An Adaptive Robust Guidance Law Designed by Non-smooth Continuous Control and Super-twisting Algorithm

Shuai Xu, Min Gao, Yi Wang, Dan Fang, Saisai Jin

***Abstract*—On the basis of non-smooth continuous control and super-twisting algorithm, an adaptive robust finite-time stable guidance law with attack angle constraint is proposed. For the nominal closed-loop system, a continuous non-smooth guidance law is presented on account of non-smooth control. By improving the fractional power function in the feedback function to speed up the convergence process, the law has a shorter convergence time. Then, super-twisting algorithm is applied to reduce the control command chattering. Besides, the parameters are adjusted by adaptive law to avoid the "excessive estimation" problem of the parameter upper limits. Simulation results under disturbance conditions indicate that the designed guidance law can effectively overcome influence of random disturbances, and obtain higher guidance precision than the law with traditional sliding mode.**

***Index Terms*—guidance law, non-smooth control, super-twisting algorithm, parameter adaptation, finite-time stability**

I. INTRODUCTION

FOR missiles with short range, the guidance time is usually short. To improve guidance precision and damage effectiveness, the LOS angle rate and LOS angle need to approach the desired values as quickly as possible before the seeker enters into dead zone. That is helpful to realize trajectory shaping in finite time and reduce the terminal aerodynamic drag and overload demand. Therefore, it is essential to design a guidance law which can reach the target above. Generally, the finite-time convergence can be achieved by discontinuous state feedback method. Via designing nonlinear sliding mode surface, within a limited time, the guidance law could drive system states on sliding mode surface to reach original point [1]-[2]. However, due to the requirement of anti-interference, the sliding mode control law includes a discontinuous switching term. The term brings high speed switching to control value, which easily generate chattering problem. Usually, continuous processing is applied to reduce the chattering, while, it will bring the system performance down at the same time. Besides, although the smooth state feedback method can obtain smooth response of closed-loop system, its performances of anti-interference and convergence have been proved to be difficult to meet the requirements. An effect way is the continuous non-smooth control. Other than common sliding mode control, the continuous non-smooth control is continuous relative to the state variable [3]-[4]. The continuous non-smooth finite time feedback controller can output continuous control quantity to realize the finite-time convergence of system states and weaken the chattering. On the basis of the analysis above, on account of the finite time homogeneity theory and Lyapunov stability theory, a finite-time convergence guidance law with attack angle constraint is proposed by continuous finite-time control technique in this paper.

II. PREPARATIONS

Consider the system as follow

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}),\ \ \boldsymbol{f}(0) = 0 \tag{1}$$

where $\boldsymbol{x} \in \boldsymbol{R}^n$, $\boldsymbol{x}(0) = \boldsymbol{x}_0$, $\boldsymbol{f} : D \to \boldsymbol{R}^n$ is continuous with respect to $\boldsymbol{x}$ in an open neighborhood D containing the original point $\boldsymbol{x} = 0$.

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Definition 1[5]: $f(x)=[f_1(x),\cdots,f_n(x)]^{\mathrm{T}}:\boldsymbol{R}^n\to\boldsymbol{R}^n$ is a continuous vector function. If $\forall\varepsilon>0$, there is $(r_1,r_2,\cdots r_n)\in\boldsymbol{R}^n$, $r_i>0\ (i=1,2,\cdots n)$, so as to

$$f_i(\varepsilon^{r_1}x_1,\varepsilon^{r_2}x_2,\cdots\varepsilon^{r_n}x_n)=\varepsilon^{k+r_i}f_i(x),\ i=1,2,\cdots n \tag{2}$$

where $k>-\min\{r_i,i=1,2,\cdots n\}$. Then $f(x)$ is considered to possess the homogeneity k with respect to the dilation $(r_1,r_2,\cdots r_n)$. If the $f(x)$ is homogeneous, the system in (1) will be a homogeneous system.

Lemma 1[6]: If the system in (1) with local homogeneity $k<0$ is global asymptotic stable, the system will be global finite-time stable.

III. Non-smooth Finite-time Convergence Sliding Mode Guidance Law

A. Missile-target Relative Motion Model

The skid-to-turn missile has axially symmetrical shape roll stabilized, thus the three channels can be decomposed into vertical plane motion and lateral plane motion. In flights at little angles of attack and side slip angles, the design method of pitch plane motion is similar to the yaw plane motion, therefore, this paper takes missile-target motion process in pitch plane as an example to analyze, which is illustrated in Fig.1.

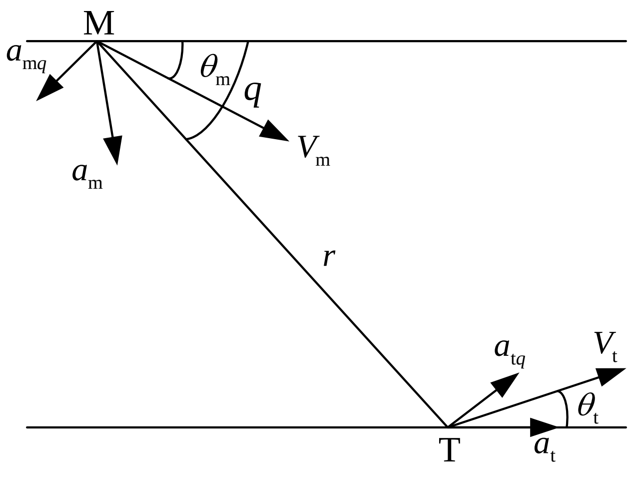

Fig.1 Missile-target relative motion in pitch plane

In Fig.1, M is the center of mass in missile and T is the target. Respectively, the missile-target relative distance and its change rate is represented by r and $\dot{r}$. V_{t} and V_{m} respect the speeds of target and missile respectively. The LOS angle is represented as q, and its derivative is LOS angle rate $\dot{q}$. The flight-path angles of target and missile are written by θ_{m} and θ_{t} respectively. $a_{\mathrm{m}q}$ and $a_{\mathrm{t}q}$ are respectively used to represent the acceleration components in normal direction with respect to LOS of missile and target. The missile-target relative motion equation is given by

$$\ddot{q}=-\frac{2\dot{r}\dot{q}}{r}-\frac{1}{r}a_{\mathrm{m}}+\frac{1}{r}a_{\mathrm{t}} \tag{3}$$

Neglecting the missile autopilot dynamics and regarding the missile body as an ideal link, the overload instruction to be designed can be set as $a_{\mathrm{c}}=a_{\mathrm{m}}$. Assume the desired attack angle constraint is q_{d}, define $x_1=q-q_{\mathrm{d}}$ and $x_2=\dot{q}$ as the state variables, a variable coefficient nonhomogeneous differential equation set is obtained as follow

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-\dfrac{2\dot{r}}{r}x_2-\dfrac{1}{r}a_{\mathrm{c}}+\dfrac{1}{r}a_{\mathrm{t}}\end{cases} \tag{4}$$

Considering the uncertainties caused by random factors, such as the measurement error of the LOS angle rate, parameter perturbation and other factors, the formula (4) is further expressed as

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=f(\bullet)+b(\bullet)a_{\mathrm{c}}+\Delta\end{cases} \tag{5}$$

where $f(\bullet)=-2x_2\dot{r}/r$ and $b(\bullet)=-1/r\neq0$ are known terms, $\Delta=\Delta f(\bullet)+\Delta b(\bullet)a_{\mathrm{c}}$ is the total uncertainty in system, $\Delta f(\bullet)$ is the complex uncertainty caused by target maneuvering acceleration a_{t}, disturbance, parameter measurement error and other factors, besides, $\Delta b(\bullet)$ is the coefficient perturbation.

The control target is to give out the control quantity a_{c} to make $x_1(t_f)$ and $x_2(t_f)$ to be zero in a finite time of t_f. In other words, it means that, before the seeker enters into dead zone and the flight control ends, the control quantity a_{c} designed by finite-time control technology can enforce $\dot{q}$ to be fully close to zero and q to be fully close to the desired angle q_{d}. During the above process, the terminal trajectory should be straight to reduce the overload demand and attack angle[7].

B. Continuous Finite-time Guidance Law Design

Considering the system in (5), the control command a_{c} is given in the form

$$a_{\mathrm{c}}=-2\dot{r}x_2-r(u_{\mathrm{n}}+u_{\mathrm{d}}) \tag{6}$$

where u_{n} is non-smooth nominal control law of double integral system, and u_{d} is the compensation control law for restraining the uncertain term. Substitute (6) into (5), we can get

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = u_n + u_d + \Delta\end{cases} \quad (7)$$

a. Non-smooth Nominal Control Law Design

Neglecting u_d and uncertainty Δ, the formula (7) can be converted to nominal double integral system as follow

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = u_n\end{cases} \quad (8)$$

By improving nonlinear feedback function, a continuous non-smooth control law u_n is given by

$$u_n = -k_1 S(\alpha_1,\lambda_1,x_1,\varepsilon_1) - k_2 S(\alpha_2,\lambda_2,x_2,\varepsilon_2) \quad (9)$$

where $S(\alpha_i,\lambda_i,x_i,\varepsilon_i)$ is expressed as

$$S(\alpha_i,\lambda_i,x_i,\varepsilon_i) = \begin{cases}|x_i|^{\alpha_i}\,\mathrm{sgn}(x_i) & |x_i| \le \varepsilon_i \\ \varepsilon_i^{\alpha_i-\lambda_i}|x_i|^{\lambda_i}\,\mathrm{sgn}(x_i) & |x_i| > \varepsilon_i\end{cases} \quad (10)$$

in which $0<\alpha_2<1$, $\alpha_1=\alpha_2/(2-\alpha_2)$, $\lambda_i \ge 1$, $k_i>0$, $\varepsilon_i>0$, $i=1,2$.

$S(\alpha,\lambda,x,\varepsilon)$ is composed by nonlinear functions $\varepsilon^{\alpha-\lambda}|x|^{\lambda}\,\mathrm{sgn}(x)$ and $|x|^{\alpha}\,\mathrm{sgn}(x)$, which can switchover at $x=\varepsilon$. When system states are away from the balance point, ε can be adjusted to adapt to different situations and quicken the approaching speed. When $|x|\ge\varepsilon$, the nonlinear function $\varepsilon^{\alpha-\lambda}|x|^{\lambda}\,\mathrm{sgn}(x)$ is adopted to speed up the velocity approaching to the balance point. When $|x|<\varepsilon$, for the sake of achieving finite-time convergence, the nonlinear function $|x|^{\alpha}\,\mathrm{sgn}(x)$ is used to make the system homogeneity to be negative. The contrast between $S(\alpha,\lambda,x,\varepsilon)$ and $|x|^{\alpha}\,\mathrm{sgn}(x)$ can be seen in Fig.2.

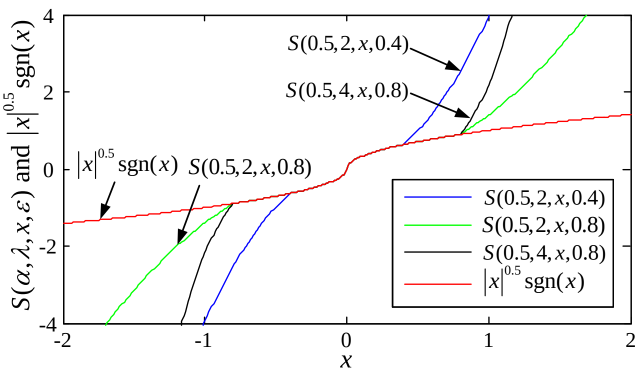

Fig.2 Comparison between $S(\alpha,\lambda,x,\varepsilon)$ and $|x|^{\alpha}\,\mathrm{sgn}(x)$

For solving the finite-time stabilization problem of system in (8), a control law was proposed in [8] as follow

$$u_n = -k_1|x_1|^{\gamma_1}\,\mathrm{sgn}(x_i) - k_2|x_2|^{\gamma_2}\,\mathrm{sgn}(x_2) \quad (11)$$

Compared with the control law in (11), the improved control law in (9) allows system states to rapidly converge far from the balance point, and overcome the defect of slow convergence in homogeneous system.

Theorem: For the system in (8), the control law in (9) can ensure that the control commands are continuous and non-smooth, and the closed-loop system can globally reach the origin within a limited time.

Proof: Constructing positive definite and continuous differentiable Lyapunov function in the form as bellow

$$V(x_1,x_2) = \frac{x_2^2}{2} + \int_0^{x_1} k_1 S(\alpha_1,\lambda_1,\tau,\varepsilon_1)d\tau \quad (12)$$

Calculating the time derivation of $V(x_1,x_2)$ along the system in (8), and considering u_n simultaneously, we get

$$\begin{aligned}\dot{V}(x_1,x_2) &= x_2\dot{x}_2 + k_1 S(\alpha_1,\lambda_1,x_1,\varepsilon_1)x_2 \\ &= -k_2 S(\alpha_2,\lambda_2,x_2,\varepsilon_2)\end{aligned} \quad (13)$$

By the LaSalle invariant set principle[9], the set $\{(x_1,x_2):\dot{V}(x_1,x_2)=0\}$ contains the coordinate axis $x_2=0$, but the unique invariant set on $x_2=0$ is $x_1=x_2=0$. Therefore, x_1 and x_2 could converge into $\Omega=\{(x_1,x_2)\,\|x_1|\le\varepsilon_1 \cap |x_2|\le\varepsilon_2\}$ within a finite time. When the system states $(x_1,x_2)\in\Omega$, there is

$$\begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = -k_1|x_1|^{\alpha_1}\,\mathrm{sgn}(x_1) - k_2|x_2|^{\alpha_2}\,\mathrm{sgn}(x_2)\end{cases} \quad (14)$$

The vector field of closed-loop system is

$$\begin{cases}f_1(x_1,x_2) = x_2 \\ f_2(x_1,x_2) = -k_1|x_1|^{\alpha_1}\,\mathrm{sgn}(x_1) - k_2|x_2|^{\alpha_2}\,\mathrm{sgn}(x_2)\end{cases} \quad (15)$$

Let homogeneous expansion operator to be $\Delta_k^r(x_1,x_2)=(x_1^{r_1},x_2^{r_2})$, where $r_1=1/\alpha_1$ and $r_2=1/\alpha_2$, we can get

$$\begin{aligned}f_1(k^{r_1}x_1,k^{r_2}x_2) &= k^{r_2}f_1(x_1,x_2) = k^{r_1+(r_2-r_1)}f_1(x_1,x_2) \\ f_2(k^{r_1}x_1,k^{r_2}x_2) &= kf_2(x_1,x_2) = k^{r_2+(1-r_2)}f_2(x_1,x_2)\end{aligned} \quad (16)$$

In combination of $\alpha_1=\alpha_2/(2-\alpha_2)$, there is $r_1-r_2=(\alpha_2-1)/\alpha_2=1-r_2$. Hence, the closed-loop system is homogeneous. Moreover, due to $0<\alpha_2<1$, the system degree of homogeneity is $m=(\alpha_2-1)/\alpha_2<0$. From the lemma 1, it is known that the system is global finite time stable.

b. Uncertainty Compensation Control Law

Considering u_{d} and uncertainty Δ, so as to ensure system states to be finite-time convergent, the control objective is to design a suitable u_{d} to restrain the influence of uncertainty Δ.

Let $e=\dot{x}_2-u_{\mathrm{n}}$, to avoid the higher order derivative with respect to the variable x_2, an auxiliary sliding mode surface s is proposed as follow

$$\begin{cases} s=\int edt=x_2+y \\ \dot{y}=-u_{\mathrm{n}} \end{cases} \tag{17}$$

The control target is to design the u_{d} to overcome the uncertainty Δ and yield $\dot{s}\to 0$. The derivative of s is $\dot{s}=u_d+\Delta$, and its partial derivative with respect to u_{d} is $\partial\dot{s}/\partial u_d=1\neq 0$. According to the definition of relative order, we can see that the relative order of system in (17) with respect to s is one. Adopting first-order sliding mode control will bring in discontinuous switching item and high frequency chattering, while, in second-order sliding mode control, symbol function of discontinuous switching term can be included in an integral link so that the control chattering can be effectively weakened.

The super-twisting algorithm is suitable for second-order system with relative order of one. It only needs the information of sliding surface s with no need for $\dot{s}$, which simplifies the guidance law design. In view of above advantages, the robust control law is designed with super-twisting algorithm [10].

Taking second-order derivative of s with respect to time, there is $\ddot{s}=\dot{u}_{\mathrm{d}}+\dot{\Delta}$. On account of the physical limit of energy and response speed in terminal guidance process, we can assume the uncertainty term Δ is globally boundary and differentiable, namely, there is an unknown normal number L_Δ satisfying $|\dot{\Delta}|<L_\Delta$.

Improving the super-twisting algorithm with a parameter adaptive law as follow

$$\begin{cases} u_{\mathrm{d}}=-\lambda|s|^{1/2}\operatorname{sgn}(s)+u_{\mathrm{d1}} \\ \dot{u}_{\mathrm{d1}}=-w\operatorname{sgn}(s) \end{cases} \tag{18}$$

$$\dot{\lambda}=\begin{cases} \rho_1|s|^{\alpha}\operatorname{sgn}(|s|-\varepsilon) & \lambda>\mu \\ \rho_2|s| & \lambda\le\mu \end{cases},\ \lambda(0)>0 \tag{19}$$

$$w<0.2\lambda \tag{20}$$

where $\lambda>0$, $\rho_1>0$, $\rho_2>0$, $\varepsilon>0$, $\mu>0$ and $0<\alpha<1$.

When $|s|<\varepsilon$, $\operatorname{sgn}(|s|-\varepsilon)$ is negative so that $\dot{\lambda}$ is negative, which make λ and w reduce gradually until $\lambda\le\mu$. If λ and w is not enough to restrain the target maneuvering, $|s|$ will become larger, and then λ and w become larger, which will drive $|s|$ converge into the interval $|s|<\varepsilon$. The value μ can guarantee that λ is always positive. The adaptive adjustment function $\operatorname{sgn}(|s|-\varepsilon)$ can ensure the global boundness of adaptive parameters λ and w. By this way, the problem about overestimating the upper bound of adaptive parameters, which is common for traditional adaptive sliding mode control [11] , can be solved well.

IV. SIMULATION ANALYSIS

So as to inspect the guidance law designed above, this section conducts mathematical simulation for the terminal guidance process. The origin values of various parameters used in simulation are chosen to be X_{m0}=0, Y_{m0}=200m, V_{m}=110m/s, θ_{m0}=5°, X_{t0}=1200m, Y_{t0}=0, V_{t0}=2m/s, θ_{t}=0°. q_{d} is set as -35°. For verifying performance in disturbance environment, we add white noise and measurement error to the r and $\dot{r}$ with ranges less than ±5% of their values, namely, there are $\Delta r\le|5\%|r$ and $\Delta\dot{r}\le|5\%|\dot{r}$. The design parameters of guidance law are carefully selected as $\alpha_1=7/13$, $\alpha_2=0.7$, $k_1=0.2$, $k_2=0.6$, $\lambda_1=\lambda_2=4$, $\varepsilon_1=\varepsilon_2=0.5$, $\rho_1=2$, $\rho_2=0.5$, $\alpha=0.1$, $\varepsilon=0.0001$, $\mu=1$ and $\lambda(0)=2$.

In combination with the requirements of design indexes and actual situations on the battlefield, the ground moving target cannot make complex motion form because of the battlefield environment limitation, so we set up three kinds of typical situations suitable in the mathematical simulation: (1) The target approaches launch point with an uniform speed of -15m/s; (2) The target is accelerated from 2m/s to 15m/s with a constant acceleration of 4m/s^2, and then it moves at a constant speed; (3) The target is accelerated from 0m/s to 25m/s with a varying acceleration $a_{tx}=4|\sin(t)|$ m/s^2, and then it moves at a constant speed. Simulation results are given as data in Table Ⅰ and curves from Fig.3 to Fig.8. In Table Ⅰ, t_{f} is the end time of guidance process.

TABLE Ⅰ
$R(t_{\mathrm{f}})$ AND $q(t_{\mathrm{f}})$ UNDER THERE TARGET MOTION SITUATIONS

Motion Situation	$R(t_{\mathrm{f}})$/m	$q(t_{\mathrm{f}})$/°
Situation 1	0.0870	-35.0065
Situation 2	0.0478	-34.9996
Situation 3	0.0962	-34.9742

As shown in Table Ⅰ, the guidance law adapts well to different target motion situations. In situation 1, because the target is approaching launch point, the missile-target relative velocity is greater, which bring out greater miss distance. In situation 3, terminal LOS angle error and miss distance are both maximal due to the varying acceleration of the target. Even so, the miss distances in three situations are all less than 0.1m. Besides, the LOS angle is very close to the desired value of -35°, and the angle deviation is less than 0.1°.

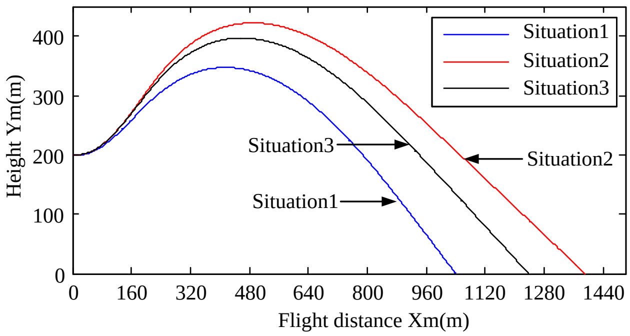

Fig.3 Curves of missile trajectories

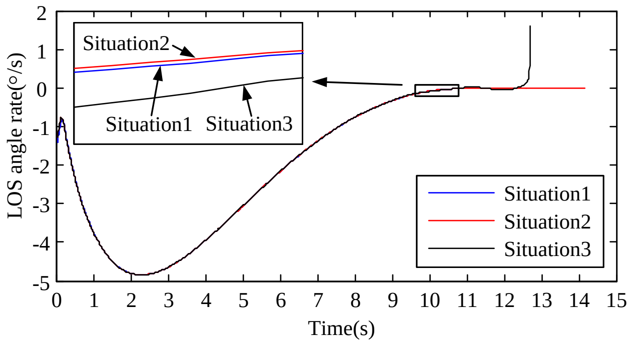

Fig.4 Curves of missile-target LOS angle change rates

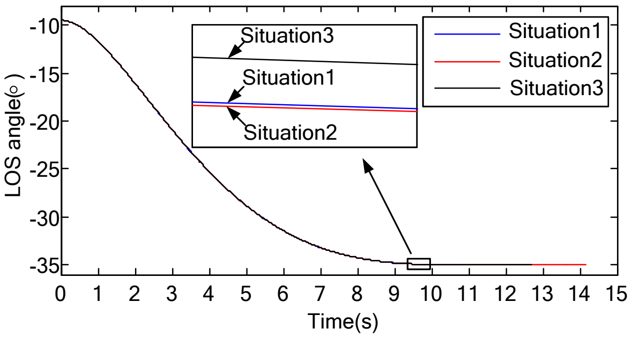

Fig.5 Curves of missile-target LOS angles

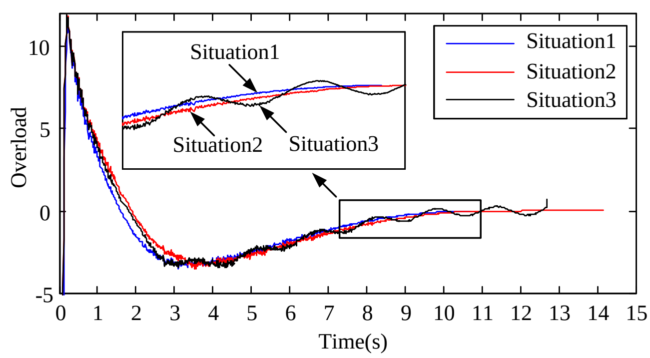

Fig.6 Curves of missile overloads

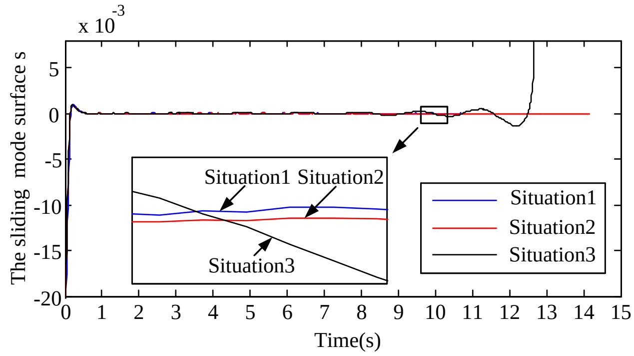

Fig.7 Curves of sliding mode surfaces

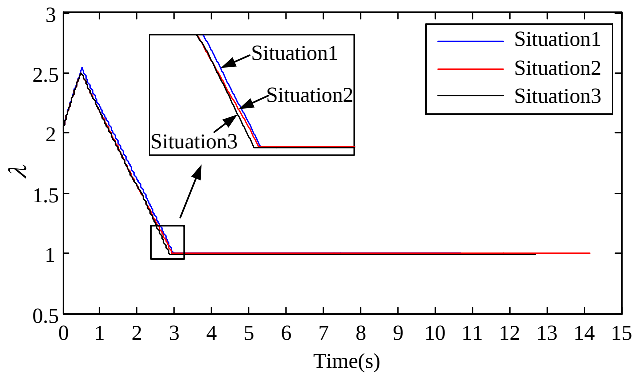

Fig.8 Adaptive parameter λ

From Fig.3 to Fig.8, we can see that in three situations, although the curves of missile trajectories are quite different, the other curves of guidance law indexes basically coincide, which indicates that missiles with the same set of control parameters can attack targets in different motion forms. In addition, in order to obtain a larger terminal LOS angle, the trajectory height is high. The missile terminal trajectories are relatively flat after the LOS angle rate approaches zero and LOS angle gets close to the desired value. Thus, the demands for overload are relatively small, which is beneficial to flight stability and guidance accuracy of missile. In Fig. 4 and Fig.5, the LOS angle rate and LOS angle both achieve the desired effect within a finite time. The influence of measurement errors is shown in Fig.6, in which, we can find that the super-twisting algorithm suppresses the disturbance to a certain extent and guarantees the guidance accuracy. In Fig.7, the super-twisting algorithm drives the auxiliary sliding surface s to converge to zero very quickly, which ensures the implementation of non-smooth control law. As can be seen in Fig.8, with the sliding surface s undulating at initial stage, the adaptive parameter λ rapidly increases to control s to be less than ε. Then, along with the decrease of s, λ decreases and stabilizes at the threshold denoted by μ, which is able to reduce the control quantity.

Next, a guidance law with non-singular terminal sliding mode in [12] is selected to compare with the guidance law proposed above. The guidance law in [12] is shown as

$$s=(q-q_{\mathrm{d}})+\beta\dot{q}^{\eta} \tag{21}$$

$$u=\frac{1}{\cos(q-\theta_{\mathrm{m}})}\left(\frac{r}{\eta\beta}\dot{q}^{2-\eta}+2\dot{r}\dot{q}\right)+\frac{M}{\mathrm{sgn}(\cos(q-\theta_{\mathrm{m}}))}\mathrm{sgmf}(s) \tag{22}$$

where $\beta=10$, $\eta=5/3$ and $M=80$. sgmf function in (23) is adopted to replace the sgn function

$$\mathrm{sgmf}(s)=\begin{cases}2\left(\dfrac{1}{1+e^{-as}}-\dfrac{1}{2}\right) & |s|\le\varepsilon_s\\ \mathrm{sgn}(s) & |s|>\varepsilon_s\end{cases} \tag{23}$$

where $\varepsilon_s=0.6$ is the boundary layer, $a>0$ and its value is inversely proportional to ε_s. In the simulation, a is set to be $8/\varepsilon_s$.

For ease of differentiation, the guidance law in [12] is denoted as "NTSM", and the guidance law derived in this paper is called "NSST". After adding the same disturbance signals and carefully adjusting the parameters, simulation results under situation 2 can be seen from Fig.9 to Fig.12.

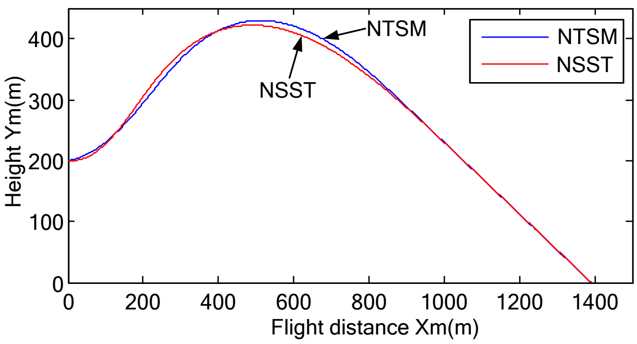

Fig.9 Curves of missile trajectories

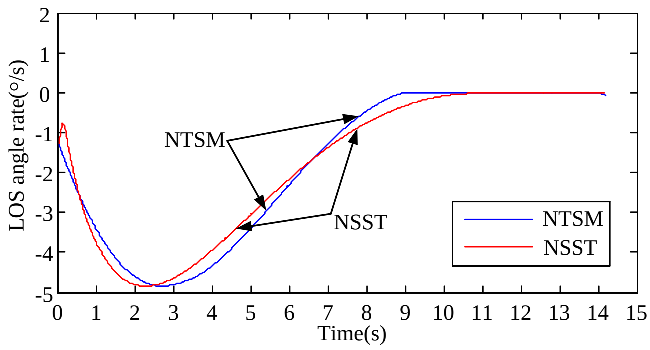

Fig.10 Curves of missile-target LOS angle rates

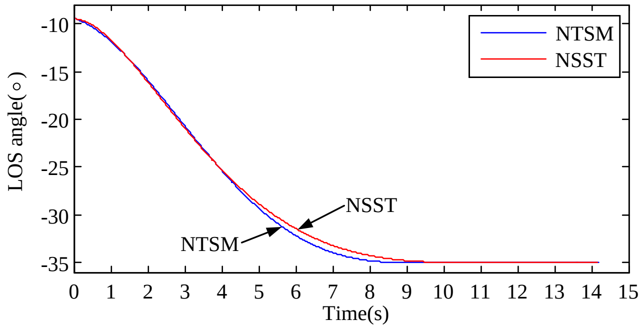

Fig.11 Curves of missile-target LOS angles

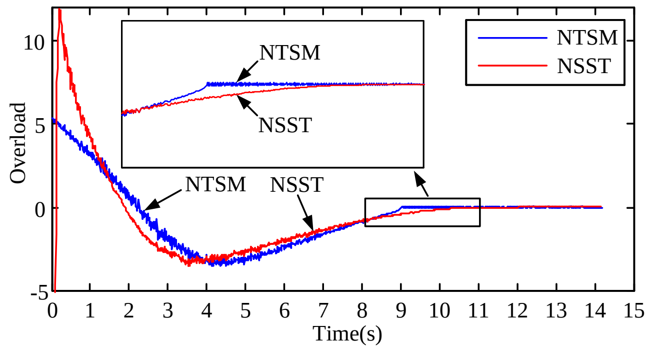

Fig.12 Curves of missile overloads

As can be observed from Fig. 9 to Fig.12, the two guidance laws differ in the ascent section of trajectory, and have similar flight trajectories in the descending section. The flight time of the missile using NTSM is 14.17s, closely, this time of the missile using NSST is 14.19s. Both guidance laws can enforce the LOS angles and LOS angle rates to approach zero in finite time to satisfy the attack angle constraint. In the meanwhile, the terminal overload requirements of these two guidance laws are both small and close to zero in the end, which is beneficial to improve the target attack accuracy. Clearly, the convergence speeds of LOS angle rate, LOS angle, and overload of NSST are all slightly slower than NTSM. However, for the process of missile attacking target, the miss distance and attack angle are the two most critical indicators that directly determine the attack effect. The miss distance and terminal LOS angle of NTSM are 0.3143m and -34.9967°, respectively, greater than 0.0478m and -39.999° of NSST. The analysis above indicates that the performance of designed guidance law yields a better attack accuracy, a smaller attack angle error, and a smoother overload demand. In addition, the overload curves in Fig.12 shows that the chattering of NSST control quantity is significantly weaker than that of NTSM, which is the main reason why the control effect of NSST is better.

V. CONCLUSION

Based on the non-smooth continuous control and super-twisting algorithm, considering effects of complex uncertain term caused by various random factors, a finite-time convergence robust guidance law with attack angle constraint is proposed. According to homogeneity theory, it is proved that the nominal system states could be driven to zero by the non-smooth control within a limited time. Simulation results show that, under different target motion situations, the guidance law in this paper can overcome the influence of disturbances and achieve finite-time convergence of LOS angle rate, which ensures that the missile could accurately hit the target with a desired LOS angle.

Otherwise, in the simulation, it is found that in order to achieve the attack angle constraint within a finite time, the missile climbs higher and the trajectory curvature is larger, which requires a better performance for the missile power. At the same time, a large trajectory curvature may cause the seeker to exceed its frame limit and easily lose the target. In addition, the complex form of algorithm presented in this paper brings out many difficulties in adjusting parameters, and improper selection can easily lead to a divergence of missile trajectories, so it is necessary to properly set guidance parameters.

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