Energy of Generalized Complements of a Graph

Sabitha D'Souza, Gowtham H. J* and Pradeep G. Bhat,

***Abstract*—Let G be a finite simple graph on n vertices. Let $P = \{V_1, V_2, V_3, \ldots, V_k\}$ be a partition of vertex set $V(G)$ of order $k \geq 2$. For all V_i and V_j in P, $i \neq j$, remove the edges between V_i and V_j in graph G and add the edges between V_i and V_j which are not in G. The graph G_k^P thus obtained is called the $k-$*complement* of graph G with respect to the partition P. Let $P = \{V_1, V_2, V_3, \ldots, V_k\}$ be a partition of vertex set $V(G)$ of order $k \geq 1$. For each set V_r in P, remove the edges of graph G inside V_r and add the edges of $\overline{G}$ (the complement of G) joining the vertices of V_r. The graph $G_{k(i)}^P$ thus obtained is called the $k(i)-$*complement of graph* G with respect to the partition P. Energy of a graph G is the sum of absolute eigenvalues of G. In this paper, we study energy of generalized complements of some families of graph. An effort is made to throw some light on showing variation in energy due to changes in the partition of the graph.**

***Index Terms*—generalized complements, spectrum, energy.**

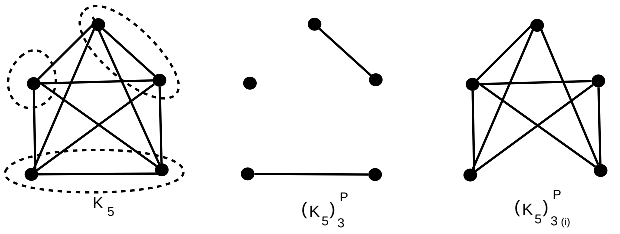
I. Introduction

LET G be a graph on n vertices and m edges. The *complement of a graph* G, denoted by $\bar{G}$ has same vertex set as that of G, but two vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in G. If G is isomorphic to $\bar{G}$ then G is said to be *self-complementary graph*. For all notations and terminologies we refer [1], [2]. E. Sampathkumar et al. in [3] introduced two types of generalized complements of a graph.

Definition 1: [3] Let $P = \{V_1, V_2, V_3, \ldots, V_k\}$ be a partition of vertex set $V(G)$ of order $k \geq 2$. For all V_i and V_j in P, $i \neq j$, remove the edges between V_i and V_j in graph G and add the edges between V_i and V_j which are not in G. The graph G_k^P thus obtained is called $k-$*complement* of graph G with respect to the partition P.

Definition 2: [3] Let $P = \{V_1, V_2, V_3, \ldots, V_k\}$ be a partition of vertex set $V(G)$ of order $k \geq 1$. For each set V_r in P, remove the edges of graph G inside V_r and add the edges of $\overline{G}$ joining vertices of V_r. The graph $G_{k(i)}^P$ thus obtained is called the $k(i)-$*complement of graph* G with respect to the partition P.

Example 3:



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The energy of a graph is defined by Ivan Gutman [4] as the sum of absolute eigenvalues of G. It represents a proper generalization of a formula valid for the total π-electron energy of a conjugated hydrocarbon as calculated by the Huckel molecular orbital (HMO) method in quantum chemistry. For recent mathematical work on the energy of a graph see [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16].

The characteristic polynomial of G is the characteristic polynomial of adjacency matrix $A(G)$ denoted as $\phi(G, \lambda)$. The set of eigenvalues of $A(G)$ given by $\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ is called spectrum of graph G. Two graphs G and H are said to be equienergetic if they have same energy. Two or more graphs are called co-spectral graphs if they have same spectra. In this paper we study the energy of generalized complements of some classes of graph. For a graph, there exists many k and $k(i)$ complements. Hence, it is interesting to study the variation of energy for these complements.

II. Preliminaries

Now we present few results on k and $k(i)$ self complementary graphs, characteristic polynomial of cluster graph $Kb_p(k)$ and complete multi-partite graph, which are extensively used to prove our main results.

Proposition 4: [3] The k-complement and $k(i)$ complement of G are related as follows:

(i) $\overline{G_k^P} \cong G_{k(i)}^P$
(ii) $\overline{G_{k(i)}^P} \cong G_k^P$

Definition 5: [17] Let f_i, $i = 1, 2, \ldots, k$, $0 \leq k \leq \lfloor \frac{p}{2} \rfloor$ be independent edges of complete graph K_p, $p \geq 3$. The graph $Kb_p(k)$ is obtained by deleting f_i, $i = 1, 2, \ldots, k$ from K_p. In addition $Kb_p(0) \cong K_p$.

Proposition 6: [17] For $p \geq 3$ and $0 \leq k \leq \lfloor \frac{p}{2} \rfloor$,

$$\phi(Kb_p(k), \lambda) = \lambda^k (\lambda + 1)^{p-2k-1} (\lambda + 2)^{k-1} [\lambda^2 - (p-3) \lambda - 2(p-k-1)].$$

Proposition 7: [10] The characteristic polynomial of complete multipartite graph $K_{n_1, n_2, \ldots, n_p}$ is

$$\phi(K_{n_1, n_2, \ldots, n_p}, \lambda) = \lambda^{n-p} (1 - \sum_{i=1}^{p} \frac{n_i}{\lambda + n_i}) \prod_{j=1}^{p} (\lambda + n_j).$$

Lemma 8: [10] Let

$$A = \begin{bmatrix} A_0 & A_1 \\ A_1 & A_0 \end{bmatrix}$$

be a 2×2 block symmetric matrix. Then eigenvalues of A are the eigenvalues of matrices $A_0 + A_1$ and $A_0 - A_1$.

III. Energy of generalized complements of classes of graphs

In this section, we find energy of generalized complements of some standard graphs like complete, complete bipartite, star, path, friendship, double star and cocktail party graph.

We also compute energy of generalized complements of graphs derived from star graphs like $K_{1,n-1}+e$, $K_{1,n-1}+2e$ and $G_s = G(K_{1,n_1-1}, K_{1,n_2-1}, \ldots, K_{1,n_r-1})$.

Theorem 9: Let $P = \{V_1, V_2, \ldots, V_k\}$ be a partition of the complete graph K_n.

(i) If $< V_i >= K_i$, for $i = 1, 2, \ldots, n$, then $E(K_n)^P_k = 2(n-k)$ and $E(K_n)^P_{k(i)} = E(K_{n_1,n_2,n_3,\ldots,n_k})$, where $|V_i| = n_i$, $i = 1, 2, \ldots, k$.

(ii) If $|V_i| = 2$ and any one partite V_i has order 1 for odd n, then $E(K_n)^P_k = \begin{cases} n, & if\ n\ is\ even \\ n-1, & if\ n\ is\ odd \end{cases}$ and

$$\phi((K_n)^P_{k(i)}; \lambda) = \lambda^{\lfloor \frac{n}{2} \rfloor} (\lambda+1)^{n-2\lfloor \frac{n}{2} \rfloor -1}$$
$$(\lambda+2)^{\lfloor \frac{n}{2} \rfloor -1} \left[\lambda^2 - (n-3)\lambda - 2(n - \lfloor \frac{n}{2} \rfloor - 1)\right].$$

Proof: For a partition $P = \{V_1, V_2, \ldots, V_k\}$, let $G = (K_n)^P_k$ be the graph.

(i) If $< V_i >= K_i$, for $i = 1, 2, \ldots, n$, then G is union of k disconnected complete subgraphs of order n_i such that $\sum_{i=1}^{n} n_i = n$. Therefore, $E(G) = \sum_{i=1}^{k} E(K_{n_i}) = 2(\sum_{i=1}^{k} n_i - k) = 2(n-k)$. Also we observe that $\overline{G} = K_{n_1,n_2,n_3,\ldots,n_k}$, complete multipartite graph. Hence $E(K_n)^P_{k(i)} = E(K_{n_1,n_2,n_3,\ldots,n_k})$, which is given by Proposition 7.

(ii) If $|V_i| = 2$, $i = 1, 2, \ldots, k$, then

$$k = \begin{cases} \frac{n}{2}, & if\ n\ is\ even \\ \frac{n+1}{2}, & if\ n\ is\ odd \end{cases}$$

By substituting the value of k in statement (i) of Theorem 9, we obtain $E(K_n)^P_k = \begin{cases} n, & if\ n\ is\ even \\ n-1, & if\ n\ is\ odd \end{cases}$

We note that $(K_n)^P_{k(i)} = Kb_n(\lfloor \frac{n}{2} \rfloor)$, a graph obtained after deleting the independent edges of K_n. Hence, from Proposition 6, we get

$$\phi((K_n)^P_{k(i)}; \lambda) = \lambda^{\lfloor \frac{n}{2} \rfloor} (\lambda+1)^{n-2\lfloor \frac{n}{2} \rfloor -1} (\lambda+2)^{\lfloor \frac{n}{2} \rfloor -1}$$
$$\left[\lambda^2 - (n-3)\lambda - 2(n - \lfloor \frac{n}{2} \rfloor - 1)\right].$$

■

Remark 10: (i) From Theorem 9, it follows that $E(K_n)^P_k$ is independent of the order of each partite set. Also, for fixed integers n and k, we obtain non-co-spectral equienergetic graphs. $E(K_n)^P_k$ decreases as order of the partition set k increases.

(ii) If $P = \{V_1, V_2\}$ is the partition of K_n with $|V_i| = n_i$, $i = 1, 2$, then energy of $k(i)$- complement of K_n is $2\sqrt{n_1 n_2}$.

Theorem 11: Let $K_{m,n}$ be complete bipartite graph with vertex set $V = \{U_m, U_n\}$ and partition $P = \{V_1, V_2\}$.

(i) If $< V_1 >= K_{s_1,s_2}$ and $< V_2 >= K_{m-s_1,n-s_2}$, where s_1, s_2 denote number of vertices of V_1 such that s_1 vertices belong to U_m and s_2 vertices belong to U_n, then $E(K_{m,n})^P_2 = 2\sqrt{(n-s_1+s_2)(m-s_2+s_1)}$ and $E(K_{m,n})^P_{2(i)} = 2(m+n-2)$.

(ii) If $|V_1| = m-1$ such that all the vertices of V_1 are from first partite set of $K_{m,n}$ and $|V_2| = n+1$, then $E(K_{m,n})^P_2 = 2\sqrt{m+n-1}$ and $E(K_{m,n})^P_{2(i)} = 2(m+n-2)$.

Proof:

(i) If $< V_1 >= K_{s_1,s_2}$ and $< V_2 >= K_{m-s_1,n-s_2}$ then $(K_{m,n})^P_2 \cong K_{n-s_1+s_2,m-s_2+s_1}$. Hence, $E(K_{m,n})^P_2 = 2\sqrt{(n-s_1+s_2)(m-s_2+s_1)}$. Also $(K_{m,n})^P_{2(i)} \cong K_m \cup K_n$. Thus, $E(K_{m,n})^P_{2(i)} = E(K_m \cup K_n) = 2(m+n-2)$.

(ii) If $|V_1| = m-1$ such that all the vertices of V_1 are from first partite set of $K_{m,n}$ and $|V_2| = n+1$, then $(K_{m,n})^P_2 \cong K_{1,m+n-1}$. Hence $E(K_{m,n})^P_2 = 2\sqrt{m+n-1}$. Also $(K_{m,n})^P_{2(i)} \cong K_1 \cup K_{m+n-1}$. Hence, $E(K_{m,n})^P_{2(i)} = 2(m+n-2)$.

■

Corollary 12: Let $P = \{V_1, V_2, \ldots, V_k\}$ be a partition of $K_{m,n}$ such that $< V_i >= K_2$ for $i = 1, 2, \ldots, k-1$ and $< V_k >$ be the union of isolated vertices. The characteristic polynomial of $(K_{m,n})^P_k$ and $(K_{m,n})^P_{k(i)}$ is $\lambda^{n-2}(\lambda+2)^{m-1}[\lambda^3 - 2(m-1)\lambda^2 + (2m^2 - 2m - mn)\lambda + m^2(m+n+1) - mn]$ and $(\lambda+1)^{n-2}(\lambda-1)^{m-1}[\lambda^3 - (n-m-1)\lambda^2 + (2m - mn + 1)\lambda + (m-1)^2(m-n-1)]$ respectively.

Theorem 13: Let $\{V_1, V_2, \ldots, V_k\}$ be a partition of path P_n. Then, $E(P_n)^P_{k(i)} = 2(k-1)$ and $E(P_n)^P_k = E(Kb_n(k-1))$ in the following cases.

(i) Any one of the pendant(non pendant) vertex is in V_1 or V_k, and remaining V_i are such that $< V_i >= K_2$, for odd path.

(ii) $< V_i >= K_2$, for even path, $i = 1, 2, \ldots k$.

Proof: We note that $(P_n)^P_{k(i)}$ is the union of $k-1$ number of $K_2's$ and in addition two isolated vertices for odd path. Hence, $E(P_n)^P_{k(i)} = 2(k-1)$. Whereas $(P_n)^P_k$ is the graph obtained from K_n by deleting all the independent edges. Thus, $E(P_n)^P_k = E(Kb_n(k-1))$, which is estimated using Proposition 6. ■

Theorem 14: Let $P = \{V_1, V_2, \ldots, V_k\}$ be a partition of cycle C_n.

1) If $< V_i >= K_2$, then $E(C_n)^P_{k(i)} = 2k$ and $E(C_n)^P_k = E(Kb_n(k))$, for even n.

2) If $< V_i >= K_2$ and one of $|V_i| = 1$, then $E(C_n)^P_{k(i)} = 2(n-k+\sqrt{2})$, for odd n.

Proof:

1) For even n, we observe that $(C_n)^P_{k(i)}$ is the union of k number of K_2's. Hence, $E(C_n)^P_{k(i)} = 2k$. Whereas $(C_n)^P_k$ is the graph obtained from K_n by deleting all the independent edges. Thus, $E(C_n)^P_k = E(Kb_n(k))$ which can be computed by Proposition 6.

2) In case of odd n, $(C_n)^P_{k(i)}$ is the union of $K_{1,2}$ and k-2 number of K_2's. Thus, $E(C_n)^P_{k(i)} = 2(n-k+\sqrt{2})$.

■

Theorem 15: Let $P = \{V_1, V_2, \ldots, V_k\}$ be a partition of star graph S_n. Then the following statements are true.

(i) For $k = 2$, if $< V_1 >= S_r$, $r \geq 2$, then $E(S_n)^P_2 = \sqrt{(r-1)(n-r+1)}$ and $E(S_n)^P_{2(i)} = 2(n-3)$.

(ii) If $|V_1| = 1$ such that the central vertex belongs to V_1 and $< V_i >= n_r K_1$, $1 \leq r \leq n-1$, $i \geq 2$ then $E(S_n)^P_k = E(K_1 \cup K_{n_2,n_3,\ldots,n_k})$, the energy of complete multipartite graph of order $n-1$.

Proof:

(i) For a partition P in (i), $(S_n)_2^P$ is the complete bipartite graph $K_{r-1,n-r+1}$. Hence $E(S_n)_2^P = \sqrt{(r-1)(n-r+1)}$ and $E(S_n)_{2(i)}^P = 2(n-3)$, since $(S_n)_{2(i)}^P$ is the union of two complete subgraphs of order $r-1$ and $n-r$.

(ii) Suppose $|V_1| = 1$ such that central vertex belongs to V_1 and $< V_i >= n_r K_1$, $1 \leq r \leq n-1$, $i \geq 2$. Then $(S_n)_k^P \cong K_1 \cup K_{n_2,n_3,\ldots,n_k}$. Hence, $E(S_n)_k^P = E(K_1 \cup K_{n_2,n_3,\ldots,n_k})$ which is given by Proposition 7. ■

Theorem 16: If $\tau = K_{1,n-1} + e$ is a unicyclic graph of order n obtained by adding an edge between two pendant vertices of star graph K_{n-1}, then

1) For partition $P = \{V_1, V_2\}$ such that $< V_1 >= C_3$ and $< V_2 >= (n-2)K_1$, $E(\tau_2^P) = E(K_{1,1,n-2})$ and $E(\tau_{2(i)}^P) = 2(n-3)$.
2) For partition $P = \{V_1, V_2\}$ such that $< V_1 >= K_2$ and $< V_2 >= (n-2)K_1$, $E(\tau_2^P) = 2\sqrt{n-2}$
3) For partition $P = \{V_1, V_2, V_3\}$ such that only central vertex is in V_1, $< V_2 >= K_2$ and $< V_3 >= (n-3)K_1$, $E(\tau_{3(i)}^P) = E(K_{1,1,n-3})$.

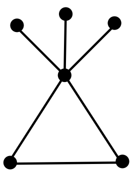

Fig. 1. $\tau = K_{1,5} + e$

Proof:

1) For the given partition P, τ_2^P results into complete multipartite graph $K_{1,1,n-2}$. Hence $E(\tau_2^P) = E(K_{1,1,n-2})$, which can be computed using Proposition 7. Note that $\tau_{2(i)}^P$ is an union of K_{n-2} and two isolated vertices. So, $E(\tau_{2(i)}^P) = 2(n-3)$.
2) For $P = \{V_1, V_2\}$ such that $< V_1 >= K_2$ and $< V_2 >= (n-2)K_1$, the resultant graph τ_2^P is an union of isolated vertex and a star graph $K_{1,n-2}$. Therefore, $E(\tau_2^P) = 2\sqrt{n-2}$.
3) For $P = \{V_1, V_2, V_3\}$ such that only central vertex is in V_1 , $< V_2 >= K_2$ and $< V_3 >= (n-3)K_1$, the graph $\tau_{3(i)}^P$ is a disconnected graph with K_1 and $K_{1,1,n-3}$ as components. Thus $E(\tau_{3(i)}^P) = E(K_{1,1,n-3})$ which can be evaluated by Proposition 7.

■

Theorem 17: Let $S(m,n)$ be double star graph with partition $P = \{V_1, V_2\}$, such that the vertices of V_1 and V_2 are of distance two. Then characteristic polynomial of $(S(m,n))_{2(i)}^P$ is $(\lambda+1)^{m+n-3}[\lambda^3 - (n+m-3)\lambda^2 + (n(m-3) + (4-3m))\lambda + (3m-4)n - (4m-4)]$ and $E(S(m,n))_2^P = 2\sqrt{(m-1)(n-1)}$.

Proof: Let $P = \{V_1, V_2\}$ be a partition of vertices of $S(m,n)$ such that the vertices of V_1 and V_2 are of distance two i. e, V_1=$\{v_1, v_2, v_3, \ldots, v_{m-1}, u_1\}$ and V_2=$\{v_m, u_2, u_3, \ldots, u_{n-1}, u_n\}$. We have $A(S(m,n))_{2(i)}^P =$

$$\left[\begin{array}{c|c|c} (J-I)_{m-1} & J_{(m-1)\times 2} & 0_{m-1\times n-1} \\ \hline J_{2\times m-1} & (J-I)_2 & J_{2\times m-1} \\ \hline 0_{n-1\times m-1} & J_{n-1\times 2} & (J-I)_{n-1} \end{array}\right]$$

Consider $\det(\lambda I - A(S(m,n)_{2(i)}^P))$

Step 1: Replacing R_i by $R_i - R_{i+1}$, for $i = v_1, v_2, v_3, \ldots, v_{m-2}, v_m$ and R_i by $R_i - R_{i-1}$, for i= $u_n, u_{n-1}, \ldots, u_4, u_3$, the determinant reduces to $(\lambda+1)^{m+n-3}\det(D)$.

Step 2: In $\det(D)$, replacing C_i by $C_i - C_{i-1}$, where $i = v_2, v_3, v_4, \ldots, v_{m-1}$ and C_i by $C_i - C_{i+1}$, for $i = u_{n-1}, u_{n-2}, \ldots, u_3, u_2$ and simplifying, it reduces to

$$\det(D) = \begin{vmatrix} \lambda-(m-2) & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 1-m & -1 & \lambda & 1-n \\ 0 & -1 & -1 & \lambda-n+2 \end{vmatrix}$$

Hence, $\det(D)$= $\lambda^3 - (n+m-3)\lambda^2 + (n(m-3) + (4-3m))\lambda + ((3m-4)n - (4m-4))$.

Thus, the characteristic polynomial of $S(m,n)_{2(i)}^P$ is

$(\lambda+1)^{m+n-3}[\lambda^3 - (n+m-3)\lambda^2 + (n(m-3) + (4-3m))\lambda + ((3m-4)n - (4m-4))]$.

We have $S(m,n)_2^P \cong K_{m-1,n-1} \cup 2K_1$.

Hence, $E(S(m,n)_2^P) = E(K_{m-1,n-1}) + 2E(K_1) = 2\sqrt{(m-1)(n-1)}$. ■

Theorem 18: Let $P = \{V_1, V_2, V_3\}$ be a partition of double star graph $S(m,n)$ such that $< V_1 >= (m-1)K_1$, $< V_2 >= K_2$ and $< V_3 >= (n-1)K_1$. Then $E(S(m,n)_3^P) = 2\sqrt{mn}$ and $E(S(m,n)_{3(i)}^P) = 2(m+n-2)$

Proof: For the partition $P = \{V_1, V_2, V_3\}$ such that $< V_1 >= (m-1)K_1$, $< V_2 >= K_2$ and $< V_3 >= (n-1)K_1$, we observe that $S(m,n)_3^P$ is a complete bipartite graph $K_{m,n}$. Hence, $E(S(m,n)_3^P) = 2\sqrt{mn}$. Also, $S(m,n)_{3(i)}^P$ is an union of two subgraphs K_m and K_n. Thus, $E(S(m,n)_{3(i)}^P) = E(K_m) + E(K_n) = 2(m+n-2)$. ■

Definition 19: [18] A graph G_s obtained by completely connecting the central vertices of r star graphs $K_{1,n_1-1}, K_{1,n_2-1}, \ldots K_{1,n_r-1}$ is called generalized star graph. It is denoted by, $G_s = G(K_{1,n_1-1}, K_{1,n_2-1}, \ldots, K_{1,n_r-1})$.

Theorem 20: Let $G_s = G(K_{1,n_1-1}, K_{1,n_2-1}, \ldots K_{1,n_r-1})$ be a generalized star graph with partition $P = \{V_1, V_2, \ldots, V_{r+1}\}$ such that $< V_j >= (n_j - 1)K_1, j = 1, 2, \ldots, r$ and $< V_{r+1} >= K_r$. Then $E(G_s)_{r+1}^P = E(K_{n_1,n_2,\ldots,n_r})$ and $E(G_s)_{(r+1)(i)}^P = 2(n_1 + n_2 + \cdots + n_r - r)$.

Proof: For partition P, $(G_s)_{r+1}^P$ results into complete multipartite graph and thus its energy is $E((G_s)_{r+1}^P) = E(K_{n_1,n_2,\ldots,n_r})$ which can be obtained using Proposition 7. On the other hand, $(G_s)_{(r+1)(i)}^P$ is an union of r complete subgraphs of order $n_1, n_2, \ldots, n_r$. So, $E(G_s)_{(r+1)(i)}^P = E(K_{n_1}) + E(K_{n_2}) + \ldots + E(K_{n_r}) = 2(n_1 + n_2 + \ldots + n_r - r)$

■

Theorem 21: If $\mathcal{H} = K_{1,n-1} + 2e$ is a bicyclic graph of order n obtained by adding an edge between a pendant vertex and a vertex of degree two of the graph $\tau = K_{1,n-1} + e$ of star graph K_{n-1}, then for partition $P = \{V_1, V_2\}$ such that $< V_1 >= (n-4)K_2$ and $< V_2 >= K_4 - e$, $E(\mathcal{H}_2^P) = E(K_{1,3,n-4})$ and $E(\mathcal{H}_{2(i)}^P) = 2(n-3)$

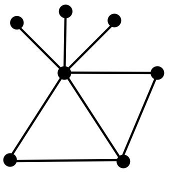

Fig. 2. $\mathcal{H} = K_{1,6} + 2e$

Proof: We observe that for the given partition P, graph $\mathcal{H}_2^P$ reduces to complete tripartite graph $K_{1,3,n-4}$. Hence, $E(\mathcal{H}_2^P) = E(K_{1,3,n-4})$ which is obtained using Proposition 7. On the other hand, $\mathcal{H}_{2(i)}^P$ will be an union of disconnected complete subgraphs of order 1,2 and $n-3$ respectively. Thus, $E(\mathcal{H}_{2(i)}^P) = E(K_{n-3}UK_1UK_3) = 2(n-4+1) = 2(n-3)$ ■

Theorem 22: Let F_n be friendship graph with partition $P = \{V_1, V_2\}$, such that central vertex belongs to V_1 and $< V_2 >= \dfrac{n}{2}K_2$. Then $E(F_n)_{2(i)}^P = 2(n+\sqrt{n^2+1}-1)$ and $E(F_n)_2^P = 2n$.

Proof: Let $P = \{V_1, V_2\}$ be a partition of vertices of F_n such that central vertex belongs to V_1 and $< V_2 >= \dfrac{n}{2}K_2$. We have

$$A(F_n)_{2(i)}^P = \left[\begin{array}{c|c} 0 & \mathbf{1}_n^T \otimes \mathbf{1}_2 \\ \hline \mathbf{1}_n \otimes \mathbf{1}_2 & (J_n - I_n) \otimes J_2 \end{array}\right].$$

The result is proved by showing $AX = \lambda X$ for certain vectors X and by making use of fact that the geometric and algebraic multiplicity of each eigenvalue λ is same, as $A(F_n)_{2(i)}^P$ is real and symmetric.

Let $X = \left[\dfrac{x}{Y \otimes Z}\right]$, $Y \in R^n$, $Z \in R^2$.

Case (i). Let $X = 0$ and Y be any vector. Let $Z = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\mathbf{1}_2^T Z = 0$. Then

$$AX = \left[\frac{\mathbf{1}_n^T Y \otimes \mathbf{1}_2^T Z}{(J_n - I_n)Y \otimes J_2 Z}\right] = \left[\frac{\mathbf{1}_n^T Y \otimes 0}{(J_n - I_n)Y \otimes 0}\right] = 0.$$

Thus, X is an eigenvector of A corresponding to eigenvalue 0. Since Y is any vector in R^n, eigenvector X can be any one of n independent vectors. Hence, eigenvalue 0 of matrix A has multiplicity at least n.

Case (ii). Let $x = 0$ and Y be a vector in the null space of J_n, $\mathbf{1}_n^T Y = 0$.

Let $Z = \mathbf{1}_2$, which implies $J_2 Z = 2Z$.

$$AX = \left[\frac{\mathbf{1}_n^T Y \otimes \mathbf{1}_2^T Z}{(J_n - I_n)Y \otimes J_2 Z}\right] = \left[\frac{0}{-Y \otimes 2Z}\right] = -2X$$

So, X is an eigenvector of A corresponding to eigenvalue -2. Since the null space of J_n has dimension $n-1$, the eigenvector Y, and hence X can be any of $n-1$ independent vectors. It implies that eigenvalue -2 of matrix A has multiplication at least $n-1$.

Case (iii). Let $\lambda = n - 1 \pm \sqrt{n^2+1}$, $x = 2n, Y = \lambda \mathbf{1}_n$and $Z = \mathbf{1}_2$.

$$AX = \left[\frac{\mathbf{1}_n^T Y \otimes \mathbf{1}_2^T Z}{X(\mathbf{1}_n \otimes \mathbf{1}_2) + (J_n - I_n)Y \otimes J_2 Z}\right]$$
$$= \left[\frac{2n\lambda}{-2n(\mathbf{1}_n \otimes \mathbf{1}_2) + (n-1)\lambda \mathbf{1}_n \otimes 2(\mathbf{1}_2)}\right]$$
$$= \left[\frac{2n\lambda}{(2n + 2(n-1)\lambda)\mathbf{1}_n \otimes \mathbf{1}_2}\right]$$

We observe that $\lambda x = 2n\lambda$ and since λ is a root of $x^2 - 2(n-1)x - 2x = 0$, we have $\lambda(Y \otimes Z) = \lambda^2(\mathbf{1}_n \otimes \mathbf{1}_2) = (2(n-1)\lambda + 2n)\mathbf{1}_n \otimes \mathbf{1}_2$. Thus, the spectrum of $A(F_n)_{2(i)}^P$ is given by $\left\{\begin{array}{cc} 0 & n \\ -2 & n-1 \\ n+\sqrt{n^2+1}-1 & 1 \\ n-\sqrt{n^2+1}-1 & 1 \end{array}\right\}$.

So, $E(F_n)_{2(i)}^P = 2(n + \sqrt{n^2+1} - 1)$. We have $(F_n)_2^P \cong nK_2 \cup K_1$. Hence, $E(F_n)_2^P = nE(K_2) + E(K_1) = 2n$. ■

Theorem 23: Let F_n be friendship graph with partition $P = \{V_1, V_2\}$, such that $< V_1 >= K_3$ and $< V_2 >= \frac{n-2}{n}K_2$. Then $E(F_n)_{2(i)}^P = 2(n-2+\sqrt{n^2-2n+2})$.

Proof: For partition $P = \{V_1, V_2\}$, such that $< V_1 >= K_3$ and $< V_2 >= \frac{n-2}{n}K_2$ of graph F_n, the adjacency matrix of $k(i)$ complement of F_n is given by,

$$A(F_n)_{2(i)}^P = \begin{bmatrix} 0_{1\times1} & 0_{\times2} & J_{1\times2} & J_{1\times2} & \dots & J_{1\times2} \\ 0_{2\times1} & 0_{2\times2} & 0_{2\times2} & 0_{2\times2} & \dots & 0_{2\times2} \\ J_{2\times1} & 0_{2\times2} & 0_{2\times2} & J_{2\times2} & \dots & 0_{2\times2} \\ J_{2\times1} & 0_{2\times2} & J_{2\times2} & 0_{2\times2} & \dots & J_{2\times2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ J_{2\times1} & 0_{2\times2} & J_{2\times2} & J_{2\times2} & \dots & 0_{2\times2} \end{bmatrix}$$

Consider $\det(\lambda I - A(F_n)_{2(i)}^P)$

Step 1: Replace R_i by $R_i - R_{i-1}$, where $i = v_{2n+1}, v_{2n}, v_{2n-1}, \dots, v_5$ and replace R_{v_4} by $R_{v_4} - R_{v_1}$. Then $\det(\lambda I - A(F_n)_{2(i)}^P)$ reduces to the form $\lambda^{n+1}\det(D)$.

Step 2: In $\det(D)$, replace R_i by $R_i - R_{i-1}$, where $i = v_{2n}, v_{2n-2}, v_{2n-4}, \dots, v_6$. We get a new determinant, let it be $\det(E)$.

Step 3: In $\det(E)$, we replace C_i by $C_i + C_{i+1}$, where $i = v_{2n}, v_{2n-1}, v_{2n-2}, \dots, v_4$. It simplifies to

$$\det(E) = (\lambda+2)^{n-2}\begin{vmatrix} \lambda & 0 & 0 & 2-2n \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\lambda-1 & 0 & 0 & \lambda+2 \end{vmatrix}$$
$$= (\lambda+2)^{n-2}\det(F).$$

Hence $\det(F)$= $(\lambda - (n-2+\sqrt{n^2-2n+2}))(\lambda - (n-2-\sqrt{n^2-2n+2}))$. So, the spectrum of $(F_n)_{2(i)}^P$ is $\left\{\begin{array}{cc} 0 & n+1 \\ -2 & n-2 \\ n-2+\sqrt{n^2-2n+2} & 1 \\ n-2-\sqrt{n^2-2n+2} & 1 \end{array}\right\}$.

Thus, $E(F_n)_{2(i)}^P = 2(n-2+\sqrt{n^2-2n+2})$. ■

Theorem 24: Let F_n be friendship graph with partition $P = \{V_1, V_2\}$, such that $< V_1 >= K_3$ and $< V_2 >= \frac{n-2}{n}K_2$. The characteristic polynomial of $(F_n)_2^P$ is $(\lambda+1)^n(\lambda-1)^{n-2}[\lambda^3 - 2\lambda^2 - (4n-3)\lambda + 2]$.

Proof: Let $P = \{V_1, V_2\}$ be a partition of vertices of F_n, where $< V_1 >= K_3$ and $< V_2 >= \frac{n-2}{n}K_2$.

$$A(F_n)_2^P = \begin{bmatrix} 0_{1\times1} & J_{1\times2} & 0_{1\times2} & 0_{1\times2} & \dots & 0_{1\times2} \\ J_{2\times1} & X & J_{2\times2} & J_{2\times2} & \dots & J_{2\times2} \\ 0_{2\times1} & J_{2\times2} & X & 0_{2\times2} & \dots & 0_{2\times2} \\ 0_{2\times1} & J_{2\times2} & 0_{2\times2} & X & \dots & 0_{2\times2} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_{2\times1} & J_{2\times2} & 0_{2\times2} & 0_{2\times2} & \dots & X \end{bmatrix}$$

where $X = (J - I)_{2\times2}$

Consider $\det(\lambda I - A(F_n)_2^P)$

Step 1: Replacing R_i by $R_i - R_{i+1}$, where $i = v_2, v_3, v_4, \dots, v_{2n}$, we obtain a new determinant say, $\det(D)$.

Step 2: In $\det(D)$, replacing C_{v_3} by $C_{v_3} - C_{v_2}$, we get a new determinant, let it be $(\lambda+1)^n\det(E)$.

Step 3: In $\det(E)$, expanding over last row, we see that, $\det(E) = (-1)^{2n+3}(-1)\det(P) + (-1)^{4n+1}(-1)\det(Q) + (-1)^{4n+2}(\lambda)\det(R)$.

(a) In $\det(P)$, replacing $C_i = C_i + C_{i+1}$, where $i = v_{2n-1}, v_{2n-2}, v_{2n-3}, \ldots, v_3$, we get a new determinant, let it be $\det(S)$.
In $\det(S)$, replacing R_i by $R_i = R_i + R_{i-1}$, where $i = v_5, v_7, v_9, \ldots, v_{2n-1}$ and by simplifying, we get $\det(S) = (-1)^{n-2}(\lambda-1)^{n-2}[2\lambda^2 + 4n\lambda - 6\lambda]$.

(b) In $\det(Q)$, replacing R_i by $R_i = R_i + R_{i-1}$, where $i = v_5, v_7, v_9, \ldots, v_{2n-1}$ and by simplifying we obtain $\det(Q) = (-1)(\lambda-1)^{n-2}[\lambda^2 + \lambda - 2]$.

(c) In $\det(R)$, replacing R_i by $R_i = R_i + R_{i-1}$, where $i = v_5, v_7, v_9, \ldots, v_{2n-1}$ and further reducing $\det(R)$, we see that $\det(R) = (\lambda - 1)^{n-2}[\lambda^2 + \lambda - 2]$.

Then from step 3, we obtain $\det(E) = (\lambda-1)^{n-2}[\lambda^3 - 2\lambda^2 - (4n-3)\lambda + 2]$. Thus, the characteristic polynomial of $(F_n)_2^P$ is $(\lambda+1)^n(\lambda-1)^{n-2}[\lambda^3 - 2\lambda^2 - (4n-3)\lambda + 2]$. ■

Theorem 25: Let $K_{n\times 2}$ be cocktail party graph with partition $P = \{V_1, V_2\}$, such that $< V_i >= K_n$, for $i = 1, 2$. Then $E(K_{n\times 2})_2^P = 4(n-1)$ and $E(K_{n\times 2})_{2(i)}^P = E(S_n^0)$.

Proof: Let $K_{n\times 2}$ be the cocktail party graph with partition $P = \{V_1, V_2\}$ such that $< V_i >= K_n$, for $i = 1, 2$. We have $A(K_{n\times 2})_2^P = \left[\begin{array}{c|c} (J-I)_n & I_n \\ \hline I_n & (J-I)_n \end{array}\right]$.

The result is proved by showing $AZ = \lambda Z$ for certain vector Z and by making use of the fact that geometric multiplicity and algebraic multiplicity of each eigenvalue λ is same, as matrix $A\left(K_{n\times 2}\right)_2^P$ is real and symmetric.
Let $Z = \begin{bmatrix} X \\ Y \end{bmatrix}$ be an eigenvector of order $2n$ partitioned conformally with $A\left(K_{n\times 2}\right)_2^P$.
Note that

$$\left(\lambda I_{2n} - A\left(K_{n\times 2}\right)_2^P\right)\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} [(\lambda+1)I - J]X - I_nY \\ -I_nX - ((\lambda+1)I - J)Y \end{bmatrix} \tag{26}$$

Case 1:Let $X = \mathbf{1}_n$ and $Y = \mathbf{1}_n$. Suppose λ is any root of the equation

$$\begin{aligned} [(\lambda+1) - n]\mathbf{1}_n - \mathbf{1}_n &= 0 \\ (\lambda - n)\mathbf{1}_n &= 0 \\ \lambda &= n \end{aligned}$$

We conclude that n is an eigenvalue of $A\left(K_{n\times 2}\right)_2^P$, with multiplicity at least one.

Case 2: Let $X = \mathbf{1}_n$ and $Y = -\mathbf{1}_n$.
Now

$$[(\lambda+1) - n]\mathbf{1}_n + \mathbf{1}_n = (\lambda - n + 2)\mathbf{1}_n = 0 \tag{27}$$

From Equation (27), we note that $\lambda - n + 2 = 0$. Thus $\lambda = n - 2$ is an eigenvalue, with multiplicity at least one.

Case 3: Let $X = X_i$ be an eigenvector with first element 1 and i^{th} element -1, for $i = 2, 3, \ldots, n$ and remaining elements equal to zero. Now $Y_i = (\lambda+1)X_i$, where λ is any root of $\lambda^2 + 2\lambda = 0$.
By noting $JX_i = 0$ and from Equation (26),

$$-(J + \lambda - 1)X_i + I(\lambda-1)X_i = -(\lambda-1)X_i + (\lambda-1)X_i = 0$$

and

$$-I_nX_i - [(\lambda+1)I_n - J](\lambda+1)X_i = (\lambda^2 + 2\lambda)X_i. \tag{28}$$

From Equation (28), we obtain $\lambda^2 + 2\lambda = 0$. Thus $\lambda = 0$ and $\lambda = -2$ are eigenvalues, each with multiplicity at least $(n-1)$, as there are $(n-1)$ independent vectors of the form X_i.
Since order of the graph is $2n$, spectrum of $(K_{n\times 2})_k^P$ is $\left\{\begin{array}{cccc} 0 & n & -2 & n-2 \\ n-1 & 1 & n-1 & 1 \end{array}\right\}$ and
$E(K_{n\times 2})_2^P = 4(n-1)$. Observe that $(K_{n\times 2})_{k(i)}^P \cong S_n^0$.
Hence, $E(K_{n\times 2})_{k(i)}^P = E(S_n^0)$. ■

Theorem 29: Let $K_{n\times 2}$ be cocktail party graph with vertex set $V = \{v_1, v_2, \cdots, v_n, u_1, u_2, \cdots, u_n\}$ and partition $P = \{V_1, V_2, \cdots, V_n\}$, such that the partition $< V_i >=< v_iu_i >$, where $i = 1, 2, \cdots, n$. Then $E(K_{n\times 2})_k^P = 0$ and $E(K_{n\times 2})_{k(i)}^P = 4n - 2$.

Proof: For the given partition, $(K_{n\times 2})_k^P$ is the totally disconnected graph. Hence, $E(K_{n\times 2})_k^P = 0$. We have $(K_{n\times 2})_{k(i)}^P \cong K_{2n}$. Hence, $E(K_{n\times 2})_{k(i)}^P = E(K_{2n}) = 4n - 2$. ■

Theorem 30: For cocktail party graph $K_{(2n+1)\times 2}$ with partition $P = \{V_1, V_2, \ldots, V_{2n+2}\}$, such that $< V_i >= K_2's\, of K_{2n+1}$ for $i = 1, 2, \ldots, 2n$ and $< V_j >= K_1$ for $j = 1, 2$. Then $E(K_{2n+1\times 2})_{(2n+2)(i)}^P = 6n - 3 + \sqrt{16n^2 + 8n + 8}$ and $E(K_{2n+1\times 2})_{(2n+2)}^P = 4n + 2$.

Proof: We have

$$A(K_{2n+1\times 2})_{(2n+2)(i)}^P = \left[\begin{array}{c|c} A & (J-I)_{2n+1} \\ \hline (J-I)_{2n+1} & A \end{array}\right]$$

$$\text{where } A = \begin{bmatrix} 0_{2\times 2} & J_{2\times 2} & \ldots & J_{2\times 2} & J_{2\times 1} \\ J_{2\times 2} & 0_{2\times 2} & \ldots & J_{2\times 2} & J_{2\times 1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ J_{2\times 2} & J_{2\times 2} & \ldots & 0_{2\times 2} & J_{2\times 1} \\ J_{1\times 2} & J_{1\times 2} & \ldots & J_{1\times 2} & 0 \end{bmatrix}_{2n+1\times 2n+1}$$

It is of the form $\left[\begin{array}{c|c} A_0 & A_1 \\ \hline A_1 & A_0 \end{array}\right]$. Hence, we can apply Lemma 8. First we shall compute spectrum of $A_0 + A_1$.

Step 1: Consider $\det(A_0 + A_1)$

(i) Replacing R_i by $R_i - R_{i+1}$, for $i = v_1, v_2, v_3, \ldots, v_{2n}$, the $\det(A_0 + A_1)$ will reduce to $(\lambda+1)^n \det(D)$.

(ii) In $\det(D)$, replacing C_i by $C_i + C_{i-1}$, for $i = v_2, v_3, v_4, \ldots, v_{2n}$, we get a new determinant $\det(E)$.

(iii) In $\det(E)$, replacing R_i by $R_i - R_{i+1}$, where $i = v_2, v_4, v_6, \ldots, v_{2n-2}$,we get
$(\lambda+3)^{n-1}\det(F)$.

(iv) Expanding $\det(F)$ over first row we obtain $\det(F) = [\lambda^2 - (4n-3)\lambda - 8n]$. Thus, we have
$det(A_0 + A_1) = (\lambda+1)^n(\lambda+3)^{n-1}[\lambda^2 - (4n-3)\lambda - 8n]$.
Spectrum of $A_0 + A_1$ is given by,
$\left\{\begin{array}{cc} \frac{4n-3+\sqrt{16n^2+8n+9}}{2} & 1 \\ \frac{4n-3+\sqrt{16n^2+8n+9}}{2} & 1 \\ -3 & n-1 \\ 1 & n \end{array}\right\}$.

To compute spectrum of $A_0 - A_1$.
Step 2: Consider $\det(A_0 - A_1)$

(i) Replacing R_i by $R_i - R_{i+1}$, for $i = v_1, v_3, v_5, \ldots, v_{2n-1}$, we see that $\det(A_0 - A_1)$ is of the form $(\lambda - 1)^n \det(D)$.

(ii) Replacing C_i by $C_i - C_{i-1}$, for $i = v_2, v_4, v_6, \ldots, v_{2n}$ in $\det(D)$, we obtain determinant of lower triangular matrix. Hence, $\det(D) = \lambda(\lambda+1)^n$. Thus, we have $\det(A_0 - A_1) = \lambda(\lambda-1)^n(\lambda+1)^n$.

Spectrum of $A_0 - A_1$ is $\begin{Bmatrix} 0 & 1 & -1 \\ 1 & n & n \end{Bmatrix}$.

Combining spectra of $A_0 + A_1$ and $A_0 - A_1$, we obtain, $E(K_{2n+1\times 2})^P_{(2n+2)(i)} = 6n - 3 + \sqrt{16n^2 + 8n + 8}$. Note that $(K_{2n+1\times 2})^P_{(2n+2)} \cong nC_4 \cup K_2$. Hence, $E(K_{2n+1\times 2})^P_{(2n+2)} = 4n + 2$. ∎

Theorem 31: For cocktail party graph $K_{2n\times 2}$ with a partition $P = \{V_1, V_2, \ldots, V_{2n}\}$, such that $< V_i >= K_2' s\ of\ K_{2n}$ for $i = 1, 2, \ldots, 2n$. Then, $E(K_{2n\times 2})^P_{2n(i)} = 10n - 6$ and $E(K_{2n\times 2})^P_k = 4n$.

Proof: Let $P = \{V_1, V_2, \ldots, V_{2n}\}$ be a partition of $K_{2n\times 2}$ such that $< V_i >= K_2' s\ of\ K_{2n}$ for $i = 1, 2, \ldots, 2n$. Then, we have

$$A(K_{2n\times 2})^P_{(2n)(i)} = \left[\begin{array}{c|c} (J_{2n} - I_{2n}) \otimes J_2 & (J-I)_{2n\times 2n} \\ \hline (J-I)_{2n\times 2n} & (J_{2n} - I_{2n}) \otimes J_2 \end{array}\right]$$

It is of the form $\left[\begin{array}{c|c} A_0 & A_1 \\ \hline A_1 & A_0 \end{array}\right]$. Hence, from Lemma 8, we get spectra of $(K_{2n\times 2})^P_{2n(i)}$ as $\begin{Bmatrix} -3 & -1 & 1 & 4n-3 \\ n-1 & 2n & n & 1 \end{Bmatrix}$. So that $E(K_{2n\times 2})^P_{2n(i)} = 10n - 6$. Also $(K_{2n\times 2})^P_{2n} \cong nC_4$. Thus, $E(K_{2n\times 2})^P_{2n} = 4n$. ∎

IV. Conclusion

The energy of a graph is one of the emerging subject within graph theory. It serves as a frontier between Chemistry and mathematics. The energy of several graphs is found in literature. In our study, we have derived explicit expression for the energy of generalized complements of some classes of graphs by taking partition of order 2, 3 and $k \geq 3$ in some cases.

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