_B^L, T_B^U], [I_B^L, I_B^U], [F_B^L, F_B^U]\rangle,$$

then the normalized Euclidean distance between A and B is

$$\begin{aligned} d(A,B) = &\frac{1}{6}\{(T_A^L - T_B^L)^2 + (T_A^U - T_B^U)^2 \\ &+ (I_A^L - I_B^L)^2 + (I_A^U - I_B^U)^2 \\ &+ (F_A^L - F_B^L)^2 + (F_A^U - F_B)^2\}^{\frac{1}{2}}. \end{aligned} \quad (2)$$

Definition 7: [28] Let A=$\langle [T_A^L, T_A^U], [I_A^L, I_A^U], [F_A^L, F_A^U]\rangle$, so the expectation of A is

$$E(A) = \frac{1}{6}\{(T_A^L + T_A^U + 1) + (I_A^L + I_A^U + 1) - (F_A^L + F_A^U)\}. \quad (3)$$

C. Single-valued neutrosophic set

Definition 8: [24] Let X be a space of points (objects). A SVNS A in X is characterized by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}.$$

That is $T_{A(x)} : X \to [0,1]$, $I_{A(x)} : X \to [0,1]$, $F_{A(x)} : X \to [0,1]$, with the condition of $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 9: [20] The complement of a neutrosophic set A is denoted by A^C and is defined as

$$A^C = \langle F_A, [1 - I_A], T_A\rangle \quad (4)$$

Definition 10: [20] Let A and B be two SVNNs, then the normalized Euclidean distance between A and B is

$$\begin{aligned} d(A,B) = &\frac{1}{3}\{(T_A - T_B)^2 + (I_A - I_B)^2 \\ &+ (F_A - F_B)^2\}^{\frac{1}{2}} \end{aligned} \quad (5)$$

Let A be a SVNSs, so the expectation of A is

$$E(A) = \frac{1}{3}\{(T_A + 1) + (I_A + 1) - F_A\}. \quad (6)$$

III. TODIM METHOD FOR MADM PROBLEM

Multiple attribute decision making is an optimal scheme to find out a certain goal from a series of alternatives with multiple attributes. It has a broad theoretical and practical background. TODIM is an effective method to solve the MADM problem. Based on TODIM method, the solution for MADM problem is given in this section.

Let alternatives are

$$A = (A_1, A_2, ..., A_m),$$

attributes are

$$G = (G_1, G_2, ..., G_n),$$

and the weights of G_j are

$$w = (w_1, w_2, ..., w_n),\ 0 \le w_j \le 1,\ \sum_{j=1}^{n} w_j = 1.$$

The weight coefficient is determined by the importance of subjective evaluation of decision-makers for each evaluation index [29]. Let a_{ij} be an attribute value of the alternative A_i under the attribute G_j, where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, and $A = (a_{ij})_{m\times n}$ be a decision matric. Furthermore, let

$$w_r = \max\{w_j | i = 1, 2, ..., n\}, \tag{7}$$

and

$$w_{jr} = \frac{w_j}{w_r}\ (j, r = 1, 2, ..., n), \tag{8}$$

where w_{jr} is the relative weight of G_j to G_r.

The following TODIM method is given in [24].

Step 1: Standardize the decision information to get the normalized decision matric. That is, normalizing $A = (a_{ij})_{m\times n}$ into $B = (b_{ij})_{m\times n}$. If the decision is an efficient factor, it cannot be changed; if the decision is a cost factor, the decision information should be changed by its complementary set.

Step 2: Choose w_r, and figure out $w_{jr} = \frac{w_j}{w_r}(j, r = 1, 2, \ldots, n)$.

Step 3: Figure out the dominance degree of B_i over every alternative B_t

$$\delta(B_i, B_t) = \sum_{j=1}^{n} \varphi_j(B_i, B_t)(i = 1, 2, \ldots, m) \tag{9}$$

In this function, $\varphi_j(B_i, B_t)$ represents the dominance degree of B_i over every alternative B_t under attribute G_j; the parameter θ is the attenuation factor of the losses. If $b_{ij} - b_{tj} > 0$, $\varphi_j(B_i, B_t)$ shows a gain; if $b_{ij} - b_{tj} < 0$, $\varphi_j(B_i, B_t)$ expresses a loss.

Step 4: Work out the overall dominance of B_i by following function

$$\xi_i = \frac{\sum_{t=1}^{m} \delta(B_i, B_t) - \min\limits_{1\le i\le m} \{\sum_{t=1}^{m} \delta(B_i, B_t)\}}{\max\limits_{1\le i\le m} \{\sum_{t=1}^{m} \delta(B_i, B_t)\} - \min\limits_{1\le i\le m} \{\sum_{t=1}^{m} \delta(B_i, B_t)\}} \tag{10}$$

Step 5: Ranking all alternatives according to the value of ξ_i. The larger the value of ξ_i, the better the alternative is.

IV. TODIM METHOD FOR INS MADM PROBLEM

In this section, we study the decision information given by the interval neutrosophic numbers. Let $A = (A_1, A_2, \ldots, A_m)$ be alternatives, $G = (G_1, G_2, \ldots, G_n)$ be attributes, and $w = (w_1, w_2, ..., w_n)$ be the weight of $G_j (j = 1, 2, \ldots, n)$, where $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$. The interval neutrosophic number a_{ij} is the value of A_i under G_j. To get normalized decision information

$$b_{ij} = \langle [T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U] \rangle,$$

we standardize decision information a_{ij}. Then, the normalized decision matric B is

$$B = (b_{ij})_{m\times n} = ([T_{ij}^L, T_{ij}^U], [I_{ij}^L, I_{ij}^U], [F_{ij}^L, F_{ij}^U])_{m\times n},$$

$i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Based on the TODIM method for MADM problem, we develop a TODIM method for INS MADM problem. The dominance degree of A_i over each alternative A_t with respect to the attribute G_j is mended by the equation (9), where $\varphi_j(B_i, B_t)$=

$$\begin{cases} \sqrt{\frac{w_{jr} d(b_{ij} - b_{tj})}{\sum_{j=1}^{n} w_{jr}}}, & \text{if } E(b_{ij}) - E(b_{tj}) > 0\ , \\ 0, & \text{if } E(b_{ij}) - E(b_{tj}) = 0\ , \\ -\frac{1}{\theta}\sqrt{\frac{(\sum_{j=1}^{n} w_{jr}) d(b_{tj} - b_{ij})}{w_{jr}}}, & \text{if } E(b_{ij}) - E(b_{tj}) < 0\ , \end{cases} \tag{11}$$

where

$$\begin{aligned} d(b_{ij}, b_{tj}) =& \frac{1}{6}\{(T_{b_{ij}}^L - T_{b_{tj}}^L)^2 + (T_{b_{ij}}^U - T_{b_{tj}}^U)^2 \\ &+ (I_{b_{ij}}^L - I_{b_{tj}}^L)^2 + (I_{b_{ij}}^U - I_{b_{tj}}^U)^2 \\ &+ (F_{b_{ij}}^L - F_{b_{tj}}^L)^2 + (F_{b_{ij}}^U - F_{b_{tj}}^U)^2\}^{\frac{1}{2}}, \end{aligned} \tag{12}$$

$$\begin{aligned} E(b_{ij}) =& \frac{1}{6}\{(T_{b_{ij}}^L + T_{b_{ij}}^U + 1) + (I_{b_{ij}}^L + I_{b_{ij}}^U + 1) \\ &- (F_{b_{ij}}^L + F_{b_{ij}}^U)\}. \end{aligned} \tag{13}$$

Then, use the equation (10) to rank all alternatives and make the optimal decision. The larger the value of ξ_i is, the better the alternative is.

V. TODIM METHOD FOR SVNS MADM PROBLEM

In this section, decision information is given by SVN-N. Let $A = (A_1, A_2, ..., A_m)$ be alternatives, $G = (G_1, G_2, ..., G_n)$ be attributes, and $w = (w_1, w_2, ..., w_n)$ be the weight of G_j $(j = 1, 2, \ldots, n)$, where $0 \le w_j \le 1$, and $\sum_{j=1}^{n} w_j = 1$. The single-valued neutrosophic number a_{ij} is the value of A_i under G_j. To get normalized decision information

$$b_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle,$$

we standardize decision information a_{ij}. So, the normalized decision matric B is

$$B = (b_{ij})_{m\times n} = (T_{ij}, I_{ij}, F_{ij})_{m\times n},$$

$i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

Based on the theory mentioned above, the dominance degree of A_i over each alternative A_t with respect to the attribute G_j can be improved by the equation (9) and (11), where

$$\begin{aligned} d(b_{ij}, b_{tj}) =& \frac{1}{3}\{(T_{ij} - T_{tj})^2 + (I_{ij} - I_{tj})^2 \\ &+ (F_{ij} - F_{tj})^2\}^{\frac{1}{2}}, \end{aligned} \tag{14}$$

$$E(b_{ij}) = \frac{1}{3}\{(T_{ij}+1)+(I_{ij}+1)-F_{ij}\}. \quad (15)$$

Then, use the equation (10) to rank all alternatives and make the optimal decision.

Likewise, the larger the value of ξ_i is, the better the alternative is.

VI. Empirical applications

A. Example 1

This subsection considers decision information given by the interval neutrosophic numbers and applies INSs to multiple attribute decision making. In order to get maximum profit, a VC firm wants to choose an innovating enterprise to invest, where $A = (A_1, A_2, A_3)$ are three enterprises and $G = (G_1, G_2, G_3)$ are three attributes of each.

These enterprises we choose here meet the following conditions

1. The enterprises are generally small and medium-sized in initial stage, and most of them are high-technology;

2. The investment cycle of VC firm is at least 3-5 years, and its way is equity investment. In general, these shares account for about 30% of all shares. The investors have no controlling rights and the enterprises do not need any collateral or mortgage;

3. Investment must be highly specialized and procedural;

4. Normally, investors take an active part in the operation and management of enterprises and provide value-added services;

5. As the aim of investment is to achieve value-added purpose and pursue excess returns, the investors may withdraw capital through listing, merger and acquisition or other forms of equity transfer.

Investing in enterprise is better than investing in project, especially investing in the leader of this enterprise. A good leader is the guarantee of success. And the industry and enterprise with growth potential are the best choice for VC firms.

So here, attributes $G = (G_1, G_2, G_3)$ represent team management, industry's outlook, and enterprise competitiveness, respectively, where G_1 is cost factor, and G_2, G_3 are efficient factors. The weight w of attribute is $w = (0.35, 0.4, 0.25)$. Besides, assume that decision maker gives the decision value by the following decision matric

$$A = \begin{bmatrix} \langle[0.2,0.3][0.3,0.4][0.2,0.5]\rangle & \langle[0.3,0.6][0.4,0.5][0.3,0.4]\rangle & \langle[0.5,0.6][0.3,0.4][0.2,0.4]\rangle \\ \langle[0.4,0.5][0.2,0.3][0.3,0.4]\rangle & \langle[0.4,0.6][0.1,0.3][0.2,0.4]\rangle & \langle[0.7,0.9][0.2,0.3][0.4,0.5]\rangle \\ \langle[0.7,0.8][0.1,0.2][0.2,0.3]\rangle & \langle[0.6,0.7][0.2,0.4][0.1,0.3]\rangle & \langle[0.6,0.7][0.3,0.4][0.8,0.9]\rangle \end{bmatrix}. \quad (16)$$

Then, get the normalized decision matric B by standardizing decision information matric A

$$B = \begin{bmatrix} \langle[0.2,0.5][0.6,0.7][0.2,0.3]\rangle & \langle[0.3,0.6][0.4,0.5][0.3,0.4]\rangle & \langle[0.5,0.6][0.3,0.4][0.2,0.4]\rangle \\ \langle[0.3,0.4][0.7,0.8][0.4,0.5]\rangle & \langle[0.4,0.6][0.1,0.3][0.2,0.4]\rangle & \langle[0.7,0.9][0.2,0.3][0.4,0.5]\rangle \\ \langle[0.2,0.3][0.8,0.9][0.7,0.8]\rangle & \langle[0.6,0.7][0.2,0.4][0.1,0.3]\rangle & \langle[0.6,0.7][0.3,0.4][0.8,0.9]\rangle \end{bmatrix}. \quad (17)$$

Next,according to the equation (12), (13)

$$\begin{gathered} E(b_{11}) = 0.583, E(b_{12}) = 0.517, E(b_{13}) = 0.533, \\ E(b_{21}) = 0.550, E(b_{22}) = 0.467, E(b_{23}) = 0.533, \\ E(b_{31}) = 0.450, E(b_{32}) = 0.583, E(b_{33}) = 0.383. \end{gathered} \quad (18)$$

$$\begin{gathered} d(b_{11}, b_{21}) = 0.058, d(b_{21}, b_{31}) = 0.078, d(b_{31}, b_{11}) = 0.131, \\ d(b_{12}, b_{22}) = 0.065, d(b_{22}, b_{32}) = 0.050, d(b_{32}, b_{12}) = 0.075, \\ d(b_{13}, b_{23}) = 0.075, d(b_{23}, b_{33}) = 0.104, d(b_{33}, b_{13}) = 0.132. \end{gathered} \quad (19)$$

When $\theta = 1$, the dominance degree of B_i over every alternative B_t under attribute G_j is

$$\begin{gathered} \varphi(b_{11}, b_{21}) = 0.142, \varphi(b_{21}, b_{31}) = 0.165, \varphi(b_{31}, b_{11}) = -0.612, \\ \varphi(b_{12}, b_{22}) = 0.161, \varphi(b_{22}, b_{32}) = -0.354, \varphi(b_{32}, b_{12}) = 0.173, \\ \varphi(b_{12}, b_{23}) = 0.000, \varphi(b_{23}, b_{33}) = 0.161, \varphi(b_{33}, b_{13}) = -0.727, \\ \varphi(b_{21}, b_{11}) = -0.406, \varphi(b_{31}, b_{21}) = -0.473, \varphi(b_{11}, b_{31}) = 0.214, \\ \varphi(b_{22}, b_{12}) = -0.402, \varphi(b_{32}, b_{22}) = 0.132, \varphi(b_{12}, b_{32}) = -0.432, \\ \varphi(b_{23}, b_{13}) = 0.000, \varphi(b_{33}, b_{23}) = -0.645, \varphi(b_{13}, b_{33}) = 0.182. \end{gathered} \quad (20)$$

The dominance degree of B_i over every alternative B_t is

$$\begin{gathered} \delta(B_1, B_2) = 0.303, \delta(B_2, B_3) = -0.665, \delta(B_3, B_1) = -1.167, \\ \delta(B_2, B_1) = -0.808, \delta(B_3, B_2) = -0.348, \delta(B_1, B_3) = -0.035. \end{gathered} \quad (21)$$

Finally,

$$\xi_1 = \frac{0.267 - (-1.515)}{0.267 - (-1.515)} = 1, \ \xi_2 = \frac{(-1.473) - (-1.515)}{0.267 - (-1.515)} = 0.024, \quad \xi_3 = \frac{(-1.515) - (-1.515)}{0.267 - (-1.515)} = 0, \quad (22)$$

implying that ξ_1 is the best choose.

B. Example 2

In this subsection, we consider that decision information is given by the single-valued neutrosophic numbers. Similarly, A VC firm wants to choose an innovating enterprise to invest, where $A = (A_1, A_2, A_3)$ are three enterprises and $G = (G_1, G_2, G_3)$ are three attributes of each. The enterprises and attributes in this example are the same with example 1. First attribute is cost factor, and the next two are efficient factors. The weight of attribute is $w = (0.35, 0.4, 0.25)$. For convenience, we take the average of maximum and minimum values of the truth-membership function $T_{A(x)}$, indeterminacy-membership function $I_{A(x)}$

and falsity-membership function $F_{A(x)}$ in the last example as reference. So

$$A = \begin{bmatrix} \langle[0.25][0.35][0.35]\rangle & \langle[0.45][0.45][0.35]\rangle & \langle[0.55][0.35][0.30]\rangle \\ \langle[0.45][0.25][0.35]\rangle & \langle[0.50][0.20][0.30]\rangle & \langle[0.80][0.25][0.45]\rangle \\ \langle[0.75][0.15][0.25]\rangle & \langle[0.65][0.30][0.25]\rangle & \langle[0.65][0.35][0.85]\rangle \end{bmatrix}, \quad (23)$$

the normalized decision matric B is

$$B = \begin{bmatrix} \langle[0.35][0.65][0.25]\rangle & \langle[0.45][0.45][0.35]\rangle & \langle[0.55][0.35][0.30]\rangle \\ \langle[0.35][0.75][0.45]\rangle & \langle[0.50][0.20][0.30]\rangle & \langle[0.80][0.25][0.45]\rangle \\ \langle[0.25][0.85][0.75]\rangle & \langle[0.65][0.30][0.25]\rangle & \langle[0.65][0.35][0.85]\rangle \end{bmatrix}. \quad (24)$$

Next, according to the equation (14), (15),

$$\begin{gathered} E(b_{11}) = 0.917, E(b_{12}) = 0.850, E(b_{13}) = 0.867, \\ E(b_{21}) = 0.883, E(b_{22}) = 0.800, E(b_{23}) = 0.867, \\ E(b_{31}) = 0.783, E(b_{32}) = 0.917, E(b_{33}) = 0.717. \end{gathered} \quad (25)$$

$$\begin{gathered} d(b_{11}, b_{21}) = 0.075, d(b_{21}, b_{31}) = 0.111, \\ d(b_{31}, b_{11}) = 0.183, d(b_{12}, b_{22}) = 0.087, \\ d(b_{22}, b_{32}) = 0.069, d(b_{32}, b_{12}) = 0.097, \\ d(b_{13}, b_{23}) = 0.103, d(b_{23}, b_{33}) = 0.146, \\ d(b_{33}, b_{13}) = 0.186. \end{gathered} \quad (26)$$

When $\theta = 1$, the dominance degree of B_i over every alternative B_t under attribute G_j is

$$\begin{gathered} \varphi(b_{11}, b_{21}) = 0.162, \varphi(b_{21}, b_{31}) = 0.197, \\ \varphi(b_{31}, b_{11}) = -0.772, \varphi(b_{12}, b_{22}) = 0.186, \\ \varphi(b_{22}, b_{32}) = -0.414, \varphi(b_{32}, b_{12}) = 0.197, \\ \varphi(b_{12}, b_{23}) = 0.000, \varphi(b_{23}, b_{33}) = 0.191, \\ \varphi(b_{33}, b_{13}) = -0.863, \varphi(b_{21}, b_{11}) = -0.461, \\ \varphi(b_{31}, b_{21}) = -0.562, \varphi(b_{11}, b_{31}) = 0.253, \\ \varphi(b_{22}, b_{12}) = -0.465, \varphi(b_{32}, b_{22}) = 0.166, \\ \varphi(b_{12}, b_{32}) = -0.493, \varphi(b_{23}, b_{13}) = 0.000, \\ \varphi(b_{33}, b_{23}) = -0.765, \varphi(b_{13}, b_{33}) = 0.216. \end{gathered} \quad (27)$$

The dominance degree of B_i over every alternative B_t is

$$\begin{gathered} \delta(B_1, B_2) = 0.348, \delta(B_2, B_3) = -0.027, \\ \delta(B_3, B_1) = -1.388, \delta(B_2, B_1) = -0.927, \\ \delta(B_3, B_2) = -1.161, \delta(B_1, B_3) = -0.024. \end{gathered} \quad (28)$$

Finally,

$$\begin{aligned} \xi_1 &= \frac{0.323 - (-2.550)}{0.323 - (-2.550)} = 1, \\ \xi_2 &= \frac{(-0.953-) - (-2.550)}{0.323 - (-2.550)} = 0.556, \\ \xi_3 &= \frac{(-2.550) - (-2.550)}{0.323 - (-2.550)} = 0, \end{aligned} \quad (29)$$

implying that ξ_1 is the best choose.

VII. Conclusion

Considering that objective things contain complex and uncertain, and human thinking is fuzzy, it is necessary to depict fuzziness accurately for solving the multiple attribute decision-making in venture capital. Based on TODIM method, this paper firstly proposes the weight information is known, and secondly studies the problem that whether the decision information is given by interval neutrosophic number or single-valued neutrosophic number, then uses the standard Euclidean distance and the expectation of neutrosophic number to measure the correlation between attributes, and finally defines a novel TODIM method to sort these enterprises and choose the best for investment. In the future, this proposed method will be further developed in venture capital to solve the MADM problem.

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