Multistability in a Novel Chaotic System with Perpendicular Lines of Equilibrium: Analysis, Adaptive Synchronization and Circuit Design

Aceng Sambas*, Sundarapandian Vaidyanathan, Sen Zhang, Wawan Trisnadi Putra, Mustafa Mamat, and Mohamad Afendee Mohamed

Abstract—**This paper reports the finding of a new chaotic system with perpendicular lines of equilibrium points. Thus, this paper makes a valuable addition to existing chaotic systems with infinite number of equilibrium points in the chaos literature. Specifically, we show that the new three-dimensional chaotic system has the y and z coordinate axes as its line equilibrium points, which are perpendicular. The period-1 attractor, period-2 attractor and chaotic attractor in the system are numerically studied by Lyapunov exponents, bifurcation diagrams and phase diagrams. In addition, the coexisting period-1 attractors and coexisting chaotic attractors of the system are presented. Adaptive controller is devised for global chaos synchronization of identical new chaotic systems with unknown parameters. An electronic circuit simulation of the new chaotic system with perpendicular lines of equilibrium points is shown using Multisim to check the feasibility of the model.**

Keywords: Chaos; chaotic systems; multistability; adaptive control; synchronization; circuit design.

I. INTRODUCTION

SOME key important topics in chaos theory are modeling and applications of nonlinear dynamical systems showing chaotic behavior [1]-[2]. Chaos has generated good interest in science and engineering applications [3]-[8]. Chaos theory has been also applied for special applications such as encryption [9]-[11], wireless communication [12], fingerprint biometric [13], robotics [14], neural networks [15], jerk systems [16]-[17], neurology [18], chemical systems [19], biology [20], oscillators [22]-[24], weather models [25]-[26], circuits [27]-[28] finance [29]-[30], memristors [31], etc.

In the literature, many scientists have studied the modeling of chaotic systems with special types of equilibrium curves such as line equilibrium [32], circle [33], axe-shaped curve [34], heart-shaped curve [35], boomerang-shaped curve [36], conic-shaped equilibrium [37], pear-shaped curve [38], etc. All these chaotic systems fall under the class of systems with hidden chaotic attractors as they possess an infinite number of equilibrium points [39]-[44].

In this paper, we report the finding of a new chaotic system with perpendicular lines of equilibrium points. Thus, this paper makes a valuable addition to existing chaotic systems with special curves of equilibrium points. Specifically, we show that the new 3-D chaotic system has the y and z coordinate axes as its line equilibrium points, which are perpendicular. The presence of coexisting chaotic and periodic attractors in the system is studied.

Synchronization of chaotic systems deals with a pair of chaotic systems called master and slave systems and the design goal is to find a suitable feedback control law attached to the slave system so as to track the signals of the master system asymptotically with time [45]-[46]. In this work, we shall use adaptive control method for the complete synchronization of the new system with itself with unknown system parameters.

Furthermore, circuit simulation of chaotic systems is an important area of research. In this work, we shall exhibit an electronic circuit simulation via Multisim for the new chaotic system with perpendicular lines of equilibrium points. We show a good matching of the theoretical simulation results via MATLAB and the electronic circuit simulation results via Multisim for the new chaotic system with perpendicular equilibrium points.

II. DYNAMICAL ANALYSIS OF THE NEW CHAOTIC SYSTEM WITH PERPENDICULAR LINES EQUILIBRIUM POINT

In this paper, we report a new 3-D chaotic system given by

$$\left\{\begin{array}{lcl} \dot{x} & = & ayz \\ \dot{y} & = & x|z| - y|x| \\ \dot{z} & = & |x| - bx^2 \end{array}\right. \tag{1}$$

where x, y, z are the states and a, b are positive parameters.

In this work, we show that the system (1) exhibits a chaotic attractor when we take the parameter values as

$$a = 1 \text{ and } b = 1 \tag{2}$$

For numerical simulations, we take the initial values as

$$x(0) = 0.2, \quad y(0) = 0.2, \quad z(0) = 0.2 \tag{3}$$

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The Lyapunov chaos exponents (LCE) are determined using MATLAB as $L_1 = 0.05239$, $L_2 = 0$ and $L_3 = -0.72753$. Since $L_1 > 0$, we conclude that the new 3-D system (1) is chaotic. Since the sum of LCE values is negative, we conclude that thew chaotic system (1) is dissipative.

The Kaplan-Yorke dimension of the new chaotic system (1) is obtained as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0720, \tag{4}$$

which indicates the chaotic nature of the new chaotic system.

The equilibrium points of the new chaotic system (1) are tracked by solving the following system:

$$ayz = 0 \tag{5a}$$

$$x|z| - y|x| = 0 \tag{5b}$$

$$|x| - bx^2 = 0 \tag{5c}$$

If we take $x = 0$, then both equations (5b) and (5c) are satisfied.

From Eq. (5a), it follows that either $y = 0$ or $z = 0$.

Thus, the equilibrium points of the new chaotic system (1) are given by the two straight lines, $S_1 = \{(x, y, z) \in \mathbf{R}^3 : x = 0, z = 0\}$, which is the y-axis and $S_2 = \{(x, y, z) \in \mathbf{R}^3 : x = 0, y = 0\}$, which is the z-axis. It is obvious that the lines S_1 and S_2 are perpendicular.

The phase portraits of the new chaotic system (1) with perpendicular lines of equilibrium points S_1 and S_2 are displayed in Figs. 1-3. The Lyapunov chaos exponents of the chaotic system (1) are displayed in Fig. 4.

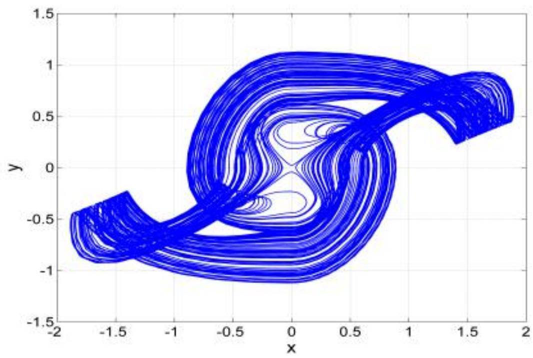

Fig. 1: MATLAB simulation of 2-D phase plot of the new chaotic system (1) in (x, y) plane for $X(0) = (0.2, 0.2, 0.2)$ and parameter values $(a, b) = (1, 1)$

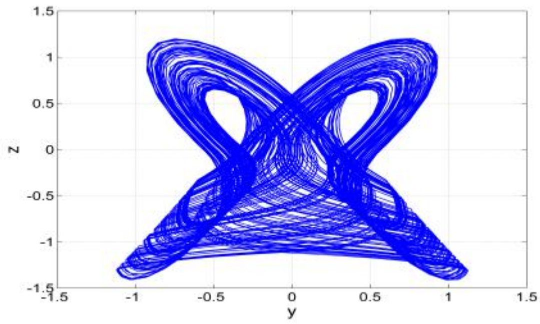

Fig. 2: MATLAB simulation of 2-D phase plot of the new chaotic system (1) in (y, z) plane for $X(0) = (0.2, 0.2, 0.2)$ and parameter values $(a, b) = (1, 1)$

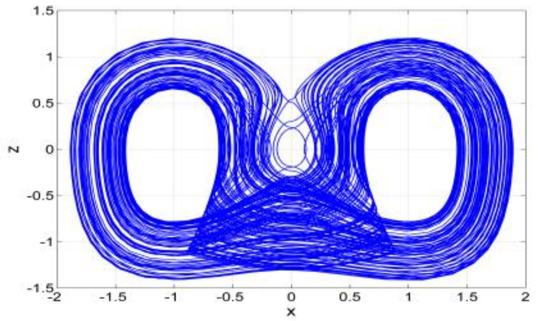

Fig. 3: MATLAB simulation of 2-D phase plot of the new chaotic system (1) in (x, z) plane for $X(0) = (0.2, 0.2, 0.2)$ and parameter values $(a, b) = (1, 1)$

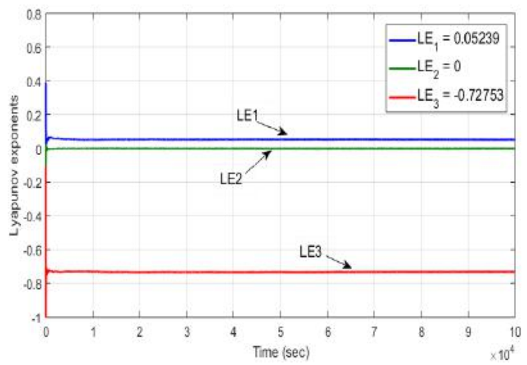

Fig. 4: Lyapunov chaos exponents (LCE) of the new chaotic system (1) for $X(0) = (0.2, 0.2, 0.2)$ and $(a, b) = (1, 1)$

It is well-known that the dynamical behaviors of a non-linear system can be explored by bifurcation diagrams and Lyapunov exponents [47]-[48]. Multistability can lead to very complex behaviors in a dynamical system, which has been reported in some chaotic systems [49]-[54]. It is very interesting that the system can exhibit multistability. Fix b = 0.5 and keep a as the control parameter. When a is varied in the region of [1, 4], the coexisting bifurcation model of the state variable of x and the corresponding Lyapunov exponents with the initial conditions (0.2, 0.2, 0.2) are plotted in Fig. 5 (a) and 5(b), respectively, where the blue orbit starts from the initial conditions (0.2, 0.2, 0.2) and the red orbit starts from the initial conditions (-0.2, -0.2, 0.2). From Fig. 5(a), we can observe several kinds of coexisting attractors with different initial conditions. Fig. 6 exhibits the coexisting periodic attractors with $a = 1.5$ and the coexisting chaotic attractors with $a = 2.5$, where the blue attractor begins with the initial conditions of (0.2, 0.2, 0.2) and the red one begins with (-0.2, -0.2, 0.2). Particularly, it can be found that the system experiences a period-doubling bifurcation route to chaos.The corresponding phase portraits are plotted in Fig. 7. From the above analysis, it can be concluded that the system in fact displays very complicated dynamics. The Poincare map new chaotic system (1) with perpendicular lines, also reflects properties of chaos (see Fig. 8). In addition, Fig. 9 shows the basin of attraction of the coexisting chaotic attractors of the new chaotic system (1) with a = 2.5.

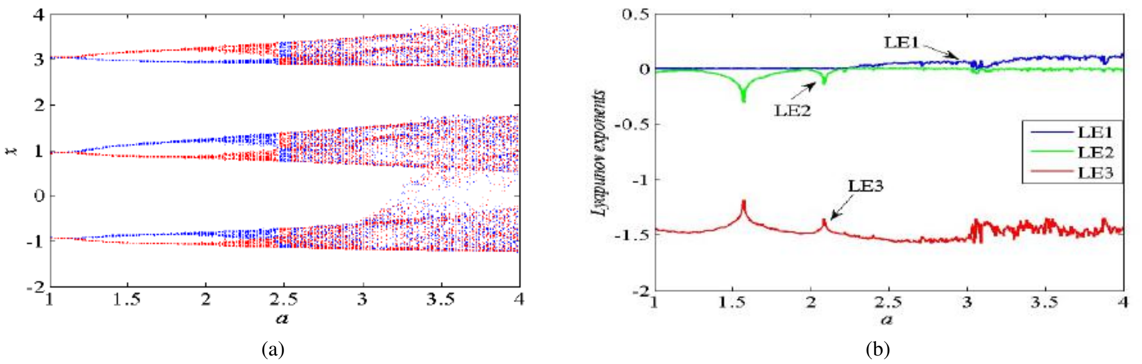

Fig. 5: Dynamics of the system: (a) the coexisting bifurcation diagrams of the state variable x with respect to the control parameter a, (b) the corresponding Lyapunov exponents spectrum with the initial conditions (0.2, 0.2, 0.2)

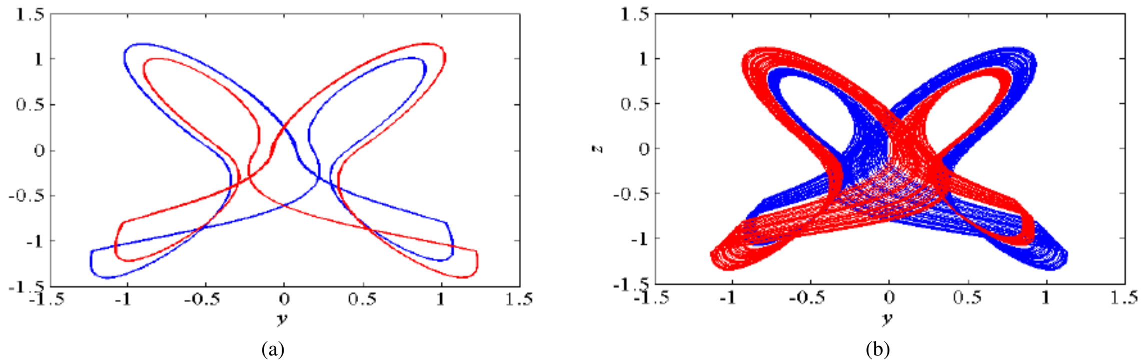

Fig. 6: Different coexisting attractors of the system: (a) coexisting period-1 attractors with a = 1.5, (b) coexisting chaotic attractors with a = 2.5

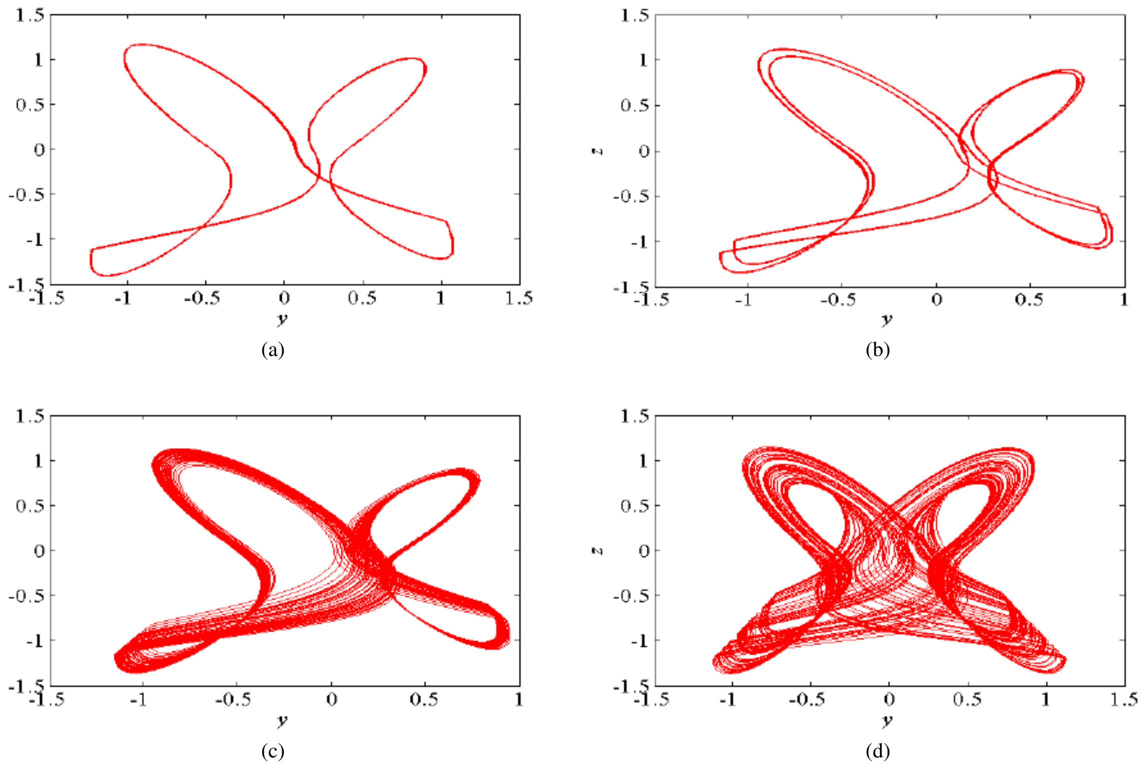

Fig. 7: Period-doubling bifurcation route to chaos: (a) period-1 attractor with a = 1.5, (b) period-2 attractor with a = 2.2, (c) chaotic attractor with a = 2.3, (d) chaotic attractor with a = 3

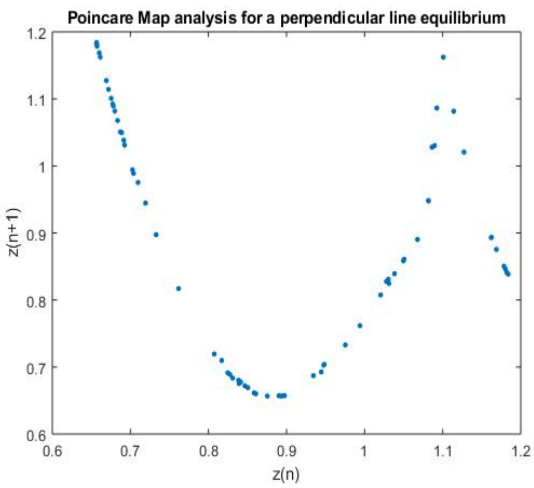

Fig. 8: Poincare map analysis of the new chaotic system (1) for $X(0) = (0.2, 0.2, 0.2)$ and $(a, b) = (1, 1)$

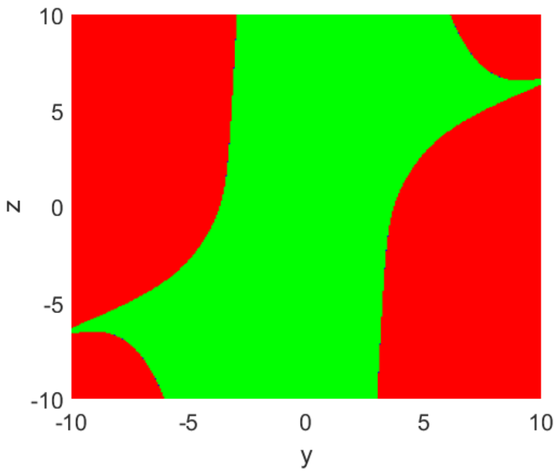

Fig. 9: The basin of attraction of the coexisting chaotic attractors of the the new chaotic system (1) in the *y*-*z* plane with *a* = 2.5

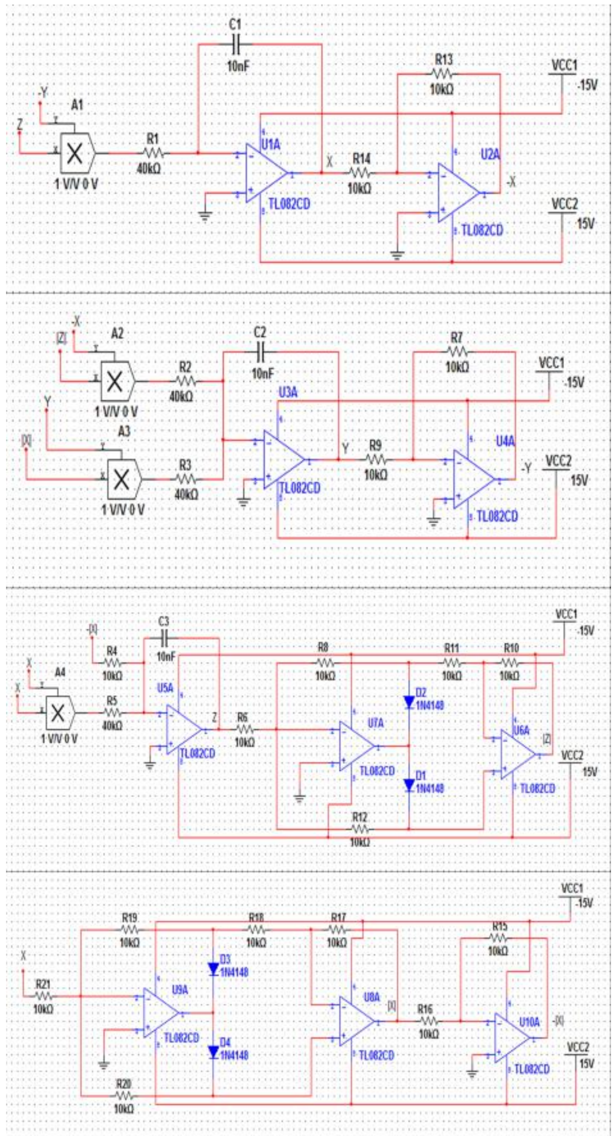

Fig. 10: The electronic circuit schematic of the new chaotic system

III. CIRCUIT DESIGN OF THE NEW CHAOTIC SYSTEM

The electronic circuits of chaotic systems has been applied in engineering applications such as wireless mobile robot, voice encryption, secure communications, image encryption process, radar system and random bits generator. The new chaotic system (1) was designed as an electronic circuit as shown in Fig. 10. As shown in Fig. 10, the electronic circuit includes 21 resistors, 3 capacitors, 10 operational amplifiers (TL082CD) and 4 analog multipliers (AD633JN).

The three state variables (x, y, z) of the new chaotic system (1) have been rescaled as $X = 4x, Y = 4y$ and $Z = 4z$. Therefore, the new chaotic system (1) is transformed into the following equivalent system:

$$\begin{cases} \dot{x} &=& \frac{ayz}{4} \\ \dot{y} &=& \frac{x|z|}{4} - \frac{y|x|}{4} \\ \dot{z} &=& |x| - \frac{bx^2}{4} \end{cases} \tag{6}$$

By applying Kirchhoff's circuit laws into the designed circuit, we get the following circuital equations:

$$\begin{cases} \dot{x} &=& \frac{1}{C_1 R_1}\, yz \\ \dot{y} &=& \frac{1}{C_2 R_2}\, x|z| - \frac{1}{C_2 R_3}\, y|x| \\ \dot{z} &=& \frac{1}{C_3 R_4}\, |x| - \frac{1}{C_3 R_5}\, x^2 \end{cases} \tag{7}$$

where the variables x, y and z correspond to the voltages in the outputs of the integrators $(U1A, U3A, U5A)$. The power supplies of all active devices are $\pm 15 volts$.

We choose the values of the circuital elements as follows: $R_1 = R_2 = R_3 = R_5 = 40$ kΩ, $R_4 = R_6 = R_7 = R_8 = R_9 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = R_{20} = R_{21} = 10$ kΩ and $C_1 = C_2 = C_3 = 10$ nF.

Multisim outputs of the scaled system are depicted in Fig. 11-13. It is easy to see the agreement between the oscilloscope outputs shown in Figs. 11-13 and the MATLAB simulations shown in Figs. 1-3 for the new chaotic system with perpendicular lines of equilibrium points.

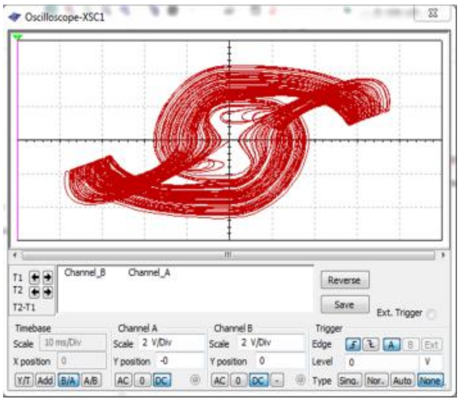

Fig. 11: Multisim outputs of the scaled new chaotic system (7) in (a) $X-Y$ plane

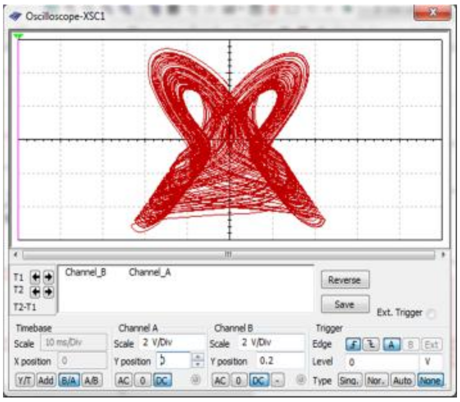

Fig. 12: Multisim outputs of the scaled new chaotic system (7) in (a) $Y-Z$ plane

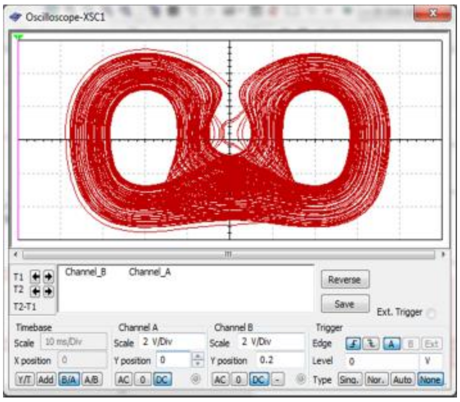

Fig. 13: Multisim outputs of the scaled new chaotic system (7) in (a) $X-Z$ plane

IV. Adaptive Synchronization of New Chaotic Systems with Unknown Parameters

In this section, we use adaptive control theory to derive a new controller for synchronizing the trajectories of a pair of new chaotic systems, considered as master and slave systems. Synchronization of chaotic systems has applications in secure communication systems.

As the master system, we consider the new chaotic system given by

$$\begin{cases} \dot{x}_1 &= ay_1z_1 \\ \dot{y}_1 &= x_1|z_1| - y_1|x_1| \\ \dot{z}_1 &= |x_1| - bx_1^2 \end{cases} \tag{8}$$

where a, b are unknown parameters and x_1, y_1, z_1 are the states.

As the slave system, we consider the new chaotic system with controls given by

$$\begin{cases} \dot{x}_2 &= ay_2z_2 + u_x \\ \dot{y}_2 &= x_2|z_2| - y_2|x_2| + u_y \\ \dot{z}_2 &= |x_2| - bx_2^2 + u_z \end{cases} \tag{9}$$

where x_2, y_2, z_2 are the states and u_x, u_y, u_z are the adaptive controls.

The synchronization error between the systems (8) and (9) is defined by

$$e_x = x_2 - x_1, \quad e_y = y_2 - y_1, \quad e_z = z_2 - z_1 \tag{10}$$

Then the synchronization error dynamics is derived as follows:

$$\begin{cases} \dot{e}_x &= a(y_2z_2 - y_1z_1) + u_x \\ \dot{e}_y &= x_2|z_2| - x_1|z_1| - y_2|x_2| + y_1|x_1| + u_y \\ \dot{e}_z &= |x_2| - |x_1| - b(x_2^2 - x_1^2) + u_z \end{cases} \tag{11}$$

We implement the adaptive controller defined by

$$\begin{cases} u_x &= -A(t)(y_2z_2 - y_1z_1) - k_xe_x \\ u_y &= -x_2|z_2| + x_1|z_1| + y_2|x_2| - y_1|x_1| - k_ye_y \\ u_z &= -|x_2| + |x_1| + B(t)(x_2^2 - x_1^2) - k_ze_z \end{cases} \tag{12}$$

where k_x, k_y, k_z are positive gains.

Implementing (12) in (11), we get the closed-loop system

$$\begin{cases} \dot{e}_x &= [a - A(t)](y_2z_2 - y_1z_1) - k_xu_x \\ \dot{e}_y &= -k_ye_y \\ \dot{e}_z &= -[b - B(t)](x_2^2 - x_1^2) - k_ze_z \end{cases} \tag{13}$$

Next, we define

$$e_a = a - A(t), \quad e_b = b - B(t) \tag{14}$$

Then we can simplify the closed-loop dynamics (13) as

$$\begin{cases} \dot{e}_x &= e_a(y_2z_2 - y_1z_1) - k_xe_x \\ \dot{e}_y &= -k_ye_y \\ \dot{e}_z &= -e_b(x_2^2 - x_1^2) - k_ze_z \end{cases} \tag{15}$$

We also note that

$$\dot{e}_a = -\dot{A}, \quad \dot{e}_b = -\dot{B} \tag{16}$$

We use Lyapunov stability theory for the main result on global chaos synchronization of new chaotic systems.

We define the candidate Lyapunov function as

$$V(e_x, e_y, e_z, e_a, e_b) = \frac{1}{2}\left(e_x^2 + e_y^2 + e_z^2 + e_a^2 + e_b^2\right), \tag{17}$$

which is quadratic and positive-definite on $\mathbf{R}^5$.

The time-derivative of V is calculated as follows:

$$\begin{aligned} \dot{V} &= -k_xe_x^2 - k_ye_y^2 - k_ze_z^2 + e_a[e_x(y_2z_2 - y_1z_1) - \dot{A}] \\ &= +e_b[-e_z(x_2^2 - x_1^2) - \dot{B}] \end{aligned} \tag{18}$$

Based on (18), we consider the parameter update law as follows:

$$\begin{cases} \dot{A} &= e_x(y_2 z_2 - y_1 z_1) \\ \dot{B} &= -e_z(x_2^2 - x_1^2) \end{cases} \tag{19}$$

Theorem 1. The new chaotic systems (8) and (9) are globally and asymptotically synchronized for all initial conditions by the adaptive control law (12) and the parameter update law (19), where k_x, k_y, k_z are positive gain constants.

Proof: We prove this result by Lyapunov stability theory [55].

First, we note that V defined by Eq. (17) is quadratic and positive definite on $\mathbf{R}^5$.

Next, after substituting the parameter update law (19) into (18), we get the time-derivative of V as

$$\dot{V} = -k_x e_x^2 - k_y e_y^2 - k_z e_z^2, \tag{20}$$

which is quadratic and negative semi-definite function on $\mathbf{R}^5$.

Hence, by Barbalat's lemma [55], it follows that the error dynamics (15) is globally asymptotically stable for all initial conditions.

This completes the proof. ■

For numerical simulations, we take the initial state of the master system (8) as $(x_1(0), y_1(0), z_1(0)) = (2.5, -3.6, 8.4)$ and the initial state of the slave system (9) as $(x_2(0), y_2(0), z_2(0)) = (6.3, 7.9, 4.8)$.

We take the initial state of the parameter estimates as $(A(0), B(0)) = (10.5, 8.7)$.

We take the gain constants as $k_x = 10, k_y = 10$ and $k_z = 10$.

Fig. 14 shows the synchronization of the new chaotic systems (8) and (9). Fig. 15 shows the time-history of the synchronization errors e_x, e_y, e_z.

V. Conclusion

This paper reported a new chaotic system with perpendicular lines of equilibrium points. Specifically, we showed that the new 3-D chaotic system has the y and z coordinate axes as its line equilibrium points, which are perpendicular. Simulation determined the coexisting attractors of the system with different parameters and initial values, such as coexisting period-1 attrators and coexisting chaotic attractors. We discussed the qualitative properties of the new chaotic system and designed an adaptive controller for global chaos synchronization of identical new chaotic systems with unknown parameters. An electronic circuit simulation of the new chaotic system with perpendicular lines of equilibrium points was displayed using Multisim to check the feasibility of the theoretical chaotic model.

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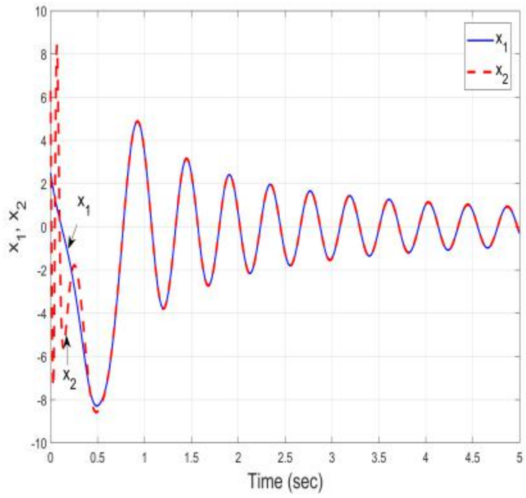
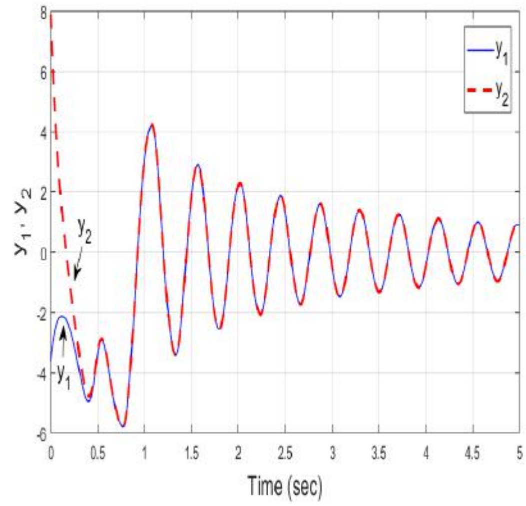


(a)



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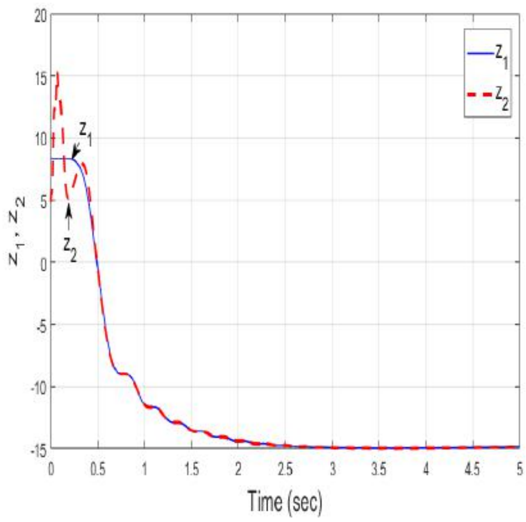

(c)

Fig. 14: Complete synchronization of the new chaotic systems (8) and (9) : (a) x_1 and x_2, (b) y_1 and y_2, (c) z_1 and z_2

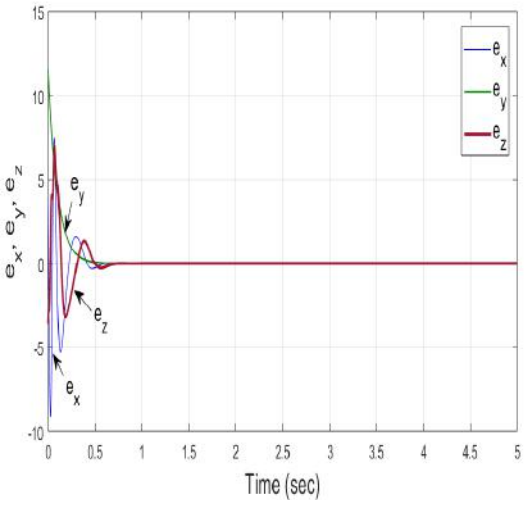

Fig. 15: Time-history of the synchronization errors e_x, e_y, e_z

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