Optimization of Simulation Parameters of Trajectory Correction Projectile Based on Orthogonal Design Method

Hanzhou Wu, Min Gao, Jingqing Xu, Yi Wang, Wendong Zhao

***Abstract*—The study of aerodynamic shape of guided projectiles is an important means to master their flight characteristics. This research describes a computational study undertaken to determine the effect of simulation parameter settings for the calculation accuracy of axial force coefficient and normal force coefficient of a kind of two-dimensional (2D) trajectory correction projectile. The turbulence model, mesh size, flow field size, y^+ value, and their interactions are regard as the factors in computational fluid dynamics simulation. Based on orthogonal design test, the error of aerodynamic simulation data is analyzed. The results show that turbulence model, mesh size, flow field size, and the interaction between turbulence model and mesh size have obviously significant influence on the calculation accuracy of axial force coefficient and normal force coefficient. The analysis shows the optimal combination scheme of parameter settings among factors. The range of aerodynamic calculation error and its confidence interval are given for the first time. The maximum error of the optimal factor level combination of calculated aerodynamic parameters determined by orthogonal test method is no more than 6% when the confidence is 90%.**

***Index Terms*—2D trajectory correction projectile, aerodynamic calculation, orthogonal design test, simulation accuracy, parameter optimization**

I. Introduction

For the guided ammunitions, well aerodynamic shape design can improve flight stability, controllability and shooting accuracy. At present, computational fluid dynamics (CFD) is widely used to design aerodynamic shape of projectiles and missiles. Based on CFD simulation, Mojtaba et al. analyzed the flow field characteristics of different aerodynamic shapes and different parts of the projectile or aircraft during flight, and the CFD results can well reflect the real flight state [1-7]. Roxan et al. analyzed the Magnus effect at different speeds and angle of attack of the projectile based on CFD simulation [8], [9]. Montgomery et al. analyzed the flow field characteristics of missiles or airfoils in different circumstances [10-14]. The feature is difficult to find or capture with other test methods. The research results provide a reference for the design of the projectile or aircraft. John et al. analyzed the aerodynamic characteristics of different shape parameters of the projectile, and the best aerodynamic shape of the projectile was selected [15-19].

Jubaraj et al. combined rigid body dynamics theory with CFD simulation to analyze the aerodynamic characteristics of the missile in different flight circumstances and achieved high simulation accuracy [20], [21]. Mojtaba et al. combined genetic algorithm with CFD simulation to optimize the airfoil shape parameters, which greatly improved the aerodynamic efficiency of the new airfoil [22], [23]. James et al. analyzed the accuracy of different turbulence models to solve the simulation model in CFD simulation, and selected the best turbulence solution model for the specific model [24-27]. Xie et al. analyzed the coupling between the canards and the fin of projectile, and pointed out that the coupling was related to the canard angle and flight speed [28], [29]. Chen analyzed the canard-controlled rocket's roll characteristics and flow field characteristics. A simulation calculation convergence criterion including convergence residuals and stable detection results was proposed [14]. Singh notes that CFD has become an important tool in the design and analysis of various aerospace vehicles in India, helping to make design process faster, more accurate, and less expensive [30].

Little research has been done on the influence of parameter setting on simulation accuracy in CFD simulation. The influence of the collaborative setting of parameters on simulation accuracy is also lack of research. At the same time, there is no relevant literature on the problem of simulation error confidence interval when simulation calculations can not be verified by experiments. In this paper, turbulence model, mesh size, flow field size, y^+ value and their interactions are regards as the factors of orthogonal design test. The variance theory is used to analyze the influence of different levels of factor combinations on the simulation precision, and the error range and error confidence interval of the optimal combination are given.

II. Selction of Factors Affect Simulation Accuracy

A. Research object

In this paper, a two-dimensional (2D) trajectory correction projectile is taken as the research object as shown in Fig. 1.

This work is supported by National Deference Pre-Research Foundation of China.

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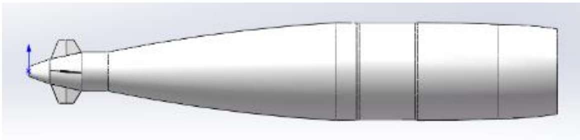
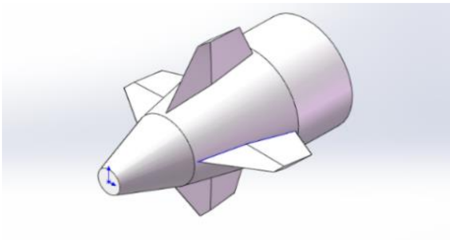
The wind tunnel test data of the 1:2 reduction model of the projectile is taken as the standard data. The test was completed by the China Academy of Aerospace Aerodynamics as shown in Fig. 2. The CFD simulation model used in this paper is similar with the model used in wind tunnel.



(a) The projectile complete model



(b) Trajectory correction module

Fig. 1. 2D trajectory correction projectile model.

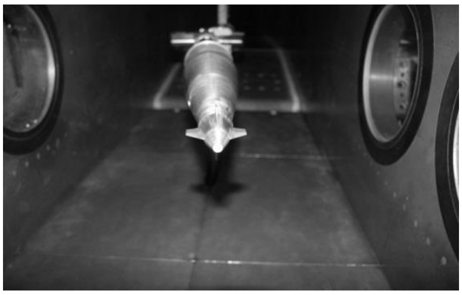


Fig. 2. Wind tunnel test photo.

B. Factor selection and parameter setting

The accuracy of CFD simulation is directly related to the settings of simulation parameters. This paper selects four factors that have great influence on the simulation accuracy: turbulence model, mesh size, flow field size, and y^+ value.

Three turbulence models commonly used in CFD simulation of projectiles are selected which are regarded as three levels in orthogonal design test, as shown in Table I.

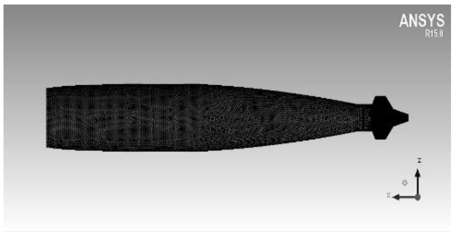
TABLE I TURBULENCE MODEL SETTING

Level	Turbulence model
1	S-A
2	k - ε
3	k-ω

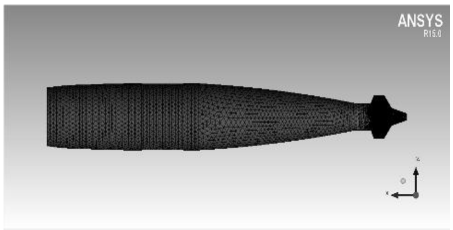
TABLE II MESH SIZE SETTING

Level	Fuze	Canards (mm)	Body (mm)	Far field (mm)
1	1	0.1	2	50
2	2	0.2	4	100
3	3	0.3	8	150

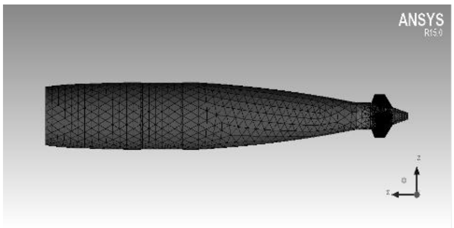
In order to reduce the number of mesh and improve the calculation efficiency, the mesh size of projectile body should be appropriately larger, and the mesh size of the fuze and the four canards should be set smaller. The maximum mesh parameter settings for different parts of the projectile are shown in Table II, and the projectile mesh is shown in Fig. 3.



(a) Level 1 mesh setting



(b) Level 2 mesh setting



(c) Level 3 mesh setting

Fig. 3. Projectile mesh size setting

The flow field parameter settings are shown in Table III, and the schematic diagram is shown as Fig. 4. Lf represents the distance between the projectile head and the flow field entrance. Lb is the distance between the tail of the projectile and the exit of the flow field. Df is the flow field diameter. L is the total length of the projectile. D is the diameter of the projectile.

TABLE III FLOW FIELD PARAMETER SETTING

Level	Lf	Lb	Df
1	4L	8L	40D
2	8L	11L	50D
3	12L	14L	60D

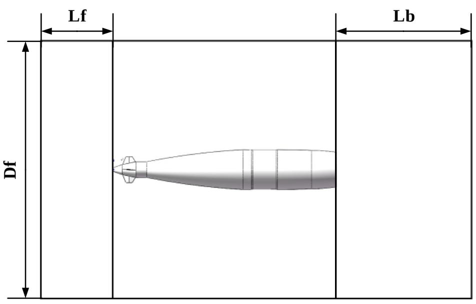
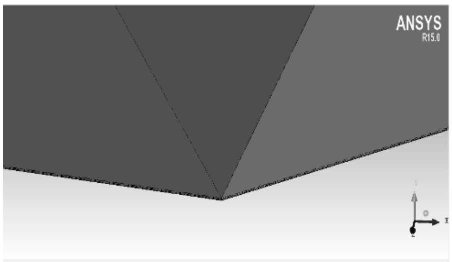


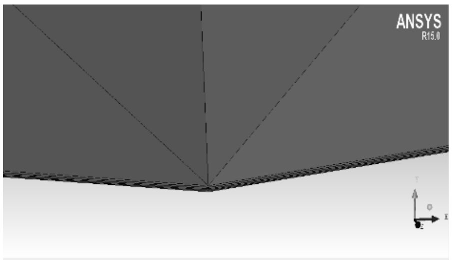
Fig. 4. Computational domain schematic diagram.

TABLE IV y^+ VALUE SETTING TABLE

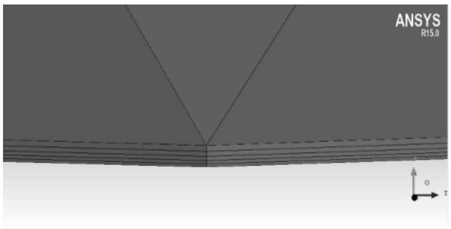
Level	y^+	Δy (mm)
1	0.9	5.81×10^{-7}
2	20	1.29×10^{-5}
3	90	5.81×10^{-5}



(a) y^+=0.9



(b) y^+=20



(c) y^+=90

Fig. 5. Boundary layer mesh of projectile.

Since y^+ is a dimensionless quantity, it needs to be converted into the first layer mesh height value Δy on the surface of the projectile before the simulation calculation. The conversion equation is $\Delta y = y^+ \cdot \mu / \sqrt{\tau_w \cdot \rho}$. Where, μ is the fluid viscosity coefficient, τ_w is the wall shear stress, and ρ is fluid density. In this paper y^+ values and Δy values are shown in Table IV. In this paper, we choose to generate 7-layer boundary layer mesh. The surface meshes of the projectile are shown in Fig. 5.

III. Orthogonal Table Design

The selected four factors are numbered as shown in Table V.

TABLE V FACTOR NUMBER TABLE

Factors	Turbulence model	Mesh size	Flow field	y^+
Number	*A*	*B*	*C*	*D*

A. Orthogonal table design

This paper selects four factors to analyze its impact on CFD simulation accuracy, and each factor has three levels. Analyzing all combination schemes of factor levels will take a lot of manpower and time. The orthogonal design test method only needs to select some typical combination schemes for experiments. Through the mathematical statistics and analysis of the test results, satisfactory test results can be obtained [31].

The interaction between the turbulence model and the flow field has an impact on the accuracy of the CFD simulation, similarly the interaction between the turbulence model and the mesh size and the interaction between the turbulence model and the y^+ value. These three interactions are analyzed as factors, and the total number of factors is 4+3=7, then the orthogonal table $L_{27}(3^{13})$ needs to be used for the experimental design. The orthogonal design table is shown in Table VI. In Table VI, the factors are arranged in columns, and the numbers in the table content area represent the factor levels. For example: the first row and the first column can be represented by [1 *A*], and the number of this position is 1, indicating that the level of factor *A* is 1 in the first test; the number in the [2 (*AC*)1] position has no guiding significance for the factor parameter setting and is only used for data analysis. For the three levels of factors, their interactions take up two columns in the orthogonal table, so (*AC*)1 represents the first interaction of factors *A* and *C*. Three blank columns were not arranged for test factors and were mainly used for test error analysis. The numbers in other locations in the table are analogous.

B. Variance analysis theory

a. Deviation square sum and degree of freedom

Suppose that *m* factors are arranged in orthogonal table, the total number of tests is *n*, the test results are $x_1, x_2, ..., x_n$, respectively, assuming n_a levels for each factor, and *a* tests for each level, then $n=a \cdot n_a$.

1. Total deviation square sum S_T

The total deviation square sum S_T is the sum of deviation square of all test data from its total average value, which indicates the total fluctuation of the test data, and its calculation formulas as follow:

$$S_T = \sum_{k=1}^{n}(x_k - \bar{x})^2 = \sum_{k=1}^{n} x_k^2 - \frac{1}{n}(\sum_{k=1}^{n} x_k)^2 = Q_T - P \quad (1)$$

Where,

$$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x_k \quad (2)$$

$$Q_T = \sum_{k=1}^{n} x_k^2 \quad (3)$$

$$P = \frac{1}{n}(\sum_{k=1}^{n} x_k)^2 \quad (4)$$

2. Deviation square sum of factors

Deviation square sum of factors reflects the fluctuation of the test data caused by the change of factor levels. Suppose S_X is the deviation square sum of factor *X*. In the orthogonal tests, x_{ij} (i=1, 2, ..., n_a; j=1, 2, ..., a) is used to represent the result of the *j*th test at the *i*th level of factor *X*, then there is

$$\sum_{i=1}^{n_a}\sum_{j=1}^{a} x_{ij} = \sum_{k=1}^{n} x_k \quad (5)$$

$$S_X = \frac{1}{a}\sum_{i=1}^{n_a}(\sum_{j=1}^{a} x_{ij})^2 - \frac{1}{n}(\sum_{i=1}^{n_a}\sum_{j=1}^{a} x_{ij})^2 = \frac{1}{a}\sum_{i=1}^{n_a} K_i^2 - \frac{1}{n}(\sum_{k=1}^{n} x_k)^2 = Q_X - P \quad (6)$$

Where,

$$Q_X = \frac{1}{a}\sum_{i=1}^{n_a} K_i^2 \quad (7)$$

$$K_i = \sum_{j=1}^{a} x_{ij} \quad (8)$$

K_i represents the sum of test data for factor X of level i. In the same way, the deviation squared sum of other factors can be calculated.

3. Factors degree of freedom

Assume that the test degree of freedom is f, then

$$f = n - 1 \quad (9)$$

Assume that the degree of freedom of factor X is f_x, then

$$f_X = n_a - 1 \quad (10)$$

The degree of freedom of interaction between factors is equal to the sum of degrees of freedom of each factor. Each column in the orthogonal table represents a factor. Columns without a scheduling factor are considered error columns of the text.

b. Significant test

Each factor is tested for significance using the F test. The F value of factor X is

$$F_X = \frac{S_X / f_X}{S_e / f_e} \quad (11)$$

Where S_e is the sum of deviations squared of the test errors, that is, the sum of the deviations squared of the blank columns, and f_e is the degree of freedom of the test error, that is, the sum of the degrees of freedom of the blank columns. F_X is a random variable subject to the F distribution with degrees of freedom (f_X, f_e). Assuming that the significance level is selected as α, the F distribution table is checked to find the critical value $F_\alpha(f_X, f_e)$. If $F_X > F_\alpha(f_X, f_e)$, the level change of the X factor has an obviously significant effect on the test result, and the confidence of the conclusion is $100(1-\alpha)\%$. α is usually chosen from 0.01 to 0.1.

C. Optimal factor level combination and confidence interval

Assumed that the optimal factor level combination is determined as $A_iB_jC_kD_l$ (i, j, k, l = 1, 2, 3). The point estimate of the simulation error of the optimal combination is

$$\hat{a}_{best} = \bar{x} + \hat{a}_i + \hat{b}_j + (\hat{a}\hat{b})_{ij} + \hat{c}_k + (\hat{a}\hat{c})_{ik} + \hat{d}_l + (\hat{a}\hat{d})_{il} \quad (12)$$

Where,

$$\begin{cases} \hat{a}_i = K_{A_i}/3 - \bar{x} \\ \hat{b}_j = K_{B_j}/3 - \bar{x} \\ \hat{c}_k = K_{C_k}/3 - \bar{x} \\ \hat{d}_l = K_{D_l}/3 - \bar{x} \\ (\hat{a}\hat{b})_{ij} = K_{A_iB_j}/3 - \hat{a}_i - \hat{b}_j - \bar{x} \\ (\hat{a}\hat{c})_{ik} = K_{A_iC_k}/3 - \hat{a}_i - \hat{c}_k - \bar{x} \\ (\hat{a}\hat{d})_{il} = K_{A_iD_l}/3 - \hat{a}_i - \hat{d}_l - \bar{x} \end{cases} \quad (13)$$

The error limit is calculated as follow:

$$\varepsilon_\alpha = \sqrt{F_\alpha(1, f_e + f_e')(S_e + S_e')/[(f_e + f_e')n/(1+f^*)]} \quad (14)$$

Where, f_e' is the sum of freedom degrees of insignificant factors; S_e' is the sum of deviation square sum of insignificant factors; f^* is the sum of freedom degrees of obviously significant factors.

The optimal combination $A_iB_jC_kD_l$ simulation precision true value range is $(\hat{x}_{best} - \varepsilon_\alpha)$ to $(\hat{x}_{best} + \varepsilon_\alpha)$, and the confidence is $100(1-\alpha)\%$.

IV. Results and Disicussion

In this paper, the flight states of subsonic (0.6Ma), transonic (1.1Ma) and supersonic (2.2Ma) are simulated. The calculation errors of axial force coefficient (C_A) and normal force coefficient (C_N) are evaluated. The simulation errors at different speeds are shown in Table VII.

The test serial number of Table VII corresponds to the test serial number in Table VI. In the following analysis, the negative data is used its absolute value.

A. Simulation results analysis at subsonic speeds

a. C_A error analysis

1. Direct analysis

Table VII shows that $A_2B_1C_3D_1$ combination calculation has the highest accuracy of the C_A at subsonic speeds.

2. Range analysis

Calculate the sum of the test data at different levels of each factor K_i (i = 1, 2, 3), and divide K_i by the numbers of test of each level in the test to obtain the average error k_i. Calculate the range R of k_i. Generally speaking, the larger the value of R, the greater the influence of this factor on the test results. The data analysis results are shown in Table VIII, and k_i of four single factors are shown in Fig. 6. The last line of Table VIII is the sort of factors according to the value of R. Table VIII shows that for the calculation of the subsonic C_A, the R of single factor is greater than that of the interaction between the factors.

Fig. 6 shows that $A_2B_1C_3D_1$ factor level combination corresponds to the smallest calculation error, which is consistent with the results in the direct analysis. Since the R of single factors are greater than the interaction factors, the determination of the optimal factor level is directly determined by single factor level settings. Then $A_2B_1C_3D_1$ is the optimal level settings combination for calculating the subsonic C_A.

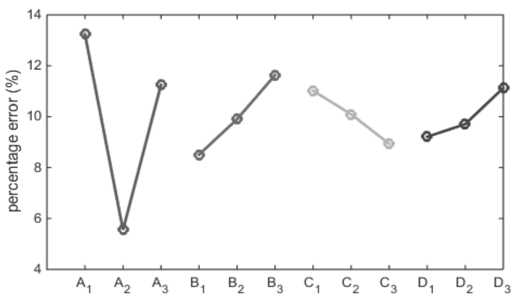


Fig. 6. k_i of four factors at subsonic speeds of C_A

TABLE VIII Error analysis of C_A at subsonic speeds

Object	A	B	C	D	$(AB)1$	$(AB)2$	$(AC)1$	$(AC)2$	$(AD)1$	$(AD)2$
k_1	13.24	8.51	11.02	9.21	10.24	10.68	10.15	9.94	9.90	9.73
k_2	5.55	9.90	10.09	9.71	10.56	9.74	10.25	10.34	9.60	10.35
k_3	11.25	11.64	8.93	11.13	9.25	9.62	9.65	9.77	10.55	9.96
R	7.68	3.13	2.08	1.92	1.43		0.69		0.95	
Rank	$A>B>C>D>AB>AD>AC$									

TABLE IX VRIANCE ANALYSIS OF FACTORS OF C_A AT SUBSONIC SPEEDS

Factors	S	f	$\bar{S}$	F	Threshold	Significant
A	286.31	2	143.16	492.09		**
B	44.26	2	22.13	76.07	$F_{0.25}(2,6)=3.31$	**
C	19.63	2	9.81	33.73	$F_{0.10}(2,6)=9.33$	**
D	17.85	2	8.93	30.68		**
AB	14.42	4	3.61	12.39	$F_{0.25}(4,6)=2.08$	**
AC	3.41	4	0.85	2.93	$F_{0.10}(4,6)=4.01$	*
AD	6.00	4	1.50	5.15		**
Errors	1.75	6	0.29			

TABLE X ERROR ANALYSIS OF C_N AT SUBSONIC SPEEDS

Object	A	B	C	D	$(AB)1$	$(AB)2$	$(AC)1$	$(AC)2$	$(AD)1$	$(AD)2$
k_1	13.68	9.14	11.76	10.20	11.01	11.03	10.46	10.29	10.98	10.68
k_2	6.18	10.72	10.53	10.32	11.27	11.09	11.14	11.28	10.08	10.89
k_3	12.13	12.13	9.70	11.47	9.71	9.87	10.39	10.42	10.93	10.42
R	7.50	2.99	2.06	1.27	1.56		0.99		0.89	
Rank	$A>B>C>AB>D>AC>AD$.									

TABLE XI VRIANCE ANALYSIS OF FACTORS OF C_N AT SUBSONIC SPEEDS

Factors	S	f	$\bar{S}$	F	Threshold	Significant
A	281.73	2	140.86	187.71		**
B	40.23	2	20.11	26.80	$F_{0.25}(2,6)=3.31$	**
C	19.35	2	9.68	12.89	$F_{0.10}(2,6)=9.33$	**
D	8.88	2	4.44	5.91		*
AB	21.26	4	5.31	7.08	$F_{0.25}(4,6)=2.08$	**
AC	8.31	4	2.08	2.77	$F_{0.10}(4,6)=4.01$	*
AD	5.57	4	1.39	1.86		-
Errors	4.50	6	0.75			

3. Variance analysis

Calculate P, Q_T, S_T respectively.

$$P=(12.56+12.81+...+12.34)^2/27=2708.34$$

$$Q_T=12.56^2+12.81^2+...+12.34^2=3101.96$$

$$S_T=Q_T-P=393.62$$

Calculate the deviation square sum of each factor.

$$S_A=Q_A-P=286.31$$

So and $S_B=44.26$, $S_C=19.63$, $S_D=17.85$, $S_e=1.75$.

Calculate the deviation square sum of interactions between factors.

$$S_{AB}=Q_{(AB)1}-P+Q_{(AB)2}-P=14.42$$

So and $S_{AC}=3.41$, $S_{AD}=6.00$.

Calculate the freedom degree of each factor, and calculate the F value of each factor. If $F_i>F_{0.1}(f_e,f_i)$ (i indicates factors), it is considered that this factor has an obviously significant influence on the calculation accuracy of C_A at subsonic speeds, and is represented by symbol "**". If $F_{0.1}(f_e,f_i)>F_i>F_{0.25}(f_e,f_i)$, it is considered that this factor has a significant influence on the accuracy calculation of C_A at subsonic speeds, and is represented by symbol "*". If $F_i<F_{0.25}(f_e,f_i)$, then this factor is considered to have an insignificant effect on the accuracy calculation, and is represented by symbol "-". The results of the above analysis are summarized into Table IX. It can be seen from Table IX:

(1) The influence of single factor on the calculation accuracy of C_A is greater than interaction between factors. The influence rank of each factor is same with the range analysis.

(2) The four single factors have obviously significant impact on the accuracy calculation of C_A, and so is interaction factors AB and AD. The interaction factor AC has significant effect on the calculation.

(3) The k-ε turbulence model has high accuracy in calculating C_A of projectiles at subsonic speeds. Decreasing the mesh size settings, increasing the calculation domain appropriately, and reducing the value of y^+ are beneficial to improve the calculation accuracy. At the same time, reasonable adjustment of the turbulence model and the parameter settings of the other three single factors can also significantly improve the calculation accuracy.

4. Confidence interval for optimal combination

The combination of the factor levels selected in above analysis is $A_2B_1C_3D_1$. The confidence interval is calculated as follow:

$\bar{x}=10.02$, $\hat{a}_2=\bar{A}_2-\bar{x}=-4.47$, so and $\hat{b}_1=-1.51$, $\hat{c}_3=-1.08$, $\hat{d}_1=-0.80$.

$(\hat{a}\hat{b})_{21}=\overline{A_2B_1}-\hat{a}_2-\hat{b}_1-\bar{x}=0.15$, so and $(\hat{a}\hat{c})_{23}=0.46$, $(\hat{a}\hat{d})_{21}=0.87$.

Calculate $\hat{x}_{best}$ according to formula (12).

$$\hat{x}_{best}=\bar{x}+\hat{a}_2+\hat{b}_1+\hat{c}_3+\hat{d}_1+(\hat{a}\hat{b})_{21}+(\hat{a}\hat{c})_{23}+(\hat{a}\hat{d})_{21}=3.64$$

As can be seen from Table IX, $f_e=6$, $f_e'=4$, $S_e=1.75$, $S_e'=3.41$, $n=27$, $f^*=16$. Checking the F distribution table, $F_{0.10}(1,10)=3.29$.

Then,

$$\varepsilon_{0.1}=\sqrt{F_{0.1}(1,f_e+f_e')(S_e+S_e')/[(f_e+f_e')n/(1+f^*)]}=1.03$$

Therefore, the optimal factor level combination $A_2B_1C_3D_1$ calculates the subsonic C_A error interval of the projectile is (3.64-1.03)% - (3.64+1.03)%, which is 2.61% - 4.67%, with a confidence of 90%.

b. C_N error analysis

1. Direct analysis

Table VII shows that the optimal combination of calculating the subsonic C_N is $A_2B_1C_3D_1$, and the calculation error is 3.86%.

2. Range analysis

Calculate k_i and R. The analysis results are shown in Table X, and the k_i value is shown in Fig. 7.

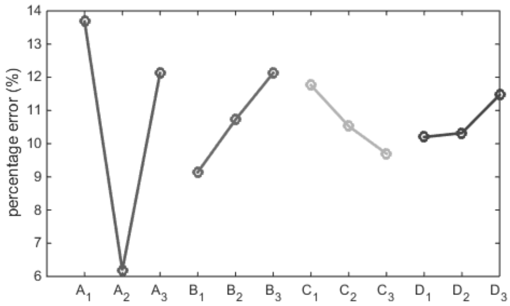


Fig. 7. k_i of four factors at subsonic speeds of C_N

Table X shows that any two single factors are ranked in front of their interaction factors, so when determining the optimal factor level combination, it is only necessary to refer to the optimal level of single factors. As can be seen from Fig. 7, the $A_2B_1C_3D_1$ factor level combination corresponds to the smallest error, which is consistent with the conclusion of direct analysis. Therefore, the optimal factor level combination for calculating the subsonic C_N is $A_2B_1C_3D_1$.

3. Variance analysis

Variance analysis of C_N is similar to C_A, and only the calculation results are given here, as shown in Table XI. As can be seen from Table XI:

(1) The S_e of C_N at subsonic speeds is significantly larger than that of C_A. Factors not considered in the test have an increased influence on the calculation of C_N. But, the error effect is still small compared to various factors.

(2) At subsonic speeds, factor D has significant effect on the calculation of C_N, and the factor AD has insignificant effect.

(3) The rank of influence of each factor is consistent with the range analysis. Therefore, $A_2B_1C_3D_1$ can be determined as the optimal combination of calculating the subsonic C_N.

4. Confidence interval for optimal combination

$\hat{x}_{best} = 3.95$. $f_e = 6$, $f_e' = 10$, $S_e = 4.50$, $S_e' = 22.76$, $n = 27$, $f^* = 10$. $F_{0.10}(1,16) = 3.05$. Then, $\varepsilon_{0.1} = 1.46$. Therefore, $A_2B_1C_3D_1$ calculates the subsonic C_N error interval of the projectile is (3.95-1.46)% - (3.95+1.46)%, which is 2.49% - 5.41%, with confidence of 90%.

B. *Analysis of simulation results at transonic speeds*

a. Direct analysis

Table VII shows that at transonic speeds, $A_2B_1C_2D_3$ is the highest accuracy in calculating C_A, the simulation error is 3.85%. The combination of $A_2B_1C_3D_1$ is the highest accuracy in calculating C_N, the simulation error is -3.91%.

b. Range analysis

Calculate k_i and R. The analysis data is shown in Table XII, and the k_i value is shown in Fig. 8 and Fig. 9.

Table XII shows that the error range of factors A, B, and C is in the top three, which have the greatest influence on the accuracy calculation of C_A at transonic speeds. Fig. 8 shows that the optimal factor level combination for calculating transonic C_A is $A_2B_1C_3D_1$, which is different from the direct analysis result. The direct analysis reference data is single and cannot be used as the main reference, the optimal combination is finally selected as $A_2B_1C_3D_1$. The C_N accuracy error range of factors AB and AC are greater than that of single factor D. Fig. 9 shows that the $A_2B_1C_3D_1$ combination calculation has the smallest error, which is consistent with the direct analysis result.

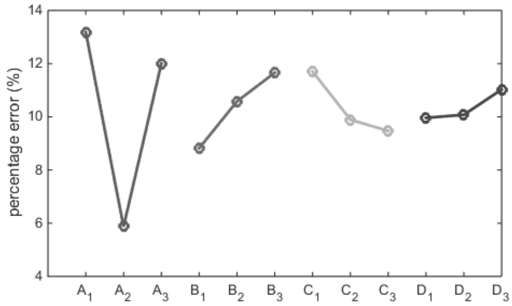


Fig. 8. k_i of four factors at transonic speeds of C_A

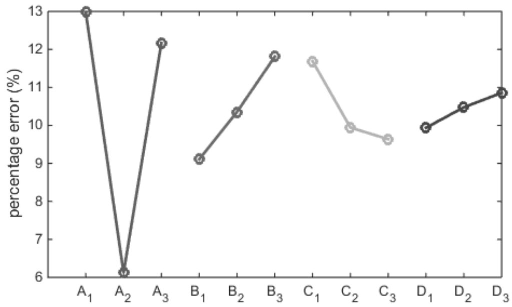


Fig. 9. k_i of four factors at transonic speeds of C_N

c. Variance analysis

Variance analysis results are shown in Table XIII. As can be seen from Table XIII:

(1) The influences of factors A, B and C on the calculation C_A are still in the top three at transonic speeds. The effect of factor AB on the transonic C_A exceeds the factor D. The rank of the influence of each factor on C_A is same with the results of range analysis.

(2) All factors have obviously significant effects on the calculation of transonic C_A, indicating that, the single factors and the interaction between factors all have great influence on the calculation of C_A.

(3) At transonic speeds, the factors A, B, C, and AB have obviously significant effect on the accuracy of C_N calculation, and the factor AD is insignificant.

(4) The rank of factor D and AC is different from the range analysis, but it does not affect the result of the optimal factor level combination obtained in the range analysis.

d. Confidence interval for optimal combination

At transonic speeds, the calculated error interval and confidence of the optimal combination are shown in Table XIV.

TABLE XII ERROR ANALYSIS AT TRANSONIC SPEEDS

Coefficient	Object	*A*	*B*	*C*	*D*	(*AB*)1	(*AB*)2	(*AC*)1	(*AC*)2	(*AD*)1	(*AD*)2
C_A	k_1	13.18	8.82	11.70	9.96	10.38	11.13	10.84	10.60	10.52	10.29
	k_2	5.87	10.58	9.89	10.08	11.09	10.20	10.26	10.48	10.00	10.69
	k_3	12.01	11.66	9.47	11.03	9.59	9.73	9.96	9.98	10.54	10.08
	R	7.31	2.84	2.23	1.07	1.54		0.88		0.69	
	Rank	*A>B>C>AB>D>AC>AD*									
C_N	k_1	12.98	9.11	11.69	9.94	10.42	10.94	10.68	10.46	10.74	10.57
	k_2	6.12	10.35	9.94	10.48	11.03	10.46	10.10	10.90	10.46	10.47
	k_3	12.17	11.81	9.64	10.86	9.82	9.87	10.49	9.91	10.07	10.23
	R	6.85	2.71	2.05	0.92	1.21		0.99		0.67	
	Rank	*A>B>C>AB>AC>D>AD*									

TABLE XIII VRIANCE ANALYSIS OF FACTORS AT TRANSONIC SPEEDS

Coefficient	Factors	*S*	*f*	$\bar{S}$	*F*	Threshold	Significant
C_A	*A*	277.51	2	138.75	1294.73		**
	B	36.97	2	18.48	172.47		**
	C	25.34	2	12.67	118.22		**
	D	6.15	2	3.08	28.70		**
	AB	19.33	4	4.83	45.08		**
	AC	5.56	4	1.39	12.97		**
	AD	3.36	4	0.84	7.84	$F_{0.25}(2,6)=3.31$	**
	Errors	0.64	6	0.11		$F_{0.10}(2,6)=9.33$	
C_N	*A*	252.49	2	126.25	282.52	$F_{0.25}(4,6)=2.08$	**
	B	33.00	2	16.50	36.92	$F_{0.10}(4,6)=4.01$	**
	C	22.09	2	11.05	24.72		**
	D	3.85	2	1.93	4.31		*
	AB	11.84	4	2.96	6.62		**
	AC	6.04	4	1.51	3.38		*
	AD	2.56	4	0.64	1.44		-
	Errors	2.68	6	0.45			

TABLE XIV THE CALCULATED ERROR INTERVAL AND CONFIDENCE AT TRANSONIC SPEEDS

Coefficient	$\hat{x}_{best}$	f_e	f'_e	S_e	S'_e	n	f^*	F	$\varepsilon_{0.1}$	Error interval	Confidence
C_A	4.30	6	0	0.64	0.00	27	20	$F_{0.10}(1,6) = 3.78$	0.56	3.74% ~ 4.86%	90%
C_N	4.01	6	10	2.68	12.46	27	10	$F_{0.10}(1,16) = 3.05$	1.08	2.93% ~ 5.09%	90%

C. Analysis of simulation results at supersonic speeds

a. Direct analysis

Table VII shows that at supersonic speeds, the combination of calculating the minimum C_A error is $A_2B_1C_3D_1$ with an error of -3.58%. The optimal combination of the calculated C_N is $A_2B_1C_3D_1$, and the calculation error is 3.51%.

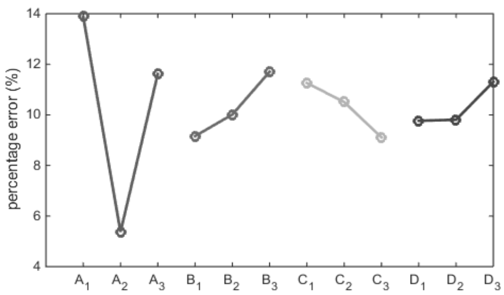


Fig. 10. k_i of four factors at supersonic speeds of C_A

b. Range analysis

Calculate k_i and R. The analysis is shown in Table XV, and the k_i value is shown in Fig. 10 and Fig. 11.

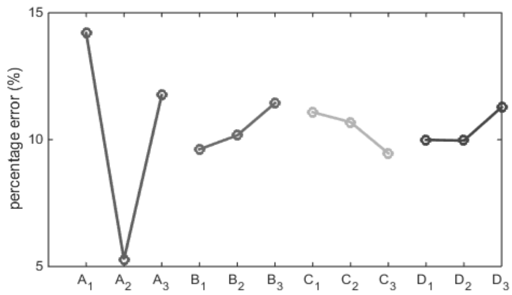


Fig. 11. k_i of four factors at supersonic speeds of C_N

Table XV shows that, in C_A calculation error analysis, the single factors *A*, *B*, and *C* are still the top three import factors. The range of the factor *AB* is greater than factor *D*. Fig. 10 shows that $A_2B_1C_3D_1$ calculates the C_A error to be the smallest, which is same with the direct analysis, and it can be considered as the optimal combination. In C_A calculation error analysis, the range of the factor *AB* is greater than factors *C* and *D*, and the range of the factor *AC* is greater than factor *D*. Fig. 11 shows that the calculation error of $A_2B_1C_3D_2$ is the smallest, and can be regarded as the optimal level combination for calculating the supersonic C_N. This combination did not appear in the orthogonal design test, which is the biggest advantage of the orthogonal design test, that is, the optimal combination can be obtained without testing all levels combination of each factor.

c. Variance analysis

Variance analysis results are shown in Table XVI. As can be seen that:

(1) Each factor has obviously significant impact on the calculation of the supersonic C_A except factor *AD*.

(2) The influence rank of each factor on supersonic C_A is different to the results of the range analysis, but does not affect the combination of the optimal factor levels obtained by the range analysis.

(3) At supersonic speeds, factors *A*, *B*, and *C* have obviously significant effect on the calculation of C_N, so and factor *AB* and factor *A*. Factor *AD* is insignificant effect.

(4) The influence rank of each factor is quite different from the range analysis, but does not affect the optimal factor level combination selected in range analysis.

d. Confidence interval for optimal combination

At supersonic speeds, the calculated error interval and confidence of the optimal combination are shown in Table XVII.

TABLE XV ERROR ANALYSIS AT SUPERSONIC SPEEDS

Coefficient	Object	*A*	*B*	*C*	*D*	(*AB*)1	(*AB*)2	(*AC*)1	(*AC*)2	(*AD*)1	(*AD*)2
C_A	k_1	13.88	9.16	11.26	9.77	10.52	11.20	10.18	10.25	10.65	10.21
	k_2	5.37	10.01	10.51	9.81	10.95	9.92	10.79	11.02	9.88	10.68
	k_3	11.62	11.71	9.11	11.31	9.41	9.76	9.91	9.61	10.35	9.99
	R	8.51	2.55	2.15	1.54	1.79		1.41		0.80	
	Rank	*A>B>C>AB>D>AC>AD*									
C_N	k_1	14.20	9.62	11.08	9.99	10.60	11.31	10.03	10.38	10.88	10.20
	k_2	5.28	10.17	10.69	9.96	10.97	10.14	11.01	11.23	10.02	10.60
	k_3	11.76	11.44	9.46	11.28	9.66	9.78	10.20	9.63	10.33	10.43
	R	8.92	1.82	1.62	1.31	1.65		1.60		0.86	
	Rank	*A>B>AB>C>AC>D>AD*									

TABLE XVI VRIANCE ANALYSIS OF FACTORS OF C_A AT SUPERSONIC SPEEDS

Coefficient	Factors	*S*	*f*	$\bar{S}$	*F*	Threshold	Significant
C_A	*A*	349.71	2	174.86	332.33		**
	B	30.22	2	15.11	28.72		**
	C	21.39	2	10.70	20.33		**
	D	13.86	2	6.93	13.17		**
	AB	22.53	4	5.63	10.71		**
	AC	12.63	4	3.16	6.00		**
	AD	5.00	4	1.25	2.38	$F_{0.25}(2,6)=3.31$	*
	Errors	3.16	6	0.53		$F_{0.10}(2,6)=9.33$	
C_N	*A*	382.79	2	191.40	302.43	$F_{0.25}(4,6)=2.08$	**
	B	15.72	2	7.86	12.42	$F_{0.10}(4,6)=4.01$	**
	C	12.89	2	6.45	10.19		**
	D	10.17	2	5.09	8.04		*
	AB	19.81	4	4.95	7.82		**
	AC	16.44	4	4.11	6.50		**
	AD	4.10	4	1.03	1.62		-
	Errors	3.80	6	0.63			

TABLE XVII THE CALCULATED ERROR INTERVAL AND CONFIDENCE AT SUPERSONIC SPEEDS

Coefficient	$\hat{x}_{best}$	f_e	f_e'	S_e	S_e'	n	f^*	F	$\varepsilon_{0.1}$	Errorinterval	Confidence
C_A	3.73	6	4	3.16	5.00	27	16	$F_{0.10}(1,10)=3.29$	1.30	2.43% ~ 5.03%	90%
C_N	3.94	6	6	3.80	14.28	27	14	$F_{0.10}(1,12)=3.17$	1.63	2.31% ~ 5.57%	90%

TABLE XVIII. INFLUENCE OF FACTORS ON CALCULATION ACCURACY OF C_A

Speed	Significant							Influence ordering	Optimal combination
	A	*B*	*C*	*D*	*AB*	*AC*	*AD*		
Subsonic	**	**	**	**	**	*	**	*A>B>C>D>AB>AD>AC*	$A_2B_1C_3D_1$
Transonic	**	**	**	**	**	**	**	*A>B>C>AB>D>AC>AD*	$A_2B_1C_3D_1$
Supersonic	**	**	**	**	**	**	*	*A>B>C>D>AB>AC>AD*	$A_2B_1C_3D_1$

TABLE XIX. INFLUENCE OF FACTORS ON CALCULATION ACCURACY OF C_N

Speed	Significant							Influence ordering	Optimal combination
	A	*B*	*C*	*D*	*AB*	*AC*	*AD*		
Subsonic	**	**	**	*	**	*	-	*A>B>C>AB>D>AC>AD*	$A_2B_1C_3D_1$
Transonic	**	**	**	*	**	*	-	*A>B>C>AB>D>AC>AD*	$A_2B_1C_3D_1$
Supersonic	**	**	**	*	**	**	-	*A>B>C>D>AB>AC>AD*	$A_2B_1C_3D_2$

D. Comprehensive analysis of results

a. Analysis of calculation results of C_A

The effects of various factors on the calculation accuracy of the C_A at different speeds are shown in Table XVIII. Table XVIII shows that:

(1) The C_A calculation accuracy of the factors *A*, *B*, *C*, *D* and *AB* are obviously significant, and their influence on C_A is ranked in the top five of all factors.

(2) The influence of interaction factors is not as great as their individual factors on calculating C_A.

(3) The optimal combination of factors for calculating the C_A of the projectile is $A_2B_1C_3D_1$ at different speeds. That is to say, k-ε turbulence model is optimal to calculate C_A, reducing the mesh size and expanding flow field is helpful to improve accuracy of calculating C_A, and the smaller the y^+ value is set, the smaller the calculation error is. However, according to statistics, the mesh parameter settings are adjusted from level 2 to level 1, the C_A calculation accuracy increases 1.23%, but the simulation calculation time increases 72%. The calculation accuracy of C_A increases 0.94% when calculation domain adjusted from level 2 to level 3, but the simulation time increases 74%.

b. Analysis of calculation results of C_N

The effects of various factors on the calculation accuracy of the C_N at different speeds are shown in Table XIX. Table XIX show that:

(1) The five factors *A*, *B*, *C*, *D* and *AB* have the influence on the calculation of the C_N in the top five, and they all have obviously significant influence on the calculation of C_N. The factor *AD* has little effect on the calculation of C_N.

(2) When calculating C_N of the projectile at supersonic speeds, the y+ value is not the smaller the calculation accuracy is higher.

(3) The influence of each factor on the calculation accuracy of C_A and C_N of the projectile is not the same, and it needs to be set differently in parameter settings.

V. CONCLUSION

In this paper, the orthogonal design test is used to analyze the influence of four factors and interaction of some factors on the calculation accuracy of C_A and C_N. The analysis results show that k-ε turbulence model is most suitable for calculating C_A and C_N of the projectile. Within a certain range of parameter settings, increasing the calculation domain, reducing the mesh size, and reducing the y^+ value is beneficial to improve the calculation accuracy of C_A and C_N. But simply increasing the computational domain and reducing the mesh size will increase the computation time a lot, and the improvement of calculation accuracy is not obvious. When calculating C_N of the projectile at supersonic speeds, the y^+ value is not the smaller the better. The interactions between the turbulence model and the flow field, mesh size and y^+ value also have great influence on the calculation of C_A and C_N. Reasonable adjustment of parameter settings between these factors can further improve the calculation accuracy.

Based on the error analysis theory, the error range and confidence interval of CFD simulation accuracy are analyzed for the first time. The maximum error of the optimal factor level combination of calculated aerodynamic parameters determined by orthogonal test method is no more than 6% when the confidence is 90%. The error range and the error confidence interval effectively solves the problem that the calculation results can not evaluated when the CFD simulation accuracy can not be verified by experiments. The concepts of error range and confidence interval of CFD simulation calculation have strong practicability in practical application, which can significantly reduce the number of experiments and save the cost of experiments.

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TABLE VI ORTHOGONAL DESIGN TABLE

Serial number	Factors												
	A	B	(AB)1	(AB)2	C	(AC)1	(AC)2	(AD)1	D	(AD)2	Blank1	Blank2	Blank3
1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	1	3	3	3	3	3	3	3	3	3
4	1	2	2	2	1	1	1	2	2	2	3	3	3
5	1	2	2	2	2	2	2	3	3	3	1	1	1
6	1	2	2	2	3	3	3	1	1	1	2	2	2
7	1	3	3	3	1	1	1	3	3	3	2	2	2
8	1	3	3	3	2	2	2	1	1	1	3	3	3
9	1	3	3	3	3	3	3	2	2	2	1	1	1
10	2	1	2	3	1	2	3	1	2	3	1	2	3
11	2	1	2	3	2	3	1	2	3	1	2	3	1
12	2	1	2	3	3	1	2	3	1	2	3	1	2
13	2	2	3	1	1	2	3	2	3	1	3	1	2
14	2	2	3	1	2	3	1	3	1	2	1	2	3
15	2	2	3	1	3	1	2	1	2	3	2	3	1
16	2	3	1	2	1	2	3	3	1	2	2	3	1
17	2	3	1	2	2	3	1	1	2	3	3	1	2
18	2	3	1	2	3	1	2	2	3	1	1	2	3
19	3	1	3	2	1	3	2	1	3	2	1	3	2
20	3	1	3	2	2	1	3	2	1	3	2	1	3
21	3	1	3	2	3	2	1	3	2	1	3	2	1
22	3	2	1	3	1	3	2	2	1	3	3	2	1
23	3	2	1	3	2	1	3	3	2	1	1	3	2
24	3	2	1	3	3	2	1	1	3	2	2	1	3
25	3	3	2	1	1	3	2	3	2	1	2	1	3
26	3	3	2	1	2	1	3	1	3	2	3	2	1
27	3	3	2	1	3	2	1	2	1	3	1	3	2

TABLE VII ORTHOGONAL DESIGN TEST DATA

Serial number	Subsonic		Transonic		Supersonic	
	C_A error (%)	C_N error (%)	C_A error (%)	C_N error (%)	C_A error (%)	C_N error (%)
1	12.56	13.25	14.39	13.52	14.48	15.22
2	12.81	13.52	11.82	12.51	14.57	15.22
3	12.49	11.85	11.16	10.49	12.58	13.05
4	14.04	14.22	15.55	14.92	14.38	13.56
5	15.70	-16.55	-14.28	12.93	-16.13	17.37
6	10.43	13.55	12.15	-12.83	11.16	-11.82
7	-16.30	15.13	-15.85	15.18	15.25	14.59
8	13.10	13.85	12.15	12.51	15.25	15.08
9	11.71	11.20	11.29	11.92	-11.16	11.88
10	4.67	5.22	5.19	5.83	4.38	-4.52
11	-4.44	-4.33	3.85	4.83	5.16	5.23
12	3.48	3.86	-4.30	-3.91	-3.58	3.51
13	6.58	7.15	7.25	6.91	6.25	-5.87
14	5.01	-4.59	5.85	5.16	4.49	4.48
15	-4.45	5.22	-5.22	-5.82	4.59	5.22
16	8.38	10.19	7.82	6.91	7.82	6.89
17	6.11	7.28	6.07	7.25	5.85	-6.22
18	6.88	7.81	7.28	8.49	6.25	5.55
19	-11.46	13.59	11.82	12.49	12.49	12.85
20	7.50	-8.12	8.30	9.19	-7.82	8.55
21	-7.18	-8.52	8.56	9.17	7.37	8.43
22	10.11	11.24	11.59	-12.82	-10.47	11.17
23	-11.31	-11.82	-11.13	10.26	10.14	9.56
24	11.46	12.15	12.18	11.52	12.49	-12.49
25	-15.08	15.88	-15.85	16.62	-15.81	-15.07
26	14.85	-14.68	15.55	14.86	15.15	14.51
27	12.34	-13.15	13.08	12.57	12.82	-13.19