Exponential Stabilization and L_2-gain for a Class of Nonlinear Switched Uncertain Systems with Mixed Delays

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***Abstract*—This paper considers the problem of exponential stabilization and L_2-gain analysis for a class of nonlinear switched uncertain systems with mixed time-varying delays and exogenous disturbance.The parameter uncertainties with unknown time-varying matrix and the mixed time delays with upper bound are firstly given. A novel multi-Lyapunov-Krasovskii functional dependent on the size of time delay is constructed blue by utilizing delay-dependent Lyapunov stability theory. Subsequently, based on Jensen's inequality, Schur complement lemma and a lower bound on the average dwell time of the switching signal, some sufficient conditions are presented to ensure the exponential stability with weighted L_2-gain performance of the nonlinear uncertain switched systems under the average dwell time (ADT) method. Finally, a numerical example and a practical example of river pollution control are given to illustrate the effectiveness of the approach proposed in this paper.**

***Index Terms*—Switched uncertain systems, Multi-Lyapunov-Krasovskii functional, L_2-gain, Mixed time delays, Average dwell time.**

I. Introduction

As we all know, switched systems belong to a class important and typical of hybrid systems in control theory and application fields, which is composed of a family of continuous-time or discrete-time subsystems and a switching scheme that orchestrates the switching among the subsystems to ensure stability and the required system performance. Over the past several decades, switched systems have attracted considerable attention of researchers and engineers due mainly to the wide range of applications. For instance, power system, flight control system, artificial intelligence system, communication system, networked control system, power electronic and automatic highway system can be modeled as multiple switching subsystems for easy research and analysis in some special cases [1-4] and the references therein. Several important advances and significant achievements have been made regarding the issue of switched systems [5-7] in recent years. With the development of research, we understand that the switching scheme determines switching from one mode to another, which is the key component of switched systems. In view of this, switched systems can be generally divided into two categories: switched systems with uncontrolled switching scheme and switched systems with controlled switching scheme. In the former, the occurrence of switching among the subsystems cannot be controlled, e.g., [8] deals with the Markovian switching with uncontrolled switching scheme. In the latter, the average dwell time (ADT) method is an available scheme to obtain a satisfied performance by designing the maximum switching numbers over a operating interval. Recently, the (ADT) method has been frequently used in many achievements. In this paper, we also use the method to solve the switching problem between subsystems.

Time delay is regarded as one of the most essential factors influencing stability and undesirable performance of a dynamical system, which leads frequently to receive signal delay[9], data out of order[10], etc. Taking into account the actual situation, the situation of time delay has appeared in many important achievements and results in recent years. For example, [11] used a Riccati type Lyapunov functional to study a switching system composed of a finite number of linear delay differential equations. [12] considered the stability problem for a class of linear switched systems with time-varying delay in the sense of Hurwitz convex combination, to name a few. However, in the actual engineering field, there are often multiple time delays in control systems, i.e., the emergence of mixed time delays. To address this issue, [13] studies the problem of H_2 and H_∞ mixed switching control of uncertain discrete switched systems with mixed time delays, and the method of dealing with time delay is also given. By following this idea, in [14], combined with the feature of mode-dependent ADT switching, the problem of reachable set estimation and synthesis for a class of discrete-time switched linear systems with time delay and bounded peak disturbance are considered. Parameter uncertainties and nonlinear are often coexisting in some practical systems, which are often give rise to instability and oscillation. In [15], a class of discrete-time switched nonlinear systems with time delay is concerned, and system stability with considering time-delay is extended to nonlinear switched systems. Moreover, [16] applies a class of nonlinear switched systems with time-varying delay to river pollution control system. Based on the above discussion, these studies have stimulated the research in this paper, which provide research foundation.

Physical systems are often subject to many external disturbances during operation. And these factors are uncertain and random. The impact of exogenous disturbance phenomena on the system can not be ignored, which is confirmed by a lot of literatures, see, e.g. [17-20]. The mean square stability and exponential mean square stability of multivariable switched stochastic systems are investigated in [21].

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The problem of adaptive tracking control for a class of switched stochastic nonlinear systems in nonstrict-feedback form with unknown nonsymmetric actuator dead-zone and arbitrary switchings is the focus of research in [22]. On the other hand, L_2-gain analysis of switched nonlinear systems is an important issue in engineering applications, and has also been widely related to many kinds of dynamic systems with different performance [23,24]. In recent years, many important results on L_2-gain analysis and controller design of switched systems have emerged. In [25], the L_2-gain analysis of switched linear systems with time-varying delay is investigated. The research field is also constantly expanding, such as delayed systems with missing measurements [26] and uncertain switched system [27]. To the best of our knowledge, many research results on the stability and L_2-gain gain performance of linear switched systems. However, at present, it is noted that few studies have reported the study of L_2-gain control problems for a class of nonlinear switched uncertain systems with mixed time-varying delays, and which constitutes the motivation of the present study.

Different from the existing results, this article focus on the study of the exponential stabilization and L_2-gain analysis of uncertain switched systems. In order to get better results, the author considers the actual working characteristics of the system and reduces the conservatism as much as possible. The distinguish feature of this paper lies in two aspects. (i) The parameter uncertainties with unknown time-varying matrix and the mixed time delays with upper bound are presented and constructing a novel Lyapunov-Krasovskii functional related to the size of mixed time delays. (ii) The stability of time-delay nonlinear switched uncertain systems is classified by the iterative relationship between subsystems, and the sufficient conditions for exponential stabilization of the system under arbitrary switching are given. This part is also the main idea of our article.

This paper studies the problem of exponential stabilization and L_2-gain for a class of switched nonlinear uncertain systems with mixed delays. The sufficient condition for the exponential stabilization with weighted L_2-gain performance are derived. The remainder of the paper is organized as follows. In Sections 2, the problem description and preliminaries are presented and some necessary lemmas are shown. Section 3 is devoted to derive the results on exponential stabilization and L_2-gain analysis for a class of switching signals with average dwell time by considering multi-Lyapunov-Krasovskii functional. In Section 4, numerical examples are carried out. The paper is concluded in Section 5.

II. Problem formulation and preliminaries

In this section, a class of switched nonlinear stochastic systems with mixed delay is considered, which is represented as follows:

$$\begin{aligned}\dot{x}(t) &= \hat{A}_{1\sigma(t)}(t)x(t) + \hat{A}_{2\sigma(t)}(t)x(t-h(t)) \\ &\quad + B_{\sigma(t)}f_{\sigma(t)}(t, x(t), x(t-\tau(t))) \\ &\quad + C_{\sigma(t)}\omega(\mathrm{t}) + E_{\sigma(t)}u(\mathrm{t}), \qquad (1)\\ x(t) &= \varphi(s), s \in [-\max(h_M, \tau_M), 0], \\ z(t) &= D_{\sigma(t)}x(t) + F_{\sigma(t)}\omega(t);\end{aligned}$$

where $x(t) \in R^n$, $\omega(t) \in R^q$, $\varphi(s) \in R^n$ and $u(t) \in R^m$ denote the state vector, exogenous disturbance, initial condition and the control input, respectively. $z(t) \in R^n$ is the measured output, the switching signal $\sigma(t) : [0, \infty] \to N = \{1, 2, \ldots, n\}$ is a piecewise continuous (from the right) function, where n is the number of subsystems. Specifically, denote, $\Sigma : \{(t_0, \sigma(t)), \cdots, (t_k, \sigma(t)), \cdots, k = 0, 1, 2, \cdots\}$, where t_0 is the initial switching instant and t_k denotes the kth switching instant.

When $t \in [t_k, t_{k+1})$, then, the ith subsystem is activated and $\sigma(t_k) = i$. $\bar{A}_{1i}(t), \bar{A}_{2i}(t)$ $(i \in N)$ is matrix with parameter uncertainties, which satisfies

$$\begin{aligned}&\hat{A}_{1i}(t) = A_{1i} + \Delta A_{1i}(t), \quad \hat{A}_{2i}(t) = A_{2i} + \Delta A_{2i}(t), \\ &[\Delta A_{1i}(t) \quad \Delta A_{2i}(t)] = H_i F_i(t)[M_{1i} \quad M_{2i}],\end{aligned} \qquad (2)$$

Here, $\Delta A_{1i}(t), \Delta A_{2i}(t)$ is the term with parameter uncertainty. H_i, M_{1i}, M_{2i} are matrices with appropriate dimension. $F_i(t)$ is an unknown time-varying matrix with Lebesgue measurable bounded elements ${F_i}^T(t)F_i(t) \leq I$. For any $i \in N, A_i, B_i, C_i, D_i, F_i$ are constant matrices. $h(t)$ and $\tau(t)$ denote the time-varying delay satisfying

$$\begin{aligned}&0 \leq h(t) \leq h_M, \quad \dot{h}(t) \leq h < 1; \\ &0 \leq \tau(t) \leq \tau_M, \quad \dot{\tau}(t) \leq \tau < 1.\end{aligned} \qquad (3)$$

$f_i(t, x(t), x(t-\tau(t)))$ is an nonlinear perturbation function, which satisfies

$$\begin{aligned}&{f_i}^T(t, x(t), x(t-\tau(t)))f_i(t, x(t), x(t-\tau(t))) \\ &\quad \leq x^T(t)V_i^T V_i x(t) + x^T(t-\tau(t))\Lambda_i^T \Lambda_i x(t-\tau(t)),\end{aligned} \qquad (4)$$

where V_i, Λ_i are known real constant matrices.

Remark 1. [16] investigates the exponential stabilization for a class of switched nonlinear uncertain systems with time-varying delay. But multiple time delays and exogenous disturbance are not considered. In this paper, the parameter uncertainties with unknown time-varying matrix and the mixed time delays with upper bound are involved. Compared with [16], the switched systems in this paper is more comprehensive and practical in engineering.

For system (1), we consider the state feedback given by

$$u(t) = K_{\sigma(t)}x(t). \qquad (5)$$

For convenience of discussion, we denote $\bar{A}_{1i} = A_{1i} + E_i K_i$. Then, the closed loop system of system (1) is denoted as:

$$\begin{aligned}\dot{x}(t) &= (\bar{A}_{1i} + H_i F_i(t) M_{1i})x(t) \\ &\quad + (A_{2i} + H_i F_i(t) M_{2i})x(t-h(t)) \\ &\quad + B_i f_i(t, x(t), x(t-\tau(t))) + C_i\omega(\mathrm{t}), \qquad (6)\\ x(s) &= \varphi(s), s \in [-\max(h_M, \tau_M), 0], \\ z(t) &= D_i x(t) + F_i\omega(\mathrm{t});\end{aligned}$$

In order to prove the main conclusions of this paper, the following definitions and lemmas are introduced.

Definition 1.([25]) *$N_\sigma(t, T)$ is the switching number of $\sigma(t)$ on an interval (t, T). For any $T > t \geq 0$, if*

$$N_\sigma(t, T) \leq N_0 + (T - t)/\tau_\alpha, \qquad (7)$$

holds for given $N_0 \geq 0, \tau_\alpha \geq 0$, then the constant τ_α is called the average dwell time. In this paper, $N_0 = 0$.

Remark 2. Average dwell time (ADT) method is an available scheme to obtain a satisfied performance by designing the maximum switching numbers over a operating interval. The concept of "average dwell time" plays a key role in switched systems, and has been used in many control problems, e.g., asymptotic behavior, asynchronous switching, L_2 gain analysis, H_∞ control and dissipativity.

Definition 2.([28]) *The equilibrium $x^* = 0$ of system (1) is said to be exponentially stable under any switching signal $\sigma(t)$, if the solution $x(t)$ of system(1) satisfies that*

$$\begin{aligned}\|x(t)\| \leq &\omega \sup_{-\max(-h_M,\tau_M)\leq\theta\leq 0} \|x(t_0+\theta)\| \, e^{-\lambda(t-t_0)}, \\ &\forall t\geq t_0, \omega\geq 1, \lambda>0,\end{aligned} \tag{8}$$

Definition 3.([29]) *For $\alpha > 0$ and $\gamma > 0$, the switched system (1) is said to have weighted L_2-gain γ, if under zero initial condition $\varphi(t) = 0$, $t \in [-h_M, 0]$, it holds that*

$$\int_0^\infty e^{-\alpha s} z^T(s)z(s)ds \leq \gamma^2 \int_0^\infty \omega^T(s)\omega(s)ds$$

Lemma 1.([30]) *For any symmetric and positive definite constant matrix $M \in R^{n\times n}$ and scalar $r > 0$, if there exists a vector function $\omega : [0, r] \to R^n$, then*

$$(\int_0^\gamma \omega(s)ds)^T M (\int_0^\gamma \omega(s)ds) \leq \gamma (\int_0^\gamma \omega^T(s) M\omega(s)ds).$$

Lemma 2.([31]) *Given constant matrices S_1, S_2, S_3, where $S_1 = S_1^T$ and $S_3 = S_3^T > 0$, then if and only if*

$$\begin{bmatrix} S_1 & S_2 \\ S_2^T & -S_3 \end{bmatrix} < 0.$$

Lemma 3.([31]) *U, V, W and X are real matrices, and $X^T = X$. If $V^T V \leq I$, then $X + UVW + W^T V^T U^T < 0$. if and only if there exists scalar $\varepsilon > 0$ such that $X + \varepsilon UU^T + \varepsilon^{-1} W^T W < 0$.*

III. Main results

In this section, the stability and L_2-gain analysis of the switched nonlinear systems (1) is shown in detail.

A. Stability analysis

Theorem 1. *For given positive constants α, h_M, τ_M and $\mu \geq 1$, if there exist positive constant ε_i and symmetric and positive definite matrices $P_i, Q_{1i}, Q_{2i}, R_{1i}, R_{2i}, K_i$ such that the following matrix inequalities hold for all $i, j \in M, i \neq j$,*

$$\begin{aligned}&P_i \leq \mu P_j, Q_{1i} \leq \mu Q_{1j}, Q_{2i} \leq \mu Q_{2j}, \\ &R_{1i} \leq \mu R_{1j},\ R_{2i} \leq \mu R_{2j},\end{aligned} \tag{9}$$

$$\Xi_i = \begin{pmatrix} \phi_{11}^i & P_iA_{2i} & 0 & P_iB_i & 0 & 0 & P_iH_i & \varepsilon_i M_{1i}^T \\ * & \phi_{22}^i & 0 & 0 & 0 & 0 & 0 & \varepsilon_i M_{2i}^T \\ * & * & \phi_{33}^i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & \phi_{55}^i & 0 & 0 & 0 \\ * & * & * & * & * & \phi_{66}^i & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_i I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_i I \end{pmatrix} < 0 \tag{10}$$

where

$$\begin{aligned}\phi_{11}^i &= P_i\bar{A}_{1i} + \bar{A}_{1i}^T P_i + Q_{1i} + Q_{2i} + h_M^2 R_{1i} + \tau_M^2 R_{2i} \\ &\quad + \alpha P_i + V_i^T V_i, \\ \phi_{22}^i &= -(1-h)e^{-\alpha h_M} Q_{1i}, \phi_{55}^i = -e^{-\alpha h_M} R_{1i}, \\ \phi_{33}^i &= \Lambda_i^T \Lambda_i - (1-\tau)e^{-\alpha\tau_M} Q_{2i}, \quad \phi_{66}^i = -e^{-\alpha\tau_M} R_{2i}.\end{aligned}$$

then system (1) is exponentially stabilizable under the feedback control (5) for any switching signal with the average dwell time satisfying

$$\tau_a > \tau_a^* = \frac{\ln\mu}{\alpha} \tag{11}$$

Proof: When $t \in [t_k, t_{k+1})$, i.e. the ith subsystem is activated, the Lyapunov-Krasovskii functional is constructed as follows:

$$\begin{aligned}V(t) =& x^T(t)P_{\sigma(t)}x(t) + \int_{t-h(t)}^t e^{\alpha(s-t)} x^T(s) Q_{1\sigma(t)} x(s)ds \\ &+ h_M \int_{-h_M}^0 \int_{t+\theta}^t e^{\alpha(s-t)} x^T(s) R_{1\sigma(t)} x(s) ds d\theta \\ &+ \int_{t-\tau(t)}^t e^{\alpha(s-t)} x^T(s) Q_{2\sigma(t)} x(s)ds \\ &+ \tau_M \int_{-\tau_M}^0 \int_{t+\theta}^t e^{\alpha(s-t)} x^T(s) R_{2\sigma(t)} x(s) ds d\theta,\end{aligned} \tag{12}$$

Derived $V(t)$ along the trajectory of the system

$$\begin{aligned}\dot{V}(t) =& 2x^T(t)P_i\dot{x}(t) + x^T(t)Q_{1i}x(t) \quad + x^T(t)Q_{2i}x(t) \\ &- (1-\dot{\tau}(t))e^{-\alpha\tau(t)} x^T(t-\tau(t)) Q_{2i} x(t-\tau(t)) \\ &+ h_M^2 x^T(t) R_{1i} x(t) + + \tau_M^2 x^T(t) R_{2i} x(t) \\ &- \alpha \int_{t-h(t)}^t e^{\alpha(s-t)} x^T(s) Q_{1i} x(s) ds \\ &- h_M \int_{-h_M}^0 e^{\alpha\theta} x^T(t+\theta) R_{1i} x(t+\theta) d\theta \\ &- \alpha \int_{t-\tau(t)}^t e^{\alpha(s-t)} x^T(s) Q_{2i} x(s) ds \\ &- \tau_M \int_{-\tau_M}^0 e^{\alpha\theta} x^T(t+\theta) R_{2i} x(t+\theta) d\theta \\ &- \alpha h_M \int_{-h_M}^0 \int_{t+\theta}^t e^{\alpha(s-t)} x^T(s) R_{1i} x(s) ds d\theta \\ &- (1-\dot{h}(t))e^{-\alpha h(t)} x^T(t-h(t)) Q_{1i} x(t-h(t)) \\ &- \alpha\tau_M \int_{-\tau_M}^0 \int_{t+\theta}^t e^{\alpha(s-t)} x^T(s) R_{2i} x(s) ds d\theta \\ \leq& x^T(t)[P_i\bar{A}_{1i} + \bar{A}_{1i}^T P_i + Q_{1i} + Q_{2i} + h_M^2 R_{1i} + \tau_M^2 R_{2i} \\ &+ (H_iF_i(t)M_{1i})^T P_i + P_iH_iF_i(t)M_{1i}]x(t) \\ &+ x^T(t-h(t))(A_{2i}^T P_i + (H_iF_i(t)M_{2i})^T P_i)x(t) \\ &+ x^T(t)(P_iA_{2i} + P_iH_iF_i(t)M_{2i})x(t-h(t)) \\ &+ x^T(t)P_iB_i f_i(t, x(t), x(t-\tau(t))) \\ &+ f_i^T(t, x(t), x(t-\tau(t)))B_i^T P_i x(t) \\ &- (1-h)e^{-\alpha h_M} x^T(t-h(t)) Q_{\cdot 1i} x(t-h(t)) \\ &- (1-\tau)e^{-\alpha\tau_M} x^T(t-\tau(t)) Q_{2i} x(t-\tau(t)) \\ &- \alpha \int_{t-h(t)}^t e^{\alpha(s-t)} x^T(s) Q_{1i} x(s) ds \\ &- \alpha \int_{t-\tau(t)}^t e^{\alpha(s-t)} x^T(s) Q_{2i} x(s) ds \\ &- \tau_M \int_{t-\tau_M}^t e^{-\alpha\tau_M} x^T(s) R_{2i} x(s) ds \\ &- h_M \int_{t-h_M}^t e^{-\alpha h_M} x^T(s) R_{1i} x(s) ds\end{aligned}$$

$$
\begin{aligned}
&-\alpha h_M\int_{-h_M}^{0}\int_{t+\theta}^{t}e^{\alpha(s-t)}x^T(s)R_{1i}x(s)dsd\theta\\
&-\alpha\tau_M\int_{-\tau_M}^{0}\int_{t+\theta}^{t}e^{\alpha(s-t)}x^T(s)R_{2i}x(s)dsd\theta.
\end{aligned}\tag{13}
$$

Inequality (4) can be deformed as follows

$$
\begin{aligned}
&x^T(t)V_i^TV_ix(t)+x^T(t-\tau(t))\Lambda_i^T\Lambda_ix(t-\tau(t))\\
&\quad-f_i^T(t,x(t),x(t-\tau(t)))f_i(t,x(t),x(t-\tau(t)))\geq 0.
\end{aligned}\tag{14}
$$

According Lemma 1, we get

$$
\begin{aligned}
&-h_M\int_{t-h_M}^{t}e^{-\alpha h_M}x^T(s)R_{1i}x(s)ds\\
&\leq -e^{-\alpha h_M}\left(\int_{t-h_M}^{t}x(s)ds\right)^T R_{1i}\left(\int_{t-h_M}^{t}x(s)ds\right);\\
&-\tau_M\int_{t-\tau_M}^{t}e^{-\alpha\tau_M}x^T(s)R_{2i}x(s)ds\\
&\leq -e^{-\alpha\tau_M}\left(\int_{t-\tau_M}^{t}x(s)ds\right)^T R_{2i}\left(\int_{t-\tau_M}^{t}x(s)ds\right).
\end{aligned}\tag{15}
$$

Combining equation (14) with (15), it is easy to obtained

$$
\begin{aligned}
&\dot V(t)+\alpha V(t)\\
&\leq x^T(t)[P_i\bar A_{1i}+\bar A_{1i}^TP_i+Q_{1i}+Q_{2i}+h_M^2R_i\\
&\quad+V_i^TV_i+\alpha P_i]x(t)+x^T(t)P_iB_if_i(t,x(t),x(t-\tau(t)))\\
&\quad+x^T(t-\tau(t))(\Lambda_i^T\Lambda_i-(1-\tau)e^{-\alpha\tau_M}Q_{2i})x(t-\tau(t))\\
&\quad-(1-h)e^{-\alpha h_M}x^T(t-h(t))Q_{1i}x(t-h(t))\\
&\quad-f_i^T(t,x(t),x(t-\tau(t)))f_i(t,x(t),x(t-\tau(t)))\\
&\quad+f_i^T(t,x(t),x(t-\tau(t)))B_i^TP_ix(t)\\
&\quad+x^T(t-h(t))(A_{2i}^TP_i+(H_iF_i(t)M_{2i})^TP_i)x(t)\\
&\quad+x^T(t)(P_iA_{2i}+P_iH_iF_i(t)M_{2i})x(t-h(t))\\
&\quad-e^{-\alpha h_M}\left(\int_{t-h_M}^{t}x(s)ds\right)^T R_{1i}\left(\int_{t-h_M}^{t}x(s)ds\right)\\
&\quad-e^{-\alpha\tau_M}\left(\int_{t-\tau_M}^{t}x(s)ds\right)^T R_{2i}\left(\int_{t-\tau_M}^{t}x(s)ds\right)
\end{aligned}\tag{16}
$$

Then,

$$
\dot V(t)+\alpha V(t)\leq \phi^T(t)\bar\Xi_i\phi(t),\tag{17}
$$

where

$$
\begin{aligned}
\phi(t)=[&x^T(t)\quad x^T(t-h(t))\quad x^T(t-\tau(t))\\
&f_i^T(t,x(t),x(t-\tau(t)))\quad \left(\int_{t-h_M}^{t}x(s)ds\right)^T\\
&\left(\int_{t-\tau_M}^{t}x(s)ds\right)^T]^T,
\end{aligned}
$$

$$
\bar\Xi_i=\begin{pmatrix}
\bar\phi_{11}^i & \phi_{12}^i & 0 & P_iB_i & 0 & 0\\
* & \phi_{22}^i & 0 & 0 & 0 & 0\\
* & * & \phi_{33}^i & 0 & 0 & 0\\
* & * & * & -I & 0 & 0\\
* & * & * & * & \phi_{55}^i & 0\\
* & * & * & * & * & \phi_{66}^i
\end{pmatrix},
$$

$$
\begin{aligned}
\bar\phi_{11}^i=&P_i\bar A_{1i}+\bar A_{1i}^TP_i+Q_{1i}+Q_{2i}+h_M^2R_{1i}+\tau_M^2R_{2i}\\
&+\alpha P_i+V_i^TV_i+P_iH_iF_i(t)M_{1i}+(H_iF_i(t)M_{1i})^TP_i,\\
\phi_{12}^i=&P_iA_{2i}+P_iH_iF_i(t)M_{2i}.
\end{aligned}
$$

By lemma 2 and 3, we can get $\bar\Xi_i<0$. Therefore,

$$
\dot V(t)-\alpha V(t)\leq 0.\tag{18}
$$

When $t\in[t_k,t_{k+1})$, simultaneously integrating from t_k to t on both sides of the above formula, we can get

$$
V(t)=V_{\sigma(t)}(t)\leq e^{-\alpha(t-t_k)}V_{\sigma(t_k)}(t_k),\quad t_k\leq t<t_{k+1}.\tag{19}
$$

Using (9), (19) and $k=N_\sigma(t,t_0)\leq(t-t_0)/\tau_a$, we obtain

$$
\begin{aligned}
V(t)&\leq e^{-\alpha(t-t_k)}\mu V_{\sigma(t_k^-)}(t_k^-)\\
&\leq\cdots\leq e^{-\alpha(t-t_0)}\mu^kV_{\sigma(t_0^-)}(t_0^-)\\
&\leq e^{-(\alpha-\ln\mu/\tau_a)(t-t_0)}V_{\sigma(t_0)}(t_0).
\end{aligned}\tag{20}
$$

Recalling (12), it is clear that

$$
\begin{aligned}
&V(t)\geq a\|x(t)\|^2,\\
&V(t_0)\leq b\sup_{-\max(h_M,\tau_M)\leq\theta\leq 0}\|x(t_0+\theta)\|^2,
\end{aligned}\tag{21}
$$

where

$$
\begin{aligned}
a=&\min_{i\in N}\lambda_{\min}(P_i),\\
b=&\max_{i\in N}\lambda_{\max}(P_i)+h_M\max_{i\in N}\lambda_{\max}(Q_{1i})\\
&+\tau_M\max_{i\in N}\lambda_{\max}(Q_{2i})+0.5h_M^3\max_{i\in N}\lambda_{\max}(R_{1i})\\
&+0.5\tau_M^3\max_{i\in N}\lambda_{\max}(R_{2i}).
\end{aligned}
$$

Hence,

$$
\|x(t)\|\leq\sqrt{\frac{b}{a}}\sup_{-(h_M,\tau_M)\leq\theta\leq 0}\|x(t_0+\theta)\|\,e^{-\frac{1}{2}(\alpha-\ln\mu/\tau_a)(t-t_0)}.\tag{22}
$$

By Definition 2, system (1) is exponentially stability.

Remark 3. In [12], based on Lyapunov stability theory, linear switched systems with time-varying delay is addressed by average dwell time (ADT) method, and some delay-dependent sufficient conditions are presented to ensure the exponential stability of linear switched systems. However, this paper considers the problem of exponential stabilization and L_2-gain analysis for a class of nonlinear switched uncertain systems with mixed time-varying delays and exogenous disturbance. Specifically, when $\omega(t)=0$ and $f(t,x(t),x(t-\tau(t)))=0$, [12] can be seen as a special case of this paper.

B. L_2-gain analysis

The exponential stabilization with L_2-gain performance for the system (1) is shown in this section.

Theorem 2. *For given positive constants α,γ,h_M and τ_M, if there exist symmetric and positive definite matrices $P_i,Q_{1i},Q_{2i},R_{1i},R_{2i}$ and K_i, such that the following matrix inequalities hold for all $i,j\in M$,*

$$
\begin{aligned}
&P_i\leq\mu P_j,\ Q_{1i}\leq\mu Q_{1j},\ Q_{2i}\leq\mu Q_{2j},\\
&R_{1i}\leq\mu R_{1j},\ R_{2i}\leq\mu R_{2j},
\end{aligned}\tag{23}
$$

$$
\tilde\Xi_i=\begin{pmatrix}
\tilde\phi_{11}^i & P_iA_{2i} & 0 & P_iB_i & 0 & 0 & \phi_{17}^i & P_iH_i & \varepsilon_iM_{1i}^T\\
* & \phi_{22}^i & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon_iM_{2i}^T\\
* & * & \phi_{33}^i & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & -I & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & \phi_{55}^i & 0 & 0 & 0 & 0\\
* & * & * & * & * & \phi_{66}^i & 0 & 0 & 0\\
* & * & * & * & * & * & \phi_{77}^i & 0 & 0\\
* & * & * & * & * & * & * & -\varepsilon_iI & 0\\
* & * & * & * & * & * & * & * & -\varepsilon_iI
\end{pmatrix}<0\tag{24}
$$

where

$$\phi_{11}^i = P_i\bar{A}_{1i} + \bar{A}_{1i}^T P_i + Q_{1i} + Q_{2i} + h_M^2 R_{1i} + \tau_M^2 R_{2i} + \alpha P_i + V_i^T V_i + D_i^T D_i,$$
$$\phi_{77}^i = F_i^T F_i - \gamma^2 I, \qquad \phi_{17}^i = P_i C_i + D_i^T F_i,$$

then the system (1) is exponentially stabilizable and has weighted L_2-gain γ under the feedback control (5) for any switching signal with the average dwell time $\tau_a > \tau_a^* = \frac{\ln\mu}{\alpha}$.

Proof: When $t \in [t_k, t_{k+1})$, we choose Lyapunov-Krasovskii functional as (12). We have

$$\begin{aligned}
&\dot{V}(t) + \alpha V(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\\
&\le x^T(t)[P_i\bar{A}_{1i} + \bar{A}_{1i}^T P_i + Q_{1i} + Q_{2i} + P_iH_iF_i(t)M_{1i}\\
&+ h_M^2 R_i + (H_iF_i(t)M_{1i})^T P_i + \tau_M^2 R_i + V_i^T V_i + \alpha P_i\\
&+ D_i^T D_i]x(t) + f_i^T(t, x(t), x(t-\tau(t)))B_i^T P_i x(t)\\
&+ x^T(t-\tau(t))(\Lambda_i^T\Lambda_i - (1-\tau)e^{-\alpha\tau_M}Q_{2i})x(t-\tau(t))\\
&+ x^T(t)P_iB_if_i(t, x(t), x(t-\tau(t)))\\
&+ x^T(t-h(t))(A_{2i}^T P_i + (H_iF_i(t)M_{2i})^T P_i)x(t)\\
&+ x^T(t)(P_iA_{2i} + P_iH_iF_i(t)M_{2i})x(t-h(t))\\
&+ \omega^T(t)(F_i^T F_i - \gamma^2 I)\omega(t)\\
&+ \omega^T(t)(C_i^T P_i + F_i^T D_i)x(t) + x^T(t)(P_iC_i + D_i^T F_i)\omega(t)\\
&- (1-h)e^{-\alpha h_M}x^T(t-h(t))Q_{1i}x(t-h(t))\\
&- f_i^T(t, x(t), x(t-d(t)))f_i(t, x(t), x(t-d(t)))\\
&- e^{-\alpha h_M}\left(\int_{t-h_M}^t x(s)ds\right)^T R_{1i}\left(\int_{t-h_M}^t x(s)ds\right)\\
&- e^{-\alpha\tau_M}\left(\int_{t-\tau_M}^t x(s)ds\right)^T R_{2i}\left(\int_{t-\tau_M}^t x(s)ds\right).
\end{aligned}$$

Defined

$$\bar{\phi}(t) = [x^T(t) \quad x^T(t-h(t)) \quad x^T(t-\tau(t)) \quad f_i^T(t, x(t), x(t-\tau(t))) \quad \left(\int_{t-h_M}^t x(s)ds\right)^T \quad \left(\int_{t-\tau_M}^t x(s)ds\right)^T \quad \omega^T(t)]^T.$$

Therefore,

$$\dot{V}(t) + \alpha V(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \le \bar{\phi}^T(t)\hat{\Xi}_i\bar{\phi}(t).$$

where

$$\hat{\Xi}_i = \begin{pmatrix}
\hat{\phi}_{11}^i & \phi_{12}^i & 0 & P_iB_i & 0 & 0 & \phi_{17}^i\\
* & \phi_{22}^i & 0 & 0 & 0 & 0 & 0\\
* & * & \phi_{33}^i & 0 & 0 & 0 & 0\\
* & * & * & -I & 0 & 0 & 0\\
* & * & * & * & \phi_{55}^i & 0 & 0\\
* & * & * & * & * & \phi_{66}^i & 0\\
* & * & * & * & * & * & \phi_{77}^i
\end{pmatrix}$$

$$\hat{\phi}_{11}^i = P_i\bar{A}_{1i} + \bar{A}_{1i}^T P_i + Q_{1i} + Q_{2i} + h_M^2 R_{1i} + \tau_M^2 R_{2i} + \alpha P_i + V_i^T V_i + D_i^T D_i + P_iH_iF_i(t)M_{1i} + (H_iF_i(t)M_{1i})^T P_i.$$

By lemma 2 and 3, $\hat{\Xi}_i < 0$ is equivalent with $\tilde{\Xi}_i < 0$. So,

$$\dot{V}(t) + \alpha V(t) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) \le 0. \tag{25}$$

When $t \in [t_k, t_{k+1})$, simultaneously integrating from t_k to t on both sides of the above formula, we can get

$$V(t) \le e^{-\alpha(t-t_k)}V(t_k) - \int_{t_k}^t e^{-\alpha(t-s)}\Lambda(s)ds. \tag{26}$$

where $\Lambda(t) = z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)$.

Considering equations (9) and (26), we have

$$\begin{aligned}
V(t) &\le e^{-\alpha(t-t_k)}V(t_k) - \int_{t_k}^t e^{-\alpha(t-s)}\Lambda(s)ds\\
&\le \mu^k V(t_0)e^{-\alpha t} - \mu^k\int_0^{t_1} e^{-\alpha(t-s)}\Lambda(s)ds\\
&- \mu^{k-1}\int_{t_1}^{t_2} e^{-\alpha(t-s)}\Lambda(s)ds\\
&- \cdots - \mu^{k-1}\int_{t_k}^t e^{-\alpha(t-s)}\Lambda(s)ds\\
&\le e^{-\alpha t + N_\sigma(0,t)\ln u}V(t_0) - \int_0^t e^{-\alpha t + N_\sigma(s,t)\ln u}\Lambda(s)ds.
\end{aligned} \tag{27}$$

Then,

$$0 \le -\int_0^t e^{-\alpha(t-s) + N_\sigma(s,t)\ln u}\Lambda(s)ds. \tag{28}$$

Using $e^{-N_\sigma(0,t)\ln\mu}$ to pre-multiply and post-multiply the left term of (28), we have

$$\begin{aligned}
&\int_0^t e^{-\alpha(t-s) - N_\sigma(0,s)\ln u}z^T(s)z(s)ds\\
&\quad \le \int_0^t e^{-\alpha(t-s) - N_\sigma(0,s)\ln u}\gamma^2\omega^T(s)\omega(s)ds.
\end{aligned} \tag{29}$$

When $N_\sigma(0,s) \le \frac{s}{\tau_a}, \tau_a > \tau_a^* = \frac{\ln u}{\alpha}$, it easy to obtain $N_\sigma(0,s)\ln u \le \alpha s$. So,

$$\int_0^t e^{-\alpha(t-s) - \alpha s}z^T(s)z(s)ds \le \int_0^t e^{-\alpha(t-s)}\gamma^2\omega^T(s)\omega(s)ds. \tag{30}$$

Integrating the above formula from 0 to ∞, then

$$\int_0^\infty e^{-\alpha s}z^T(s)z(s)ds \le \int_0^\infty \gamma^2\omega^T(s)\omega(s)ds.$$

The proof is completed.

Remark 4. In this paper, a novel multi-Lyapunov-Krasovskii functional dependent on the size of time delay is constructed by utilizing delay-dependent Lyapunov stability theory. Nevertheless, a common Lyapunov functional(CLF) is often employed to characterize stability in some results. For example, [9] deals with the stabilization of switched linear systems with time-varying delay in switching occurrence detection by a common Lyapunov functional (CLF) under online and offline feedback mechanisms. But, a common Lyapunov functional approach might become too conservative when stabilization of switched systems is addressed in certain circumstances. Therefore, multiple Lyapunov-Krasovskii functional plays an important role in switched system analysis and control synthesis. Compared with [9], the conservativeness of our results is lower.

IV. Numerical examples

The following a numerical example and a practical example are presented to confirm the effectiveness of the proposed approach.

Example 1. Consider system (1) composed of two subsystems with the following parameters:

$$A_{11}=\begin{bmatrix}-2.2 & 0\\ 0 & -2.3\end{bmatrix},\ A_{21}=\begin{bmatrix}-1.2 & 0\\ 0.3 & -1.8\end{bmatrix},$$
$$A_{12}=\begin{bmatrix}-2.4 & 0\\ 0 & -2.5\end{bmatrix},\ A_{22}=\begin{bmatrix}-1.5 & 0\\ 0.1 & -1.2\end{bmatrix},$$
$$B_1=\begin{bmatrix}0.3 & 0\\ 0 & 0.2\end{bmatrix},\quad B_2=\begin{bmatrix}0.2 & 0\\ 0 & 0.3\end{bmatrix}.$$
$$C_1=\begin{bmatrix}-0.4 & 0.1\\ 0 & -0.2\end{bmatrix},\ C_2=\begin{bmatrix}-0.2 & 0.1\\ 0 & -0.4\end{bmatrix},$$
$$D_1=\begin{bmatrix}-0.4 & 0.1\\ 0.1 & -0.6\end{bmatrix},\ D_2=\begin{bmatrix}-0.3 & 0.1\\ 0.1 & -0.3\end{bmatrix},$$
$$F_1=\begin{bmatrix}-0.3 & 0.5\\ 0.3 & -0.4\end{bmatrix},\ F_2=\begin{bmatrix}-0.2 & 0.4\\ 0.3 & -0.3\end{bmatrix},$$
$$V_1=\begin{bmatrix}-0.2 & 0.1\\ 0.2 & -0.4\end{bmatrix},\ V_2=\begin{bmatrix}-0.1 & 0.1\\ 0.2 & -0.2\end{bmatrix},$$
$$\Lambda_1=\begin{bmatrix}-0.4 & 0.1\\ 0.1 & -0.6\end{bmatrix},\ \Lambda_2=\begin{bmatrix}-0.4 & 0.2\\ 0.4 & -0.6\end{bmatrix}.$$
$$H_1=\begin{bmatrix}0.1 & 0.4\\ 0.2 & 0.2\end{bmatrix},\quad H_2=\begin{bmatrix}0.3 & 0.4\\ 0.1 & 0.1\end{bmatrix},$$
$$M_{11}=\begin{bmatrix}-0.1 & 0.1\\ 0.1 & 0.2\end{bmatrix},\ M_{21}=\begin{bmatrix}-0.5 & 0.6\\ 0.2 & 0.1\end{bmatrix},$$
$$M_{12}=\begin{bmatrix}-0.2 & 0.1\\ 0.3 & 0.6\end{bmatrix},\ M_{22}=\begin{bmatrix}-0.6 & 0.5\\ 0.2 & 0.1\end{bmatrix},$$
$$E_1=\begin{bmatrix}0.3\\ 0.4\end{bmatrix},\quad E_2=\begin{bmatrix}0.3\\ 0.2\end{bmatrix}.$$

Choose α= 0.4, $h_M = 0.8$, $\tau_M = 0.5, h = 0.6$, $\tau = 0.2, \mu = 1.9, \gamma = 0.4, \varepsilon_1 = \varepsilon_2 = 0.3, h(t) = 0.2+0.6\sin(t)$, $\tau(t) = 0.3 + 0.2\cos(t)$, then the average dwell time is $\tau_a > 1.61$.

Suppose

$$\omega_1(t)=(0.1e^{-2t}\ \ 0.2\cos t)^T,\omega_2(t)=(0.2\sin t\ \ e^{-3t})^T,$$
$$f_1(t,x(t),x(t-\tau(t)))=\begin{pmatrix}0.1(\sin(x_1(t)+\cos t)\\ 0.1\sin(x_2(t-\tau(t)))\end{pmatrix},$$
$$f_2(t,x(t),x(t-\tau(t)))=\begin{pmatrix}0.2(\cos(x_1(t)+\sin t)\\ 0.2\cos(x_2(t-\tau(t)))\end{pmatrix}.$$

By solving (23) and (24), we can get

$$P_1=\begin{bmatrix}1.0986 & -0.2581\\ -0.2581 & 2.4007\end{bmatrix},$$
$$P_2=\begin{bmatrix}4.0239 & -2.1765\\ -2.1765 & 5.2809\end{bmatrix},$$
$$Q_{11}=\begin{bmatrix}0.8648 & 0.0638\\ 0.0638 & 1.8921\end{bmatrix},$$
$$Q_{21}=\begin{bmatrix}0.5830 & -0.1727\\ -0.1727 & 1.0523\end{bmatrix},$$
$$R_{11}=\begin{bmatrix}0.5967 & -0.1600\\ -0.1600 & 1.0986\end{bmatrix},$$
$$R_{21}=\begin{bmatrix}0.6637 & -0.1506\\ -0.1506 & 1.2250\end{bmatrix},$$
$$Q_{12}=\begin{bmatrix}0.8779 & -0.3854\\ -0.3854 & 0.9377\end{bmatrix},$$
$$Q_{22}=\begin{bmatrix}1.4513 & -0.4700\\ -0.4700 & 1.6580\end{bmatrix},$$
$$R_{12}=\begin{bmatrix}2.2820 & -0.5920\\ -0.5920 & 2.3275\end{bmatrix},$$
$$R_{22}=\begin{bmatrix}1.6272 & -0.4477\\ -0.4477 & 1.9051\end{bmatrix}.$$

Then, the controller gains are

$$K_1=\begin{bmatrix}0.7825 & 1.5995\end{bmatrix},\quad K_2=\begin{bmatrix}1.7965 & -0.8958\end{bmatrix}.$$

Switching signal and state response diagrams are shown in Figure 1 and 2 with the initial state is $x(0) = (-1.5, 1.5)^T$, which shown the exponential stabilization with L_2-gain property for the system system (1).

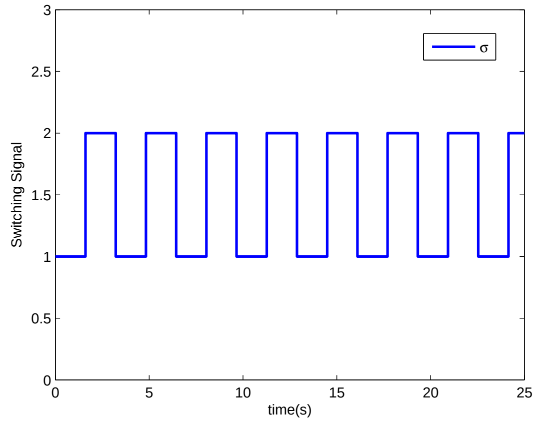

Fig. 1: The switching law.

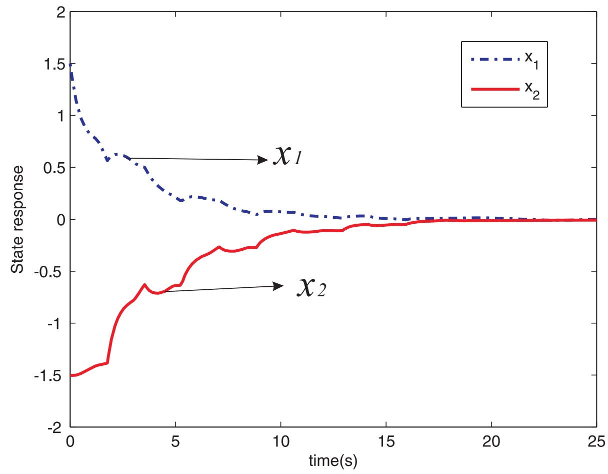

Fig. 2: State response of the closed-loop system.

Example 2. The problem of water pollution is an important issue facing every country, and its development is of great significance to social development. In this section, a practical example of applying switched uncertain systems to water pollution control systems will be demonstrated.

To facilitate the creation of models for water pollution control systems, we denote $r(t)$ and $q(t)$ as the concentrations per unit volume of biochemical oxygen demand and dissolved oxygen, respectively. Simultaneously, let r^* and q^* be the expected steady values of $r(t)$ and $q(t)$ in a reach of a polluted river, respectively. Given by the following definition:

$$x_1(t)=r(t)-r^*,\ x_2(t)=q(t)-q^*,\ x(t)=[x_1^T(t)\ x_2^T(t)]^T$$

As a result, the water pollution dynamic equation for $x(t)$

can be expressed as:

$$\dot{x}(t) = \hat{A}_1(t)x(t) + \hat{A}_2(t)x(t-h(t)) + Eu(t) + \omega(t) + Bf(t, x(t), x(t-\tau(t))) \quad (31)$$

where

$$\hat{A}_1(t) = A_1 + \Delta A_1(t), \quad \hat{A}_2(t) = A_2 + \Delta A_2(t),$$
$$[\Delta A_1(t) \quad \Delta A_2(t)] = HF(t)[M_1 \quad M_2],$$

$$A_1 = \begin{bmatrix} -p_1 - \epsilon_1 - \epsilon_2 & 0 \\ -p_3 & -p_2 - \epsilon_1 - \epsilon_2 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} \epsilon_2 & 0 \\ 0 & \epsilon_2 \end{bmatrix}, B = \begin{bmatrix} \epsilon_1 \\ 1 \end{bmatrix},$$

$p_i(i = 1, 2, 3), \epsilon_1$ and ϵ_2 are known constants. $u(t) = \left[u_1^T(t)\ u_2^T(t)\right]^T$, $\omega(t), \Delta A_1(t), \Delta A_2(t)$ denote the control variable, disturbance input, uncertainty of river pollution system, respectively. Moreover, we can learn the engineering significance of these parameters from [36] and [37]. This paper assumes that system actuators have good performance or failure, and according to the actual situation, we know that at least one actuator can ensure the normal operation of the river pollution system. In addition, for simulation of our purposes, we considered disturbance input and nonlinear term in this paper. As a consequence, the river pollution system (31) can be modeled as a switched system consisting of two subsystems:

$$\begin{cases} \dot{x}(t) = \hat{A}_{11}(t)x(t) + \hat{A}_{21}(t)x(t-h(t)) + Eu(t) + \omega(t) \\ \qquad + Bf(t, x(t), x(t-\tau(t))),\ no\ failures\ occur; \\ \dot{x}(t) = \hat{A}_{12}(t)x(t) + \hat{A}_{22}(t)x(t-h(t)) + Eu(t) + \omega(t) \\ \qquad + Bf(t, x(t), x(t-\tau(t))),\ failures\ occur. \end{cases} \quad (32)$$

Next, we choose $p_1 = 1.4, p_2 = 0.7, p_3 = 1.6, \epsilon_1 = 0.3, \epsilon_2 = 0.2$, and get that

$$A_1 = \begin{bmatrix} -1.9 & 0 \\ -1.6 & -1.2 \end{bmatrix}, A_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, E = \begin{bmatrix} 0.3 \\ 1 \end{bmatrix}.$$

Choose

$$H_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, H_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}, M_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.3 \end{bmatrix},$$

$\alpha = 0.55, h_M = 0.7, \tau_M = 0.4, h = 0.5, \tau = 0.3, \mu = 1.25, \gamma = 0.4, \varepsilon_1 = \varepsilon_2 = 0.3, h(t) = 0.3 + 0.4\sin(t)$, $\tau(t) = 0.2 + 0.1\cos(t), \omega(t) = (0.1e^{-4t}\ 0.2e^{-5t})^T$,

$$f(t, x(t), x(t-\tau(t))) = \begin{pmatrix} 0.02\sin(x_1(t)) \\ 0.01\sin(x_2(t-\tau(t))) \end{pmatrix},$$

By solving (23) and (24), we can get

$$P_1 = \begin{bmatrix} 0.8562 & 0.1726 \\ 0.1726 & 1.1362 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 1.1553 & 0.2658 \\ 0.2658 & 1.4221 \end{bmatrix},$$
$$Q_{11} = \begin{bmatrix} 1.0482 & 0.1227 \\ 0.1227 & 1.2356 \end{bmatrix},$$
$$Q_{21} = \begin{bmatrix} 0.9266 & 0.2639 \\ 0.2639 & 1.1356 \end{bmatrix},$$
$$R_{11} = \begin{bmatrix} 1.2973 & -0.1232 \\ -0.1232 & 1.0263 \end{bmatrix},$$
$$R_{21} = \begin{bmatrix} 1.0376 & -0.1063 \\ -0.1063 & 1.3253 \end{bmatrix},$$
$$Q_{12} = \begin{bmatrix} 1.2368 & 0.2448 \\ 0.2448 & 1.0792 \end{bmatrix},$$
$$Q_{22} = \begin{bmatrix} 1.3543 & -0.1407 \\ 0.1407 & 1.4857 \end{bmatrix},$$
$$R_{12} = \begin{bmatrix} 1.3043 & -0.2631 \\ -0.2631 & 1.9331 \end{bmatrix},$$
$$R_{22} = \begin{bmatrix} 1.3586 & -0.3258 \\ -0.3258 & 1.7263 \end{bmatrix}.$$

Then, the controller gains of switched systems (31) are

$$K_1 = \begin{bmatrix} 1.3852 & 2.1275 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.8225 & 1.2764 \end{bmatrix}.$$

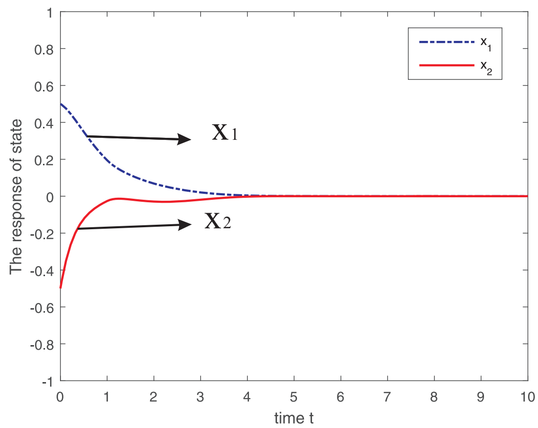

Fig. 3: State response of the subsystem 1.

Fig.3 describes state response of the subsystem 1 with the initial condition $x(0) = (0.5, -0.5)^T$, and we can see that the subsystem 1 is unstable. Simultaneously, Fig.4 describes state response of the subsystem 2 with the initial condition $x(0) = (0.5, -0.5)^T$. Through the designed switching signal and our approach, we can get that the system (32) with the initial condition $x(0) = (0.5, -0.5)^T$ is exponentially stabilizable for any switching signal under the feedback control form Fig. 5. As a consequence, this verifies the effectiveness of our results in the control of river pollution process.

Remark 5. In Example 2, a practical example of river pollution control is carried out to demonstrate the effectiveness of the proposed method. According to the actual situation, we know that the system actuators have good performance or failure. Based on the switching theory, we can guarantee that at least one actuator can ensure the normal operation of the

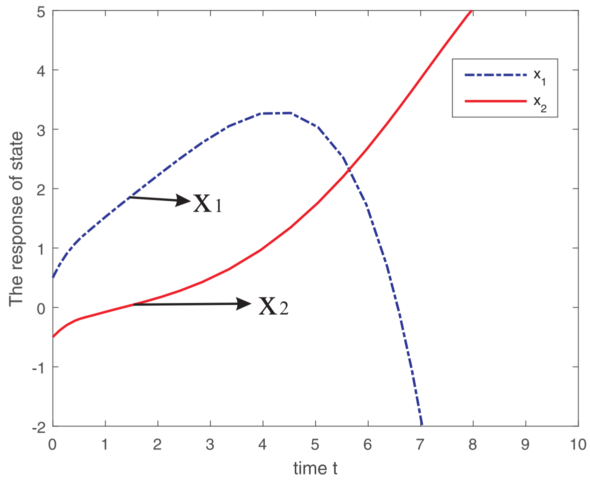

Fig. 4: State response of the subsystem 2.

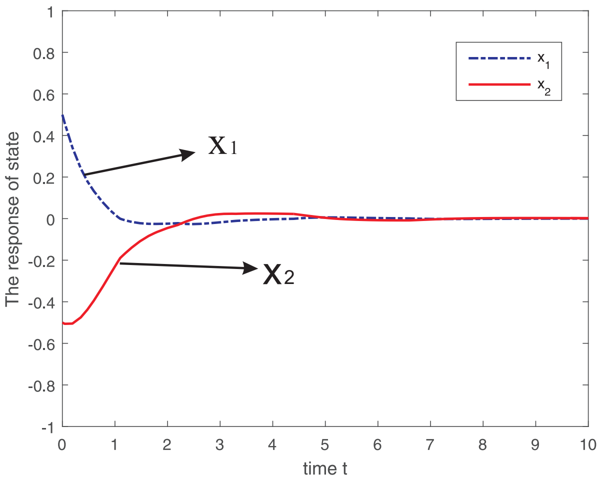

Fig. 5: State response of the closed-loop system (32).

river pollution system. Therefore, the river pollution system (31) can be modeled as a switched system. Moreover, the disturbance input and nonlinear term are considered in this paper. Compared with [36] and [37], our approaches have a larger application range in engineering.

V. Conclusions

This paper mainly studies the exponential stabilization and L_2-gain analysis of a class of nonlinear switched uncertain systems with mixed time-varying delays and exogenous disturbance. Due to the occurrence of parameter uncertainties and mixed time-varying delays in switched systems, a novel delay-dependent multi-Lyapunov-Krasovskii functional is constructed to reduce conservation caused by mixed time-varying delays. Based on Lyapunov stability theory, some delay-dependent sufficient conditions of exponential stabilization with weighted L_2-gain performance for nonlinear uncertain time-delay switched systems are obtained by utilizing the average dwell time method under arbitrary switching signals. Finally, a second-order numerical example and a practical example of river pollution control are carried out, which is solved by Matlab simulation. At the same time, the result of state response diagram for the switched systems are provided to show the effectiveness of the method.

Through the research of this paper, it is important to derive a less conservative condition for exponential stabilization and L_2-gain disturbance attenuation performance for nonlinear switched systems with delay, and expand theoretical to other fields[32-35]. As a part of future work, the optimization of uncertainties and reduction of conservation is my next research plans.

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