Optimal Policies in a Green Supply Chain with Socially Responsible Manufacturer

Shengju Sang

Abstract—**This paper explores the influences of the social responsibility of the manufacturer on the pricing and greening level decisions in a two-echelon green supply chain with one manufacturer and one retailer. The manufacturer has social concerns in addition to economic goal and cares about the consumer surplus. Three game theory models with Manufacturer-Stackelberg (MS) game, Retailer -Stackelberg (RS) game and Vertical-Nash (VN) game are developed, and their optimal solutions are also derived. Finally, the results of the proposed game models are analyzed via a numerical example. Finally, the results of the proposed game models are analyzed via a numerical example. The results show that the greening level of the product increases and the retail price decreases with the increasing social responsibility of the manufacturer. Also, the retailer and the consumer can benefit, while the manufacturer can suffer with consideration of the social responsibility of the manufacturer in the three games.**

Index Terms—**green supply chain, corporate social responsible, game theory**

I. Introduction

NOWADAYS, green supply chain has received a significant concern from governments, industries and consumers around the world. A large group of consumers prefer to buy green products which are more environmentally friendly. Therefore, it is a vital issue for the manufacturer and the retailer how to make their optimal pricing and greening level decisions in the channel.

Over the past decades, there are several studies which have explored optimal policies in a green supply chain. For example, Ghosh and Shah [1] studied the pricing and greening strategies with both decentralized channel policy and cooperative policy. Chen et al. [2] explored the pricing and greening strategies in a duopoly green supply chain with vertical and horizontal competition, which included a green manufacturer, a traditional manufacturer and a common retailer. Taleizadeh1 and Heydarian1 [3] developed the pricing and refund optimization problem with green product and non-green product under both non-cooperative and cooperative strategies in a two stage supply chain. Hafezalkotob [4] developed the price-energy-saving competition and cooperation models for two green supply chains under government financial intervention. Yang et al. [5] studied the pricing and carbon emission reduction decisions in two competitive supply chains with vertical and horizontal cooperation. They found that the manufacturers' horizontal cooperation would damage retailers' profit and consumers' welfare. Liu and Yi [6] studied the pricing policies of green supply chain considering targeted advertising and product green degree in the Big Data environment. Basiri and Heydari [7] investigated three decision scenarios including decentralized scenario, integrated scenario and collaborative scenario in a green supply chain. Sang [8] analyzed the influences of reference price effect and fairness concerns on the pricing and greening level decisions in a two stage green supply chain. Ma et al. [9] proposed six game models to explore the optimal pricing strategies of green supply chain with two competitive manufacturers and one retailer. Recently, some researchers have studied the optimal policies of green supply chain in an uncertain environment. For example, Yang and Xiao [10] studied the pricing and greening level decisions of a green supply chain with governmental interventions under fuzzy uncertainties. Sang [11] developed three different decentralized decision models of green supply chain, in which the production cost and market demand were fuzzy. Sang [12] also studied the greening level and pricing decisions with a risk averse retailer in a green supply chain under uncertain demand environment.

Some scholars also studied the coordination mechanisms of green supply chain. For example, Swami and Shah [13] studied the channel coordination of the supply chain by a two part tariff. In a competitive dual channel green supply chain, Li et al. [14] examined the pricing policies in both centralized and decentralized cases and also used a two part tariff contract to coordinate the decentralized dual-channel green supply chain Zhang et al. [15] designed a two part tariff contract to coordinate the decentralized green supply chain in a dynamic environment. Ghosh and Shah [16] adopt the cost sharing contract for coordinating a green supply chain. They showed that cost sharing contracts resulted in high supply chain profits, but failed to reach the optimal profits of the integrated setting. Zhu et al. [17] explore the coordination mechanism of cost sharing for green food production and marketing between a food producer and a supplier who both contribute to the sales of green food. Song and Gao [18] proposed a retailer led revenue sharing contract game model and a bargaining revenue sharing contract game model to coordinate green supply chain. Taleizadeh et al. [19] used three coordination contracts including wholesale price contract, cost sharing contract and buyback contract to enhance the performance of the green supply chain. Raj et al. [20] used five different contract types, namely wholesale price, linear two part tariff, greening cost sharing, revenue sharing, and revenue and greening cost sharing contracts to study the coordination issues of a green supply chain. Hong and Guo [21] analyzed price-only contract, green-marketing cost-sharing contract

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and two-part tariff contract and investigated their environmental performance. They found that cooperation between manufacturers and retailers would not always benefit all members.

Our work is also related to previous research on corporate social responsibility (CSR) in a supply chain. In global business environment, CSR is now a determining factor in choice of consumers that cannot be ignored by the supply chain members. Panda [22] used the revenue sharing contract to coordinate a socially responsible supply chain with a corporate social responsible retailer or corporate social responsible manufacturer. Panda et al. [23] also analyzed profit distribution and channel coordination through contract bargaining process with a social responsible manufacturer in a three stage supply chain. Modak et al. [24] explored pricing policy and channel coordination with two competitive retailers and a socially responsible manufacturer in a two stage supply chain. In a closed-loop supply chain, Panda et al. [25] analyzed the effects of corporate social responsibility and explored channel coordination though a revenue sharing contract. Sinayi and Rasti-Barzoki [26] studied the pricing, greening and social welfare policies in a supply chain with government intervention. They showed that the different government policies had a greater effect on the pricing, greening level and profit. Raza [27] proposed a supply chain coordination scheme for pricing inventory, and corporate social responsibility investments decisions with one manufacturer and one retailer in a supply chain. Liu et al. [28] investigated the pricing and environmental governance efficiency decisions and channel coordination of a dyadic tourism supply chain with corporate social responsibility.

To our knowledge, no one has studied the pricing and greening level decisions with different power structures in a socially responsible green supply chain. Therefore, in this paper, we consider that the manufacturer has social concerns and cares about the social welfare of the consumers. We mainly analyze the conditions where the manufacturer and the retailer pursue three non-cooperative games: Manufacturer-Stackelberg (MS) game, Retailer-Stackelberg (RS) game and Vertical-Nash (VN) game. We try to find how the socially responsible of the manufacturer affects the optimal decisions of the supply chain members.

The rest of this paper is organized as follows. The problem description and assumptions related to this paper are described in Section II. Three different kinds of non-cooperative game models with a socially responsible manufacturer in a green supply chain are developed in Section III, and then the numerical example is shown in Section IV. Concluding remarks and some further research ideas are provided in Section V.

II. Model Description

We consider a two stage green supply chain with one manufacturer and one retailer. The manufacturer sells green products to the retailer directly at a wholesale price, and then the retailer sells products to consumers at a retail price. Consumers are sensitive to the green products, and need to consider both the retail price and the greening level when buying products.

The basic notations are shown in Table I.

TABLE I
PARAMETERS AND ITS MEANING

Parameter	Meaning
α	The market potential
β	The retail price sensitivity of the consumers
γ	The greening level sensitivity of the consumers
p	The retail price
θ	The greening level
η	The cost coefficient of the investments
q	The market demand
w	The wholesale price
m	The profit margin
c	The cost of the producing green product
μ	the fraction of the consumer surplus of the socially responsible manufacturer's concern
CS	The consumer surplus
Π_R	The profit of the retailer
Π_M	The pure profit of the manufacturer
V_M	The total profit of the manufacturer
Π_{SC}	The profit of the supply chain

To formulate the problem, some assumptions are made:

Assumption 1. The demand faced by the manufacturer and the retailer is a function of the retailer price p and the greening level of the product θ, thus the demand function is $q=\alpha-\beta p+\gamma\theta$, in which $\alpha>0$ is the market potential, $\beta>0$ is the retail price sensitivity of the consumers and $\gamma>0$ is the greening level sensitivity of the consumers. Since the retail price p equals to the wholesale price w plus the profit margin m, then the demand function can be rewritten as $q=\alpha-\beta(w+m)+\gamma\theta$.

Assumption 2. The production of the green product doesn't affect the manufacturer's traditional producing cost. To produce the green product, the manufacturer must make extra investments to get the green innovation based on the original production process. The cost of achieving green innovation is a quadratic function of the greening level of the product θ. The investment is $\frac{1}{2}\eta\theta^2$, in which η is the cost coefficient.

Assumption 3. The manufacturer has social concerns and cares about the social welfare of the consumers, therefore the manufacturer considers consumer surplus as an index of social welfare in its profit function. Consumer surplus is the difference between the total amount that consumers are willing and able to pay for a product and the total amount that they actually pay. Thus the consumer surplus is

$$CS=\int_{p_{\min}}^{p_{\max}} q\,\mathrm{d}\,p=\int_{\frac{\alpha+\gamma\theta-q}{\beta}}^{\frac{\alpha+\gamma q}{\beta}}\left(\alpha-\beta p+\gamma\theta\right)\mathrm{d}\,p$$

$$=\frac{1}{2\beta}\left(\alpha-\beta p+\gamma\theta\right)^2=\frac{q^2}{2\beta} \quad (1)$$

If $\mu\in[0,1]$ is the fraction of the consumer surplus that the socially responsible manufacturer's concern, then the amount of consumer surplus incorporated in the manufacturer's profit is

$$\mu CS = \frac{\mu}{2\beta}(\alpha - \beta p + \gamma\theta)^2 = \frac{\mu q^2}{2\beta} \quad (2)$$

Note that, when $\mu = 0$, the manufacturer is pure profit maximized, whereas $\mu = 1$ indicates that the manufacturer is a perfect welfare maximized.

Assumption 4. To ensure the existence of the optimal solutions, we assume $\frac{\beta\eta}{\gamma^2} > 1$.

Under the model setting, the profit function of the retailer, purer and total profit functions of the manufacturer are given below:

$$\pi_R = m\left[\alpha - \beta(w+m) + \gamma\theta\right] \quad (3)$$

$$\pi_M = (w-c)\left[\alpha - \beta(w+m) + \gamma\theta\right] - \frac{1}{2}\eta\theta^2 \quad (4)$$

$$\begin{aligned} V_M &= \pi_M + \mu CS \\ &= (w-c)\left[\alpha - \beta(w+m) + \gamma\theta\right] - \frac{1}{2}\eta\theta^2 \\ &\quad + \frac{\mu}{2\beta}\left[\alpha - \beta(w+m) + \gamma\theta\right]^2 \end{aligned} \quad (5)$$

III. Models Analysis

In this section, we discuss the supply chain members how to set their optimal policies when the manufacturer has social concerns and cares about the social welfare of the consumers with different power structures. We mainly analyze the conditions where they pursue three non-cooperative games: the manufacturer dominates the supply chain, the retailer dominates the supply chain, and they have the same power. In the following discussion, we use superscripts *MS*, *RS*, and *VN* to denote that the corresponding quantities are for the MS (Manufacturer-Stackelberg), RS (Retailer-Stackelberg) and VN (Vertical-Nash) cases, respectively.

A. MS game model

The MS (Manufacturer-Stackelberg) game model arises in the market where the size of the retailer is smaller compared to the manufacturer. In this case, the manufacturer is the leader, and the retailer is the follower. That is, firstly, the manufacturer sets the greening level θ and the wholesale price w using the retailer's reaction function to maximize his total profit. Then, the retailer sets the profit margin m so as to maximize his profit. Thus, the MS game model can be given as follows

$$\begin{aligned} \max_{\theta,w} V_M &= (w-c)\left[\alpha - \beta(w+m) + \gamma\theta\right] - \frac{1}{2}\eta\theta^2 \\ &\quad + \frac{\mu}{2\beta}\left[\alpha - \beta(w+m) + \gamma\theta\right]^2 \end{aligned} \quad (6)$$

$$s.t. \begin{cases} m = \arg\max \pi_R \\ \max_m \pi_R = m\left[\alpha - \beta(w+m) + \gamma\theta\right] \end{cases}$$

Theorem 1. In the MS game model, the optimal policies of the manufacturer and the retailer are as follows

$$\theta^{MS} = \frac{\gamma(\alpha - \beta c)}{(4-\mu)\beta\eta - \gamma^2} \quad (7)$$

$$w^{MS} = \frac{(2-\mu)\eta(\alpha - \beta c)}{(4-\mu)\beta\eta - \gamma^2} + c \quad (8)$$

$$m^{MS} = \frac{\eta(\alpha - \beta c)}{(4-\mu)\beta\eta - \gamma^2} \quad (9)$$

Proof. First we solve the profit function of the retailer as follows

$$\max_m \pi_R = m\left[\alpha - \beta(w+m) + \gamma\theta\right] \quad (10)$$

The first order condition is

$$\frac{\partial \pi_R}{\partial m} = -2\beta m - \beta w + \alpha + \gamma\theta$$

Then the second order condition is

$$\frac{\partial^2 \pi_R}{\partial m^2} = -2\beta$$

Note that the second order condition of π_R is negative definite, since $\beta > 0$. Consequently, π_R is strictly concave in *m*. Hence, the optimal reaction function of the retailer can be obtained by solving the first order condition as below

$$-2\beta m - \beta w + \alpha + \gamma\theta = 0 \quad (11)$$

Solving (11), we can obtain the optimal response function of the retailer as

$$m^{MS}(\theta, w) = \frac{\gamma\theta - \beta w + \alpha}{2\beta} \quad (12)$$

Next we solve the total profit function of the manufacturer

$$\begin{aligned} \max_{\theta,w} V_M &= (w-c)\left[\alpha - \beta(w+m) + \gamma\theta\right] - \frac{1}{2}\eta\theta^2 \\ &\quad + \frac{\mu}{2\beta}\left[\alpha - \beta(w+m) + \gamma\theta\right]^2 \end{aligned} \quad (13)$$

Substituting $m^{MS}(\theta, w)$ into (13), we get

$$\begin{aligned} \max_{\theta,w} V_M &= \frac{1}{2}(w-c)(\alpha - \beta w + \gamma\theta) - \frac{1}{2}\eta\theta^2 \\ &\quad + \frac{\mu}{8\beta}(\alpha - \beta w + \gamma\theta)^2 \end{aligned} \quad (14)$$

The first order conditions are

$$\frac{\partial V_M}{\partial \theta} = -\frac{4\beta\eta - \mu\gamma^2}{4\beta}\theta + \frac{(2-\mu)\gamma}{4}w + \frac{\gamma(\mu\alpha - 2\beta c)}{4\beta}$$

$$\frac{\partial V_M}{\partial w} = -\frac{(4-\mu)\beta}{4}w + \frac{(2-\mu)\gamma}{4}\theta + \frac{2-\mu}{4}\alpha + \frac{1}{2}\beta c$$

Therefore, the Hessian matrix of V_M is

$$\mathrm{H} = \begin{bmatrix} \frac{\partial^2 V_M}{\partial \theta^2} & \frac{\partial^2 V_M}{\partial \theta \partial w} \\ \frac{\partial^2 V_M}{\partial w \partial \theta} & \frac{\partial^2 V_M}{\partial w^2} \end{bmatrix} = \begin{bmatrix} -\frac{(4-\mu)\beta}{4} & \frac{(2-\mu)\gamma}{4} \\ \frac{(2-\mu)\gamma}{4} & -\frac{4\beta\eta - \mu\gamma^2}{4\beta} \end{bmatrix}$$

Note that the Hessian matrix of V_M is negative definite, since $0 \le \mu \le 1$, $\beta > 0$, $\gamma > 0$ and $\frac{\beta\eta}{\gamma^2} > 1$. Consequently, V_M is strictly jointly concave in θ and w. Hence, the optimal policies of the manufacturer can be obtained by solving the first order conditions as below

$$-\frac{4\beta\eta-\mu\gamma^2}{4\beta}\theta+\frac{(2-\mu)\gamma}{4}w+\frac{\gamma(\mu\alpha-2\beta c)}{4\beta}=0 \quad (15)$$

$$-\frac{(4-\mu)\beta}{4}w+\frac{(2-\mu)\gamma}{4}\theta+\frac{2-\mu}{4}\alpha+\frac{1}{2}\beta c=0 \quad (16)$$

Solving (15) and (16) simultaneously, we can obtain the optimal greening level θ^{MS} and the wholesale price w^{MS} of the manufacturer as follows

$$\theta^{MS}=\frac{\gamma(\alpha-\beta c)}{(4-\mu)\beta\eta-\gamma^2}$$

$$w^{MS}=\frac{(2-\mu)\eta(\alpha-\beta c)}{(4-\mu)\beta\eta-\gamma^2}+c$$

Substituting θ^{MS} and w^{MS} into (12), we can obtain the optimal margin profit m^{MS} of the retailer as follows

$$m^{MS}=\frac{\eta(\alpha-\beta c)}{(4-\mu)\beta\eta-\gamma^2}$$

The proof of Theorem 1 is completed.

Then, the optimal retail price p^{MS} and market demand q^{MS} can be obtained as

$$p^{MS}=w^{MS}+m^{MS}=\frac{(3-\mu)\eta(\alpha-\beta c)}{(4-\mu)\beta\eta-\gamma^2}+c \quad (17)$$

$$q^{MS}=\alpha-\beta\left(w^{MS}+m^{MS}\right)+\gamma\theta^{MS}=\frac{\beta\eta(\alpha-\beta c)}{(4-\mu)\beta\eta-\gamma^2} \quad (18)$$

By combining (7), (8) and (9) with (3), (4) and (5), we derive the retailer's optimal profit, the manufacturer's optimal purer and total profits in the MS game model as follows

$$\pi_R^{MS}=\frac{\beta\eta^2(\alpha-\beta c)^2}{\left[(4-\mu)\beta\eta-\gamma^2\right]^2} \quad (19)$$

$$\pi_M^{MS}=\frac{\left[2(2-\mu)\beta\eta-\gamma^2\right]\eta(\alpha-\beta c)^2}{2\left[(4-\mu)\beta\eta-\gamma^2\right]} \quad (20)$$

$$V_M^{MS}=\frac{\eta(\alpha-\beta c)^2}{2\left[(4-\mu)\beta\eta-\gamma^2\right]} \quad (21)$$

The profit of the supply chain system and the consumer surplus are

$$\pi_{SC}^{MS}=\pi_R^{MS}+\pi_M^{MS}=\frac{\left[2(3-\mu)\beta\eta-\gamma^2\right]\eta(\alpha-\beta c)^2}{2\left[(4-\mu)\beta\eta-\gamma^2\right]^2} \quad (22)$$

$$CS=\frac{1}{2\beta}\left[\alpha-\beta\left(w^{MS}+m^{MS}\right)+\gamma\theta^{MS}\right]^2$$
$$=\frac{\beta\eta^2(\alpha-\beta c)^2}{2\left[(4-\mu)\beta\eta-\gamma^2\right]^2} \quad (23)$$

Proposition 1. In the MS game model

1) $\frac{\partial\theta^{MS}}{\partial\mu}>0$, $\frac{\partial m^{MS}}{\partial\mu}>0$, $\frac{\partial q^{MS}}{\partial\mu}>0$, $\frac{\partial\pi_R^{MS}}{\partial\mu}>0$, $\frac{\partial V_M^{MS}}{\partial\mu}>0$, $\frac{\partial\pi_{SC}^{MS}}{\partial\mu}>0$, $\frac{\partial CS^{MS}}{\partial\mu}>0$.

2) $\frac{\partial w^{MS}}{\partial\mu}<0$, $\frac{\partial p^{MS}}{\partial\mu}<0$, $\frac{\partial\pi_M^{MS}}{\partial\mu}<0$.

Proof. The first derivatives of the greening level, the margin profit, the market demand, the profit of the retailer, the total profit of the manufacturer, the profit of the supply chain system and the consumer surplus with respect to μ are as follows

$$\frac{\partial\theta^{MS}}{\partial\mu}=\frac{\beta\gamma\eta(\alpha-\beta c)}{\left[(4-\mu)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\partial m^{MS}}{\partial\mu}=\frac{\beta\eta^2(\alpha-\beta c)}{\left[(4-\mu)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\partial q^{MS}}{\partial\mu}=\frac{\beta^2\eta^2(\alpha-\beta c)}{\left[(4-\mu)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\partial\pi_R^{MS}}{\partial\mu}=\frac{2\beta^2\eta^3(\alpha-\beta c)^2}{\left[(4-\mu)\beta\eta-\gamma^2\right]^3}>0$$

$$\frac{\partial V_M^{MS}}{\partial\mu}=\frac{\beta\eta^2(\alpha-\beta c)^2}{\left[(4-\mu)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\partial\pi_{SC}^{MS}}{\partial\mu}=\frac{(2-\mu)\beta^2\eta^3(\alpha-\beta c)^2}{\left[(4-\mu)\beta\eta-\gamma^2\right]^3}>0$$

$$\frac{\partial CS^{MS}}{\partial\mu}=\frac{(2-\mu)\beta^2\eta^3(\alpha-\beta c)^2}{\left[(4-\mu)\beta\eta-\gamma^2\right]^3}>0$$

Also, the first derivatives of the wholesale price, the retail price, and the profit of the manufacturer with respect to μ are as follows

$$\frac{\partial w^{MS}}{\partial\mu}=-\frac{\left[2(3-\mu)\beta\eta-\gamma^2\right]\eta(\alpha-\beta c)}{\left[(4-\mu)\beta\eta-\gamma^2\right]^2}<0$$

$$\frac{\partial p^{MS}}{\partial\mu}=-\frac{\left[(5-2\mu)\beta\eta-\gamma^2\right]\eta(\alpha-\beta c)}{\left[(4-\mu)\beta\eta-\gamma^2\right]^2}<0$$

$$\frac{\partial\pi_M^{MS}}{\partial\mu}=-\frac{\mu\beta^2\eta^3(\alpha-\beta c)^2}{\left[(4-\mu)\beta\eta-\gamma^2\right]^3}<0$$

The proof of Proposition 1 is completed.

Proposition 1 shows that the greening level, the margin profit, the market demand, the profit of the retailer, the total profit of the manufacturer, the profit of the supply chain system and the consumer surplus will increase with the rise of the manufacturer's CSR activity. Also, if the manufacturer puts more emphasis on CSR then the wholesale price, the retail price and the profit of the manufacturer will all decrease in the MS game model.

B. RS game model

The RS (Retailer-Stackelberg) game model arises in the market where the size of the retailer is larger compared to the manufacturer. In this case, the retailer is the leader, and the manufacturer is the follower. That is, firstly, the retailer sets the profit margin m using the manufacturer's reaction functions to maximize his profit. Then, the manufacturer sets the greening level θ and the wholesale price w so as to maximize his total profit. Thus, the RS game model can be given as follows

$$\max_{m} \pi_R = m\left[\alpha-\beta\left(w+m\right)+\gamma\theta\right]$$

$$s.t.\begin{cases} \theta, w = \arg\max V_M \\ \max_{\theta,w} V_M = \left(w-c\right)\left[\alpha-\beta\left(w+m\right)+\gamma\theta\right]-\frac{1}{2}\eta\theta^2 \\ \qquad +\frac{\mu}{2\beta}\left[\alpha-\beta\left(w+m\right)+\gamma\theta\right]^2 \end{cases} \tag{24}$$

Theorem 2. In the RS game model, the optimal policies of the manufacturer and the retailer are as follows

$$\theta^{RS}=\frac{\gamma\left(\alpha-\beta c\right)}{2\left[\left(2-\mu\right)\beta\eta-\gamma^2\right]} \tag{25}$$

$$w^{RS}=\frac{\left(1-\mu\right)\eta\left(\alpha-\beta c\right)}{2\left[\left(2-\mu\right)\beta\eta-\gamma^2\right]}+c \tag{26}$$

$$m^{RS}=\frac{\alpha-\beta c}{2\beta} \tag{27}$$

Proof. First we solve the total profit function of the manufacturer as follows

$$\max_{\theta,w} V_M = \left(w-c\right)\left[\alpha-\beta\left(w+m\right)+\gamma\theta\right]-\frac{1}{2}\eta\theta^2 +\frac{\mu}{2\beta}\left[\alpha-\beta\left(w+m\right)+\gamma\theta\right]^2 \tag{28}$$

The first order conditions are

$$\frac{\partial V_M}{\partial\theta}=-\frac{\beta\eta-\mu\gamma^2}{\beta}\theta+\left(1-\mu\right)\gamma w-\left(1-\mu\right)\beta m+\frac{\gamma\left(\mu\alpha-\beta c\right)}{\beta}$$

$$\frac{\partial V_M}{\partial w}=-\left(2-\mu\right)\beta w+\left(1-\mu\right)\gamma\theta-\left(1-\mu\right)\beta m+\left(1-\mu\right)\alpha+\beta c$$

Therefore, the Hessian matrix of V_M is

$$\mathrm{H}=\begin{bmatrix} \frac{\partial^2 V_M}{\partial\theta^2} & \frac{\partial^2 V_M}{\partial\theta\partial w} \\ \frac{\partial^2 V_M}{\partial w\partial\theta} & \frac{\partial^2 V_M}{\partial w^2} \end{bmatrix}=\begin{bmatrix} -\left(2-\mu\right)\beta & \left(1-\mu\right)\gamma \\ \left(1-\mu\right)\gamma & -\frac{\beta\eta-\mu\gamma^2}{\beta} \end{bmatrix}$$

Note that the Hessian matrix of V_M is negative definite, since $0\le\mu\le1$, $\beta>0$, $\gamma>0$ and $\frac{\beta\eta}{\gamma^2}>1$. Consequently, V_M is strictly jointly concave in θ and w. Hence, the optimal reaction functions of the manufacturer can be obtained by solving the first order conditions as below

$$-\frac{\beta\eta-\mu\gamma^2}{\beta}\theta+\left(1-\mu\right)\gamma w-\left(1-\mu\right)\beta m+\frac{\gamma\left(\mu\alpha-\beta c\right)}{\beta}=0 \tag{29}$$

$$-\left(2-\mu\right)\beta w+\left(1-\mu\right)\gamma\theta-\left(1-\mu\right)\beta m+\left(1-\mu\right)\alpha+\beta c=0 \tag{30}$$

Solving (29) and (30) simultaneously, we can obtain the optimal response functions of the manufacturer as follows

$$\theta^{RS}\left(m\right)=\frac{\gamma\left(-\beta m+\alpha-\beta c\right)}{\left(2-\mu\right)\beta\eta-\gamma^2} \tag{31}$$

$$w^{RS}\left(m\right)=\frac{\left(1-\mu\right)\eta\left(-\beta m+\alpha-\beta c\right)}{\left(2-\mu\right)\beta\eta-\gamma^2}+c \tag{32}$$

Next we solve the profit function of the retailer

$$\max_{m} \pi_R = m\left[\alpha-\beta\left(w+m\right)+\gamma\theta\right] \tag{33}$$

Substituting $\theta^{RS}\left(m\right)$ and $w^{RS}\left(m\right)$ into (33), we get

$$\max_{m} \pi_R = \frac{\beta\eta}{\left(2-\mu\right)\beta\eta-\gamma^2}m\left[-\beta m+\alpha-\beta c\right]$$

The first order condition is

$$\frac{\partial\pi_R}{\partial m}=\frac{\beta\eta}{\left(2-\mu\right)\beta\eta-\gamma^2}\left(-2\beta m+\alpha-\beta c\right)$$

Then the second order condition is

$$\frac{\partial^2\pi_R}{\partial m^2}=-\frac{2\beta^2\eta}{\left(2-\mu\right)\beta\eta-\gamma^2}$$

Note that the second order condition of π_R is negative definite, since $\frac{\beta\eta}{\gamma^2}>1$. Consequently, π_R is strictly concave in m. Hence, the optimal profit margin of the retailer can be obtained by solving the first order condition as below

$$\frac{\beta\eta}{\left(2-\mu\right)\beta\eta-\gamma^2}\left(-2\beta m+\alpha-\beta c\right)=0 \tag{34}$$

Solving (34), we can obtain the optimal profit margin of the retailer as

$$m^{RS}=\frac{\alpha-\beta c}{2\beta}$$

Substituting m^{RS} into (31) and (32), we can obtain the optimal greening level θ^{RS} and the wholesale price w^{RS} of the manufacturer as follows

$$\theta^{RS}=\frac{\gamma\left(\alpha-\beta c\right)}{2\left[\left(2-\mu\right)\beta\eta-\gamma^2\right]}$$

$$w^{RS}=\frac{\left(1-\mu\right)\eta\left(\alpha-\beta c\right)}{2\left[\left(2-\mu\right)\beta\eta-\gamma^2\right]}+c$$

The proof of Theorem 2 is completed.

Then, the optimal retail price p^{RS} and market demand q^{RS} can be obtained as

$$p^{RS}=w^{RS}+m^{RS}=\frac{\left[\left(3-2\mu\right)\beta\eta-\gamma^2\right]\left(\alpha-\beta c\right)}{2\beta\left[\left(2-\mu\right)\beta\eta-\gamma^2\right]}+c \tag{35}$$

$$q^{RS}=\alpha-\beta\left(w^{RS}+m^{RS}\right)+\gamma\theta^{RS}=\frac{\beta\eta\left(\alpha-\beta c\right)}{2\left[\left(2-\mu\right)\beta\eta-\gamma^2\right]} \tag{36}$$

By combining (25), (26) and (27) with (3), (4) and (5), we derive the retailer's optimal profit, the manufacturer's optimal purer and total profits in the RS game model as follows

$$\pi_R^{RS}=\frac{\eta\left(\alpha-\beta c\right)^2}{4\left[\left(2-\mu\right)\beta\eta-\gamma^2\right]} \tag{37}$$

$$\pi_M^{RS}=\frac{\left[2\left(1-\mu\right)\beta\eta-\gamma^2\right]\eta\left(\alpha-\beta c\right)^2}{8\left[\left(2-\mu\right)\beta\eta-\gamma^2\right]^2} \tag{38}$$

$$V_M^{RS}=\frac{\eta\left(\alpha-\beta c\right)^2}{8\left[\left(4-\mu\right)\beta\eta-\gamma^2\right]} \tag{39}$$

The profit of the supply chain system and the consumer surplus are

$$\pi_{SC}^{RS}=\pi_R^{RS}+\pi_M^{RS}=\frac{\left[2(3-2\mu)\beta\eta-3\gamma^2\right]\eta(\alpha-\beta c)^2}{8\left[(4-\mu)\beta\eta-\gamma^2\right]^2} \quad (40)$$

$$CS=\frac{1}{2\beta}\left[\alpha-\beta\left(w^{RS}+m^{RS}\right)+\gamma\theta^{RS}\right]^2$$
$$=\frac{\beta\eta^2(\alpha-\beta c)^2}{8\left[(2-\mu)\beta\eta-\gamma^2\right]^2} \quad (41)$$

Proposition 2. In the RS game model

1) $\frac{\partial m^{RS}}{\partial\mu}=0$.

2) $\frac{\partial\theta^{RS}}{\partial\mu}>0$, $\frac{\partial q^{RS}}{\partial\mu}>0$, $\frac{\partial\pi_R^{RS}}{\partial\mu}>0$, $\frac{\partial V_M^{RS}}{\partial\mu}>0$, $\frac{\partial CS^{RS}}{\partial\mu}>0$.

3) $\frac{\partial w^{RS}}{\partial\mu}<0$, $\frac{\partial p^{RS}}{\partial\mu}<0$, $\frac{\partial\pi_M^{RS}}{\partial\mu}<0$.

4) $\frac{\partial\pi_{SC}^{RS}}{\partial\mu}<0$, if $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{2(1-\mu)}$, and $\frac{\partial\pi_{SC}^{RS}}{\partial\mu}>0$, If $\frac{\beta\eta}{\gamma^2}>\frac{1}{2(1-\mu)}$.

Proof. The first derivative of the margin profit with respect to μ is

$$\frac{\partial m^{RS}}{\partial\mu}=\frac{\partial\left(\frac{\alpha-\beta c}{2\beta}\right)}{\partial\mu}=0$$

The first derivatives of the greening level, the market demand, the profit of the retailer, the total profit of the manufacturer, and the consumer surplus with respect to μ are as follows

$$\frac{\partial\theta^{RS}}{\partial\mu}=\frac{\beta\gamma\eta(\alpha-\beta c)}{2\left[(2-\mu)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\partial q^{RS}}{\partial\mu}=\frac{\beta^2\eta^2(\alpha-\beta c)}{2\left[(2-\mu)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\partial\pi_R^{RS}}{\partial\mu}=\frac{\beta\eta^2(\alpha-\beta c)^2}{4\left[(2-\mu)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\partial V_{VM}^{RS}}{\partial\mu}=\frac{\beta\eta^2(\alpha-\beta c)^2}{8\left[(2-\mu)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\partial CS^{RS}}{\partial\mu}=\frac{\beta^2\eta^3(\alpha-\beta c)^2}{8\left[(4-\mu)\beta\eta-\gamma^2\right]^3}>0$$

The first derivatives of the wholesale price, the retail price, and the profit of the manufacturer with respect to μ are as follows

$$\frac{\partial w^{RS}}{\partial\mu}=-\frac{\left(\beta\eta-\gamma^2\right)\eta(\alpha-\beta c)}{2\left[(2-\mu)\beta\eta-\gamma^2\right]^2}<0$$

$$\frac{\partial p^{RS}}{\partial\mu}=-\frac{\left(\beta\eta-\gamma^2\right)\eta(\alpha-\beta c)}{2\left[(2-\mu)\beta\eta-\gamma^2\right]^2}<0$$

$$\frac{\partial\pi_M^{RS}}{\partial\mu}=-\frac{\mu\beta^2\eta^3(\alpha-\beta c)^2}{4\left[(2-\mu)\beta\eta-\gamma^2\right]^3}<0$$

The first derivative of the profit of the supply chain system with respect to μ is

$$\frac{\partial\pi_{SC}^{RS}}{\partial\mu}=\frac{\left[2(1-\mu)\beta\eta-\gamma^2\right]\beta\eta^2(\alpha-\beta c)^2}{4\left[(2-\mu)\beta\eta-\gamma^2\right]^3}$$

When $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{2(1-\mu)}$, we have $\frac{\partial\pi_{SC}^{RS}}{\partial\mu}<0$, and when $\frac{\beta\eta}{\gamma^2}>1$, we have $\frac{\partial\pi_{SC}^{RS}}{\partial\mu}>0$.

The proof of Proposition 2 is completed.

Proposition 2 shows that the margin profit of the retailer is independent of μ. The greening level, the market demand, the profit of the retailer, the total profit of the manufacturer and the consumer surplus will increase with the rise of the manufacturer's CSR activity. Also, if the manufacturer puts more emphasis on CSR then the wholesale price, the retail price and the profit of the manufacturer will all decrease. The profit of the supply chain system increases first and then decreases with the increasing of μ, and when $\mu=1-\frac{\gamma^2}{2\beta\eta}$, the supply chain system obtains his largest total profit.

C. VN game model

The VN (Vertical-Nash) game model arises in the market where the manufacturer and the retailer have equal market power. In this case the manufacturer determines the greening level and the wholesale price, and the retailer makes the profit margin m simultaneously and independently, so as to maximize their profits. Thus, the VN game model can be given as follows

$$\begin{cases}\max\limits_{\theta,w}V_M=(w-c)\left[\alpha-\beta(w+m)+\gamma\theta\right]-\frac{1}{2}\eta\theta^2\\ \qquad+\frac{\mu}{2\beta}\left[\alpha-\beta(w+m)+\gamma\theta\right]^2\\ \max\limits_{m}\pi_R=m\left[\alpha-\beta(w+m)+\gamma\theta\right]\end{cases} \quad (42)$$

Theorem 3. In the VN game model, the optimal policies of the manufacturer and the retailer are as follows

$$\theta^{VN}=\frac{\gamma(\alpha-\beta c)}{(3-\mu)\beta\eta-\gamma^2} \quad (43)$$

$$w^{VN}=\frac{(1-\mu)\eta(\alpha-\beta c)}{(3-\mu)\beta\eta-\gamma^2}+c \quad (44)$$

$$m^{VN}=\frac{\eta(\alpha-\beta c)}{(3-\mu)\beta\eta-\gamma^2} \quad (45)$$

Proof. First we solve the total profit function of the manufacturer as follows

$$\max_{\theta,w}V_M=(w-c)\left[\alpha-\beta(w+m)+\gamma\theta\right]-\frac{1}{2}\eta\theta^2+\frac{\mu}{2\beta}\left[\alpha-\beta(w+m)+\gamma\theta\right]^2 \quad (46)$$

The first order conditions are

$$\frac{\partial V_M}{\partial \theta}=-\frac{\beta\eta-\mu\gamma^2}{\beta}\theta+(1-\mu)\gamma w-(1-\mu)\beta m+\frac{\gamma(\mu\alpha-\beta c)}{\beta}$$

$$\frac{\partial V_M}{\partial w}=-(2-\mu)\beta w+(1-\mu)\gamma\theta-(1-\mu)\beta m+(1-\mu)\alpha+\beta c$$

Therefore, the Hessian matrix of V_M is

$$\mathrm{H}=\begin{bmatrix}\frac{\partial^2 V_M}{\partial\theta^2} & \frac{\partial^2 V_M}{\partial\theta\partial w}\\ \frac{\partial^2 V_M}{\partial w\partial\theta} & \frac{\partial^2 V_M}{\partial w^2}\end{bmatrix}=\begin{bmatrix}-(2-\mu)\beta & (1-\mu)\gamma\\ (1-\mu)\gamma & -\frac{\beta\eta-\mu\gamma^2}{\beta}\end{bmatrix}$$

Note that the Hessian matrix of V_M is negative definite, since $0\le\mu\le 1$, $\beta>0$, $\gamma>0$ and $\frac{\beta\eta}{\gamma^2}>1$. Consequently, V_M is strictly jointly concave in θ and w. Hence, the optimal reaction functions of the manufacturer can be obtained by solving the first order conditions as below

$$-\frac{\beta\eta-\mu\gamma^2}{\beta}\theta+(1-\mu)\gamma w-(1-\mu)\beta m+\frac{\gamma(\mu\alpha-\beta c)}{\beta}=0 \quad (47)$$

$$-(2-\mu)\beta w+(1-\mu)\gamma\theta-(1-\mu)\beta m+(1-\mu)\alpha+\beta c=0 \quad (48)$$

Solving (47) and (48) simultaneously, we can obtain the optimal response functions of the manufacturer as follows

$$\theta^{VN}(m)=\frac{\gamma(-\beta m+\alpha-\beta c)}{(2-\mu)\beta\eta-\gamma^2} \quad (49)$$

$$w^{VN}(m)=\frac{(1-\mu)\eta(-\beta m+\alpha-\beta c)}{(2-\mu)\beta\eta-\gamma^2}+c \quad (50)$$

Next we solve the profit function of the retailer

$$\max_m \pi_R=m\left[\alpha-\beta(w+m)+\gamma\theta\right] \quad (51)$$

The first order condition is

$$\frac{\partial\pi_R}{\partial m}=-2\beta m-\beta w+\alpha+\gamma\theta$$

Then the second order condition is

$$\frac{\partial^2\pi_R}{\partial m^2}=-2\beta$$

Note that the second order condition of π_R is negative definite, since $\beta>0$. Consequently, π_R is strictly concave in *m*. Hence, the optimal reaction function of the retailer can be obtained by solving the first order condition as below

$$-2\beta m-\beta w+\alpha+\gamma\theta=0 \quad (52)$$

Solving (52), we can obtain the optimal response function of the retailer as

$$m^{VN}(\theta,w)=\frac{\gamma\theta-\beta w+\alpha}{2\beta} \quad (53)$$

Substituting $\theta^{VN}(m)$ and $w^{VN}(m)$ into (53), we can obtain the optimal profit margin m^{VN} of the retailer as follows

$$m^{VN}=\frac{\eta(\alpha-\beta c)}{(3-\mu)\beta\eta-\gamma^2} \quad (54)$$

Substituting m^{VN} into (49) and (50), we can obtain the optimal greening level θ^{VN} and the wholesale price w^{VN} of the manufacturer as follows

$$\theta^{VN}=\frac{\gamma(\alpha-\beta c)}{\left[(2-\mu)\beta\eta-\gamma^2\right]}$$

$$w^{VN}=\frac{(1-\mu)\eta(\alpha-\beta c)}{(2-\mu)\beta\eta-\gamma^2}+c$$

The proof of Theorem 3 is completed.

Then, the optimal retail price p^{VN} and market demand q^{VN} can be obtained as

$$p^{VN}=w^{VN}+m^{VN}=\frac{(2-\mu)\eta(\alpha-\beta c)}{(3-\mu)\beta\eta-\gamma^2}+c \quad (55)$$

$$q^{VN}=\alpha-\beta\left(w^{VN}+m^{VN}\right)+\gamma\theta^{VN}=\frac{\beta\eta(\alpha-\beta c)}{(3-\mu)\beta\eta-\gamma^2} \quad (56)$$

By combining (43), (44) and (45) with (3), (4) and (5), we derive the retailer's optimal profit, the manufacturer's optimal purer and total profits in the VN game model as follows

$$\pi_R^{VN}=\frac{\beta\eta^2(\alpha-\beta c)^2}{\left[(3-\mu)\beta\eta-\gamma^2\right]^2} \quad (57)$$

$$\pi_M^{VN}=\frac{\left[2(1-\mu)\beta\eta-\gamma^2\right]\eta(\alpha-\beta c)^2}{2\left[(3-\mu)\beta\eta-\gamma^2\right]^2} \quad (58)$$

$$V_M^{VN}=\frac{\left[(2-\mu)\beta\eta-\gamma^2\right]\eta(\alpha-\beta c)^2}{2\left[(3-\mu)\beta\eta-\gamma^2\right]^2} \quad (59)$$

The profit of the supply chain system and the consumer surplus are

$$\pi_{SC}^{VN}=\pi_R^{VN}+\pi_M^{VN}=\frac{\left[2(2-\mu)\beta\eta-\gamma^2\right]\eta(\alpha-\beta c)^2}{2\left[(3-\mu)\beta\eta-\gamma^2\right]^2} \quad (60)$$

$$CS=\frac{1}{2\beta}\left[\alpha-\beta\left(w^{VN}+m^{VN}\right)+\gamma\theta^{VN}\right]^2$$
$$=\frac{\beta\eta^2(\alpha-\beta c)^2}{2\left[(3-\mu)\beta\eta-\gamma^2\right]^2} \quad (61)$$

Proposition 3. In the VN game model

1) $\frac{\partial\theta^{VN}}{\partial\mu}>0$, $\frac{\partial m^{VN}}{\partial\mu}>0$, $\frac{\partial q^{VN}}{\partial\mu}>0$, $\frac{\partial\pi_R^{VN}}{\partial\mu}>0$, $\frac{\partial\pi_{SC}^{VN}}{\partial\mu}>0$, $\frac{\partial CS^{VN}}{\partial\mu}>0$.

2) $\frac{\partial w^{VN}}{\partial\mu}<0$, $\frac{\partial p^{VN}}{\partial\mu}<0$, $\frac{\partial\pi_M^{VN}}{\partial\mu}<0$.

3) $\frac{\partial V_M^{VN}}{\partial\mu}<0$, if $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, and $\frac{\partial V_M^{VN}}{\partial\mu}>0$, if $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$.

Proof. The first derivatives of the greening level, the margin profit, the market demand, the profit of the retailer, the profit of the supply chain system, and the consumer surplus with respect to μ are as follows

$$\frac{\partial\theta^{VN}}{\partial\mu}=\frac{\beta\gamma\eta(\alpha-\beta c)}{\left[(3-\mu)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\partial m^{VN}}{\partial \mu}=\frac{\beta\eta^2(\alpha-\beta c)}{\left[(3-\mu)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\partial q^{VN}}{\partial \mu}=\frac{\beta^2\eta^2(\alpha-\beta c)}{\left[(3-\mu)\beta\eta-\gamma^2\right]^2}>0$$

$$\frac{\partial \pi_R{}^{VN}}{\partial \mu}=\frac{2\beta^2\eta^3(\alpha-\beta c)^2}{\left[(3-\mu)\beta\eta-\gamma^2\right]^3}>0$$

$$\frac{\partial \pi_{SC}{}^{VN}}{\partial \mu}=\frac{\beta^2\eta^3(\alpha-\beta c)^2}{\left[(3-\mu)\beta\eta-\gamma^2\right]^3}>0$$

$$\frac{\partial CS^{VN}}{\partial \mu}=\frac{\beta^2\eta^3(\alpha-\beta c)^2}{\left[(3-\mu)\beta\eta-\gamma^2\right]^3}>0$$

The first derivatives of the wholesale price, the retail price, and the profit of the manufacturer with respect to μ are as follows

$$\frac{\partial w^{VN}}{\partial \mu}=-\frac{(2\beta\eta-\gamma^2)\eta(\alpha-\beta c)}{\left[(3-\mu)\beta\eta-\gamma^2\right]^2}<0$$

$$\frac{\partial p^{VN}}{\partial \mu}=-\frac{(\beta\eta-\gamma^2)\eta(\alpha-\beta c)}{\left[(3-\mu)\beta\eta-\gamma^2\right]^2}<0$$

$$\frac{\partial \pi_M{}^{VN}}{\partial \mu}=-\frac{\beta^2\eta^3(\alpha-\beta c)^2}{\left[(3-\mu)\beta\eta-\gamma^2\right]^3}<0$$

The first derivative of the total profit of the manufacturer with respect to μ is

$$\frac{\partial V_M^{VN}}{\partial \mu}=\frac{\left[(1-\mu)\beta\eta-\gamma^2\right]\beta\eta^2(\alpha-\beta c)^2}{2\left[(3-\mu)\beta\eta-\gamma^2\right]^3}$$

When $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, we have $\frac{\partial V_M^{VN}}{\partial \mu}<0$, and when $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$, we have $\frac{\partial V_M^{VN}}{\partial \mu}>0$.

The proof of Proposition 3 is completed.

Proposition 3 shows that the greening level, the margin profit, the market demand, the profit of the retailer, the total profit of the manufacturer and the consumer surplus will increase with the rise of the manufacturer's CSR activity. Also, if the manufacturer puts more emphasis on CSR then the wholesale price, the retail price and the profit of the manufacturer will all decrease. The total profit of the manufacturer increases first and then decreases with the increasing of μ, and when $\mu=1-\frac{\gamma^2}{\beta\eta}$, the manufacturer obtains his largest total profit.

D. Models comparison

On the basis of the above three models, the optimal policies of the manufacturer and the retailer are compared, and the following five propositions are proposed.

Proposition 4. The optimal greening levels satisfy the following

If $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, then $\theta^{RS}>\theta^{VN}>\theta^{MS}$;

If $\frac{\beta\eta}{\gamma^2}=\frac{1}{1-\mu}$, then $\theta^{RS}=\theta^{VN}>\theta^{MS}$;

If $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$, then $\theta^{VN}>\theta^{RS}>\theta^{MS}$.

Proof. It is easy to verify that

$$\theta^{RS}-\theta^{MS}=\frac{(\mu\beta\eta+\gamma^2)\gamma(\alpha-\beta c)}{2\left[(4-\mu)\beta\eta-\gamma^2\right]\left[(2-\mu)\beta\eta-\gamma^2\right]}>0$$

$$\theta^{VN}-\theta^{MS}=\frac{\beta\eta\gamma(\alpha-\beta c)}{\left[(4-\mu)\beta\eta-\gamma^2\right]\left[(3-\mu)\beta\eta-\gamma^2\right]}>0$$

$$\theta^{VN}-\theta^{RS}=\frac{\left[(1-\mu)\beta\eta-\gamma^2\right]\gamma(\alpha-\beta c)}{2\left[(3-\mu)\beta\eta-\gamma^2\right]\left[(2-\mu)\beta\eta-\gamma^2\right]}$$

When $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, we obtain $\theta^{RS}>\theta^{VN}$;

when $\frac{\beta\eta}{\gamma^2}=\frac{1}{1-\mu}$, we obtain $\theta^{RS}=\theta^{VN}$;

when $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$, we obtain $\theta^{VN}>\theta^{RS}$.

The proof of Proposition 4 is completed.

Proposition 5. The optimal wholesale prices satisfy the following

If $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, then $w^{MS}>w^{RS}>w^{VN}$;

If $\frac{\beta\eta}{\gamma^2}=\frac{1}{1-\mu}$, then $w^{MS}>w^{RS}=w^{VN}$;

If $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$, then $w^{MS}>w^{VN}>w^{RS}$.

Proof. It is easy to verify that

$$w^{MS}-w^{RS}=\frac{\left[(4-3\mu+\mu^2)(\beta\eta-\gamma^2)+(1-\mu)^2\gamma^2\right]\eta(\alpha-\beta c)}{2\left[(4-\mu)\beta\eta-\gamma^2\right]\left[(2-\mu)\beta\eta-\gamma^2\right]}>0$$

$$w^{MS}-w^{VN}=\frac{(2\beta\eta-\gamma^2)\eta(\alpha-\beta c)}{\left[(4-\mu)\beta\eta-\gamma^2\right]\left[(3-\mu)\beta\eta-\gamma^2\right]}>0$$

$$w^{VN}-w^{RS}=\frac{(1-\mu)\left[(1-\mu)\beta\eta-\gamma^2\right]\eta(\alpha-\beta c)}{2\left[(4-\mu)\beta\eta-\gamma^2\right]\left[(2-\mu)\beta\eta-\gamma^2\right]}$$

When $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, we obtain $w^{RS}>w^{VN}$;

when $\frac{\beta\eta}{\gamma^2}=\frac{1}{1-\mu}$, we obtain $w^{RS}=w^{VN}$;

when $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$, we obtain $w^{VN}>w^{RS}$.

The proof of Proposition 5 is completed.

Proposition 6. The optimal margin profits satisfy the following

If $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, then $m^{VN}>m^{RS}>m^{MS}$;

If $\frac{\beta\eta}{\gamma^2}=\frac{1}{1-\mu}$, then $m^{VN}=m^{RS}>m^{MS}$;

If $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$, then $m^{RS}>m^{VN}>m^{MS}$.

Proof. It is easy to verify that

$$m^{VN}-m^{MS}=\frac{\beta\eta^2(\alpha-\beta c)}{\left[(4-\mu)\beta\eta-\gamma^2\right]\left[(3-\mu)\beta\eta-\gamma^2\right]}>0$$

$$m^{RS}-m^{MS}=\frac{\left[(2-\mu)\beta\eta-\gamma^2\right](\alpha-\beta c)}{2\beta\left[(4-\mu)\beta\eta-\gamma^2\right]}>0$$

$$m^{RS}-m^{VN}=\frac{\left[(1-\mu)\beta\eta-\gamma^2\right](\alpha-\beta c)}{2\beta\left[(3-\mu)\beta\eta-\gamma^2\right]}>0$$

When $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, we obtain $m^{VN}>m^{RS}$;

when $\frac{\beta\eta}{\gamma^2}=\frac{1}{1-\mu}$, we obtain $m^{VN}=m^{RS}$;

when $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$, we obtain $m^{RS}>m^{VN}$.

The proof of Proposition 6 is completed.

Proposition 7. The optimal retail prices satisfy the following

If $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, then $p^{MS}>p^{VN}>p^{RS}$;

If $\frac{\beta\eta}{\gamma^2}=\frac{1}{1-\mu}$, then $p^{MS}>p^{VN}=p^{RS}$;

If $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$, then $p^{MS}>p^{RS}>p^{VN}$.

Proof. It is easy to verify that

$$p^{MS}-p^{VN}=\frac{\left(\beta\eta-\gamma^2\right)\eta(\alpha-\beta c)}{\left[(4-\mu)\beta\eta-\gamma^2\right]\left[(3-\mu)\beta\eta-\gamma^2\right]}>0$$

$$p^{MS}-p^{RS}=\frac{\left(\beta\eta-\gamma^2\right)\left(\beta\eta+\gamma^2\right)(\alpha-\beta c)}{2\beta\left[(4-\mu)\beta\eta-\gamma^2\right]\left[(2-\mu)\beta\eta-\gamma^2\right]}>0$$

$$p^{RS}-p^{VN}=\frac{\left[(1-\mu)\beta\eta-\gamma^2\right]\left(\beta\eta-\gamma^2\right)(\alpha-\beta c)}{2\beta\left[(3-\mu)\beta\eta-\gamma^2\right]\left[(2-\mu)\beta\eta-\gamma^2\right]}$$

When $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, we obtain $p^{VN}>p^{RS}$;

when $\frac{\beta\eta}{\gamma^2}=\frac{1}{1-\mu}$, we obtain $p^{VN}=p^{RS}$;

when $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$, we obtain $p^{RS}>p^{VN}$.

The proof of Proposition 7 is completed.

Proposition 8. The optimal market demands satisfy the following

If $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, then $q^{RS}>q^{VN}>q^{MS}$;

If $\frac{\beta\eta}{\gamma^2}=\frac{1}{1-\mu}$, then $q^{RS}=q^{VN}>q^{MS}$;

If $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$, then $q^{VN}>q^{RS}>q^{MS}$.

Proof. It is easy to verify that

$$q^{RS}-q^{MS}=\frac{\left[\mu\beta\eta+\gamma^2\right]\beta\eta(\alpha-\beta c)}{2\left[(4-\mu)\beta\eta-\gamma^2\right]\left[(2-\mu)\beta\eta-\gamma^2\right]}>0$$

$$q^{VN}-q^{MS}=\frac{\beta^2\eta^2(\alpha-\beta c)}{\left[(4-\mu)\beta\eta-\gamma^2\right]\left[(3-\mu)\beta\eta-\gamma^2\right]}>0$$

$$q^{VN}-q^{RS}=\frac{\left[(1-\mu)\beta\eta-\gamma^2\right]\beta\eta(\alpha-\beta c)}{2\left[(3-\mu)\beta\eta-\gamma^2\right]\left[(2-\mu)\beta\eta-\gamma^2\right]}$$

When $1<\frac{\beta\eta}{\gamma^2}<\frac{1}{1-\mu}$, we obtain $q^{RS}>q^{VN}$;

when $\frac{\beta\eta}{\gamma^2}=\frac{1}{1-\mu}$, we obtain $q^{RS}=q^{VN}$;

when $\frac{\beta\eta}{\gamma^2}>\frac{1}{1-\mu}$, we obtain $q^{VN}>q^{RS}$.

The proof of Proposition 8 is completed.

IV. NUMERICAL EXAMPLE

In this section, we tend to further elucidate the proposed three game models with a numerical example. We will analyze that the effective of the manufacturer's CSR activity μ on the optimal policies.

The following parameters are used for illustration:

$\alpha=200$, $\beta=10.0$, $\gamma=8.0$, $\eta=16.0$, and $c=6.0$.

Based on the analysis showed in the Section III, the optimal policies with different of μ in the MS, RS and VN game models are showed in Table II.

TABLE II

THE OPTIMAL POLICIES WITH DIFFERENT μ

	μ	θ	w	m	p	q
MS	0	1.94	13.78	3.89	17.67	38.89
	0.20	2.06	13.41	4.12	17.53	41.18
	0.40	2.19	13.00	4.38	17.38	43.75
	0.60	2.33	12.53	4.67	17.20	46.67
	0.80	2.50	12.28	5.00	17.00	50.00
	1.00	2.69	11.38	5.38	16.77	53.85
RS	0	2.19	10.38	7.00	17.38	43.75
	0.20	2.50	10.00	7.00	17.00	50.00
	0.40	2.92	9.50	7.00	16.50	58.33
	0.60	3.50	8.80	7.00	15.80	70.00
	0.80	4.38	7.75	7.00	14.75	87.50
	1.00	5.83	6.00	7.00	13.00	116.67
VN	0	2.69	11.38	5.38	16.77	53.85
	0.20	2.92	10.67	5.83	16.50	58.33
	0.40	3.18	9.82	6.36	16.18	63.64
	0.60	3.50	8.80	7.00	15.80	70.00
	0.80	3.88	7.56	7.78	15.33	77.78
	1.00	4.38	6.00	8.75	14.75	87.50

Based on the results showed in Table II, we find:

1) As the manufacturer's CSR activity increases, the greening level and the market demand increase in the

three games. In this case, when 0≤μ<0.60, the greening level and the market demand are the highest in the VN game, followed by the RS game and then MS game. When 0.60<μ≤1, the greening level and the market demand are the highest in the RS game, followed by the VN game and then the MS game. When μ=0.60, the greening level and the market demand in the RS game are equal to that in the VN game, and both are larger than that in the MS game.

2) When the manufacturer puts more weight on CSR, it reduces the wholesale price and the retail price. When 0.60<μ≤1, the wholesale price is the highest in the MS game and the lowest in RS game. When 0.60<μ≤1, the wholesale price is the highest in the MS game and the lowed in VN game. When μ=0.60 and μ=1, the wholesale price in the RS game is equal to that in the VN game, and is smaller than that in the MS game. When 0≤μ<0.60, the retail price is the highest in the MS game, followed by the RS game and then VN game. When0.60<μ≤1, the retail price is the highest in the MS game, followed by the VN game and then MS game. When μ=0.60, the retail price in the RS game is equal to that in the VN game, and is smaller than that in the MS game.

3) The margin profit of the manufacturer will increases by increasing the manufacturer's CSR activity in the MS and VN games. While in the RS game, the margin profit is not varying with the increasing of μ . When 0≤μ<0.60, the margin profit is the highest in the RS game, followed by the VN game and then the MS game. When0.60<μ≤1, the margin profit is the highest in the RS game, followed by the VN game and then the MS game. When μ=0.60, the margin profit in the RS game is equal to that in the VN game, and is larger than that in the MS game.

4) The consumer will benefit from the manufacturer's CSR activity this is because under this case, the retail price is lower and the greening level is higher.

The retailer's profit, manufacturer's pure profit, manufacturer's total profit, supply chain system's profit and consume surplus with different of μ are showed in following Figurers.

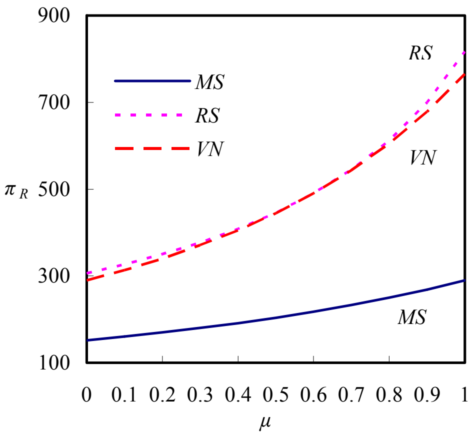

Fig 1: Retailer's profit with μ

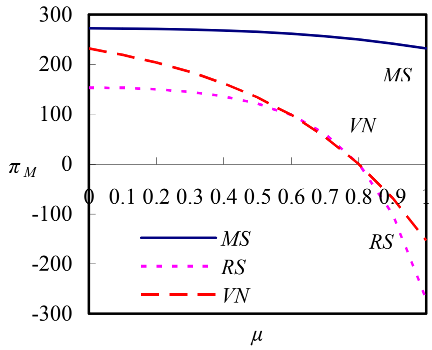

Fig 2: Manufacturer's profit with μ

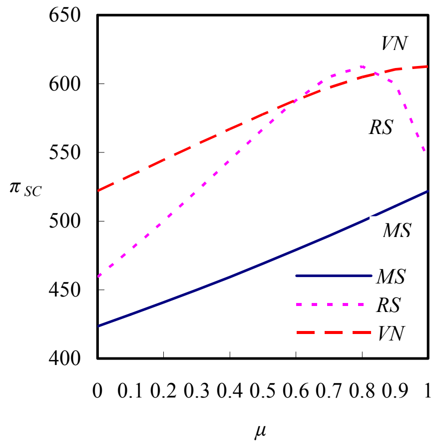

Fig.3: Supply chain's profit with μ

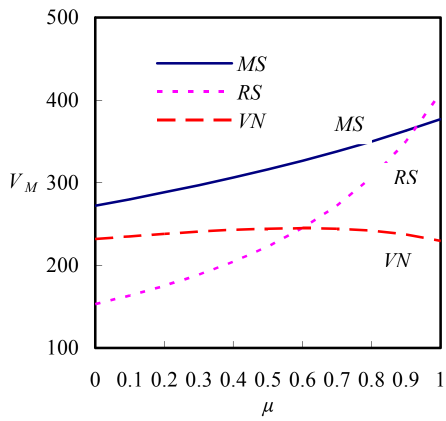

Fig.4: Manufacturer's total profit with μ

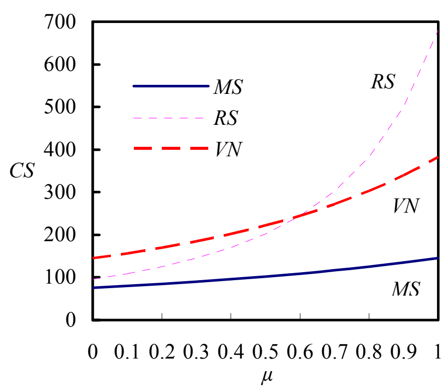

Fig.5: Consumer surplus with μ

Based on the above figurers, we find:

5) Fig. 1 shows that by increasing μ the profit of the retailer will increase in the three games. In addition, the retailer makes his largest profits in the RS game, and the smallest in the MS game.
6) Fig. 2 shows that by increasing μ the pure profit of the manufacturer will decrease in the three games. The pure profit of the manufacturer is the highest in the MS game, followed by the VN game and then RS game. In addition, when μ=0.80, the pure profit is zero, and when 0.80<μ≤1, the pure profit is negative.
7) Fig. 3 shows that by increasing μ the profit of the supply chain system will increase in the MS and VN games, and will increase first and then decrease in the RS game. When μ=0.80, the supply chain makes the largest profit in the RS game. When 0≤μ<0.60 and 0.86<μ≤1, the profit is the highest in the VN game, followed by the RS game and then the MS game. When 0.60<μ<0.86 the profit is the highest in the RS game, followed by the VN game and then the MS game.
8) Fig. 4 shows that by increasing μ the total profit of the manufacturer will increase in the MS and RS games, and will increase first and then decrease in the VN game. When 0≤μ<0.60, the profit is the highest in the MS game and lowest in the RS game. When 0.60<μ<0.93, the profit is the highest in the MS game and lowest in the VN game. When 0.93<μ≤1, the profit is the highest in the RS game and the lowest in the VN game.
9) Fig. 5 shows that by increasing μ the consumer surplus will increase in the three games. When 0≤μ<0.60, the consumer surplus is the highest in the VN game and the lowest in the MS game. When 0.60<μ≤1, the consumer surplus is the highest in the RS game and the lowest in the MS game.

V. CONCLUSION

This paper explores the greening level and the pricings issues in a socially responsible green supply chain by proposing three different games. We consider the conditions in which the socially responsible manufacturer and the retailer have different power structures: Manufacturer-Stackelberg game, Retailer-Stackelberg game and Vertical-Nash game. We also analyze the effect of the manufacturer's CSR activity on the greening level, wholesale price, margin profit, retail price, market demand, retailer's profit, manufacturer's pure and total profits, supply chain system's profit and consumer surplus.

This paper has some limitations. First, we only consider that the manufacturer has social concerns. Therefore, one possible extension work is to study that the manufacturer and the retailer both have social concerns. Second, we only consider one manufacturer and one retailer. Then multiple competing manufacturers or retailers with corporate social responsibility can be considered. Third, we assume that the demand function of the green product is a linear demand function. In future, he other types of the demand function can be employed.

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