Transmission Network Expansion Planning Considering Security Constraints through Nodal Indexes

Sergio D. Saldarriaga-Zuluaga, Jesús M. López-Lezama, and Fernando Villada.

Abstract—This paper presents a modelling approach for the Transmission Network Expansion Planning (TNEP) problem that considers security constraints through Weighted Transmission Loading Relief (WTLR) nodal indexes. Small scale generation was also included as a complement to candidate solutions to the TNEP. Additionally, this work describes a multi-objective approach for minimizing operating and investment costs while maximizing network security. The latter is modeled through WTLR nodal indexes that allow to measure the severity of overloads under normal operating conditions as well as under contingencies. Two different metaheuristics are implemented and compared: Non-dominated Sorting Genetic Algorithm II (NSGA-II) and Pareto Envelope-based Selection Algorithm II (PESA-II). Both techniques enable to find a set of solutions that represent a trade-off between the two objective functions (costs vs. security), over which the system planner is able to make decisions according to a given budget and security target. The results of the Garver’s system and a benchmark IEEE power system support the applicability of the proposed model and the effectiveness of both metaheuristics.

Index Terms—Transmission expansion planning, nodal indexes, security constraints, multi-objective optimization.

I. NOMENCLATURE

The nomenclature used throughout the paper is presented below for quick reference.

A. Variables

\[ f_1, f_2 \] Objective functions 1 and 2

\[ w_i \] New line \( i \)

\[ z_k \] New generator \( k \)

\[ NAD_i \] Non-attended demand at node \( i \) [MW]

\[ g_{k_i} \] Active power supplied by generator \( k \) connected at node \( i \) [MW]

\[ \theta_i \] Phase angle at bus \( i \) [rad]

\[ WTLR_i \] WTLR index for node \( i \)

\[ N_{viol} \] Number of overloads in normal operation and under contingencies

\[ OL_{sys} \] Sum of all system overloads in normal operation and under contingencies.

\[ PCO_l \] Overload of line \( l \) under normal operating conditions [MW]

\[ PCO_{l,c} \] Overload in line \( l \) under contingency of line \( c \) [MW]

\[ f_{l} \] Initial power flow of line \( l \) prior to contingency \( c \) [MW]

\[ f_{ij} \] Power flow on line \( l \) connected between nodes \( i \) and \( j \) under normal operating conditions [MW]

\[ f_{ij,c} \] Power flow on line \( l \) connected between nodes \( i \) and \( j \) under contingency \( c \) [MW]

\[ ISF_l^i \] Sensibility to load flow changes in line \( l \) with respect to a power injection at node \( i \) under normal operating conditions

\[ ISF_{l,c}^i \] Sensibility to load flow changes in line \( l \) with respect to a power injection at node \( i \) under contingency of line \( c \)

\[ LODF_{l,c} \] Sensibility to load flow changes in line \( l \) under contingency \( c \)

B. Parameters

\[ d_i \] Demand in bus \( i \) [MW]

\[ \bar{g}_k \] Maximum active generation limit of generator \( k \) [MW]

\[ c_l \] Investment cost of line \( l \) [$]

\[ c_k \] Investment cost of generator \( k \) [$]

\[ c_{o_k} \] Operating cost of generator \( k \) [$/MW]

\[ f_l \] Maximum active power flow limit in line \( l \) [MW]

\[ x_{l}^{pu} \] Reactance of line \( l \) [p.u]

\[ S_{base} \] Base power [MW]

\[ \bar{\theta} \] Maximum phase angle [rad]

\[ C_{DNA} \] Cost of non-attended demand [$/MW]

C. Sets

\[ \Omega_b \] Set of buses

\[ \Omega_l \] Set of existing lines

\[ \Omega_g \] Set of existing generators

\[ \Omega_n \] Set of new lines

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\( \Omega_{\Omega} \) Set of new generators
\( \Omega \) Set of contingences

II. INTRODUCTION

The Transmission Network Expansion Planning (TNEP) problem consists in determining a new set of transmission assets that are needed to satisfy a forecasted demand in a power system at a minimum investment cost while complying with a set of constraints [1]. Such constraints consider the power balance in buses and power limits of generation units and transmission lines, as well as the nature of decision variables. Recent studies have also investigated transmission congestion, carbon capture operating constraints [2], transmission switching, and pumped-storage [3]. Due to its long-lasting effect, the TNEP problem is one of the main strategic decisions in power systems; therefore, several models and solution techniques have been proposed to tackle this issue. A comprehensive approach to the TNEP problem must consider an AC modeling of the transmission network that takes into account the effect of both active and reactive power injections [4]. AC models also offer the advantage of employing other types of studies in expansion planning, such as voltage stability and the impact of FACTS devices. However, since reactive planning can be completed at a later stage and it is mainly considered a short-term local problem, most approaches to the medium and long-term TNEP are implemented using linearized mathematical models. Such models, as classified in [5] and [6], only consider the active power and phase angles; in addition, they include transport, hybrid lineal, DC, and disjunctive models.

The methods generally adopted to solve the TNEP problem can be broadly classified into two groups: classic mathematical programing and metaheuristic approaches. The first type includes techniques such as linear programing [7], mixed integer linear programing [8], dynamic programing [9], Benders decomposition [10], and branch and cut methods [11]. The second kind encompasses ant colony optimization algorithms [12], genetic algorithms [13], simulated annealing [14], differential evolution [15], and tabu search [16], among others. The main advantage of classical mathematical methods lies in the fact that, under convexity conditions, they are able to find global optimal solutions to the TNEP problem. Nevertheless, they are usually time-consuming when dealing with large-scale systems; additionally, the power system model must be converted into a set of linear equations. Thus, only static studies can be used and dynamical studies (such as stability analyses) cannot be conducted. Regarding metaheuristic techniques, these methods are easy to implement and very straightforward; furthermore, recasting the power system model into an optimization programing set is not necessary. Power system analyses, such as power flows, optimal power flows or stability studies, can be carried out in independent software and subsequently be fed into the optimization algorithm. Notwithstanding, they do not guarantee the global optimality of the solutions that are found, and the quality of the same depends on the settings of several parameters. A more detailed classification of models and solution techniques applied to the TNEP problem can be consulted in [5], [17] and [6].

The unbundling of electricity markets has also posed new challenges to find solutions to the TNEP problem. Within said market environments, expansion planning is often modeled as a two-level optimization problem: the upper-level model represents the planner’s decision-making process, while its lower-level counterpart accounts for the market clearing problem. This process is performed by a market operator that seeks to maximize social welfare. Discrete decisions such as which reinforcements to introduce in the network are considered at the upper level, anticipating the reaction of the market clearing procedure. Usually, the lower-level optimization problem is represented as a linear program, as shown in [18]; nevertheless, recent studies consider nonlinear formulations, such as the second-order cone power flow models proposed in [19].

In traditional planning, the objective of the TNEP problem is to minimize the investment cost. However, said planning might be carried out with different objectives in mind, e.g., the reduction of carbon emissions [2] and security enhancements [20]. The latter issue is usually taken into account through the N-1 criterion, which establishes that the power system must continue to operate within allowed limits after any single contingency takes place. As a result, the TNEP becomes more difficult and time consuming to solve. In [21], the authors are forced to limit the maximum number of lines per corridor and reduce the search space by constraining the possibility of adding new lines in all corridors. When considering security criteria, the expansion problem is usually approached in two phases [22]. Initially, the problem is solved without contingencies; afterward, new lines are added any time a contingency makes the system operation unfeasible. The main drawback of such approach lies in the fact that it does not guarantee the optimality of the solutions that are found. Consequently, the problem must consider the whole set of possible contingencies. This approach is implemented in [23] through Mixed-Integer Linear Programing and in [24] by building an equivalent power system formed by islands that represent all contingencies; notwithstanding, for medium-size and large power systems, the time required to solve the problem becomes prohibitive. In this paper, such inconvenience is avoided by resorting to Weighted Transmission Loading Relief (WTLR) nodal indexes that capture the effect of contingencies and are computed through shift and power distribution factors, which allows the model to implicitly account for security constraints [20]. The use of WTLR indexes enables a multi-objective approach to the TNEP problem that considers security and costs. In this work, two different metaheuristics are implemented to find a set of non-dominated feasible solutions that represent a trade-off between both objectives. Additionally, the proposed model considers the possibility of including small-scale generation as part of the expansion plans.

To summarize, this work makes three contributions: (1) it proposes a multi-objective TNEP model that considers the trade-offs between investment costs and network security through WTLR nodal indexes; (2) it implements and compares two optimization algorithms, namely the NSGA-II and the PSEA-II; and (3) it includes small-scale generation as complementary elements of the solution candidates in
TNEP.

The remaining of this paper is organized as follows: Section III presents the mathematical modeling of the TNEP problem. Section IV describes the proposed metaheuristics to approach the TNEP problem. Section V details the tests and results obtained with both techniques in Garver’s system and IEEE 24-bus power system. Finally, Section VI proposes conclusions and final remarks.

III. MATHEMATICAL MODELLING

The mathematical formulation of the multi-objective TNEP problem proposed in this paper is given by (1)-(21) [25].

\[
\begin{align*}
\text{Min } f_1 & = \sum_{i=1}^{n} c_i w_i + \sum_{k=1}^{m} c_k z_k + \sum_{k=1}^{m} c_{i,k} g_{i,k} z_k \\
+ & \sum_{l=1}^{L} c_{l} g_{l} + \sum_{l=1}^{L} NAD_{l}\text{,} (1) \\
\text{Min } f_2 & = \text{MAX}_{\text{ref}} \left| WTLR \right|, (2) \\
\text{Subject to}
\end{align*}
\]

\[
\begin{align*}
WTLR & = \frac{\text{Nviol}}{\text{OL}_{\text{sys}}}, (3) \\
\text{PCO} & = \sum_{l \in \Omega_{\text{sys}}} \left( \frac{f_{lj} - \bar{f}_l}{\bar{f}_l} \right) \iff f_{lj} > \bar{f}_l, (4) \\
\text{PCO}_{\text{sys}} & = \sum_{i \in \Omega_{\text{sys}}} \sum_{l \in \Omega_{\text{sys}}} \text{ISF}_{i,l} \text{PCO}_{i,l}, (5) \\
\text{f}_{li} & = f_{li} + \text{LODF}_{i,l} f_{\text{ref}}, (6) \\
\text{OL}_{\text{sys}} & = \sum_{l \in \Omega_{\text{sys}}} \text{PCO}_{\text{sys}} + \sum_{i \in \Omega_{\text{sys}}} \sum_{l \in \Omega_{\text{sys}}} \text{PCO}_{i,l}, (7) \\
0 \leq g_{li} & \leq \bar{g}_{li}, \forall k \in \Omega_{\text{sys}} \quad (15) \\
g_{li} & = 0 \iff z_k = 0, \forall k \in \Omega_{\text{sys}} \quad (16) \\
0 \leq g_{li} & \leq \bar{g}_{li} \iff z_k = 1, \forall k \in \Omega_{\text{sys}} \quad (17) \\
-\bar{\theta} & \leq \theta_i \leq \bar{\theta}, \forall i \in \Omega_{\text{sys}} \quad (18) \\
w_i & = \text{binary}, \forall i \in \Omega_{\text{sys}} \quad (19) \\
z_i & = \text{binary}, \forall i \in \Omega_{\text{sys}} \quad (20) \\
\theta_i & = 0, \forall i \in \Omega_{\text{sys}}/i = \text{ref} \quad (21)
\end{align*}
\]

Equation (1) is the first objective function and consists of five terms. The first two terms indicate the cost of adding new transmission lines and small-scale generators, respectively, to the system. The third and fourth terms denote the operation costs of existing and new generators, respectively. Finally, the fifth term represents the cost of non-attended demand.

Equation (2) is the second objective function that considers the minimization of the WTLR indexes, which are defined by (3). In this study, the indexes are computed once the new lines and generators are specified. The terms used to compute the WTLR indexes are given by (4)–(7). Constraints (4) and (5) are the power flow limits of lines under normal operating conditions. Equations (6) and (7) define the power flow limits of lines under contingency. Note that overloads of up to 120% of the maximum transmission capacity limit are allowed.

The WTLR indexes measure the change in the total overload of the system considering normal and contingency states that result from a marginal injection of 1 MW into a given bus [20]. Such indexes may take either positive or negative values. The receiving ends of overloaded elements have negative WTLR indexes, which indicates that injecting power into those nodes produces counter flows that would relieve the overload. Conversely, the emitting ends of overloaded elements have positive indexes, which indicates that injecting power into those nodes would worsen the overload. To reduce overloads under both normal and contingency conditions, new elements must be added to the existing transmission network in such a way that the magnitudes of the WTLR indexes are reduced. That is to say, if said indexes equal zero, there are no overloads under normal operating conditions or contingencies.

Equation (8) enables the computation of post-contingency power flows for each line under each contingency through Line Outage Distribution Factors (LODFs). Such factors indicate the sensitivity of the change in power flow in each line for each contingency. Constraint (9) represents the Injection Shift Factor (ISF) of each line with respect to each node for each contingency. The constraint given by (10) is used to calculate the total system overload. Equation (11) defines the nodal power balance constraint for each node. Equation (12) models the power flows in existing lines, while (13) and (14) account for power flows of the expansion candidate lines. Constraint (15) indicates generation limits of existing generators, while (16) and (17) do the same for new generators. Equation (18) sets the maximum limits on phase angles for each bus. Equations (19) and (20) consider the binary nature of the decision variables for lines and generators, respectively. Finally, constraint (21) indicates that the angle of the reference bus must be zero.
IV. SOLUTION APPROACH

From the standpoint of computational complexity, the TNEP problem is classified as NP hard [8], a type of problem that is better handled by metaheuristic techniques than by classic optimization methods [26], [27]. In this study, a candidate solution to the TNEP problem is represented by means of a binary vector that indicates whether a new line or generator must be added to the network. The length of the vector corresponds to the number of candidate lines in corridors and generators in load buses. If a given position of the vector is zero, it indicates that the corresponding element is not considered in the expansion plan. The two multi-objective optimization metaheuristics implemented in this work to approach the TNEP problem are explained below.

A. Non-dominated Sorting Genetic Algorithm II (NSGA-II)

The objective of NSGA-II is to improve the adaptive fit of a population of candidate solutions to a Pareto front constraint by conflicting objective functions. Fig. 1 is a scheme of the NSGA-II. It starts with a population of $P_t$ parents (N individuals), each of them representing a candidate solution to the TNEP problem. A descendant population $Q_t$ is created with the same number of individuals. Those two populations combined form the set $R_t$ with 2N candidate solutions. Subsequently, a non-dominated sorting of the $R_t$ set is performed to classify it into different Pareto fronts. The new population is created from the configuration of non-dominated fronts. It starts with the best non-dominated solutions (F1), followed by the solutions of the second front (F2), and so on until a new population of N individuals is created. The best half of the population is selected to be the parents of the next generation [28].

![Fig. 1. Scheme of the NSGA-II.](image)

The flowchart in Fig. 2 represents the NSGA-II implemented in this work. Given an initial set of candidate solutions, their fitness is computed for both objective functions, and the concept of dominance is applied to classify them. The initial solution must go through the stages of tournament, selection, crossover, and mutation to generate a new set of descendants or new solutions. A non-dominated sorting of the combined population is carried out (Fig. 2). The process is repeated until a stopping criterion is met. A more in-depth description of the NSGA-II can be consulted in [25] and [29].

![Fig. 2. Flowchart of the NSGA-II implemented in this work.](image)

B. Pareto Envelope-based Selection Algorithm II (PESA-II)

PESA-II is a classic evolutionary multi-objective algorithm that features a grid-based fitness assignment strategy in its selection stage. This metaheuristic was proposed in [30], and it has been applied in different fields such as supply chain programing [31] and evolutionary computation [32]. PESA-II considers two population-based parameters: the size of the internal population (IP) and the size of the archive, i.e., the external population (EP). More specifically, PESA-II follows four steps:

- **Step 1.** Generate and evaluate the fitness function of the IP and initialize the EP to the empty set.
- **Step 2.** Incorporate the non-dominated members of IP into EP, one by one.
- **Step 3.** Check the stopping criterion; if it is met, stop, and retrieve the set of solutions in EP as the result. Otherwise, delete the current content of IP and repeat the following until IP new candidate solutions are generated:
  - With probability $p_c$, select two parents from EP, produce a single offspring via crossover, and apply mutation. With probability $(1-p_c)$, select one parent and mutate it to produce an offspring.
- **Step 4.** Return to Step 2.

In step 2, a candidate solution may enter the archive if it is non-dominated within the IP. Once a candidate solution is included in the archive, the members of the archive dominated by this solution are removed. Furthermore, if the addition of a given candidate solution renders the archive overfull, a current member of the EP must be removed.

A region-based selection is performed during Step 3. This is one of the main highlights of the PESA-II. In region-based selection, the unit of selection is a hyper-box, rather than an individual. This concept is illustrated in Fig. 3, which presents a set of candidate solutions for a two-objective minimization problem.
The circles represent non-dominated solutions, while the small squares denote dominated solutions. Both types of solutions might be in the current initial population. The crowding strategy works by drawing an implicit hyper-grid that divides the space into hyper-boxes (Fig. 3). In this case, the problem is two-dimensional and, therefore, the hyper-boxes are squares. Each solution candidate is associated with a particular hyper-box. The number of solutions in a given hyper-box is known as the squeeze factor. For example, the squeeze factor of A is greater than the squeeze factor of C. Said factor is used for selective fitness, and region-based selection favors isolated solutions to conserve the diversity of the population. Consequently, the candidate solution in C has a greater probability of being selected than those in B, which, in turn, are more likely to be selected than those in A. A more in-depth description of PESA-II can be consulted in [30].

![Fig. 3. Illustration of hyper-boxes and crowding strategy of the PESA-II.](image)

**V. TESTS AND RESULTS**

To confirm the applicability of the model and the effectiveness of the proposed solution techniques, several tests were performed with a Garver’s 6-bus test system and an IEEE 24-bus reliability test system. Two scenarios were considered for comparative purposes. Scenario 1 features high investment costs for transmission lines, as given in [33]. Scenario 2 considers lower investment costs than Scenario 1, as given in [34]. Small-scale generation units of 10, 20, and 30 MW were included as additional candidates to be considered in the TNEP in all load buses. The investment cost for generators was set at 1 million USD/MW.

**A. Results with Garver’s 6-bus Test System**

This system comprises 6 buses, 6 lines, and 5 load buses that add up a future demand of 820 MW [35]. To carry out the tests with the proposed algorithms, all existing corridors were considered. The installation of up to 2 additional lines per corridor was allowed. Fig. 4 presents the results obtained in Scenario 1 for both algorithms. Every gray square and black rhombus represents an expansion plan. As expected, reducing the maximum WTLR index to guarantee network security increases the costs of the expansion plans. Note that, for low-investment expansion plans, the solutions found by the PSEA-II clearly dominate those found by NSGA-II; however, as investment costs increase, the solutions found by both algorithms are similar. The best solutions offered by the NSGA-II were found using a population of 30 individuals, 100 generations, and crossover and mutation rates of 90% and 10%, respectively. In turn, the best solutions offered by PSEA-II were found with an initial population of 30 individuals, 100 generations, and crossover and mutation rates of 70% and 30%, respectively. The average computation time of the NSGA-II for this scenario was 3740s. On the other hand, the PSEA-II took approximately half the time to solve the same problem.

![Fig. 4. Pareto fronts obtained with the NSGA-II and the PESA-II in Scenario 1.](image)

Details of the two expansion plans marked with arrows in Fig.4 are presented in Table I, where $f_1$ denotes the investment cost and $f_2$, the maximum WTLR index of the system. $L_{i-j}$ indicates the addition of a line to corridor $i-j$; and the field *Bus number (capacity of the generator in MW)* specifies location and size of a small-scale generator proposed by the algorithm. For example, N2(20) means that a 20-MW generator is proposed to be installed in bus 2. Note that the solutions found by the PESA-II involve more small-scale generation units than those found by NSGA-II. This result highlights the importance of such resources in the TNEP.

**TABLE I**

| TRANSMISSION AND GENERATION EXPANSION PLANS FOR SCENARIO 1 (GARVER’S SYSTEM) |
|----------|----------|----------|
|          | NSGA-II  | PESA-II  |
| $f_1$ ([M$\$]) | 270      | 260      |
| $f_2$ Max (WTRL) | 8.33E-6  | 6.28E-6  |
| Transmission Lines | L1-5, L2-3, L2-3 | L1-2, L1-5, L2-3 |
| Bus number (capacity of the generator in MW) | N2(20), N3(10), N3(30), N4(10), N4(10), N5(10), N6(20), N6(30) | N2(20), N2(30), N4(10), N4(30), N5(10), N5(20), N6(10), N6(20), N6(30) |

The best solutions found for Scenario 2 with both algorithms are plotted in Fig. 5. The settings of both methods were kept the same for this scenario. The average computation time of the NSGA-II was 3848s, while the PESA-II took approximately half that time. It should be noted that computation time is not an essential issue in the TNEP, since decisions are made over a time horizon that

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considers several years ahead. As in Scenario 1, the solutions found by the PSEA-II dominate those found by the NSGA-II for low-investment expansion plans; however, as the security of the system is enhanced (more expensive expansion plans) the solutions found by both algorithms are similar. For illustrative purposes, Table II presents the details of the two expansion plans marked with arrows in Fig. 5.

![Fig. 5. Pareto fronts obtained with the NSGA-II and the PESA-II in Scenario 2.](image)

**TABLE II**

<p>| Transmission and Generation Expansion Plans for Scenario 2 (Garver’s System) |</p>
<table>
<thead>
<tr>
<th>NSGA-II</th>
<th>PESA-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$ ([M$$])</td>
<td>194</td>
</tr>
<tr>
<td>$f_2$ Max (WTRL)</td>
<td>3.45E-12</td>
</tr>
<tr>
<td>Transmission Lines</td>
<td>L1-3, L1-6, L2-3, L2-6, L2-6, L3-5</td>
</tr>
<tr>
<td>Generators Bus (MW)</td>
<td>N2(20), N3(20), N4(20), N4(30), N5(30), N6(30)</td>
</tr>
</tbody>
</table>

The solutions presented in Table I for the NSGA-II and the PESA-II are depicted in Figs. 6 and 7, respectively, for illustrative purposes. Dashed lines and generators represent new elements to be installed in the network.

![Fig. 6. Expansion plan for Scenario 1, presented in Table I (NSGA-II).](image)

**B. Comparison of Results for Garver’s 6-bus Test System**

The TNEP problem can also be solved using a classical modeling approach. In [36], the authors proposed a Mixed-Integer Linear Programming (MILP) model to solve the TNEP problem. In that case, they only considered one objective function, namely, the minimization of the expansion cost subject to N-1 security constraints. As in this paper, they also included small-scale generation units as complementary expansion options. The results obtained for Scenario 1 with the approach proposed in [36] are reported in Table III. It should be noted that the MILP approach provides a single expansion plan as the global optimal solution; in contrast, metaheuristic techniques such as those implemented in this paper provide a set of possible solutions among which a planner can decide. In this study, five new transmission lines are proposed along with eight new small-scale generators. Fig. 8 depicts the expansion plan reported in [36].

![Fig. 7. Expansion plan for Scenario 1, presented in Table I (PESA-II).](image)

**TABLE III**

<p>| Transmission and Generation Expansion Plans for Scenario 1 Using MILP (Garver’s System) |</p>
<table>
<thead>
<tr>
<th>MILP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$ ([M$$])</td>
</tr>
<tr>
<td>Transmission Lines</td>
</tr>
<tr>
<td>Bus number (capacity of the generator in MW)</td>
</tr>
</tbody>
</table>

![Fig. 8. Expansion plan for Scenario 1 using MILP.](image)
Note that the expansion plan found by the MILP approach (see Table III) is more expensive than the ones proposed by the NSGA-II and the PSEA-II in Table I. This is due to the fact that the aforementioned metaheuristics treat security as a soft constraint, while the MILP model enforces it as a hard constraint. Therefore, the plans offered by MILP will be slightly more secure but more costly.

C. Results with IEEE 24-Bus Reliability Test System

This system comprises 24 buses, 38 lines, and 17 load buses that add up a future demand of 8550 MW. To carry out the tests with the proposed algorithm, all existing corridors, plus 7 more as indicated in [33], were considered. Additionally, the installation of up to 2 additional lines per corridor was allowed. Fig. 9 presents the results obtained in Scenario 1 for both algorithms. Every gray square and black rhombus represents an expansion plan. As expected, reducing the maximum WTLR index to guarantee network security increases the costs of the expansion plans. The best solutions suggested by the NSGA-II were found with a population of 60 individuals, 100 generations, and crossover and mutation rates of 90% and 10%, respectively. On the other hand, the best solutions produced by the PESA-II were found with an initial population of 40 individuals, 100 generations, and crossover and mutation rates of 70% and 30%, respectively. The average computation time of the NSGA-II for this scenario was 9531s. On the other hand, the best solutions produced by the PESA-II were found with a population of 60 individuals, 100 generations, and crossover and mutation rates of 90% and 10%, respectively. On the other hand, the best solutions produced by the PESA-II took approximately half the time to solve the same problem.

Note that most solutions found by the NSGA-II are dominated by those of the PESA-II. This indicates that the PESA-II is able to find more secure expansion plans that are less expensive than those proposed by the NSGA-II. For example, guaranteeing a maximum WTLR index near zero (i.e., no overloads under normal operating conditions or any single contingency) requires an investment of nearly 800 million U.S. dollars when the TNEP problem is solved by the PESA-II, and more than 1 billion U.S. dollars when using the NSGA-II.

The details of the two expansion plans marked with arrows in Fig. 9 are presented in Table IV. Note that the solutions found by the PESA-II involve more small-scale generation units than those found by the NSGA-II. This results in fewer transmission lines and, therefore, a less expensive expansion plan.

<table>
<thead>
<tr>
<th>NSGA-II</th>
<th>PESA-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1 (M$)</td>
<td>1270</td>
</tr>
<tr>
<td>f2 Max (WTRL)</td>
<td>1.74E-12</td>
</tr>
</tbody>
</table>

Transmission Lines

<table>
<thead>
<tr>
<th>NSGA-II</th>
<th>PESA-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1-2, L1-2, L1-3, L1-5, L2-4, L4-9, L5-10, L5-10, L6-10, L6-10, L15-16, L16-17</td>
<td></td>
</tr>
</tbody>
</table>

Bus number (capacity of the generator in MW)

<table>
<thead>
<tr>
<th>NSGA-II</th>
<th>PESA-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>N3(20), N11(10), N12(30), N16(10), N22(10), N24(20)</td>
<td>N5(10), N11(10), N13(30), N17(10), N23(10), N23(10)</td>
</tr>
<tr>
<td>N5(10), N11(10), N12(30), N16(10), N22(10), N24(20)</td>
<td>N3(30), N4(10), N4(10), N6(30), N6(30), N6(30), N6(30), N6(30), N6(30)</td>
</tr>
</tbody>
</table>

The best solutions found by both algorithms for Scenario 2 are plotted in Fig. 10. The settings of both methods were kept the same for this scenario. The average computation time of the NSGA-II was 10843s, while the PESA-II took approximately half that time.

In Scenario 2, the superiority of the PESA-II over the NSGA-II is not as evident as in Scenario 1. Note that, at the beginning of the Pareto fronts (left side of Fig. 10), some solutions produced by the PESA-II are dominated by those found by the NSGA-II. That is, some solutions found by the NSGA-II guarantee higher security at lower costs. However, as the investment costs are increased to reduce the maximum WTLR index, the PESA-II clearly outperforms the NSGA-II. Finally, on the right side of the Pareto fronts, the quality of solutions is similar. Table V presents the details of the two expansion plans marked with arrows in Fig. 10. Note that the difference in costs (f1) is not as significant as in Scenario 1. Nevertheless, the PESA-II was able to find a less expensive expansion plan than the NSGA-II with a similar security level (f2).

The solutions presented in Table IV for the NSGA-II and the PESA-II are depicted in Figs. 11 and 12, respectively, for illustrative purposes. Dashed lines and generators represent new elements to be installed in the network.
TABLE V
TRANSMISSION AND GENERATION EXPANSION PLANS FOR SCENARIO 2
(IEEE-24 BUS RELIABILITY TEST SYSTEM)

<table>
<thead>
<tr>
<th>NSGA-II</th>
<th>PESA-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>f1 (M$)</td>
<td>892</td>
</tr>
<tr>
<td>f2 Max  (WTRL)</td>
<td>2E-13</td>
</tr>
</tbody>
</table>

Transmission Lines

Bus number (capacity of the generator in MW)
- NSGA-II: N2(10), N3(20), N4(30), N5(20), N6(10), N7(20), N9(10), N18(10), N20(10), N22(10)
- PESA-II: N1(20), N2(20), N3(30), N4(10), N4(30), N5(30), N6(30), N8(20), N9(10), N9(20), N10(20), N11(10), N11(30), N13(10), N14(2), N19(20), N20(30), N23(20), N24(10)

D. Comparison of Results for the IEEE 24-Bus Reliability Test System
In order to stress the importance of including small-scale generation in the TNEP problem, several tests were carried out considering only transmission lines as candidate solutions to the expansion plan. Fig. 13 details the Pareto front obtained with the NSGA-II for Scenario 1. Note that the optimal Pareto front presents more expensive solutions than those found when also considering small-scale generation (Fig. 9). In this case, the minimum investment cost that guarantees WTLR indexes near zero (approximately 1.8 billion U.S. dollars) is much higher than that of solutions that integrate small-scale generation (around 1 billion U.S. dollars) (Fig. 9). Details of the expansion plan marked with an arrow in Fig 13 are presented in Table VI. Note that 36 new transmission lines are needed to guarantee a secure operation. In contrast, only 21 lines are needed when small-scale generation is included as a complementary option in the expansion plan (see Table IV). The expansion plan presented in Table VI is detailed in Fig 14 for illustrative purposes.
TABLE VI
TRANSMISSION EXPANSION PLAN FOR SCENARIO 1 (IEEE-24 BUS RELIABILITY TEST SYSTEM)

<table>
<thead>
<tr>
<th>NSGA-II</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1 (\text{MS}))</td>
<td>1806</td>
</tr>
<tr>
<td>(f_2 \text{ Max (WTRL)})</td>
<td>1.15E-12</td>
</tr>
</tbody>
</table>

Transmission Lines:

Fig. 14. Expansion plan for Scenario 1 presented in Table VI (NSGA-II).

VI. CONCLUSIONS

This paper presented a multi-objective approach to the TNEP problem considering the minimization of investment and operating costs in addition to the maximization of security levels. The latter objective was modeled through WTLR nodal indexes, and it constitutes one of the main contributions of this work. WTLR indexes measure overloads under normal operating conditions and contingencies. Reducing such indexes also guarantees a secure operation. WTLR indexes are expressed as a function of power distribution factors, and they not only indicate the level of network security but also identify the most sensitive buses to power injections in terms of post-contingency power flows.

Furthermore, two multi-objective optimization techniques were proposed to address the TNEP problem: the NSGA-II and the PESA-II. Both methods allowed to find a set of expansion plans that represent a trade-off between two objective functions; system planners can use such plans to make decisions. Considering two different scenarios, the tests confirmed that, in most cases, the PESA-II is able to find better solutions than the NSGA-II in less computation time. Additionally, the presence of small-scale generation was found to result in less expensive and more secure expansion plans, avoiding the construction of new lines. Future works might take into account a more detailed model modeling of small-scale generation units, as well as the assessment of other multi-objective metaheuristic techniques.

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