Analytical Solutions of a Continuous Arithmetic Asian Model for Option Pricing using Projected Differential Transform Method

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Abstract—In financial mathematics, the procedure for finding the solutions of the Black-Scholes model defined in terms of Arithmetic Asian Option (AAO) is one of the most tasking issues when considering its corresponding analytical solutions. In this paper, such analytical solution of a continuous arithmetic Asian option is obtained through the application of a proposed semi-analytical approach known as Projected Differential Transform Method (PDTM). The Asian option model in continuous arithmetic form witnesses the application of PDTM for the first time in literature (to the authors’ best of knowledge). The PDTM entails less computational work, even without neglecting the high level of accuracy. The obtained solution is in agreement with those in literature via other solution methods. In terms of recommendation, the proposed solution method will be of great interest for related versions or forms of Asian option pricing models (geometric) likewise other financial nonlinear differential models.

Index Terms— Option pricing, Asian option, Black-Scholes model, Adomian polynomials, Analytical solution

I. INTRODUCTION

Option pricing is a basic aspect of financial fields such as financial mathematics, financial engineering, and financial physics. Asian option is a special form of option contracts. The payoff functions associated with Asian options depend significantly on the average underlying price over some specified period of time usually referred to as the option life time [2, 3]. They are thus, classified as path dependent options since the payoff function depends on the sequence average of the prices of the underlying asset over some specified period of time. This indeed is a different case compared to the usual American and European style of options, where the option payoff function depends on the price of the underlying stock at exercise [4-7]. Asian options are therefore referred to as one of the main forms of exotic options. Asian options are advantageous because they minimize the risk of market influence of the underlying stock at maturity, and also encompass relative cost compared to American or European options [8].

Averages in option are of two forms namely: geometric and arithmetic. Both of these can be structured as either calls or puts. It is noted that the geometric Asian option has a closed form solution, therefore the ease of pricing [9]. The arithmetic type happens to be the most frequently used but it seems challenging for pricing in terms of closed form solution since the average of a class of lognormal random variables (LN RVs) is not log-normally distributed [10]. Providing a simple analytical expression for the valuation of Asian (arithmetical) options has posed a lot of problems in financial mathematics [1].

In a bid to handle this, some researchers have developed and adopted variety of solution techniques: German and Yor [10], consider Asian option price via Laplace transform but for some cases based on the configuration region of the transform. Rogers and Shi [1], consider the problem of valuing the Asian option in two ways by reducing the problem to a two-variable parabolic partial differential equation (PDE) using scaling property, and thereafter provide a lower bound formulae on the basis of some zero-mean Gaussian variable. In [11], Vecer builds his approach on traded account by providing a PDE for Asian option in one-dimensional form. Zhang [12], considers Asian option pricing via a continuously-sampled theory in line with a perturbation technique.

Chen and Lyuu [13], extended the work of Rogers and Shi [1] by including general maturity terms instead of one year with regard to the lower bound pricing formulae. Elshegmani, Ahmad and Zakaria [14] using Fourier Transform, present a simple solution for the Asian option PDE proposed by Rogers and Shi [1]. Kumar, Waikos and Chakrabarty [15], obtain a standard partial differential equation for the price of arithmetic average strike Asian call option via the derivation of a Crank-Nicolson implicit equation for the price of arithmetic average strike Asian call option. Rogers and Shi [1] by including general maturity terms instead of one year with regard to the lower bound pricing formulae. Elshegmani, Ahmad and Zakaria [14] using Fourier Transform, present a simple solution for the Asian option PDE proposed by Rogers and Shi [1]. Kumar, Waikos and Chakrabarty [15], obtain a standard partial differential equation for the price of arithmetic average strike Asian call option via the derivation of a Crank-Nicolson implicit approach and a higher order compact finite difference scheme for the considered pricing problem at various values of risk-free and volatility rates. Zhang, Yu, and Wang [16], study the numerical solution for the Delta of Asian arithmetic option and provide a simple, fast and reliable solution on the basis of Monte Carlo Simulation (MCS). Fadugba [17], applies the Mellin transform method for analytical solution of geometric Asian option and obtain a closed form solution for the continuous geometric version of the Asian option. Elshegmani, Ahmad [18], obtain a closed form solution for a continuous arithmetic Asian (CAA) option via PDE by Mellin transform. Recently, Elshegmani and Ahmad [2],
apply the PDE approach coupled with Laplace transform as an efficient method for pricing an arithmetic Asian option of continuous type. They [2] do this by transforming the PDE of the Asian option from three dimensional-form to two dimensional ordinary differential equation (ODE).

In this paper, PDTM which is an improved form of the standard differential transform method (DTM) is basically applied to a CAA option pricing model for analytical solutions.

In terms of structure, in the rest sections of the paper are organised as follows: we have a concise note as regards Asian option pricing model in section 2; section 3 is on PDTM- the proposed method of solution; applications and examples are considered in section 4 while section 5 is on concluding remarks.

II. ASIAN OPTION PRICING MODEL VIA THE TWO-DIMENSIONAL BLACK-SCHOLES EQUATION

Suppose the price of the stock, \( S(t) \) follows a geometric (exponential) Brownian Motion (BM) governed by the stochastic differential (dynamical) equation of the form:

\[
dS(t) = S(t)(r dt + \sigma dW(t)), \quad t \in \mathbb{R}^+
\]

where \( r \) is a drift term (average rate of growth), \( \sigma \) is a volatility coefficient, \( 0 \leq t \leq T \), and \( W(t) \) a BM. Then the payoff function for an Asian option associated with an arithmetic average strike (AAS) [19-21] is defined and denoted by:

\[
\Lambda(T) = \left( -\frac{1}{T} \int_0^T S(\zeta) \, d\zeta + S(T), \, 0 \right). \tag{2.2}
\]

The price of the option at \( 0 \leq t \leq T \) is an equivalent martingale pricing formula (risk-neutral form) defined and denoted by [19] as:

\[
\Lambda(t) = E\left( e^{-r(T-t)} \Lambda(T) \mid F_t \right), \tag{2.3}
\]

where \( E(\cdot) \) and \( F_t \) represent expected-value operator and filtration respectively.

Now let us consider \( \Lambda(S, I, t) \) as the price function and \( I = I(t) \) the path dependent variable be defined as:

\[
I(t) = \int_0^t h(S(\zeta), \zeta) \, d\zeta \tag{2.4}
\]

where \( h(\cdot) \) is a specific function for any possible path-dependent option under consideration. It is obvious that \( \Lambda = \Lambda(S, I, t) \) is a function of three variables since \( I = I(t) \) does not depend on \( S \) (the current asset price).

Hence,

\[
\frac{dI}{dt} = h(S(t), t). \tag{2.5}
\]

As such, applying the multidimensional Ito’s lemma on \( \Lambda(S, I, t) \) gives:

\[
d\Lambda = \left( \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 \Lambda}{\partial S^2} + rS \frac{\partial \Lambda}{\partial S} + \frac{\partial \Lambda}{\partial t} \right) dt + h(S, t) \frac{\partial \Lambda}{\partial t} \, dW \tag{2.6}
\]

Suppose we construct a delta-hedge portfolio, \( \Pi \), by longing a contingent claim value, and shorting a delta unit of the asset price, then we have:

\[
\begin{align*}
\Pi &= \Lambda - \Delta S \\
\text{d}\Pi &= \text{d}\Lambda - \Delta \text{d}S.
\end{align*}
\]

So, (2.1) and (2.6) in (2.7) gives:

\[
\begin{align*}
\text{d}\Pi &= \text{d}\Lambda - \Delta \text{d}S \\
&= \left\{ \left( \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 \Lambda}{\partial S^2} + rS \frac{\partial \Lambda}{\partial S} + \frac{\partial \Lambda}{\partial t} + h(S, t) \frac{\partial \Lambda}{\partial t} \right) \right\} dt + \sigma S \frac{\partial \Lambda}{\partial S} \, dW \\
&= \left\{ \left( \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 \Lambda}{\partial S^2} + rS \frac{\partial \Lambda}{\partial S} + \frac{\partial \Lambda}{\partial t} + h(S, t) \frac{\partial \Lambda}{\partial t} \right) \right\} dt + \sigma S \frac{\partial \Lambda}{\partial S} \, dW \\
&= \Lambda\left( rS(t) \, dt + \sigma S(t) \, dW(t) \right).
\end{align*}
\]

We set \( \Delta = \frac{\partial \Lambda}{\partial S} \) in order to eliminate the stochastic term from the portfolio, hence,

\[
\text{d}\Pi = \left( \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 \Lambda}{\partial S^2} + \frac{\partial \Lambda}{\partial t} + h(S, t) \frac{\partial \Lambda}{\partial t} \right) dt. \tag{2.9}
\]

For adopting the notion of no arbitrage opportunities, (it implies that change in the portfolio needs to coincide with the change of the corresponding monetary value as deposit in the bank account for a risk-free interest rate). Thus,

\[
\begin{align*}
\text{d}\Pi &= r \Pi \text{dt} \\
&= r \left( \Lambda - \Delta S \right) \text{dt} \\
&= r \left( \Lambda - \frac{\partial \Lambda}{\partial S} S \right) \text{dt}.
\end{align*}
\]

So, combining (2.9) and (2.10) gives:

\[
\begin{align*}
\left( \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 \Lambda}{\partial S^2} + \frac{\partial \Lambda}{\partial t} + h(S, t) \frac{\partial \Lambda}{\partial t} \right) dt &= r \left( \Lambda - \frac{\partial \Lambda}{\partial S} S \right) dt.
\end{align*}
\]

That implies that:

\[
\left( \frac{\partial \Lambda}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 \Lambda}{\partial S^2} + rS \frac{\partial \Lambda}{\partial S} + h(S, t) \frac{\partial \Lambda}{\partial t} \right) - r \Lambda = 0. \tag{2.12}
\]

In order to consider the Black-Scholes model for Asian option, the payoff \( \Lambda(T) \) is based the trajectory past knowledge (path dependent). Thus, a stochastic process in (2.13) is introduced [19]:

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\[ I(t) = \frac{1}{t^\alpha} S(t) d\omega \quad (2.13) \]

whose first derivative w.r.t. time, \( t \), gives \( h(\cdot) \), and \( I(t) \) denotes the strike price running total. Hence,

\[ \frac{dl}{dt} = S(t) = h(S,t). \quad (2.14) \]

So, using (2.14) in (2.12) gives the associated option price of the Asian call described by the following model:

\[ \left( \frac{\partial \Lambda}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 \Lambda}{\partial S^2} + rS \frac{\partial \Lambda}{\partial S} + S \frac{\partial \Lambda}{\partial t} \right) - r \Lambda = 0 \quad (2.15) \]

solved by \( \Lambda(S,I,t) \) for \( t \in [0,\infty) \) and \( S \in (0,\infty) \). It is obvious that (2.15) is comparable to the classical form of the time-fractional Black-Scholes (BS) option pricing model at \( \alpha = 1 \) but for the averaging term denoted by \( \left( \frac{S \frac{\partial \Lambda}{\partial t}}{\partial \Lambda} \right) \) [22-23]. In most cases, modification may be required leading to the adoption of numerical-analytical or semi-analytical methods [24-29]. Though, the associated volatility in (2.5) can be further viewed from the aspect of transaction cost inclusion, and stochastic point while adopting a simple and a class control variate methods [30-32]. Rehurek [33] considered (2.15) by discussing some numerical techniques as well as their corresponding variations. Recently, analytical solutions and the associated Greek (sensitivities) parameters of a CAA option pricing model were considered [34, 35]. It is obvious that (2.15) will finally yield a problematic issue in terms of computational since it is of (3) dimensional. Therefore, the notion of reduction to a lower level in dimension is needed with aid of transformation variables [2, 36]:

\[
\begin{align*}
\Lambda(S,I,t) &= Sg(t,\psi), \\
\psi S &= k - \frac{I}{T}.
\end{align*}
\]

(2.16)

Hence, (2.5) becomes:

\[
\begin{align*}
\frac{\partial \psi}{\partial t} + \frac{1}{2} \sigma^2 \psi^2 \frac{\partial^2 \psi}{\partial \psi^2} \left( \frac{1}{T} + r\psi \right) \frac{\partial \psi}{\partial \psi} &= 0, \\
g(T,\psi) &= \varphi(\psi).
\end{align*}
\]

(2.17)

It can thus be remarked that (2.17) is now in two dimensional form. The solution of this (2.17) will yield the price of the Asian option via the relation in (2.16).

III. THE DTM AND PROJECTED DTM

A. Analysis of a Two-Dimensional DTM

Here, we make remarks on the preliminaries of two dimensional version of the DTM as follows.

Suppose the two-variable function: \( u(x,y) \) is analytic function at \( (x_s,y_s) \) w.r.t. the Domain, \( D \), so, the differential transform of \( u(x,y) \) is presented as follows:

\[ \bar{u}(k,h) = \left( \frac{1}{k!h!} \right) \frac{\partial^{k+h} u(x,y)}{\partial x^k \partial y^h} \]

while the differential inverse transform of \( \bar{u}(k,h) \) is:

\[ u(x,y) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \bar{u}(k,h)(x-x_s)^k (y-y_s)^h, \]

where the capital and the small letters denote the transformed and the original functions respectively.

B. Projected Differential Transform Method

We present a brief but concise introduction of the proposed method (PDTM) with emphasis on the basic features and properties [37, 38].

Let \( z(\omega,t) \) be considered analytic w.r.t \( (\omega,t) \) in a given domain \( D \), then by the Taylor series expansion of \( z(\omega,t) \), with a preference to some variables, say; \( s_p = t \) rather than all the concerned variables as presumed in the standard DTM. Therefore, the projected DTM of \( z(\omega,t) \) with respect to \( t \) at \( t = t_s \) is defined and denoted by:

\[ Z(\omega,\varepsilon) = \left[ \frac{\partial^s z(\omega,t)}{\partial t^s} \right]_{t=t_s} \quad (3.1) \]

and consequently:

\[ z(\omega,\varepsilon) = \sum_{\varepsilon=0}^{\infty} Z(\omega,\varepsilon)(-t_s + t)^\varepsilon. \quad (3.2) \]

Equation (3.2) is the inverse PDTM of \( Z(\omega,\varepsilon) \) in (3.1).

Note, at \( t = t_s = 0 \), (3.1) helps in realizing \( Z(x,0) \) from the given initial condition(s) \( z(x,0) \), where the capital and the small letters denote the transformed and the original functions respectively.

C. Properties of the proposed method: PDTM

Let \( z(\omega,t) \) be as defined above, then the following hold:

a: \[ z(\omega,t) = \alpha z_s(\omega,t) + \beta z_b(\omega,t) \]

\[ \Rightarrow Z(\omega,\varepsilon) = \alpha Z_s(\omega,\varepsilon) + \beta Z_b(\omega,\varepsilon), \]

b: \[ z(\omega,t) = \beta \frac{\partial^s z_s(\omega,t)}{\partial t^s} \]

\[ \Rightarrow Z(\omega,\varepsilon) = \beta \frac{(\varepsilon + n)!}{\varepsilon!} Z_s(\omega,\varepsilon + n), \]
\[
\begin{align*}
 z(\omega, t) &= \alpha \frac{\partial z}{\partial t}, \\
 &\Rightarrow Z(\omega, \varepsilon) = \alpha \frac{(\varepsilon + 1)!Z(\omega, \varepsilon + 1)}{\varepsilon!}, \\
 &= \alpha (\varepsilon + 1)Z(\omega, \varepsilon + 1)
\end{align*}
\]

\[
\begin{align*}
 z(\omega, t) &= p(\omega) \frac{\partial^n z(\omega, t)}{\partial \omega^n}, \\
 &\Rightarrow Z(\omega, \varepsilon) = p(\omega) \frac{\partial^n Z(\omega, \varepsilon)}{\partial \omega^n}, \\
 z(\omega, t) &= p(\omega)z^2(\omega, t)
\end{align*}
\]

\[
\begin{align*}
 Z(\omega, h) &= p(\omega) \sum_{r=0}^{h} Z_*(\omega, r)Z_*(\omega, h-r), \\
 l(\omega, y) &= \omega^n y^n \\
 &\Rightarrow L(k, h) = \delta(k-n, h-n^*) \\
 &= \delta(k-n)\delta(h-n^*), \\
 \delta(k-n) &= \begin{cases} 1, & \text{if } k = n \\ 0, & \text{if } k \neq n \end{cases}, \\
 \delta(k-n^*) &= \begin{cases} 1, & \text{if } k = n^* \\ 0, & \text{if } k \neq n^* \end{cases}
\]

where

IV. APPLICATIONS / ILLUSTRATIVE EXAMPLES

In this section, analytical solutions the Asian option pricing model are considered based on the PDTM with regard to three cases: I, II and III as follows:

A. Case I

Consider (2.15) via (2.16-2.17) in the following form:

\[
\begin{align*}
 \frac{\partial^2 g}{\partial t^2} &= \frac{1}{T} \frac{\partial G(\psi, h)}{\partial \psi} \\
 g(0, \psi) &= \varphi(\psi)
\end{align*}
\]

(4.1)

Taking the PDTM of (4.1) gives:

\[
G(\psi, h+1) = \frac{1}{h+1} \left\{ \frac{1}{T} + r\psi \right\} \frac{\partial G(\psi, h)}{\partial \psi} - \frac{1}{2} \sigma^2 \psi^2 \frac{\partial^2 G(\psi, h)}{\partial \psi^2}
\]

(4.2)

\[
G(\psi, 0) = \frac{1}{rT} (1-e^{-rT}) - \psi e^{-rT}
\]

(4.3)

Hence, for \( h \geq 0 \), we have:

\[
G(\psi, 1) = \left\{ \frac{1}{T} + r\psi \right\} \frac{\partial G(\psi, 0)}{\partial \psi} - \frac{1}{2} \sigma^2 \psi^2 \frac{\partial^2 G(\psi, 0)}{\partial \psi^2}
\]

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Hence,
\[ g(\psi, t) = \sum_{m=0}^{\infty} G(\psi, m) r^m \]
\[ = \left[ \frac{1}{rT} (1 - e^{-rt}) - \psi e^{-rt} \right] \]
\[ - \left( t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots \right) \left( \frac{1}{T} + ry \right) e^{-rt} \]
\[ = \frac{1}{rT} (-e^{-rt} + 1) - \psi e^{-rt} \]
\[ = \frac{e^{-rt}}{r} \left( -1 + \sum_{i=0}^{\infty} \frac{(rt)^i}{i!} \right) \left( \frac{1}{T} + ry \right) \]
\[ = \frac{1}{rT} (-e^{-(\psi t)} + 1) - \psi e^{-(\psi t)} \]
\[ = \frac{e^{-(\psi t)}}{r} \left( -1 + e^{\psi t} \right) \left( \frac{1}{T} + ry \right) \]
\[ = \frac{1}{rT} (-e^{-(\psi t)} + 1) - \psi e^{-(\psi t)} \], \( \psi \leq 0 \). \( \quad \) (4.6)

But from (2.16),
\[ \psi S = \frac{I}{T} - k. \]

Therefore,
\[ \Lambda(S, I, t) = \left( \frac{S}{T} \right) \left( -e^{-(\psi t)} + 1 \right) - \left( k - \frac{I}{T} \right) e^{-(\psi t)}. \] \( \quad \) (4.7)

Equation (4.7) is the analytical solution of (2.15) corresponding to the CAA option pricing model. This can be validated as follows. From (4.5), we have for \( \Lambda(S, I, t) = \Lambda \):
\[ \frac{\partial \Lambda}{\partial t} = \left[ \frac{-Se^{-(\psi t)}}{T} - r \left( k - \frac{I}{T} \right) e^{-(\psi t)} \right]. \] \( \quad \) (4.8)

Taking the PDTM of (4.12) yields:
\[ G(\psi, h+1) = \frac{1}{h+1} \left( \frac{1}{T} + ry \right) \frac{\partial G(\psi, h)}{\partial \psi} - \frac{1}{2} \sigma^2 \psi^2 \frac{\partial^2 G(\psi, h)}{\partial \psi^2}, \] \( \quad \) (4.13)

Thus, following the approach in Case I, the following are obtained:
\[ G(\psi, 1) = \left( \frac{1}{T} + ry \right) rSe^{-rt}. \]
\[ G(\psi, 2) = \frac{1}{2!} \left( \frac{1}{T} + ry \right) r^2 Se^{-rt}. \]
\[ G(\psi, 3) = \frac{1}{3!} \left( \frac{1}{T} + ry \right) r^3 Se^{-rt}. \]
\[ G(\psi, 4) = \frac{1}{4!}\left(1 + r\psi\right) r^4 Se^{-rT}, \]
\[ G(\psi, 5) = \frac{1}{5!}\left(1 + r\psi\right) r^5 Se^{-rT}, \]
\[ G(\psi, 6) = \frac{1}{6!}\left(1 + r\psi\right) r^6 Se^{-rT}, \]
\[ G(\psi, 7) = \frac{1}{7!}\left(1 + r\psi\right) r^7 Se^{-rT}, \]
\[ G(\psi, 8) = \frac{1}{8!}\left(1 + r\psi\right) r^8 Se^{-rT}, \]
\[ G(\psi, 9) = \frac{1}{9!}\left(1 + r\psi\right) r^9 e^{-rT}, \]
\[ \cdots G(\psi, p) = \frac{1}{p!}\left(1 + r\psi\right) r^p Se^{-rT}, \quad p \in \mathbb{N}. \]

So, we have:

\[
g(\psi, t) = \sum_{m=0}^{\infty} G(\psi, m) t^m \]

\[
\left[ \left(\psi r - \frac{1}{rT}\right) Se^{-rT} + \left(\frac{rt + (rt)^2}{2!} + \frac{(rt)^3}{3!} + \frac{(rt)^4}{4!} + \cdots \left(\frac{1}{rT}\right)^n Se^{-rT} \right) \right]
\[
= \left(\psi r - \frac{1}{rT}\right) Se^{-rT} + \left(-1 + \sum_{n=0}^{\infty} \frac{(rt)^n}{n!} \left(\frac{1}{rT}\right) Se^{-rT} \right)
\]

\[
= \left(\psi r - \frac{1}{rT}\right) + (-1 + e^r) \left(\frac{1}{rT} + r\psi\right) Se^{-rT}. \]

Thus, we have:

\[
\Lambda(S, I, t) = Sg(t, \psi), \]
\[
\psi S = k - \frac{I}{T}. \]
\[
\Lambda = \left(\psi r - \frac{1}{rT}\right) + (-1 + e^r) \left(\frac{1}{rT} + r\psi\right) S^2 e^{-rT} \]
\[
\Lambda(S, I, t) = \Lambda. \quad (4.14) \]

Equation (4.14) represents the analytical solution of (2.15) subject to (4.11).

C. Case III

Suppose the initial condition associated with (2.15) via (2.16-2.17) is

\[ g(\psi, 0) = \left(1 + \frac{1}{rT}\right) \psi S^2 e^{-rT} \quad (4.15) \]

then we have:

\[ \frac{\partial g}{\partial t} = \left(1 + r\psi\right) \frac{\partial g}{\partial \psi} - \frac{1}{2} \sigma^2 \psi^2 \frac{\partial^2 g}{\partial \psi^2} \]
\[ g(\psi, 0) = \left(1 + \frac{1}{rT}\right) \psi S^2 e^{-rT}. \quad (4.16) \]

Taking the PDTM of (4.16) gives:

\[ G(\psi, h + 1) = \frac{1}{h + 1} \left[ \frac{1}{T + r\psi} \frac{\partial G(\psi, h)}{\partial \psi} - \frac{1}{2} \sigma^2 \psi^2 \frac{\partial^2 G(\psi, h)}{\partial \psi^2} \right] \]
\[ G(\psi, 0) = \left(1 + \frac{1}{rT}\right) \psi S^2 e^{-rT}. \quad (4.17) \]

Hence, for \( h \geq 0 \), we have:

\[ G(\psi, 0) = \left(1 + \frac{1}{rT}\right) \psi S^2 e^{-rT} \]
\[ G(\psi, 1) = \frac{1}{T + r\psi} \left[ \frac{1}{T + r\psi} \left(1 + \frac{1}{rT}\right) S^2 e^{-rT} \right] \]
\[ G(\psi, 2) = \frac{1}{2!} \frac{1}{T + r\psi} \left(1 + \frac{1}{rT}\right) rS^2 e^{-rT} \]
\[ G(\psi, 3) = \frac{1}{3!} \frac{1}{T + r\psi} \left(1 + \frac{1}{rT}\right) r^2 S^2 e^{-rT} \]
\[ G(\psi, 4) = \frac{1}{4!} \frac{1}{T + r\psi} \left(1 + \frac{1}{rT}\right) r^3 S^2 e^{-rT} \]
\[ G(\psi, 5) = \frac{1}{5!} \frac{1}{T + r\psi} \left(1 + \frac{1}{rT}\right) r^4 S^2 e^{-rT} \]
\[ G(\psi, 6) = \frac{1}{6!} \frac{1}{T + r\psi} \left(1 + \frac{1}{rT}\right) r^5 S^2 e^{-rT} \]
\[ G(\psi, 7) = \frac{1}{7!} \frac{1}{T + r\psi} \left(1 + \frac{1}{rT}\right) r^6 S^2 e^{-rT} \]
\[ G(\psi, 8) = \frac{1}{8!} \frac{1}{T + r\psi} \left(1 + \frac{1}{rT}\right) r^7 S^2 e^{-rT} \]
\[ G(\psi, 9) = \frac{1}{9!} \frac{1}{T + r\psi} \left(1 + \frac{1}{rT}\right) r^8 S^2 e^{-rT} \]
\[ \cdots G(\psi, p) = \frac{1}{p!} \frac{1}{T + r\psi} \left(1 + \frac{1}{rT}\right) r^p S^2 e^{-rT}, \quad p \geq 1. \]
In this work, we have successfully obtained analytical solutions of the CAA option pricing model via the application of a proposed PDTM. The method involved less computational work without compromising the level of accuracy. Three cases were considered as regards illustration whose results ascertained the method’s efficiency and effectiveness. The works of Rogers & Shi [1], and Elshegmani & Ahmad [2] serve as benchmarks to this current work. The proposed semi-analytical approach is therefore endorsed for obtaining analytical and/or approximate solutions of similar forms of Asian option pricing models likewise other financial pricing models. In addition, other financial PDEs resulting from stochastic dynamics can be considered via this approach.

CONFLICT OF INTERESTS
Conflict of interest is not declared by the authors as regards the publication of this paper.

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REFERENCES


