

Empirical Bayes in Skip Lot Sampling Plan V by Variables Sampling Plan

Khanittha Tinochai, Saowanit Sukparungsee, Katechan Jampachaisri, Yupaporn Areepong

Abstract— Skip lot sampling plans (SkSP) are widely used in industry to reduce sample size and, therefore, save costs for inspecting products in the lot. In this paper, we propose a new technique for an acceptance sampling plan for lot inspection of the skip lot sampling plan V (SkSP-V) by variables with Empirical Bayes (EB) as a reference plan for specified by two points (AQL, RQL). This compares the traditional approach as a single sampling plan (SSP) by variables to SkSP-V which uses a single sampling plan by variables as a reference plan, with data normally distributed under a known mean but unknown variance and an unknown mean and known variance. The probability of acceptance (P_a) and average sample number (ASN) are considered as comparison criteria. Results indicated that the proposed plan yielded the smallest average sample size with the highest probability of acceptance of the lot.

Index Terms— Empirical Bayes, Single Sampling Plan, Skip lot Sampling Plan V

I. INTRODUCTION

Acceptance sampling plans have been widely used in industry for quality assurance inspection to reduce sample size and cost of inspecting products in the lot. Acceptance sampling plans can be classified into two types as attributes sampling plans and variables sampling plans. Variables sampling plans provide more information regarding production in the lots than the attributes ones, and using a small sample size [1]. Sampling plans include various schemes types as a single, double, multiple, sequential and skip lot plans (SkSP). Advantages of the SkSP include providing a smaller sample size for lot inspection than the single sampling plan, thereby reducing the cost of inspecting products in the lot [2].

The SkSP was developed by Dodge (1955) for bulk production inspection. Later, the skip lot sampling plan 2 (SkSP-2) was proposed by Dodge and Perry (1971), followed by the SkSP-V by Balamurali and Jun [3], which

was designed to reduce the number of lots at the normal inspection stage [4]. Aslam et al. [5] exhibited the SkSP as a variable resampling plan when data are assumed to follow a normal distribution with both known and unknown variances, and finding optimal parameters of a single sampling plan (SSP) and SkSP. The P_a and the ASN at acceptable quality level (AQL) and rejectable quality level (RQL) were considered as criteria of comparison. Balamurali and Subramani [6] studied SkSP-2 by variables using SSP by variables as a reference plan when data are normally distributed under both known and unknown variances. The P_a , ASN, average outgoing quality (AOQ) and average total inspection (ATI) were used as criteria of comparison. Muthulakshmi and Lakshmi (2012) compared SSP, SkSP-2 and SkSP-V by attributes using P_a , ASN, AOQ and ATI. Balamurali et al. [7] illustrated SkSP in a resampling plan using attributes and compared SSP with SkSP-2. The operating characteristic (OC) functions, ASN and ATI were used as criteria for comparison. Koatpoothon and Sudasna-na-Ayudhya [8] compared P_a , ASN, AOQ and ATI of SkSP-2 and SkSP-V by attributes. Aslam et al. [9] demonstrated SkSP-V by attributes, using a double sampling plan (DSP) as a reference and compared P_a and ASN at AQL and RQL to determine optimal parameters of DSP and SkSP-V. Aslam et al. [10] considered SkSP-V in a variable sampling plan based on the process capability index when data are normally distributed. Their proposed plan aimed to minimize ASN under both symmetric and asymmetric fraction of defective units. For more details about SkSP-V, it can see in [11], [12], [13], [14], [15], [16] and [17].

The Bayesian approach is applied widely in several sampling plans which are extensively applied in statistical inferences as an alternative or accompanying with traditional approaches. In addition, an application of Bayesian method in sampling plans used the posterior probability distribution function of defective proportion to obtain the acceptance probability in the lots [18]. If the posterior distribution functions are complicated then it can be used numerical methods. The Bayesian principle is to incorporate historical information regarding the parameters through a prior distribution, assuming a known form of distribution. The parameters of prior distribution, called hyper-parameters, are usually assumed to be known or can be estimated regardless of the observed data. However, when the hyper-parameters are unknown and estimated from observed data, this is called the EB approach [19]. Research involving Bayesian and EB has been conducted by many authors including [20], [21], [22], [23], [24], [25],

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[26], [27] and [28]. For more details about Bayesian in SkSP, it can refer to [29] developed Bayesian risks in acceptance sampling that compared with classical method when data follows a Weibull distribution. Suresh and Umamaheswari [30] studied Bayesian methods in SkSP under the Poisson model for destructive testing to obtain optimal parameters at AQL and RQL when the fraction of defective units is assumed to follows a gamma prior. Rajeswari and Jose [31] developed SkSP-2 with a Bayesian modified chain sampling plan as a reference. The proportion of defective units, considered as gamma prior distribution, was applied to determine the probability of acceptance of the lots. Nirmala and Suresh [32] studied Bayesian methods in SkSP-V with multiple deferred states (0, 2) as a reference plan when the proportion of defective unit is defined by gamma prior distribution. Seifi et al. [33] considered a process capability index with the Bayesian method for sampling variables of resubmitted lots, using the posterior probability distribution function of the process capability index to obtain the acceptance probability in the lot. Their objective was to assess optimal parameters for various producer and consumer risks. The proposed plan was compared with SSP, DSP, multiple deferred states (MDS) and repetitive group sampling (RGS) when ASN was the criterion for comparison. Craig and Bland [34] considered the EB method in variables sampling plans for normally distributed data with unknown prior distribution. Delgadillo and Bremer [35] applied the EB method, combined with a specified cost function to test the destructiveness of high-quality products in the Poisson process compared with traditional methods.

However, there is no research regarding the EB method for SkSP-V by variables is currently available. Here, we propose a new technique to obtain an acceptance sampling plan which utilizes EB in SkSP-V by variables when data are normally distributed under two cases, an unknown mean but known variance, and a known mean but unknown variance. Our proposed plan is compared with traditional approaches as SSP by variables and SkSP-V with SSP as the reference. Details about SSP by variables and SkSP-V with SSP by variables as a reference plan are provided in section 2 and section 3, respectively. Section 4 describes the use of EB in SkSP-V. Section 5 covers simulation and comparison methods, and the example is provided in section 6. Conclusions are drawn in section 7.

II. SINGLE SAMPLING PLAN (SSP BY VARIABLES)

The single sampling plan by variables is performed by taking a random sample size n , X_1, X_2, \dots, X_n , the quality characteristic of interest has upper specification limits (USL) and then calculate $z = (USL - \bar{X})/\sigma$ when σ is known and $z = (USL - \bar{X})/s$ when σ is unknown. The lot is accepted if $z \geq K$ and rejected if $z < K$ where K is acceptance criterion [4]. The SSP by variables is based on two parameters (n, K) which can be calculated as follows.

$$K = \frac{Z_{p_2} Z_\alpha + Z_{p_1} Z_\beta}{Z_\alpha + Z_\beta}, \quad (1)$$

and

$$n = \begin{cases} \left(\frac{Z_\alpha + Z_\beta}{Z_{p_1} - Z_{p_2}} \right)^2, & \text{where } \sigma \text{ is known} \\ \left(1 + \frac{K}{2} \right) \left(\frac{Z_\alpha + Z_\beta}{Z_{p_1} - Z_{p_2}} \right)^2, & \text{where } \sigma \text{ is unknown} \end{cases} \quad (2)$$

III. SKIP LOT SAMPLING PLAN V (SKSP-V)

For the acceptance of a sampling plan by variables, the quality characteristics are measured by a continuous scale and then the data are assumed a normal distribution [1]. In this paper, the SkSP-V plan uses the SSP by variables as the reference plan. The SkSP-V plan depends on four parameters in which i is consecutive lots on the reference plan, f is the proportion of lots ($0 < f < 1$), k is consecutive lots on skipping inspection when $k < i$, $k = i$, $k > i$ and c or x is the number of the lots being reduced from the stage of skipping inspection (clearance number) when $c < i$ [9]. Thus, the procedure of the SkSP-V plan is shown in Fig 1.

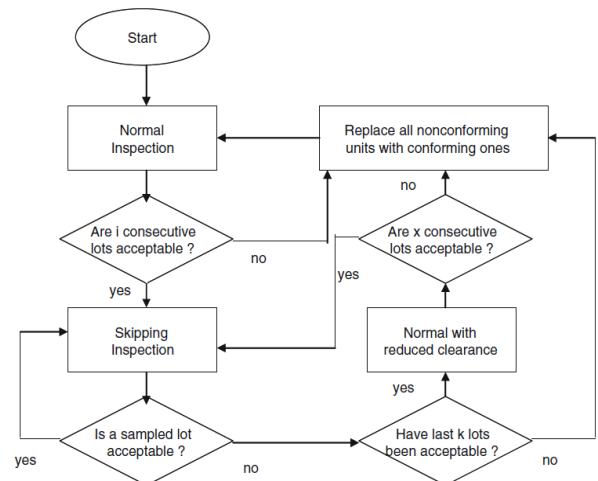


Fig. 1. The procedure of the SkSP-V sampling plan [9].

Let p be the proportion of defective units in the lot and Q is the probability of acceptance a lot with the reference plan or SSP by variables [7]. The criterion for comparison of the SkSP-V is the probability of acceptance a lot (P_a) and average sample number (ASN). Generally, the P_a is considered with two points as follows: the producer's risk (α) and the consumer's risk (β) that are called AQL, denoted by p_1 and RQL, denoted by p_2 , respectively. Thus, it can be shown as

$$P_a(p) = \frac{fQ + (1-f)Q^i + fQ^{k+1}(Q^i - Q^c)}{f(1+Q^{i+k} - Q^{k+c}) + (1-f)Q^i}, \quad (4)$$

and the P_a at the AQL and RQL are given as

$$P_a(p_1) \geq 1 - \alpha, \tag{5}$$

$$P_a(p_2) \leq \beta. \tag{6}$$

The ASN is provided by

$$ASN(p) = \frac{nf + nf(Q^{i+k} - Q^{k+c})}{f(1 + Q^{i+k} - Q^{k+c}) + (1-f)Q^i}. \tag{7}$$

IV. EMPIRICAL BAYES (EB) APPROACH IN SKSP-V

In Bayesian approach, the unknown parameters δ are considered as a random variable, depending on information in the history of parameters, called prior probability density function, assuming known prior distribution, $\pi(\delta|\omega)$, and known hyper-parameter ω . Thus, inference concerning δ is performed using Bayes' theorem which can be expressed up to proportionality as the product of likelihood function, $L(\delta)$, and the prior distribution, $\pi(\delta|\omega)$. The posterior distribution, $h(\delta|\underline{x})$, is determined by

$$h(\delta|\underline{x}) = \frac{L(\delta|\underline{x}) \cdot \pi(\delta|\omega)}{M(\underline{x})} \propto L(\delta|\underline{x}) \cdot \pi(\delta), \tag{8}$$

where $M(\underline{x})$ denotes the marginal distribution of \underline{x} .

The EB approach is involved, when the unknown hyper-parameter (ω) is estimated from the observed data which do not conform to Bayesian concept. The hyper-parameter can be calculated from the marginal distribution of \underline{x} , given by

$$M(\underline{x}|\varpi) = \int_{\theta} f(\underline{x}|\delta) \cdot \pi(\delta|\varpi) d\delta. \tag{9}$$

where the observed data \underline{x} are continuous random sample [16].

In this paper, we propose the use of EB in SkSP-V by variables when data are normally distributed, $X \sim N(\mu, \sigma^2)$, with two cases as follows: case 1: unknown mean μ but known variance σ_0^2 and case 2: known mean μ_0 but unknown variance σ^2 . The defective proportion of samples in a lot is given by

$$p = P(X > USL | \mu) = 1 - F[(USL - \mu)/\sigma] \tag{10}$$

Suppose that $w = \frac{(USL - \mu)}{\sigma}$, $F(w) = \int_{-\infty}^w \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$ is a cumulative distribution function of standard normal distribution, $z \sim N(0,1)$ [36].

A. Case 1: unknown mean μ but known variance σ_0^2 .

Let μ be unknown parameter, assuming μ is defined on informative prior: $\mu \sim N(\theta, \tau^2)$ and hyper-parameters θ and τ^2 are unknown. The hyper-parameters can be estimated from the marginal likelihood distribution as follows.

$$\begin{aligned} M(\underline{x}|\theta, \tau^2) &= \int_{-\infty}^{\infty} f(\underline{x}|\mu) \cdot \pi(\mu|\theta, \tau^2) d\mu, \\ &= \int_{-\infty}^{\infty} \frac{1}{(2\pi\sigma_0^2)^{n/2}} e^{-\frac{1}{2\sigma_0^2} \sum_{i=1}^n (x_i - \mu)^2} \cdot \frac{1}{(2\pi\tau^2)^{1/2}} e^{-\frac{1}{2\tau^2}(\mu - \theta)^2} d\mu, \\ &= \frac{(\sigma_0^2)^{1/2} e^{-\frac{1}{2(n\tau^2 + \sigma_0^2)} \sum_{i=1}^n (x_i - \theta)^2 - \frac{n\tau^2}{2\sigma_0^2(n\tau^2 + \sigma_0^2)} \left(\sum_{i=1}^n x_i^2 - n\bar{x}^2 \right)}}{\left[(2\pi\sigma_0^2)^n (n\tau^2 + \sigma_0^2) \right]^{1/2}} \end{aligned} \tag{11}$$

Then, the likelihood function is provided by $L(\theta, \tau^2 | \underline{x}) = M(\underline{x}|\theta, \tau^2)$, the maximum likelihood (ML) estimator of θ is $\hat{\theta} = \bar{x}$ and the ML estimator of τ^2 is $\hat{\tau}^2 = \sigma_0^2/n$. After that, the estimators $\hat{\theta}$ and $\hat{\tau}^2$ will be substituted into the posterior distribution function.

The posterior distribution function is determined as follows.

$$\begin{aligned} h(\mu|\underline{x}) &\propto L(\mu|\underline{x}) \cdot \pi(\mu), \\ &= \frac{1}{(2\pi\sigma_0^2)^{n/2}} e^{-\frac{1}{2\sigma_0^2} \sum_{i=1}^n (x_i - \mu)^2} \cdot \frac{1}{(2\pi\hat{\tau}^2)^{1/2}} e^{-\frac{1}{2\hat{\tau}^2}(\mu - \hat{\theta})^2}. \end{aligned}$$

With rearrangement, we obtain

$$h(\mu|\underline{x}) \propto \exp \left\{ -\frac{1}{2\sigma_0^2 \hat{\tau}^2} (n\hat{\tau}^2 + \sigma_0^2) \left(\mu - \frac{(\hat{\theta}\sigma_0^2 + n\bar{x}\hat{\tau}^2)}{(n\hat{\tau}^2 + \sigma_0^2)} \right)^2 \right\} \cdot C, \tag{12}$$

$$\text{where } C = \exp \left[-\frac{1}{2\sigma_0^2 \hat{\tau}^2} \left(\sigma_0^2 \hat{\theta}^2 + \hat{\tau}^2 \sum_{i=1}^n x_i^2 - \frac{(\hat{\theta}\sigma_0^2 + n\bar{x}\hat{\tau}^2)^2}{(n\hat{\tau}^2 + \sigma_0^2)} \right) \right].$$

Thus, the posterior distribution is normal distributed as $\mu|\underline{x} \sim N(M, H)$, where $M = \left(\hat{\theta}\sigma_0^2 + n\bar{x}\hat{\tau}^2 \right) / (n\hat{\tau}^2 + \sigma_0^2)$ and $H = \sigma_0^2 \hat{\tau}^2 / (n\hat{\tau}^2 + \sigma_0^2)$.

Specification $Q_1 = F(p) = \int_{-\infty}^p h(\mu|\underline{x}) d\mu$ is a cumulative posterior distribution function of μ when the proportion of defective units (p) is defined. Therefore, the probability of acceptance the lot is given by Q_1 which can

be obtained using the posterior probability distribution function of μ [18].

B. Case 2: known mean μ_0 but unknown variance σ^2 .

Assuming σ^2 is defined on informative prior: $\sigma^2 \sim IG(a, b)$ with hyper-parameters a and b are unknown. Similarly, the hyper-parameters can be calculated from the marginal likelihood distribution which can be written as

$$M(\underline{x} | a, b) = \int_0^\infty f(\underline{x} | \sigma^2) \cdot \pi(\sigma^2 | a, b) d\sigma^2, \\ = \int_0^\infty \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma^2} \sum_{i=1}^n (x_i - \mu_0)^2} \cdot \frac{b^a}{\Gamma(a)} (\sigma^2)^{-(a+1)} e^{-\frac{b}{\sigma^2}} d\sigma^2,$$

Then,

$$M(\underline{x} | a, b) = \frac{b^a \cdot \Gamma\left(a + \frac{n}{2}\right)}{(2\pi)^{n/2} \Gamma(a) \left[\frac{1}{2} \sum_{i=1}^n (x_i - \mu_0)^2 + b\right]^{\left(a + \frac{n}{2}\right)}}. \quad (13)$$

Next, let $L(a, b | \underline{x}) = M(\underline{x} | a, b)$, which is not a closed form, causing difficulty in solving a problem. The hyper-parameters alternatively are obtained using numerical method. In this study, we utilized the Newton Raphson method [37]. Then, the estimators \hat{a} and \hat{b} will be replaced into the posterior distribution function.

The posterior distribution function of σ^2 is provided as

$$h(\sigma^2 | \underline{x}) \propto L(\sigma^2 | \underline{x}) \cdot \pi(\sigma^2), \\ = \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma^2} \sum_{i=1}^n (x_i - \mu_0)^2} \cdot \frac{\hat{b}^{\hat{a}}}{\Gamma(\hat{a})} (\sigma^2)^{-(\hat{a}+1)} e^{-\frac{\hat{b}}{\sigma^2}}.$$

After some calculation, we obtain

$$h(\sigma^2 | \underline{x}) \propto (\sigma^2)^{-\left(\hat{a} + \frac{n}{2} + 1\right)} e^{-\frac{1}{\sigma^2} \left[\frac{1}{2} \sum_{i=1}^n (x_i - \mu_0)^2 + \hat{b}\right]}. \quad (14)$$

Therefore, the posterior distribution of σ^2 has inverse gamma distribution that is

$$\sigma^2 | \underline{x} \sim IG\left(\hat{a} + \frac{n}{2}, \hat{b} + \frac{1}{2} \sum_{i=1}^n (x_i - \mu_0)^2\right).$$

Determination $Q_2 = F(p) = \int_0^p h(\sigma^2 | \underline{x}) d\sigma^2$ is a cumulative posterior distribution function of σ^2 when the proportion of defective units is determined. Thus, the probability of acceptance the lot is specified by Q_2 which

can be obtained using the posterior probability distribution function of the σ^2 .

V. THE CRITERION FOR COMPARISON IN THE EB IN SKSP-V

The P_a and ASN of the EB in SkSP-V for the case 1 and 2 are given as follows.

$$P_a(p) = \frac{fQ_j + (1-f)Q_j^i + fQ_j^{k+1}(Q_j^i - Q_j^c)}{f(1+Q_j^{i+k} - Q_j^{k+c}) + (1-f)Q_j^i}, \quad (15)$$

and

$$ASN(p) = \frac{nf + nf(Q_j^{i+k} - Q_j^{k+c})}{f(1+Q_j^{i+k} - Q_j^{k+c}) + (1-f)Q_j^i}. \quad (16)$$

where $j = 1$ and 2 .

VI. SIMULATION AND COMPARISON METHODS

The data are generated from $X \sim N(0,1)$, under two cases as follows: case 1: unknown mean μ but known variance σ_0^2 , by assuming informative prior on μ : $\mu \sim N(\theta, \tau^2)$ and case 2: known mean μ_0 but unknown variance σ^2 , by assuming informative prior on σ^2 : $\sigma^2 \sim IG(a, b)$, where θ, τ^2, a and b denote hyper-parameters. The number of iterations is given by $t = 1,000$, $\alpha = 0.05$ and $\beta = 0.10$. The proportion of defective at AQL(p_1) is 0.01, at RQL(p_2) is 0.02 and the sample sizes (n) are defined as $n = 115$ for σ known and $n = 388$ for σ unknown. The parameters of the SkSP-V are specified by two situations as follows: (1) $i = 5, f = 1/3, 1/5, k = 3, 5, 10, c = 3, 2$ and (2) $i = 10, f = 1/5, 1/10, k = 5, 10, 15, c = 8, 5$, respectively.

In this paper, the EB in SkSP-V is compared with traditional methods, SSP by variables and SkSP-V with SSP by variables as a reference plan and the P_a and ASN at p_1 and p_2 are considered as the criteria for comparison. The result of simulation can be as shown in Table I to Table IV.

Table I shows P_a at the p_1 and p_2 of the SSP by variables, SkSP-V and EB in SkSP-V, with an unknown mean μ but known variance σ_0^2 , where the values of the hyper-parameters in case 1 are $\hat{\theta} = 0.0007, \hat{\tau}^2 = 0.0087$, respectively. The P_a of SkSP-V and EB in SkSP-V can be determined by equation (15). At p_1 , the value of P_a for the proposed plan is highest, varying from 0.9952 to 0.9991 for all cases. In addition, the value of P_a in SkSP-V is higher than those in SSP by variables, with values of P_a falling between 0.9810 and 0.9931, whereas the value of P_a in SSP by variables is 0.95. At p_2 , the value of P_a for EB in SkSP-V is smallest falling between 0.0017 and 0.0095, which is smaller than those in SkSP-V, and SSP by variables.

Table II illustrates ASN comparison at p_1 and p_2 of three methods, which can be determined by equation (16). The EB in SkSP-V gives the smallest ASN, about 12 to 39 per

lot at p_1 and 12 to 40 at p_2 , whereas SSP by variables gives the highest ASN, about 115 per lot for all cases. The ASN of SkSP-V is between 16 and 44 per lot at p_1 and 25 to 51 per lot at p_2 , respectively.

For the case of known mean μ_0 but unknown variance σ^2 , where the values of the hyper-parameters in case 2 are $\hat{a} = 0.0998$ and $\hat{b} = 0.1496$, respectively. The result shown in Table III and Table IV. The P_a of EB in SkSP-V can be determined by equation (16). At p_1 , the value of P_a for EB in SkSP-V is highest, between 0.9963 and 0.9990. The value of P_a for SkSP-V is higher than those in SSP by variables. At p_2 , the value of P_a for EB in SkSP-V is smallest between 0.00215 and 0.00693. All value of P_a for SSP by variables is higher than those for SkSP-V.

Table IV displays ASN of three approaches at p_1 and p_2 . The ASN of EB in SkSP-V is obtained by equation (17). At p_1 , the proposed plan gives smaller values of ASN than SSP by variables and SkSP-V, which lies at between 40 and 132 per lot. In contrast, the SSP by variables gives the largest ASN at 388 per lot for all cases. The ASN of the SkSP-V lies between 54 and 147 per lot. At p_2 , the EB in SkSP-V yields the smallest ASN, about 42 to 135 per lot. The ASN of SkSP-V is also smaller than those of SSP by variables, about 83 to 173 per lot.

Furthermore, Fig. 2 and Fig. 3 illustrate that the EB in SkSP-V case 1 and 2 provide higher the P_a than the SSP by variables and SkSP-V where the proportion of defective units is higher than 0.1.

Fig. 4 and Fig. 5 show the ASN curves for three sampling plans. The charts give that the two proposed plans provide smaller the ASN than as compared to the traditional approaches where the proportion of defective units is higher than 0.05.

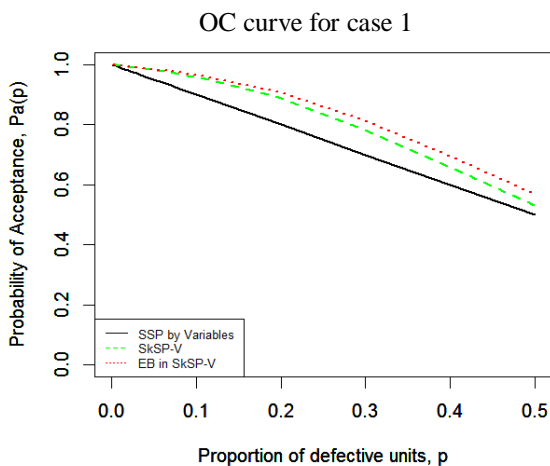


Fig. 2. The comparison P_a of the EB in SkSP-V case 1, SSP by variables and SkSP-V where $i = 5, f = 1/3, k = 3, c = 2$.

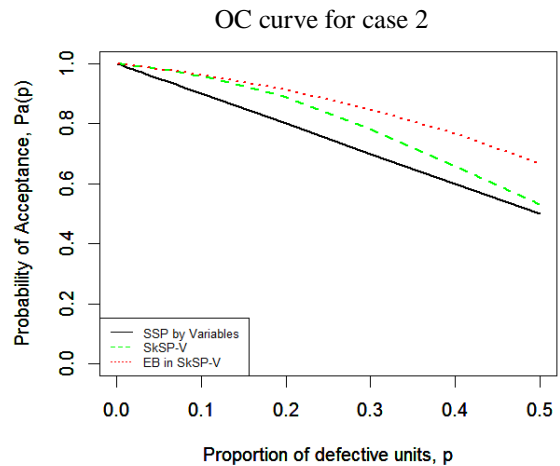


Fig. 3. The comparison P_a of the EB in SkSP-V case 2, SSP by variables and SkSP-V where $i = 5, f = 1/3, k = 3, c = 2$.

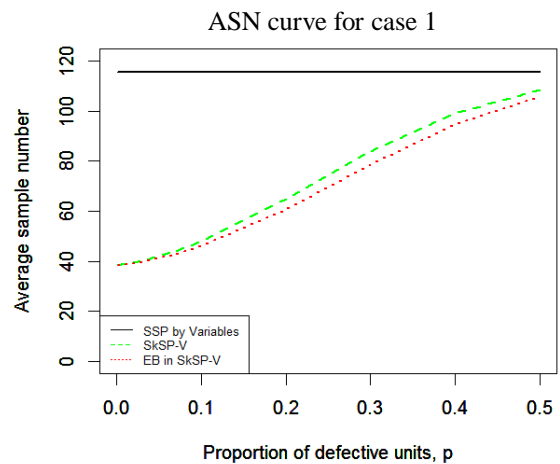


Fig. 4. The comparison ASN of the EB in SkSP-V case 1, SSP by variables and SkSP-V where $i = 5, f = 1/3, k = 3, c = 2$.

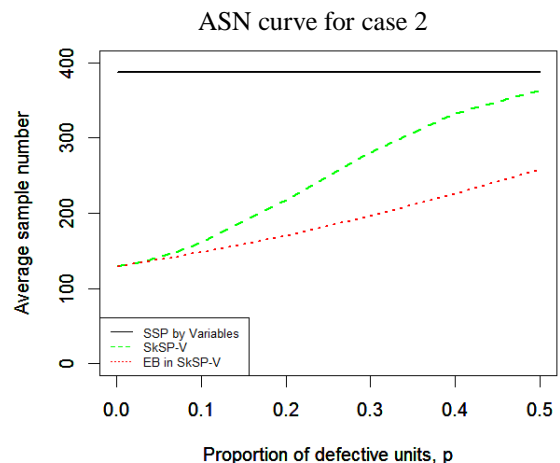


Fig. 5. The comparison ASN of the EB in SkSP-V case 2, SSP by variables and SkSP-V where $i = 5, f = 1/3, k = 3, c = 2$.

TABLE I
 THE P_a AT AQL(P_1) AND RQL(P_2) OF THE SSP BY VARIABLES, SKSP-V AND EB IN SKSP-V IN CASE OF UNKNOWN MEAN μ BUT KNOWN VARIANCE σ_0^2 .

Parameters				Unknown μ , known σ_0^2					
i	f	k	c	$P_a(p_1)$			$P_a(p_2)$		
				SSP	SkSP-V	EB in SkSP-V	SSP	SkSP-V	EB in SkSP-V
5	1/3	3	3	0.9500	0.9813	0.9969	0.0999	0.0432	0.0064
			2	0.9500	0.9818	0.9966	0.0999	0.0415	0.0068
		5	3	0.9500	0.9812	0.9971	0.0999	0.0437	0.0065
			2	0.9500	0.9816	0.9965	0.0999	0.0424	0.0067
		10	3	0.9500	0.9810	0.9967	0.0999	0.0446	0.0085
			2	0.9500	0.9813	0.9952	0.0999	0.0438	0.0057
	1/5	3	3	0.9500	0.9886	0.9975	0.0999	0.0275	0.0043
			2	0.9500	0.9888	0.9981	0.0999	0.0262	0.0051
		5	3	0.9500	0.9884	0.9977	0.0999	0.0280	0.0043
			2	0.9500	0.9887	0.9985	0.0999	0.0269	0.0066
		10	3	0.9500	0.9883	0.9972	0.0999	0.0287	0.0039
			2	0.9500	0.9886	0.9978	0.0999	0.0281	0.0032
10	1/5	5	8	0.9500	0.9858	0.9975	0.0999	0.0405	0.0052
			5	0.9500	0.9867	0.9986	0.0999	0.0380	0.0042
		10	8	0.9500	0.9857	0.9984	0.0999	0.0410	0.0046
			5	0.9500	0.9864	0.9976	0.0999	0.0396	0.0068
		15	8	0.9500	0.9855	0.9980	0.0999	0.0413	0.0095
			5	0.9500	0.9861	0.9972	0.0999	0.0405	0.0038
	1/10	5	8	0.9500	0.9925	0.9989	0.0999	0.0233	0.0027
			5	0.9500	0.9931	0.9987	0.0999	0.0214	0.0022
		10	8	0.9500	0.9924	0.9988	0.0999	0.0236	0.0017
			5	0.9500	0.9928	0.9985	0.0999	0.0226	0.0022
		15	8	0.9500	0.9923	0.9989	0.0999	0.0238	0.0024
			5	0.9500	0.9927	0.9991	0.0999	0.0232	0.0019

TABLE II
 THE ASN AT AQL(P_1) AND RQL(P_2) OF THE SSP BY VARIABLES, SKSP-V AND EB IN SKSP-V IN CASE OF UNKNOWN MEAN μ BUT KNOWN VARIANCE σ_0^2 .

Parameters				Unknown μ , known σ_0^2					
i	f	k	c	ASN(p_1)			ASN(p_2)		
				SSP	SkSP-V	EB in SkSP-V	SSP	SkSP-V	EB in SkSP-V
5	1/3	3	3	115.2914	43.2265	39.0752	115.2914	49.8199	39.8802
			2	115.2914	42.0828	38.8915	115.2914	47.9082	39.5089
		5	3	115.2914	43.4292	39.0253	115.2914	50.4176	39.9057
			2	115.2914	42.4044	38.9306	115.2914	48.9056	39.5453
		10	3	115.2914	43.8500	39.1429	115.2914	51.4355	40.5803
			2	115.2914	43.0689	39.2335	115.2914	50.5785	39.4282
	1/5	3	3	115.2914	26.5998	23.6959	115.2914	31.7754	24.2501
			2	115.2914	25.7389	23.3806	115.2914	30.2363	24.0836
		5	3	115.2914	26.7535	23.6557	115.2914	32.2632	24.2668
			2	115.2914	25.9799	23.2663	115.2914	31.0353	24.5582
		10	3	115.2914	27.0736	23.8444	115.2914	33.1016	24.2107
			2	115.2914	26.4805	23.4858	115.2914	32.3951	23.7439
10	1/5	5	8	115.2914	32.7391	24.6980	115.2914	46.7372	26.7491
			5	115.2914	30.5768	23.6677	115.2914	43.8616	25.1826
		10	8	115.2914	33.0174	24.1951	115.2914	47.3098	26.3896
			5	115.2914	31.3654	24.2367	115.2914	45.6679	26.9352
		15	8	115.2914	33.2315	24.4380	115.2914	47.6436	29.9098
			5	115.2914	31.9656	24.5257	115.2914	46.6925	25.1538
	1/10	5	8	115.2914	17.2762	12.3277	115.2914	26.8101	13.6495
			5	115.2914	15.9380	12.2252	115.2914	24.7185	12.7810
		10	8	115.2914	17.4508	12.5964	115.2914	27.2355	12.9413
			5	115.2914	16.4222	12.3352	115.2914	26.0236	12.8952
		15	8	115.2914	17.5856	12.7505	115.2914	27.4849	13.4918
			5	115.2914	16.7937	12.0145	115.2914	26.7770	12.7062

TABLE III

THE P_a AT AQL(P_1) AND RQL(P_2) OF THE SSP BY VARIABLES, SKSP-V AND EB IN SKSP-V IN CASE OF KNOWN MEAN μ_0 BUT UNKNOWN VARIANCE σ^2 .

Parameters				Known μ_0 , unknown σ^2					
				$P_a(p_1)$			$P_a(p_2)$		
i	f	k	c	SSP	SkSP-V	EB in SkSP-V	SSP	SkSP-V	EB in SkSP-V
5	1/3	3	3	0.9500	0.9814	0.9969	0.0999	0.0432	0.00698
			2	0.9500	0.9818	0.9965	0.0999	0.0415	0.00685
		5	3	0.9500	0.9813	0.9966	0.0999	0.0437	0.00693
			2	0.9500	0.9820	0.9964	0.0999	0.0424	0.00680
		10	3	0.9500	0.9811	0.9963	0.0999	0.0446	0.00692
			2	0.9500	0.9812	0.9962	0.0999	0.0438	0.00682
	1/5	3	3	0.9500	0.9888	0.9980	0.0999	0.0275	0.00419
			2	0.9500	0.9889	0.9978	0.0999	0.0262	0.00413
		5	3	0.9500	0.9885	0.9979	0.0999	0.0280	0.00416
			2	0.9500	0.9887	0.9977	0.0999	0.0269	0.00411
		10	3	0.9500	0.9884	0.9976	0.0999	0.0287	0.00418
			2	0.9500	0.9886	0.9977	0.0999	0.0281	0.00412
10	1/5	5	8	0.9500	0.9858	0.9975	0.0999	0.0407	0.00445
			5	0.9500	0.9868	0.9979	0.0999	0.0380	0.00430
		10	8	0.9500	0.9856	0.9978	0.0999	0.0410	0.00442
			5	0.9500	0.9864	0.9980	0.0999	0.0396	0.00428
		15	8	0.9500	0.9857	0.9974	0.0999	0.0413	0.00439
			5	0.9500	0.9865	0.9979	0.0999	0.0405	0.00427
	1/10	5	8	0.9500	0.9928	0.9989	0.0999	0.0234	0.00226
			5	0.9500	0.9932	0.9991	0.0999	0.0214	0.00217
		10	8	0.9500	0.9925	0.9990	0.0999	0.0236	0.00224
			5	0.9500	0.9929	0.9988	0.0999	0.0226	0.00216
		15	8	0.9500	0.9924	0.9987	0.0999	0.0238	0.00222
			5	0.9500	0.9927	0.9994	0.0999	0.0232	0.00215

TABLE IV

THE ASN AT AQL(P_1) AND RQL(P_2) OF THE SSP BY VARIABLES, SKSP-V AND EB IN SKSP-V IN CASE OF KNOWN MEAN μ_0 BUT UNKNOWN VARIANCE σ^2 .

Parameters				Known μ_0 and unknown σ^2					
				ASN(p_1)			ASN(p_2)		
i	f	k	c	SSP	SkSP-V	EB in SkSP-V	SSP	SkSP-V	EB in SkSP-V
5	1/3	3	3	387.5267	145.2963	131.8950	387.5267	167.4587	134.4099
			2	387.5267	141.4521	131.0722	387.5267	161.0327	132.8403
		5	3	387.5267	145.9777	131.8787	387.5267	169.4675	134.3481
			2	387.5267	142.5331	131.0722	387.5267	164.3854	132.8403
		10	3	387.5267	147.3922	131.8393	387.5267	172.8890	134.2040
			2	387.5267	144.7664	131.0722	387.5267	170.0083	132.8403
	1/5	3	3	387.5267	89.4094	79.4517	387.5267	106.8059	81.2841
			2	387.5267	86.5157	78.8555	387.5267	101.6325	80.1388
		5	3	387.5267	89.9259	79.4398	387.5267	108.4457	81.2389
			2	387.5267	87.3259	78.8550	387.5267	104.3181	80.1385
		10	3	387.5267	91.0020	79.4113	387.5267	111.2638	81.1335
			2	387.5267	89.0084	78.8553	387.5267	108.8890	80.1380
10	1/5	5	8	387.5267	110.0452	82.2447	387.5267	157.0969	86.3140
			5	387.5267	102.7774	80.5850	387.5267	147.4314	83.3512
		10	8	387.5267	110.9809	82.0804	387.5267	159.0214	85.7315
			5	387.5267	105.4279	80.5007	387.5267	153.5024	83.0455
		15	8	387.5267	111.7006	81.9239	387.5267	160.1434	85.2030
			5	387.5267	107.4453	80.4204	387.5267	156.9468	82.7685
	1/10	5	8	387.5267	58.0702	41.4323	387.5267	90.1162	43.7714
			5	387.5267	53.5722	40.4872	387.5267	83.0858	42.0653
		10	8	387.5267	58.6573	41.3385	387.5267	91.5462	43.4347
			5	387.5267	55.1997	40.4392	387.5267	87.4725	41.8901
		15	8	387.5267	59.1102	41.2492	387.5267	92.3844	43.1297
			5	387.5267	56.4483	40.3937	387.5267	90.0050	41.7317

VII. AN EXAMPLE

The real data in thin film transistor liquid crystal display (TFT-LCD) [38] is utilized, where $USL = 25 \mu m$ and $n = 46$. The defective proportions are 0.04 and 0.09 at p_1 and p_2 , respectively. $\alpha = 0.05$ and $\beta = 0.10$. The observations are shown as follows.

11.6015	15.0628	9.9393	14.4734	15.7765	16.2799
16.0025	13.3181	10.0423	12.0464	11.0135	17.1608
13.4624	14.2235	10.9065	17.9988	12.5331	13.8901
10.1995	16.1308	12.8635	9.8069	12.0955	9.0961
9.2012	10.6522	13.9687	10.8885	14.9516	12.6782
10.2634	10.8754	14.4964	16.7877	11.3240	12.5248
15.0168	15.0026	13.3096	11.1455	15.1508	11.9452
9.8483	15.6493	11.3775	10.2598		

The sample mean is $\bar{x} = 12.8966$ and standard deviation is $s = 2.3902$. Suppose that $i = 5$, $f = 1/3$, $k = 3$, $c = 2$, and then the values of P_a at p_1 in SSP by variables, SkSP-V and EB in SkSP-V are 0.8564, 0.9292 and 0.9856, respectively. However, the values of P_a at p_2 in SSP by variables, SkSP-V and EB in SkSP-V are 0.2034, 0.1191 and 0.0351, respectively. In addition, the ASNs at p_1 in SSP by variables, SkSP-V and EB in SkSP-V are 46, 23 and 17 per lot. The ASNs at p_2 in SSP by variables, SkSP-V and EB in SkSP-V are 46, 27 and 18 per lot.

Therefore, it is clear that the values of P_a in proposed plan at p_1 are higher than those in SSP by variables and SkSP-V whereas the values of P_a in proposed plan at p_2 is smaller than those in SSP by variables and SkSP-V. The ASNs of proposed plan at two points is the smallest.

Fig. 6 shows the OC curve for three sampling plans for real data analysis. It can see that the proposed plan provides higher the P_a than the classical methods where the proportion of defective units is higher than 0.15.

Fig. 7 gives the graph that the proposed plan provide smaller the ASN than as compared to the traditional approaches where the proportion of defective units is higher than 0.03.

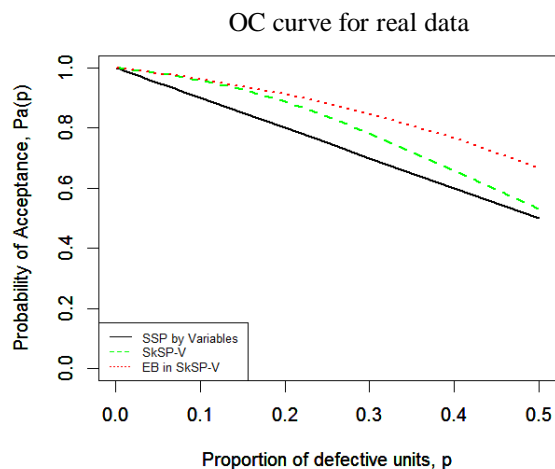


Fig. 6. The comparison P_a of the EB in SkSP-V case 2, SSP by variables and SkSP-V for real data.

ASN curve for real data

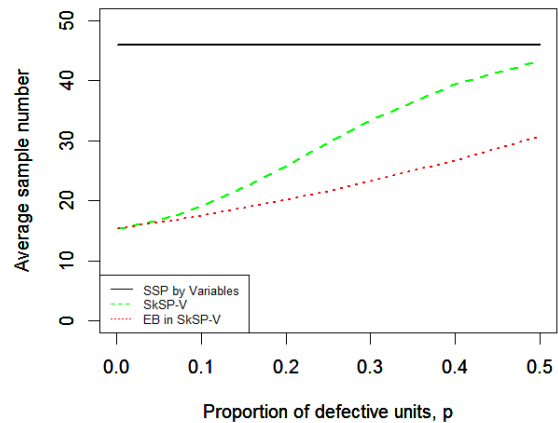


Fig. 7. The comparison ASN of the EB in SkSP-V case 2, SSP by variables and SkSP-V for real data.

VIII. CONCLUSIONS

In this paper, we propose the SkSP-V with EB approach as a reference plan which incorporates prior information about parameters in computation of P_a and ASN. The proposed plans are divided into two cases, according to an unknown mean but known variance and known mean but unknown variance. The P_a and ASN for the proposed plan are then compared with classical approaches as the SSP by variables and SkSP-V with a SSP by variables as a reference plan for specified values of p_1 and p_2 . For both cases, the results of simulation indicate that the proposed method provides a higher P_a and a smaller ASN than the classical methods. The proposed method also reduces the number of average sample sizes for products inspection in the lot and lower both producer risk and consumer risk. In addition, we apply the proposed plan to real data, thin film transistor liquid crystal, which yields consistent results with those in simulation.

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