The Use of Principal Component Analysis for Cross-efficiency Aggregation

Peng Liu and Li-Fang Wang

***Abstract*—Existing theoretical researches about data envelopment analysis (DEA) cross-efficiency evaluation pay more attention to the calculation of DEA cross-efficiency matrix than aggregation process. The most commonly used aggregation method is to aggregate them with equal weight. This paper focuses on the DEA cross-efficiency aggregation process and proposes the use of principal component analysis to aggregate them. In this study, we view the cross efficiencies calculated by the same set of weights determined by each DMU as attribute values of different DMUs. The cross-efficiency values will transform to be the various attribute values of DMUs and then use of principal component analysis aggregates cross-efficiencies to provide the ultimate weighted average cross-efficiency for each DMU. Finally, two numerical examples are illustrated to show that the proposed method is suitable to aggregate cross-efficiencies.**

***Index Terms*—DEA Cross-efficiency evaluation, Cross-efficiency aggregation, Principal component analysis**

I. Introduction

TO improve the discrimination power of the DEA traditional models, DEA cross-efficiency evaluation has been proposed as an extension to DEA [1]. Unlike DEA traditional models, which use a self-evaluation mode, DEA cross-efficiency evaluation employs both self-evaluation and peer-evaluation mode instead. In DEA cross-efficiency evaluation, each DMU will obtain a self-evaluated efficiency value determined by its own most favorable weights and $n-1$ peer-evaluated efficiencies using the optimal sets of weights of other $n-1$ DMUs. Then, all these n efficiencies for each DMU are averaged into a value to arrive at its cross-efficiency score. Cross-efficiency evaluation made some significant contributions to DEA. Firstly, it can produce a unique rank order to DMUs [2]. Secondly, it can eliminate unrealistic weight schemes without incorporating weights restrictions [3]. Thirdly, it effectively discriminates the good and poor performers among the DMUs [4]. Due to these advantages, it has been widely applied in many areas such as measuring economic-environmental disparities for regional development in China [5] and so on.

However, the optimal set of weights solution in DEA traditional models for each DMU is usually not unique that will damage the use of DEA cross-efficiency evaluation [1,6]. To deal with the problem of non-uniqueness of optimal weights for DMUs, Sexton, Silkman, and Hogan proposed to use secondary goal models [1]. Inspired by the idea, many secondary goal models were introduced. The rank models are to optimize the rank order of the DMU under evaluation [7, 8]. In reality, each input or output is critical in the production process, so the weights dissimilarity level should be not very large of different inputs or outputs. Based on this idea, the weight-balanced models are proposed to lessen large differences in weighted data [9]. Among all the secondary goal models, the aggressive and benevolent models are most widely used. The aggressive (benevolent) model minimizes (maximizes) the average cross-efficiency value of other DMUs while keeping efficiency value of DMU under evaluation on its CCR efficiency level [10].

The existing researches on DEA cross-efficiency

Manuscript received December 6, 2018; This work was substantially supported by the National Natural Sciences Foundation of China under Grant No. 71601155, and Nature Science Foundation of Shaanxi Province, China, under Grant No. 2017JQ7001.

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(Advance online publication: 12 August 2019)

evaluation are mainly focused on calculation of cross-efficiency matrix. Little attention has been paid to the aggregation process of cross-efficiencies. The most commonly used method is to aggregate them with equal weights. However, this method does not consider the difference among cross-efficiencies, so the ultimate average cross-efficiency results sometimes cannot reflect the actual performances of DMUs fairly [11]. Our literature review reveals that few researchers proposed non-equal weights methods considering their difference in their aggregation. Wu et al. argued that average cross-efficiency using equal weights is not good enough because it is not a Pareto solution. Recognizing this shortcoming, they eliminate the average assumption for determining the ultimate cross efficiency scores, and DMUs are regarded as the players in a cooperative game to generate the weights to determine the ultimate cross efficiency scores in light of nucleolus solution or Shapley values in cooperative game [12, 13]. Besides these, they proposed the methods based on information entropy theory to determining the ultimate cross efficiency scores [14, 15]. However, except for the abstract reason that average cross-efficiency with equal weights is not a Pareto solution, Wu's methods did not provide any concrete reasons why the cross-efficiencies should be aggregated by different weights [11]. Wang et al. argue that using equal weights for cross-efficiency aggregation has a significant shortfall that the weights allocated to self-evaluated efficiencies are only $1/n$ and the self-evaluated efficiencies fail to play a sufficient role in final overall assessment. To address this issue, they propose the use of ordered weighted averaging (OWA) operator for cross-efficiency aggregation, where the self-evaluated efficiencies can be weighted to play a significant role in ultimate cross-efficiency in terms of decision maker's optimism level [16]. The shortfalls of this method are that weights are determined by the orness degree α (decision maker's optimism level), and different orness degree values will lead to different results [15]. Especially, it is difficult to measure the optimism level (orness degree value) of the decision maker in practice. Besides, Wang et al. consider that the cross-efficiency matrix is determined by n sets of weights. N sets of weights are from different ideas and are thus of different importance, so cross-efficiencies corresponding to different set of weights will be allocated to different weight to reflect their importance [11]. Based on this idea, they introduce three alternative approaches to determine the relative importance weights for cross-efficiency aggregation. However, the n sets of weights are determined by a given secondary goal model and the viewpoint of given secondary goal model is fixed, so the n sets of weights are from same viewpoint. Lianlian Song and Fan Liu [17] prove that the weights generated by use of Shannon entropy for cross-efficiency aggregation [15] will break Zeleny's rule that if all available alternatives scores are about equal with respect to a given attribute, then such an attribute will be judged by unimportant by most decision makers. Such an attribute does not help in making a decision [18]. To address this issue, they propose a variance coefficient method based on the Shannon entropy to improve Wu's method. However, they still did not give any concrete reasons why the cross-efficiencies should be aggregated by different weights.

To overcome these shortfalls, we suggest the use of principal component analysis for cross-efficiency aggregation. The proposed method views the cross-efficiency calculated by the same set of weights as an attribute. The cross-efficiency values will transform to be the different attribute values of DMUs. Because the attributes play different roles to reflect the system information (evaluated objects information), so they should be allocated different weights while aggregating them. In this paper, we use principal component analysis to generate the relative importance weights for determining the ultimate cross-efficiency for each DMU.

The rest of paper is organized as follows: Section 2 gives a brief introduction to cross-efficiency evaluation. The cross-efficiency aggregation based on principal component analysis is shown in Section 3. Section 4 presents two illustrative examples and conclusions are made in Section 5.

II. Cross-efficiency evaluation and aggregation

n DMUS are to be evaluated where m inputs are consumed to produce s outputs. The inputs and outputs value of $DMU_j (j=1,\cdots,n)$ are denoted by $x_{ij} (i=1,\ldots,m)$ and $y_{rj} (r=1,\ldots,s)$. The CCR efficiency of DMU_k can be measured by the following model:

$$
\begin{aligned}
&\textit{Maximize} \quad \theta_{kk} = \sum_{r=1}^{s} u_{rk} y_{rk} \\
&\textit{Subject to} \quad \sum_{i=1}^{m} v_{ik} x_{ik} = 1 \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0,\ j = 1,\ldots,n, \\
&\qquad u_{rk}, v_{ik} \ge 0,\ r = 1,\ldots,s;\ i = 1,\ldots,m
\end{aligned} \tag{1}
$$

Where $DMU_k \in \{DMU_1 \ldots, DMU_n\}$ is the decision-making unit (DMU) under evaluation, $v_{ik}\,(i = 1,\ldots,m)$ and $u_{rk}\,(r = 1,\ldots,s)$ are the inputs and outputs weights. If $u^*_{rk}\,(r = 1,\ldots,s)$ and $v^*_{ik}\,(i = 1,\ldots,m)$ are the optimal solution to the above CCR model, the $\theta^*_{kk} = \sum_{r=1}^{s} u^*_{rk} y_{rk}$ will be the CCR efficiency value of DMU_k. If θ^*_{kk} is equal to 1, the DMU_k is referred to be CCR efficient; Otherwise, it will be non-CCR efficient. $\theta_{jk} = \sum_{r=1}^{s} u^*_{rk} y_{rj} / \sum_{i=1}^{m} v^*_{ik} x_{ij}$ is viewed as the cross-efficiency of DMU_j evaluated by the weights determined by DMU_k, $j = 1,\ldots,n,\ j \ne k$.

CCR model (1) is solved for each of n DMUS respectively. Each DMU will obtain a set of weights that is most favorable to itself. Based on n sets of weights, n cross-efficiencies shown in table I including one self-evaluated efficiency value and $n-1$ peer-evaluated efficiency values will be generated for each DMU. The traditional way to aggregate them is to simply aggregate them with equal weights. However, average cross-efficiency (ACE) did not consider the difference among cross-efficiencies, and ACE results sometimes cannot reflect the actual performances of DMUs fairly. To eliminate the shortfall of ACE, the cross-efficiencies should be weighted before aggregation. The ultimate efficiency score for each DMU will become weighted average cross-efficiency (WACE) score:

$$\bar{\theta}_i = \sum_{k=1}^{n} w_k \theta_{ik},\ i = 1,\ldots,n, \tag{2}$$

where $w_1,\ldots,w_n$ are weights, satisfying $w_k \ge 0 (k = 1,\ldots,n)$.

It is noticed that the optimal solution for CCR model maybe not unique that will damage the use of cross-efficiency evaluation. To handle this problem, many secondary goal models were proposed. Among them, aggressive and benevolent secondary goals are most widely used. Their formulations are as follow respectively:

$$
\begin{aligned}
&\textit{Minimize} \quad \sum_{r=1}^{s} u_{rk} \left(\sum_{j=1, j\ne k}^{n} y_{rj} \right) \\
&\textit{Subject to} \quad \sum_{i=1}^{m} v_{ik} \left(\sum_{j=1, j\ne k}^{n} x_{ij} \right) = 1 \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rk} - \theta^*_{kk} \sum_{i=1}^{m} v_{ik} x_{ik} = 0 \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \quad j = 1,\ldots,n;\ j \ne k, \\
&\qquad u_{rk} \ge 0, \quad r = 1,\ldots,s, \\
&\qquad v_{ik} \ge 0, \quad i = 1,\ldots,m,
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
&\textit{Maximize} \quad \sum_{r=1}^{s} u_{rk} \left(\sum_{j=1, j\ne k}^{n} y_{rj} \right) \\
&\textit{Subject to} \quad \sum_{i=1}^{m} v_{ik} \left(\sum_{j=1, j\ne k}^{n} x_{ij} \right) = 1 \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rk} - \theta^*_{kk} \sum_{i=1}^{m} v_{ik} x_{ik} = 0 \\
&\qquad \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \quad j = 1,\ldots,n;\ j \ne k, \\
&\qquad u_{rk} \ge 0, \quad r = 1,\ldots,s, \\
&\qquad v_{ik} \ge 0, \quad i = 1,\ldots,m,
\end{aligned} \tag{4}
$$

However, the unique set of weights selected by aggressive formulation usually contains many zero weights where some inputs or outputs information are ignored while calculating cross-efficiencies. To avoid this, we will use the cross-efficiency matrix determined by benevolent formulation to aggregate, but the proposed aggregation method is also applicable to cross-efficiencies determined by aggressive and other formulations.

TABLE I

CROSS-EFFICIENCY MATRIX AND WACE

DMUs	Target DMU				Weighted Average
	1	2	$\cdots$	n	Cross-efficiency (WACE)
1	θ_{11}	θ_{12}	$\cdots$	θ_{1n}	$\sum_{k=1}^{n} w_k \theta_{1k}$
2	θ_{21}	θ_{22}	$\cdots$	θ_{2n}	$\sum_{k=1}^{n} w_k \theta_{2k}$
…	…	…	…	…	…
n	θ_{n1}	θ_{n2}	$\cdots$	θ_{nn}	$\sum_{k=1}^{n} w_k \theta_{nk}$

III. Determination of Ultimate Cross-Efficiency Using Principal Component Analysis

To provide the WACE for each DMU, the main issue is to determine the non-equal weights for cross-efficiency aggregation. This paper proposes the use of principal component analysis to determine them. The proposed method views the cross-efficiencies calculated by the same set of weights as an attribute values of DMUs. The cross-efficiencies will transform to be attribute values of DMUs. Because of their importance difference, they should be given different weights. Because the different attributes usually are linear correlated, it cause the difficulty to aggregate them. The principal component analysis is a commonly used evaluation method to handle linear correlated multiple attributes. It provides a comprehensive evaluation value for evaluated objects through transforming the linear correlated multiple attributes to some principal components that are linear uncorrelated each other. The calculation procedure of using principal component analysis is as follows. Step 1 standardizes the original attributes values. Step 2 calculates the correlation matrix R. Step 3 calculates the characteristic roots ($\lambda_1,...\lambda_n$) of R and component matrix. Step 4 selects the principal component whose characteristic root is more than 1. Step 5 determines the formulation of the selected principal components through their characteristic roots and factor loadings to original attributes shown in the component matrix. Step 6 provides the comprehensive results for evaluated objects after setting the individual variance contribution rate to be the weight of the selected principal components.

IV. NUMERICAL EXAMPLES

In this part, two numerical examples are illustrated to show how the cross-efficiencies will be aggregated using principal component analysis.

Example 1: Five DMUs are evaluated where two inputs are consumed to produce one output [11]. The output is normalized. It clearly shows that four of them are CCR efficient and efficient DMUs cannot be distinguished further from table II. To handle this question, DEA cross-efficiency evaluation is an alternative. Table III and IV show the unique set of weights selected by benevolent model and corresponding cross-efficiencies matrix. The cross-efficiencies are aggregated by equal weights in table IV but the ACE results ignore the difference among cross-efficiencies. If we view the cross-efficiencies calculated by the same set of weights as an attributes values of DMUs, the ACE treatment simply considers that all attributes can reflect the system information (evaluated objects information) equally. This straightforward and rough treatment is to be questioned easily. The table V and VI show the OWA operator weights for cross-efficiency aggregation and corresponding aggregation results. They clearly show that different optimism level of DM leads to different results. At the same time, it is difficult to measure actual optimism level of DM. In the next, we will provide the weights for cross-efficiency aggregation through principal component analysis. This method views the cross-efficiency determined by the same set of weights as an attribute. The cross-efficiencies will be transformed to be attributes values for DMUs. Because the attributes play different roles to reflect the system information (evaluated objects information), so they should be allocated different weights while aggregating them. We view the cross-efficiencies calculated by the set of weights determined by DMU1, DMU2, DMU3, DMU4, DMU5 as attributes x_1, x_2, x_3, x_4, x_5 values. The cross-efficiency matrix in table IV will be transformed to attributes values shown in table VII. We can obtain the standard attributes values, the correlation matrix R shown in table VIII, the characteristic roots ($\lambda_1,...\lambda_n$) of R and component matrix shown in table IX through calculation software SPSS. From the correlation matrix R, it clearly shows that the attributes have a high correlation coefficient each other, and this numerical example is suitable for the use of principal component analysis. Through the selection criterion whose characteristic root (variance) is more than 1, we select two principal components F_1, F_2 whose characteristic roots are 2.718, 2.232 respectively. Though the component matrix and the characteristic roots of two principal components, we can attain the formulation of F_1, F_2. The coefficient of F_i to x_i is calculated by $V/\sqrt{\lambda_i}$ (V is F_i's factor load to x_i shown in the component matrix). Through the formulation, we can attain that: $F_1 = -0.4076x_1 + 0.2384x_2 + 0.2384x_3 + 0.5999x_4 + 0.5999x_5$,

$F_2 = 0.4826x_1 + 0.6145x_2 + 0.6145x_3 - 0.0803x_4 - 0.0803x_5$.

The ultimate cross-efficiency for each DMU shown in table X is attained through the formulation: $C = 0.54361F_1 + 0.44647F_2$, where 0.54361 and 0.44647 are the individual variance contribution rate of F_1 and F_2. It also means that the weights allocated to x_1, x_2, x_3, x_4, x_5 (different cross-efficiencies) are -0.0061, 0.4040, 0.4040, 0.2903, 0.2903 respectively that reflects their different roles in reflecting the system information. From table X where the attributes values (cross-efficiencies) are standardized, it shows that the rank is different from ACE results. Quite evidently, the rank between DMU2 and DMU3 is reversed. The rank between DMU1 and DMU5 is also reversed. Meantime, the aggregation result by PCA is fixed and unique.

TABLE II

DMUS WITH TWO INPUTS AND ONE NORMALIZED OUTPUT AND THEIR CCR EFFICIENCIES

DMUS	Input1	Input2	Output	CCR-efficiency
1	2	12	1	1
2	2	8	1	1
3	5	5	1	1
4	10	4	1	1
5	10	6	1	0.75

TABLE III

INPUTS AND OUTPUTS WEIGHTS DETERMINED BY EACH DMU THROUGH BENEVOLENT FORMULATION (4)

DMUS	Input1	Input2	Output
1	0.037037	0	0.074074
2	0.018519	0.018519	0.185185
3	0.018519	0.018519	0.185185
4	0.005747	0.028736	0.172414
5	0.006098	0.030488	0.182927

TABLE IV

BENEVOLENT CROSS-EFFICIENCY MATRIX AND AVERAGE CROSS-EFFICIENCY RESULTS

DMUS	Target DMU					ACE	Rank
	1	2	3	4	5		
1	1.00	0.71	0.71	0.48	0.48	0.68	4
2	1.00	1.00	1.00	0.72	0.72	0.89	1
3	0.40	1.00	1.00	1.00	1.00	0.88	2
4	0.20	0.71	0.71	1.00	1.00	0.72	3
5	0.20	0.63	0.63	0.75	0.75	0.59	5

Example 2: Efficiency evaluation of 7 colleges in one university with three inputs and three outputs [20]. It clearly shows that six of them are CCR efficient from table XI and they cannot be distinguished further though CCR model. DEA cross-efficiency evaluation can give a unique rank order for each DMU. Table XII shows the benevolent cross-efficiency matrix and average cross efficiency (ACE) which shows DMU6 performs best. Table XIII and XIV show the OWA operator weights and aggregation results. They still show that different optimism level of DM leads to different results.

Considering the different roles of cross-efficiencies in reflecting system information (evaluated objects information), we use principal component analysis to aggregate cross-efficiencies. The results are shown in table XV where the attributes values (cross-efficiencies) are standardized. It is clearly shown that the rank of DMU1 and DMU2 and DMU7 is different from their ranks based on ACE. Different from the non-uniqueness by OWA operator, the aggregation results are unique by PCA.

TABLE V

OWA OPERATOR WEIGHTS FOR CROSS-EFFICIENCY AGGREGATION

Ranked Position	Optimism Level of DM					
	α=1	α=0.9	α=0.8	α=0.7	α=0.6	α=0.5
1st	1	0.6333	0.4600	0.3600	0.28	0.2
2nd	0	0.3333	0.3200	0.2800	0.24	0.2
3rd	0	0.0333	0.1800	0.20	0.2	0.2
4th	0	0	0.0400	0.12	0.16	0.2
5th	0	0	0	0.04	0.12	0.2

TABLE VI

CROSS-EFFICIENCY AGGREGATION BY OWA OPERATOR WEIGHTS

DMUS	Optimism Level of DM					
	α=1	α=0.9	α=0.8	α=0.7	α=0.6	α=0.5
1	1	0.8875	0.8365	0.7803	0.7298	0.6793
2	1	1	0.9886	0.9543	0.9200	0.8857
3	1	1	1	0.9760	0.9280	0.8800
4	1	0.9904	0.9371	0.8766	0.8011	0.7257
5	0.75	0.7458	0.7225	0.6880	0.6390	0.5900

TABLE VII

THE 5 ATTRIBUTES VALUE OF DMUS

DMUS	Attributes				
	x_1	x_2	x_3	x_4	x_5
1	1.0000	0.7143	0.7143	0.4839	0.4839
2	1.0000	1.0000	1.0000	0.7143	0.7143
3	0.4000	1.0000	1.0000	1.0000	1.0000
4	0.2000	0.7143	0.7143	1.0000	1.0000
5	0.2000	0.6250	0.6250	0.7500	0.7500

V. CONCLUSIONS

To improve the discrimination power of DEA traditional models, DEA Cross-efficiency evaluation was proposed. T-

TABLE VIII

CORRELATION MATRIX

	x_1	x_2	x_3	x_4	x_5
x_1	1.000	0.387	0.387	-0.737	-0.737
x_2	0.387	1.000	1.000	0.274	0.274
x_3	0.387	1.000	1.000	0.274	0.274
x_4	-0.737	0.274	0.274	1.000	1.000
x_5	-0.737	0.274	0.274	1.000	1.000

TABLE IX

COMPONENT MATRIX

	Components	
	1	2
x_1	-0.672	0.721
x_2	0.393	0.918
x_3	0.393	0.918
x_4	0.989	-0.120
x_5	0.989	-0.120

TABLE X

CROSS-EFFICIENCY AGGREGATION THOUGH PRINCIPAL COMPONENT ANALYSIS

DMUs	x_1	x_2	x_3	x_4	x_5	F_1	F_2	WACE	Rank
1	1.0735	-0.5460	-0.5460	-1.4057	-1.4057	-2.3845	0.0728	-1.2637	5
2	1.0735	1.0719	1.0719	-0.3464	-0.3464	-0.3421	1.8911	0.6584	2
3	-0.3904	1.0719	1.0719	0.9672	0.9672	1.8307	0.9736	1.4299	1
4	-0.8783	-0.5460	-0.5460	0.9672	0.9672	1.2581	-1.2502	0.1257	3
5	-0.8783	-1.0517	-1.0517	-0.1823	-0.1823	-0.3622	-1.6871	-0.9501	4

he scholars mainly focused on the calculation of cross-efficiency matrix but pay little attention to cross-efficiency aggregation process. They simply aggregate them with equal weights to provide the average cross-efficiency score for each DMU. However, this treatment to cross-efficiencies did not consider their difference and that will result in ultimate efficiency score for each DMU cannot reflect the true performance of DMUs fairly. To eliminate the shortfall, cross-efficiencies need to be weighted before aggregation. In this paper, we view the cross-efficiencies calculated by the same set of weights as an attribute values of DMUs. Because different attributes will play a distinguished role to reflect the whole system information (evaluated objects information), so the attributes values (cross-efficiencies) should be weighted differently while aggregating them. In this paper, we propose the use of principal component analysis to generate distinguished weights for cross-efficiency aggregation.

Compared with other non-equal weights determination methods of Wu et al. [12-15], Wang et al. [11], Lianlian Song and Fan Liu [17], our proposed approach is more reasonable and clearer in modeling mechanism where we p-

TABLE XI

DATA AND CCR-EFFICIENCY FOR 7 ACADEMIC COLLEGES IN A UNIVERSITY

DMUs	Inputs			Outputs			CCR-efficiency
	x_1	x_2	x_3	y_1	y_2	y_3	
1	12	400	20	60	35	17	1
2	19	750	70	139	41	40	1
3	42	1500	70	225	68	75	1
4	15	600	100	90	12	17	0.8197
5	45	2000	250	253	145	130	1
6	19	730	50	132	45	45	1
7	41	2350	600	305	159	97	1

TABLE XII

BENEVOLENT CROSS-EFFICIENCY MATRIX AND ACE OF ACADEMIC COLLEGES IN A UNIVERSITY

DMUs	Target DMU							ACE	Rank
	1	2	3	4	5	6	7		
1	1.0000	0.9219	1.0000	0.6875	1.0000	1.0000	1.0000	0.9442	3
2	0.9812	1.0000	0.8510	1.0000	0.8461	0.9812	0.9812	0.9486	2
3	0.7690	0.7719	1.0000	0.7349	0.6651	0.7690	0.7690	0.7827	6
4	0.6411	0.7013	0.4542	0.8197	0.4135	0.6411	0.6411	0.6160	7
5	0.9382	0.8990	0.4950	0.7650	1.0000	0.9382	0.9382	0.8534	5
6	1.0000	1.0000	1.0000	0.9506	0.9104	1.0000	1.0000	0.9801	1
7	1.0000	1.0000	0.2941	1.0000	1.0000	1.0000	1.0000	0.8992	4

TABLE XIII

OWA OPERATOR WEIGHTS FOR CROSS-EFFICIENCY AGGREGATION

Ranked Position	Optimism Level of DM					
	α=1	α=0.9	α=0.8	α=0.7	α=0.6	α=0.5
1st	1	0.5333	0.36	0.2714	0.2071	0.1428
2	0	0.3333	0.28	0.2285	0.1857	0.1428
3	0	0.1333	0.2	0.1856	0.1643	0.1428
4	0	0	0.12	0.1427	0.1429	0.1428
5	0	0	0.04	0.0998	0.1215	0.1428
6	0	0	0	0.0569	0.1001	0.1428
7	0	0	0	0.014	0.0787	0.1428

TABLE XIV

CROSS-EFFICIENCY AGGREGATION BY OWA OPERATOR WEIGHTS

DMUS	Optimism Level of DM					
	α=1	α=0.9	α=0.8	α=0.7	α=0.6	α=0.5
1	1	1	1	1	0.9679	0.9442
2	1	0.9974	0.9932	0.9802	0.9652	0.9486
3	1	0.8931	0.8530	0.8281	0.8060	0.7827
4	0.8197	0.7563	0.7223	0.6888	0.6528	0.6160
5	1	0.9711	0.9589	0.9340	0.8943	0.8534
6	1	1	1	0.9948	0.9883	0.9801
7	1	1	1	0.9890	0.9447	0.8992

TABLE XV

CROSS-EFFICIENCY AGGREGATION THOUGH PRINCIPAL COMPONENT ANALYSIS

DMUs	x_1	x_2	x_3	x_4	x_5	x_6	x_7	F_1	F_2	WACE	Rank
1	0.67	0.19	0.89	-1.25	0.75	0.67	0.67	1.10	1.67	1.10	2
2	0.54	0.84	0.41	1.14	0.06	0.54	0.54	1.36	-0.44	0.89	4
3	-0.95	-1.06	0.89	-0.89	-0.76	-0.95	-0.95	-2.16	1.09	-1.35	6
4	-1.84	-1.65	-0.90	-0.24	-1.90	-1.84	-1.84	-4.07	-0.88	-3.08	7
5	0.24	-0.01	-0.77	-0.66	0.76	0.24	0.24	0.45	0.005	0.323	5
6	0.67	0.84	0.89	0.76	0.35	0.67	0.67	1.62	0.23	1.20	1
7	0.67	0.84	-1.43	1.13	0.75	0.67	0.67	1.71	-1.67	0.92	3

rovide the concrete reasons why cross-efficiencies should be allocated different weights for aggregation. Meanwhile, using our proposed method it does not need to measure the optimism level of decision maker and the results are fixed. Although the proposed method has many advantages, it is only applicable to the situations where there exists correlation relation among cross-efficiencies. Finally, the proposed approach further enriches DEA cross-efficiency aggregation theory.

ACKNOWLEDGMENT

The authors are indebted to the editors and the reviewers for their constructive comments and suggestions which significantly improved the quality of the paper.

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