Applications of Left Almost Subhypersemigroups to Fuzzy Subsets

Pairote Yairayong*

Abstract— **In this paper, we introduce and analyze a new type of fuzzy LA -hypersemigroups, as a generalization of fuzzy hypersemigroup and left almost semihypergroups.Then we discuss the relations between the fuzzy LA -hypersemigroups and the fuzzy hypersemigroups. We introduce the notion of fuzzy LA -subhypersemigroup, left fuzzy hyperideal (right fuzzy hyperideal, fuzzy hyperideal), left fuzzy hypersimple and fuzzy hyper bi-ideals of fuzzy LA -hypersemigroups and obtain their basic properties. Finally, we obtain some characterizations of left fuzzy hyperideal (right fuzzy hyperideal, fuzzy hyperideal), left fuzzy hypersimple and fuzzy hyper bi-ideals were obtained.**

Index Terms—**fuzzy LA -hypersemigroup, fuzzy LA -subhypersemigroup, left fuzzy hyperideal, left fuzzy hypersimple, fuzzy hyper bi-ideal.**

I. Introduction

In 2008, Sen et al. [19] introduced the notion of fuzzy hypersemigroup, fuzzy hypergroup, fuzzy hyperideal, homomorphism, fuzzy homomorphism, hyper congruence, fuzzy hypercongruence. In 2009, Fotea and Davvaz [10] defined the concept of fuzzy hyperrings. In 2010, Yin et al. [22] introduced and studied the L-fuzzy hypermodules over an L-fuzzy hyperring. The idea of fuzzy join n-ary spaces and fuzzy canonical n-ary hypergroups first introduced by Fotea in 2010 [9], as a generalization of join spaces and canonical hypergroups. In 2011, Ameri and Nozari [4] formulated and studied the notion of fuzzy regular (fuzzy strongly regular) relations of hyperalgebras. Corsini et al. [7] introduced the notion of prime (semiprime) hyperideals and prime (semiprime) fuzzy hyperideals in semihypergroups and studied basic properties of them. Yin et al. [20] defined the concept of (weak) L-fuzzy polygroups. The concepts of hypercongruence on hyperlattices and the fuzzy (strong) hypercongruence on fuzzy hyperlattice were introduced and discussed by He and Xin [11]. The idea of fuzzy hypervector spaces first introduced by Ameri and Motameni [3], as a generalization of fuzzy vector spaces. In 2013, Ameri and Sadeghi [5] introduced and studied the fuzzy R_{Γ}-hypermodules. In 2016, Motameni et al. [15] studied this structure under the name of prime fuzzy hyperideals and maximal fuzzy hyperideals in fuzzy hyperrings. Ameri and Nozari [16] formulated and studied the notion of commutative fundamental relation in fuzzy hypersemigroups. In 2018, Ameri et al. [2] have shown that a fuzzy geometric space is strongly transitive on hypergroups, while it is not strongly transitive on hypersemigroups.

The idea of locally associative LA -semihypergroups first introduced by Amjad et al. [1], as a generalization of locally associative LA -semigroups. In 2017, Rehman et al. [17] introduced the notion of hyperideals in LA-hyperrings and studied basic properties of them. Khan et al. characterized regular and intra-regular LA -semihypergroups by their fuzzy hyperideals also see [13]. In 2018, Azar et al. [6] introduced the notion of fuzzy ordered LA -semihypergroups and studied basic properties of them. Some authors studied similar types of fuzzy subsets of other algebraic structures seen in [18].

Now in this paper we introduced and study fuzzy left almost hypersemigroups (fuzzy LA -hypersemigroup) as generalization of LA -semihypergroup as well as fuzzy hypersemigroups. The paper has been prepared in 5 sections. In section 2 we recall some basic notions and results on fuzzy hypersemigroups and LA -semihypergroups. In section 3, we introduce some definitions and results of fuzzy LA -hypersemigroups which we need to developing our paper. In section 4, we introduced and study fuzzy LA -subhypersemigroup and left fuzzy hyperideals (right fuzzy hyperideal, fuzzy hyperideal) of fuzzy LA -hypersemigroups and obtain its basic results. In section 5, we introduced and study fuzzy hyper bi-ideals of fuzzy LA -hypersemigroups.

II. Preliminaries

In this section, we summarize some basic concepts (see [14, 20]) which will be used throughout the paper and introduce and study fuzzy hypersemigroups and LA -semihypergroups.

Definition 2.1. [14] Let S be a non empty set and $\mathrm{P}^*(S)$ denotes the set of all non empty subsets of $S.$ A **hyperoperation or join operation** on S is a mapping $\circ : S \times S \to \mathrm{P}^*(S)$ written as $(a,b) \mapsto a \circ b.$ A non empty S together with a hyperoperation "$\circ$"is called a **hypergroupoid**.

Let A and B be two non empty subsets of a non empty set S and $x \in S.$ Then we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, x \circ B = \{x\} \circ B$$

Manuscript received February 22, 2019; revised October 20, 2019.
P. Yairayong* is with the Department of Mathematics, Faculty of Science and Technology, Pibulsongkram Rajabhat University, Phitsanulok 65000, Thailand (e-mail: pairote0027@hotmail.com).

And $A\circ x = A\circ\{x\}$.

Definition 2.2. [12] A hypergroupoid $(S,\circ)$ is called a **left almost semihypergroup** (LA-semihypergroup) if

$$(x\circ y)\circ z = (z\circ y)\circ x$$

for all $x,y,z\in S$.

Sen et al. [19] introduced the notion of fuzzy hypersemigroups as a generalization of semigroups, fuzzy semigroups and fuzzy subsets. Let us see now what subhypermodules are.

Definition 2.3. [19] Let S be a non empty set and $\mathrm{F}(S)$ denotes the set of all fuzzy subset of S. A **fuzzy hyperoperation** on S is a mapping

$$\circ : S\times S \to \mathrm{F}(S)$$

written as $(a,b)\mapsto a\circ b$. A non empty S together with a fuzzy hyperoperation "$\circ$" is called a **fuzzy hypergroupoid**.

It is natural to speak now about hyperoperation on $\mathrm{F}(S)$.

Definition 2.4. [19] Let $(S,\circ)$ be a fuzzy hypergroupoid and $\mu,\nu\in\mathrm{F}(S)$. Then we define $\mu\circ\nu$ by

$$(\mu\circ\nu)(a) = \bigcup_{x,y\in S}\left(\mu(x)\wedge(x\circ y)(a)\wedge\nu(y)\right)$$

for all $a\in S$.

III. Fuzzy LA-hypersemigroups

The notions of fuzzy hypersemigroups and LA-semihypergroups are introduced by Sen et al. in [19] and Marty in [14], respectively. In a similar way, we give the definitions of fuzzy left almost hypersemigroups (fuzzy LA-hypersemigroup) as follows.

Definition 3.1. A fuzzy hypergroupoid $(S,\circ)$ is called a **fuzzy left almost hypersemigroup** (fuzzy LA-hypersemigroup) if for all $x,y,z\in S$, $(x\circ y)\circ z = (z\circ y)\circ x$, where for any $\mu\in\mathrm{F}(S)$

$$(x\circ\mu)(a) = \begin{cases}\bigcup_{s\in S}(x\circ s)(a)\wedge\mu(s) & ;\text{if}\mu\neq 0\\ 0 & ;\text{otherwise}\end{cases}$$

and

$$(\mu\circ x)(a) = \begin{cases}\bigcup_{s\in S}\mu(s)\wedge(s\circ x)(a) & ;\text{if}\mu\neq 0\\ 0 & ;\text{otherwise.}\end{cases}$$

In what follows let S denote a fuzzy LA-hypersemigroup unless otherwise specified. The following four theorems provide us some examples of fuzzy LA-hypersemigroups.

Theorem 3.2. Let S be a non empty set. Define a fuzzy hyperoperation "$\circ$" on S by $x\circ y = \chi_{\{x,y\}}$ for all $x,y\in S$, where $\chi_{\{x,y\}}$ denotes the characteristic function of the set $\{x,y\}$. Then $(S,\circ)$ is a fuzzy LA-hypersemigroup.

Proof. Let $a,x,y,z\in S$. We divide our proof into two cases.

Case 1 $a\in\{x,y,z\}$. By assumption,

$$\begin{aligned}((x\circ y)\circ z)(a) &= \left(\chi_{\{x,y\}}\circ z\right)(a)\\ &= \bigcup_{s\in S}\left(\chi_{\{x,y\}}(s)\wedge(s\circ z)(a)\right)\\ &= \chi_{\{x,y,z\}}(a)\\ &= 1\end{aligned}$$

and

$$\begin{aligned}((z\circ y)\circ x)(a) &= \left(\chi_{\{z,y\}}\circ x\right)(a)\\ &= \bigcup_{s\in S}\left(\chi_{\{z,y\}}(s)\wedge(s\circ x)(a)\right)\\ &= \chi_{\{z,y,x\}}(a)\\ &= 1\end{aligned}$$

It follows that $(x\circ y)\circ z = (z\circ y)\circ x$.

Case 2 $a\notin\{x,y,z\}$. It is easy to see that,

$$\begin{aligned}((x\circ y)\circ z)(a) &= \chi_{\{x,y,z\}}(a)\\ &= 0\\ &= \chi_{\{z,y,x\}}(a)\\ &= ((z\circ y)\circ x)(a).\end{aligned}$$

Hence $(S,\circ)$ is a fuzzy LA-hypersemigroup.

Theorem 3.3. Let S be an LA-semigroup and $0\neq\mu\in\mathrm{F}(S)$. Define a fuzzy hyperoperation "$\circ$" on S by

$$(x\circ y)(a) = \begin{cases}\mu(x)\wedge\mu(y) & ;\text{if } a = xy\\ 0 & ;\text{otherwise}\end{cases}$$

for all $x,y\in S$. If μ is a fuzzy LA-subsemigroup on S, then $(S,\circ)$ is a fuzzy LA-hypersemigroup.

Proof. Let $a,x,y,z\in S$. It is easy to see that,

$$\begin{aligned}((x\circ y)\circ z)(a) &= \bigcup_{s\in S}\left((x\circ y)(s)\wedge(s\circ z)(a)\right)\\ &= (\mu(x)\wedge\mu(y))\wedge(xy\circ z)(a)\end{aligned}$$

and

$$\begin{aligned}((z\circ y)\circ x)(a) &= \bigcup_{s\in S}\left((z\circ y)(s)\wedge(s\circ x)(a)\right)\\ &= (\mu(z)\wedge\mu(y))\wedge(zy\circ x)(a)\end{aligned}$$

for all $a,x,y,z\in S$. If $a=(xy)z$, then $a=(zy)x$. By assumption,

$$\begin{aligned}((x\circ y)\circ z)(a) &= (\mu(x)\wedge\mu(y))\wedge(xy\circ z)(a)\\ &= \mu(x)\wedge\mu(y)\wedge\mu(xy)\wedge\mu(z)\\ &= \mu(x)\wedge\mu(y)\wedge\mu(z)\\ &= \mu(z)\wedge\mu(y)\wedge\mu(zy)\wedge\mu(x)\\ &= (\mu(z)\wedge\mu(y))\wedge(zy\circ x)(a)\end{aligned}$$

$= ((z \circ y) \circ x)(a).$

Assume that $a \neq (xy)z$. Thus $a \neq (zy)x$. Clearly, $((x \circ y) \circ z)(a) = 0 = ((z \circ y) \circ x)(a)$. Hence $(S, \circ)$ is a fuzzy LA-hypersemigroup.

Theorem 3.4. Let $S = \mathbf{Z}^- \cup \{0,1,\ldots,n\}$. Define a fuzzy hyperoperation "$\circ$" on S by $x \circ y = \chi_{max\{x,y\}}$ for all $x, y \in S$. Then $(S, \circ)$ is a fuzzy LA-hypersemigroup.

Proof. Let $a, x, y, z \in S$. Clearly,

$$\begin{aligned} ((x \circ y) \circ z)(a) &= \left(\chi_{max\{x,y\}} \circ z\right)(a) \\ &= \bigcup_{s \in S}\left(\chi_{max\{x,y\}}(s) \wedge (s \circ z)(a)\right) \\ &= (max\{x, y\} \circ z)(a) \\ &= \chi_{max\{max\{x,y\},z\}}(a) \\ &= \chi_{max\{max\{z,y\},x\}}(a) \\ &= (max\{z, y\} \circ x)(a) \\ &= \bigcup_{s \in S}\left(\chi_{max\{z,y\}}(s) \wedge (s \circ x)(a)\right) \\ &= \left(\chi_{max\{z,y\}} \circ x\right)(a) \\ &= ((z \circ y) \circ x)(a). \end{aligned}$$

Consequently, $(S, \circ)$ is a fuzzy LA-hypersemigroup.

The following theorem presents the connection between LA-semigroups and fuzzy LA-hypersemigroups.

Theorem 3.5. Let S be a LA-semigroup. Define a fuzzy hyperoperation "$\circ$" on S by $x \circ y = \chi_{\{xy\}}$ for all $x, y \in S$, where $\chi_{\{xy\}}$ denotes the characteristic function of the set $\{xy\}$. Then $(S, \circ)$ is a fuzzy LA-hypersemigroup.

Proof. Let $a, x, y, z \in S$. Then

$$\begin{aligned} ((x \circ y) \circ z)(a) &= \left(\chi_{\{xy\}} \circ z\right)(a) \\ &= \bigcup_{s \in S}\left(\chi_{\{xy\}}(s) \wedge (s \circ z)(a)\right) \\ &= ((xy) \circ z)(a) \\ &= \chi_{\{(xy)z\}}(a) \\ &= \chi_{\{(zy)x\}}(a) \\ &= ((zy) \circ x)(a) \\ &= \bigcup_{s \in S}\left(\chi_{\{zy\}}(s) \wedge (s \circ x)(a)\right) \\ &= \left(\chi_{\{zy\}} \circ x\right)(a) \\ &= ((z \circ y) \circ x)(a). \end{aligned}$$

Hence $(S, \circ)$ is a fuzzy LA-hypersemigroup.

The next theorem show a connection between hyperoperations of characteristic functions and LA-hypersemigroups.

Theorem 3.6. Let S be a fuzzy LA-hypersemigroup. Then the following statements hold:

1. $\chi_x \circ \chi_y = x \circ y$ for all $x, y \in S$.
2. For every $x \in S, \chi_S \circ x = S \circ x$ and $x \circ \chi_S = x \circ S$.
3. For every $\mu \in F(S), \chi_S \circ \mu = S \circ \mu$ and $\mu \circ \chi_S = \mu \circ S$.

Proof. 1. By assumption,

$$\begin{aligned} \chi_x \circ \chi_y(a) &= \bigcup_{r,s \in S}\left(\chi_x(r) \wedge (r \circ s)(a) \wedge \chi_y(s)\right) \\ &= 1 \wedge (x \circ y)(a) \wedge 1 \\ &= (x \circ y)(a) \end{aligned}$$

for all $a, x, y \in S$. Hence $\chi_x \circ \chi_y = x \circ y$.

2. Let $a, x \in S$. By assumption,

$$\begin{aligned} (\chi_S \circ x)(a) &= \bigcup_{s \in S}\left(\chi_S(s) \wedge (s \circ x)(a)\right) \\ &= \bigcup_{s \in S}\left(1 \wedge (s \circ x)(a)\right) \\ &= \bigcup_{s \in S}(s \circ x)(a) \\ &= (S \circ x)(a). \end{aligned}$$

Hence $\chi_S \circ x = S \circ x$. Similarly we can show that $x \circ \chi_S = x \circ S$.

3. Let $\mu \in F(S)$ and $a \in S$. It is easy to see that,

$$\begin{aligned} (\chi_S \circ \mu)(a) &= \bigcup_{r,s \in S}\left(\chi_S(r) \wedge (r \circ s)(a) \wedge \mu(s)\right) \\ &= \bigcup_{r,s \in S}\left((r \circ s)(a) \wedge \mu(s)\right) \\ &= \bigcup_{r \in S}\left((r \circ \mu)(a)\right) \\ &= (S \circ \mu)(a). \end{aligned}$$

Hence $\chi_S \circ \mu = S \circ \mu$. Similarly we can show that $\mu \circ \chi_S = \mu \circ S$.

Next, let us give some theorem which are useful for the following theorems.

Theorem 3.7. Let S be a fuzzy LA-hypersemigroup and $\mu, \nu, \lambda \in F(S)$. Then the following statements hold:

1. For every $x, y \in S, (x \circ y) \circ \mu = (\mu \circ y) \circ x$.
2. For every $x, y \in S, (x \circ \mu) \circ y = (y \circ \mu) \circ x$.
3. For every $x, y \in S, (\mu \circ x) \circ y = (y \circ x) \circ \mu$.
4. For every $x \in S, (\mu \circ \nu) \circ x = (x \circ \nu) \circ \mu$.
5. For every $x \in S, (\mu \circ x) \circ \nu = (\nu \circ x) \circ \mu$.
6. $(\mu \circ \nu) \circ \lambda = (\lambda \circ \nu) \circ \mu$.

Proof. 1. Let $a, x, y \in S$ and $\mu \in F(S)$. By assumption,

$$((x \circ y) \circ \mu)(a) = \bigcup_{r \in S}\left(((x \circ y) \circ r)(a) \wedge \mu(r)\right)$$

$$= \bigcup_{r\in S}\big(((r\circ y)\circ x)(a)\wedge \mu(r)\big)$$
$$= \bigcup_{r,s\in S}((r\circ y)(s)\wedge (s\circ x)(a) \wedge\mu(r)$$
$$= \bigcup_{r,s\in S}(\mu(r)\wedge (r\circ y)(s)\wedge (s\circ x)(a))$$
$$= \bigcup_{s\in S}\big((\mu\circ y)(s)\wedge (s\circ x)(a)\big)$$
$$= ((\mu\circ y)\circ x)(a).$$

Hence $(x\circ y)\circ \mu = (\mu\circ y)\circ x.$

2. Let $a, x, y \in S$ and $\mu \in \mathrm{F}(S).$ By assumption,

$$((x\circ \mu)\circ y)(a) = \bigcup_{r\in S}\big((x\circ \mu)(r)\wedge (r\circ y)(a)\big)$$
$$= \bigcup_{r,s\in S}((x\circ s)(r)\wedge \mu(r)\wedge (r\circ y)(a))$$
$$= \bigcup_{r,s\in S}((x\circ s)(r)\wedge (r\circ y)(a) \wedge\mu(r))$$
$$= \bigcup_{r,s\in S}\big(((x\circ s)\circ y)(a)\wedge \mu(r)\big)$$
$$= \bigcup_{r,s\in S}\big(((y\circ s)\circ x)(a)\wedge \mu(r)\big)$$
$$= \bigcup_{r,s\in S}((y\circ s)(r)\wedge (r\circ x)(a) \wedge\mu(r))$$
$$= \bigcup_{r\in S}\big((y\circ \mu)(r)\wedge (r\circ x)(a)\big)$$
$$= ((y\circ \mu)\circ x)(a).$$

Hence $(x\circ \mu)\circ y = (y\circ \mu)\circ x.$

3. Let $a, x, y \in S$ and $\mu \in \mathrm{F}(S).$ By assumption,

$$((\mu\circ x)\circ y)(a) = \bigcup_{r\in S}\big((\mu\circ x)(r)\wedge (r\circ y)(a)\big)$$
$$= \bigcup_{r,s\in S}(\mu(s)\wedge (s\circ x)(r) \wedge(r\circ y)(a))$$
$$= \bigcup_{s,r\in S}((s\circ x)(r)\wedge (r\circ y)(a) \wedge\mu(s))$$
$$= \bigcup_{s\in S}\big(((s\circ x)\circ y)(a)\wedge \mu(s)\big)$$
$$= \bigcup_{s\in S}\big(((y\circ x)\circ s)(a)\wedge \mu(s)\big)$$
$$= ((y\circ x)\circ \mu)(a).$$

Hence $(x\circ \mu)\circ y = (y\circ \mu)\circ x.$

4. Let $a, x \in S$ and $\mu, \nu \in \mathrm{F}(S).$ By assumption,

$$((\mu\circ \nu)\circ x)(a) = \bigcup_{r\in S}\big((\mu\circ \nu)(r)\wedge (r\circ x)(a)\big)$$
$$= \bigcup_{r,s\in S}(\mu(s)\wedge (s\circ \nu)(r) \wedge(r\circ x)(a))$$
$$= \bigcup_{s\in S}\big(\mu(s)\wedge ((s\circ \nu)\circ x)(a)\big)$$
$$= \bigcup_{s\in S}\big(\mu(s)\wedge ((x\circ \nu)\circ s)(a)\big)$$
$$= \bigcup_{r,s\in S}(\mu(s)\wedge (x\circ \nu)(r)\wedge (r\circ s)(a))$$
$$= \bigcup_{r,s\in S}((x\circ \nu)(r)\wedge (r\circ s)(a) \wedge\mu(r))$$
$$= \bigcup_{r\in S}\big((x\circ \nu)(r)\wedge (r\circ \mu)(a)\big)$$
$$= ((x\circ \nu)\circ \mu)(a).$$

Hence $(x\circ \mu)\circ y = (y\circ \mu)\circ x.$

5. The proof is similar to part 4.

6. Let $a \in S$ and $\mu, \nu, \lambda \in \mathrm{F}(S).$ By assumption,

$$((\mu\circ \nu)\circ \lambda)(a) = \bigcup_{r\in S}\big((\mu\circ \nu)(r)\wedge (r\circ \lambda)(a)\big)$$
$$= \bigcup_{r,s\in S}(\mu(s)\wedge (s\circ \nu)(r)\wedge (r\circ \lambda)(a))$$
$$= \bigcup_{s\in S}\big(\mu(s)\wedge ((s\circ \nu)\circ \lambda)(a)\big)$$
$$= \bigcup_{s\in S}\big(\mu(s)\wedge ((\lambda\circ \nu)\circ s)(a)\big)$$
$$= \bigcup_{s,r\in S}(\mu(s)\wedge (\lambda\circ \nu)(r)\wedge (r\circ s)(a))$$
$$= \bigcup_{r,s\in S}((\lambda\circ \nu)(r)\wedge (r\circ s)(a) \wedge \mu(s))$$
$$= \bigcup_{r\in S}\big((\lambda\circ \nu)(r)\wedge (r\circ \mu)(a)\big)$$
$$= ((\lambda\circ \nu)\circ \mu)(a).$$

Hence $(\mu\circ \nu)\circ \lambda = (\lambda\circ \nu)\circ \mu.$

Remark. Let S be a fuzzy LA-hypersemigroup and $\mathrm{P}^*(S)$ denotes the set of all non empty subsets of $S.$ Then by Theorem 3.7 (6), $(\mathrm{P}^*(S), \circ)$ is a LA-semihypergroup.

As a consequence of Theorem 3.7, we obtain the following result.

Corollary 3.8. Let S be a fuzzy LA-hypersemigroup and $\mu, \nu, \lambda, \xi \in \mathrm{F}(S).$ Then the following statements hold:

1. $(x\circ y)\circ (z\circ \mu) = (x\circ z)\circ (y\circ \mu)$ for every $x, y, z \in S.$
2. $(x\circ y)\circ (\mu\circ z) = (x\circ \mu)\circ (y\circ z)$ for every $x, y, z \in S.$
3. $(\mu\circ x)\circ (y\circ z) = (\mu\circ y)\circ (x\circ z)$ for every $x, y, z \in S.$
4. $(x\circ y)\circ (\mu\circ \nu) = (x\circ \mu)\circ (y\circ \nu)$ for every $x, y, z \in S.$
5. $(\mu\circ \nu)\circ (x\circ y) = (\mu\circ x)\circ (\nu\circ y)$ for every $x, y, z \in S.$
6. $(x\circ \mu)\circ (\nu\circ y) = (x\circ \nu)\circ (\mu\circ y)$ for every $x, y, z \in S.$

7. $(x\circ\mu)\circ(\nu\circ\lambda)=(x\circ\nu)\circ(\mu\circ\lambda)$ for every $x,y,z\in S$.

8. $(\mu\circ x)\circ(\nu\circ\lambda)=(\mu\circ\nu)\circ(x\circ\lambda)$ for every $x,y,z\in S$.

9. $(\mu\circ\nu)\circ(\lambda\circ x)=(\mu\circ\lambda)\circ(\nu\circ x)$ for every $x,y,z\in S$.

10. $(\mu\circ\nu)\circ(\lambda\circ\xi)=(\mu\circ\lambda)\circ(\nu\circ\xi)$.

Proof. 1. Let $x,y,z\in S$ and $\mu\in \mathrm{F}(S)$. By Theorem 3.8

$$\begin{aligned}(x\circ y)\circ(z\circ\mu)&=((z\circ\mu)\circ y)\circ x\\&=((y\circ\mu)\circ z)\circ x\\&=(x\circ z)\circ(y\circ\mu).\end{aligned}$$

2-10. The proof is similar to part 1.

In what follows, we consider a first connection between LA -semihypergroups and fuzzy LA -hypersemigroups, using the α -cuts of a fuzzy subset.

Let S be an LA -semihypergroup, endowed with a fuzzy LA -hypersemigroups "$\circ$" and for all $x,y\in S$, consider the α -cuts

$$(x\circ y)_\alpha:=\{s\in S:(x\circ y)(s)\geq\alpha\}$$

of $x\circ y$, where $\alpha\in[0,1]$. For all $\alpha\in[0,1]$, we define the following crisp hyperoperation on S :

$$x\circ_\alpha y:=(x\circ y)_\alpha.$$

Theorem 3.9. Let S be an LA -semihypergroup and $x\in S$. Then $\chi_S=x\circ S$ if and only if $S=x\circ_\alpha S$ for all $\alpha\in[0,1]$.

Proof. Suppose that $\chi_S=x\circ S$ for all $x\in S$. Let $a\in S$ and $\alpha\in[0,1]$. By assumption,

$$\bigcup_{s\in S}(x\circ s)(a)=\chi_S(a)=1\geq\alpha.$$

Then there exists $r\in S$ such that $(x\circ r)(a)\geq\alpha$, which means that $a\in x\circ_\alpha r$. Hence $S=x\circ_\alpha S$.

Conversely assume that, $S=x\circ_\alpha S$ for all $\alpha\in[0,1]$. By assumption, $S=x\circ_1 S$. Then there exists $s\in S$ such that $a\in x\circ_1 s$ for all $a\in S$, which means that $(x\circ s)(a)=1$. Consequently, $\chi_S=x\circ S$.

Connections between fuzzy hyperoperations and the above associated hyperoperations have been considered by Sen et al. [19] in the context of semihypergroups. They have shown that if $(S,\circ)$ is a fuzzy hypergroup, then $(S,\circ_\alpha)$ is a hypergroup while $(S,\circ)$ is a fuzzy semihypergroup if and only if $(S,\circ_\alpha)$ is a semihypergroup (see [8]).

The next theorem establishes a similar result for LA - semihypergroup.

Theorem 3.10. Let S be an LA -semihypergroup and $x\in S$. Then $(S,\circ)$ is a fuzzy LA -semihypergroup if and only if $(S,\circ_\alpha)$ is an LA -semihypergroup.

Proof. Let $a,x,y,z\in S$ and $\alpha\in[0,1]$. It is easy to see that $((x\circ y)\circ z)(a)=\bigcup_{s\in S}\big((x\circ y)(s)\wedge(s\circ z)(a)\big)\geq\alpha$ if and only if there exists $r\in S$ such that $r\in x\circ_\alpha y$ and $a\in r\circ_\alpha z$, which means that $(x\circ y)(r)\geq\alpha$ and $(r\circ z)(a)\geq\alpha$. Hence $a\in(x\circ_\alpha y)\circ_\alpha z$. This complete the proof.

Let S be a fuzzy LA -hypersemigroup. We say that an element e of S is **left identity** if for all $r,s\in S$, then $(e\circ s)(s)=1$.

Theorem 3.11. Let S be an LA -semigroup with left identity. Define a fuzzy hyperoperation "$\circ$" on S by $x\circ y=\chi_{\{xy\}}$ for all $x,y\in S$, where $\chi_{\{xy\}}$ denotes the characteristic function of the set $\{xy\}$. Then $(S,\circ)$ is a fuzzy LA -hypersemigroup with left identity.

Proof. By Theorem 5, $(S,\circ)$ is a fuzzy LA - hypersemigroup. Let $x\in S$. By assumption,

$$\begin{aligned}(e\circ x)(x)&=\chi_{\{ex\}}(x)\\&=\chi_{\{x\}}(x)\\&=1.\end{aligned}$$

Hence $(S,\circ)$ is a fuzzy LA -hypersemigroup with left identity.

Next, let us give some lemma which are useful for the following theorems.

Lemma 3.12. Let S be a fuzzy LA -hypersemigroup with left identity. Then the following statements hold:

1. $e\circ\mu=\mu$ for all $\mu\in\mathrm{F}(S)$.
2. $e\circ(x\circ y)=x\circ y$ for all $x,y\in S$.
3. $x\circ(e\circ y)=x\circ y$ for all $x,y\in S$.
4. $(e\circ x)\circ(y\circ z)=x\circ(y\circ z)$ for all $x,y,z\in S$.

Proof. 1. Let $a\in S$ and $\mu\in\mathrm{F}(S)$. By assumption, $(e\circ\mu)(a)=\bigcup_{r\in S}\big((e\circ r)(a)\wedge\mu(r)\big)=\mu(a)$. Consequently, $e\circ\mu=\mu$.

2. The proof is similar to part 1.

3. Let $a,x,y\in S$. Then

$$\begin{aligned}(x\circ(e\circ y))(a)&=\bigcup_{s\in S}\big((x\circ s)(a)\wedge(e\circ y)(s)\big)\\&=(x\circ y)(a).\end{aligned}$$

Consequently, $x\circ(e\circ y)=x\circ y$.

4. Clearly,

$$(e\circ x)\circ(y\circ z)(a)=\bigcup_{r,s\in S}((e\circ x)(r)\wedge (r\circ s)(a)\wedge(y\circ z)(s))$$
$$=\bigcup_{s\in S}\big((x\circ s)(a)\wedge(y\circ z)(s)\big)$$
$$=(x\circ(y\circ z))(a)$$

for all $a,x,y,z\in S$. Thus $(e\circ x)\circ(y\circ z)=x\circ(y\circ z)$.

As a direct consequence, we have the following result.

Theorem 3.13. Let S be a fuzzy LA-hypersemigroup with left identity and $\mu,\nu,\lambda\in \mathrm{F}(S)$. Then the following statements hold:

1. For every $x\in S, \mu\circ(x\circ y)=x\circ(\mu\circ y)$.
2. For every $x\in S, \mu\circ(x\circ y)=x\circ(\mu\circ y)$.
3. For every $x\in S, \mu\circ(\nu\circ x)=\nu\circ(\mu\circ x)$.
4. For every $x\in S, \mu\circ(\nu\circ x)=\nu\circ(\mu\circ x)$.
5. For every $x\in S, \mu\circ(\nu\circ\lambda)=\nu\circ(\mu\circ\lambda)$.

Proof. 1. Let $a,x,y\in S$ and $\mu\in \mathrm{F}(S)$. By Lemma 3.12,

$$\begin{aligned}\mu\circ(x\circ y)&=(e\circ\mu)\circ(x\circ y)\\&=((x\circ y)\circ\mu)\circ e\\&=((\mu\circ y)\circ x)\circ e\\&=(e\circ x)\circ(\mu\circ y).\end{aligned}$$

2. Let $a,x,y,z\in S$. By Lemma 3.12,

$$\begin{aligned}x\circ(y\circ z)&=(e\circ x)\circ(y\circ z)\\&=((y\circ z)\circ x)\circ e\\&=((x\circ z)\circ y)\circ e\\&=(e\circ y)\circ(x\circ z)\\&=y\circ(x\circ z).\end{aligned}$$

3. Let $x\in S$ and $\mu,\nu\in \mathrm{F}(S)$. By Lemma 3.12,

$$\begin{aligned}\mu\circ(\nu\circ x)&=(e\circ\mu)\circ(\nu\circ x)\\&=((\nu\circ x)\circ\mu)\circ e\\&=((\mu\circ x)\circ\nu)\circ e\\&=(e\circ\nu)\circ(\mu\circ x)\\&=\nu\circ(\mu\circ x).\end{aligned}$$

Next, as the following example shows, the LA-hypersemigroup is a fuzzy hypersemigroup.

Example 1. Let $S=\{1,2,3,4\}$, the binary operation "$\cdot$" on S be defined as follows:

$\cdot$	1	2	3	4
1	1	2	3	4
2	1	2	3	4
3	1	4	2	3
4	1	3	4	2

Clearly, S is an LA-semigroup (see (Yaqoob et al.; 2013)). By Theorem 3.5, $(S,\circ)$ is a fuzzy LA-hypersemigroup. Then it can be easily verify that

$$\begin{aligned}((3\circ 4)\circ 2)(2)&=\chi_{\{(3\cdot 4)\cdot 2\}}(2)\\&=\chi_{\{2\}}(2)\\&=1\\&\neq 0\\&=\chi_{\{4\}}(2)\\&=\chi_{\{3\cdot(4\cdot 2)\}}(2)\\&=(3\circ(4\circ 2))(2).\end{aligned}$$

Thus $(S,\circ)$ is not a fuzzy hypersemigroup.

The following theorem presents the connection between fuzzy hypersemigroups and fuzzy LA-hypersemigroups.

Theorem 3.14. A fuzzy LA-hypersemigroup S is a fuzzy hypersemigroup if and only if $x\circ(y\circ z)=(z\circ y)\circ x$ holds for all $x,y,z\in S$.

Proof. Suppose that S is a fuzzy hypersemigroup. Let $x,y,z\in S$. Then $(x\circ y)\circ z=(z\circ y)\circ x=z\circ(y\circ x)$.

Conversely assume that, $x\circ(y\circ z)=(z\circ y)\circ x$ holds for all $x,y,z\in S$. By assumption,

$$(x\circ y)\circ z=(z\circ y)\circ x=x\circ(y\circ z).$$

Hence S is a fuzzy LA-hypersemigroup.

IV. Left Fuzzy Hyperideals

In this section we shall introduce and analyze the notions of fuzzy LA-subhypersemigroups, left fuzzy hyperideals, right fuzzy hyperideals and right fuzzy hyperideals in fuzzy LA-hypersemigroups. In particular, we shall analyze the left fuzzy hypersimple of fuzzy LA-hypersemigroups.

Definition 4.1. Let S be a fuzzy LA-hypersemigroup. A fuzzy subset μ of S is called a **fuzzy left almost subhypersemigroup** (fuzzy LA-subhypersemigroup) of S if $\mu\circ\mu\subseteq\mu$.

As a direct consequence, we have the following result.

Theorem 4.2. If μ and ν are two fuzzy LA-subhypersemigroups of a fuzzy LA-hypersemigroup S, then $\mu\cap\nu$ is also a fuzzy LA-subhypersemigroup of S.

Proof. It is straight forward.

Next, we shall introduce the left fuzzy hyperideals, right fuzzy hyperideals and fuzzy hyperideals of fuzzy LA-hypersemigroups, as follows:

Definition 4.3. A fuzzy subset μ of a fuzzy LA-hypersemigroup S is called a **left fuzzy hyperideal** (right fuzzy hyperideal) of S if

$$s\circ\mu\subseteq\mu(\mu\circ s\subseteq\mu)$$

for all $s\in S$. A fuzzy subset μ of S is called a **fuzzy hyperideal** of S if it is a left and a right fuzzy hyperideal.

Obviously, every left fuzzy hyperideal (right fuzzy hyperideal, fuzzy hyperideal) of a fuzzy LA-hypersemigroup is a fuzzy LA-subhypersemigroup of S. The converse is not true, in general, that is, a left fuzzy hyperideal may not be fuzzy LA-subhypersemigroup the following example shows.

Example 2. Consider the fuzzy LA-hypersemigroup given in Example 1. Then, the fuzzy subset μ of S defined by $\mu(1)=1=\mu(2)$ and $\mu(3)=\mu(4)=0$. It is easy to see that, μ is a fuzzy LA-subhypersemigroup of S but μ is a left fuzzy hyperideal of S, because

$$3\circ\mu(4)=\bigcup_{s\in S}(3\circ s)(4)\wedge\mu(s)=1\not\leq 0=\mu(4).$$

Now the following theorem is one of the prominent characterization of the left fuzzy hyperideal (right fuzzy hyperideal, fuzzy hyperideal) in fuzzy LA-hypersemigroups.

Theorem 4.4. Let S be a fuzzy LA-hypersemigroup and $\mu\in \mathrm{F}(S)$. Then the following statements hold:

1. Then μ is a left fuzzy hyperideal of S if and only if $S\circ\mu\subseteq\mu$.
2. Then μ is a right fuzzy hyperideal of S if and only if $\mu\circ S\subseteq\mu$.
3. Then μ is a fuzzy hyperideal of S if and only if $S\circ\mu\subseteq\mu$ and $\mu\circ S\subseteq\mu$.

Proof. 1. Suppose that μ is a left fuzzy hyperideal of S. Then

$$(S\circ\mu)(a)=\bigcup_{s\in S}(s\circ\mu)(a)\leq\bigcup\mu(a)=\mu(a).$$

Hence $S\circ\mu\subseteq\mu$.

Conversely, assume that $S\circ\mu\subseteq\mu$. Let $s\in S$. Then

$$(s\circ\mu)(a)\leq\bigcup_{s\in S}(s\circ\mu)(a)\leq\mu(a).$$

Hence μ is a left fuzzy hyperideal of S.

2, 3. The proof is similar to part 1.

The next two theorems show connections between the left fuzzy hyperideals and fuzzy hyperideals of fuzzy LA-hypersemigroups.

Theorem 4.5. Let S be a fuzzy LA-hypersemigroup with $S\circ S=S$. Then every right fuzzy hyperideal of S is a fuzzy hyperideal.

Proof. Suppose that μ is a right fuzzy hyperideal of S. Then

$$\begin{aligned} S\circ\mu &= (S\circ S)\circ\mu \\ &= (\mu\circ S)\circ S \\ &\subseteq \mu\circ S \\ &\subseteq \mu. \end{aligned}$$

Hence μ is a fuzzy hyperideal of S.

Theorem 4.6. Let S be a fuzzy LA-hypersemigroup with left identity. Then every right fuzzy hyperideal of S is a fuzzy hyperideal.

Proof. Suppose that μ is a right fuzzy hyperideal of S. Then by Lemma 3.6,

$$\begin{aligned} S\circ\mu &= \chi_S\circ\mu \\ &= (e\circ\chi_S)\circ\mu \\ &= (\mu\circ\chi_S)\circ e \\ &\subseteq \mu\circ e \\ &\subseteq \mu. \end{aligned}$$

Hence μ is a fuzzy hyperideal of S.

As a direct consequence, we have the following result.

Theorem 4.7. Let μ and ν be two left fuzzy hyperideals of a fuzzy LA-hypersemigroup S. Then the following statements hold:

1. $\mu\cap\nu$ is a left fuzzy hyperideal of S.
2. $\mu\cup\nu$ is a left fuzzy hyperideal of S.

Proof. 1. Let μ and ν be two left fuzzy hyperideals of S and $a,s\in S$. Then

$$\begin{aligned} (s\circ(\mu\cap\nu))(a) &= \bigcup_{r\in S}\big((s\circ r)(a)\wedge(\mu\cap\nu)(r)\big) \\ &= \bigcup_{r\in S}\big((s\circ r)(a)\wedge(\mu(r)\wedge\nu(r))\big) \\ &= \bigcup_{r\in S}(((s\circ r)(a)\wedge\mu(r))\wedge \\ &\quad ((s\circ r)(a)\wedge\nu(r))) \\ &= (s\circ\mu)(a)\wedge(s\circ\nu)(a) \\ &\leq \mu(a)\wedge\nu(a) \\ &= (\mu\cap\nu)(a). \end{aligned}$$

Consequently, $\mu\cap\nu$ is a left fuzzy hyperideal of S.

2. Let μ and ν be two left fuzzy hyperideals of S and $a,s\in S$. Then

$$\begin{aligned} (s\circ(\mu\cup\nu))(a) &= \bigcup_{r\in S}\big((s\circ r)(a)\wedge(\mu\cup\nu)(r)\big) \\ &= \bigcup_{r\in S}\big((s\circ r)(a)\wedge(\mu(r)\vee\nu(r))\big) \\ &= \bigcup_{r\in S}(((s\circ r)(a)\wedge\mu(r)) \\ &\quad \vee((s\circ r)(a)\wedge\nu(r))) \\ &= (s\circ\mu)(a)\vee(s\circ\nu)(a) \\ &\leq \mu(a)\vee\nu(a) \\ &= (\mu\cup\nu)(a). \end{aligned}$$

Consequently, $\mu\cup\nu$ is a left fuzzy hyperideal of S.

The following is the corollary of Theorem 4.7.

Corollary 4.8. Let μ and ν be two right fuzzy hyperideals of a fuzzy LA-hypersemigroup S. Then the following statements hold:

1. $\mu \cap \nu$ is a right fuzzy hyperideal of S.
2. $\mu \cup \nu$ is a right fuzzy hyperideal of S.

Proof. Follows from the Theorem 4.7.

Theorem 4.9. Let S be a fuzzy LA-hypersemigroup and let μ and ν be two fuzzy hyperideals of S. Then the following statements hold:

1. $\mu \cap \nu$ is a fuzzy hyperideal of S.
2. $\mu \cup \nu$ is a fuzzy hyperideal of S.

Proof. Follows from the Theorem 4.7 and Theorem 4.8.

The next theorem establishes a similar result for fuzzy LA-hypersemigroups.

Theorem 4.10. Let S be a fuzzy LA-hypersemigroup. Then the following statements hold:

1. χ_S is a fuzzy hyperideal of S.
2. For every $x \in S$ if S is a fuzzy LA-hypersemigroup with left identity, then $S \circ x$ is a left fuzzy hyperideal of S.
3. For every $x \in S$ if S is a fuzzy LA-hypersemigroup with left identity, then $x \circ S$ is a left fuzzy hyperideal of S.
4. For every $\mu \in \mathrm{F}(S)$ if S is a fuzzy LA-hypersemigroup with left identity, then $S \circ \mu$ is a left fuzzy hyperideal of S.
5. For every $\mu \in \mathrm{F}(S)$ if S is a fuzzy LA-hypersemigroup with left identity, then $\mu \circ S$ is a left fuzzy hyperideal of S.

Proof. 1. It is straight forward.

2. Let $s, x \in S$. Then by Theorem 3.6,

$$\begin{aligned} s \circ (S \circ x) &= s \circ (\chi_S \circ x) \\ &= (e \circ s) \circ (\chi_S \circ x) \\ &= \chi_S \circ ((e \circ s) \circ x) \\ &= \chi_S \circ ((x \circ s) \circ e) \\ &= (x \circ s) \circ (\chi_S \circ e) \\ &= ((\chi_S \circ e) \circ s) \circ x \\ &\subseteq \chi_S \circ x \\ &= S \circ x. \end{aligned}$$

Hence $S \circ x$ is a left fuzzy hyperideal of S.

3. Let $s, x \in S$. Then by Theorem 3.6,

$$\begin{aligned} s \circ (x \circ S) &= s \circ (x \circ \chi_S) \\ &= x \circ (s \circ \chi_S) \\ &\subseteq x \circ \chi_S \\ &= x \circ S. \end{aligned}$$

Hence $x \circ S$ is a left fuzzy hyperideal of S.

4. The proof is similar to part 2.
5. The proof is similar to part 3.

The following is the corollary of Theorem 4.10.

Corollary 4.11. Let S be a fuzzy LA-hypersemigroup with left identity. Then the following statements hold:

1. For every $x \in S$ if μ is a left fuzzy hyperideal of S, then $\mu \circ x$ is a left fuzzy hyperideal of S.
2. For every $x \in S$ if μ is a left fuzzy hyperideal of S, then $x \circ \mu$ is a left fuzzy hyperideal of S.
3. For every $\nu \in \mathrm{F}(S)$ if μ is a left fuzzy hyperideal of S, then $\nu \circ \mu$ is a left fuzzy hyperideal of S.
4. For every $\nu \in \mathrm{F}(S)$ if μ is a left fuzzy hyperideal of S, then $\mu \circ \nu$ is a left fuzzy hyperideal of S.

Proof. Follows from the Theorem 4.10.

Theorem 4.12. Let S be a fuzzy LA-hypersemigroup with left identity. If $\mu \in \mathrm{F}(S)$, then $S \circ \mu$ is a smallest left fuzzy hyperideal of S containing μ.

Proof. By Theorem 4.10, $S \circ \mu$ is a left fuzzy hyperideal of S containing μ. Let μ be a left fuzzy hyperideal of S containing μ. Then $\mu \subseteq \nu$. Thus by Theorem 4.4, $S \circ \mu \subseteq S \circ \nu \subseteq \nu$. Hence $S \circ \mu$ is a smallest left fuzzy hyperideal of S containing μ.

Now, we introduce the notion of the left fuzzy hypersimple, as follows.

Definition 4.13. A fuzzy LA-hypersemigroup S is called a **left fuzzy hypersimple** if $\mu \circ x = \mu$ for all left fuzzy hyperideal μ of S and $x \in S$.

Theorem 4.14. Let S be a fuzzy LA-hypersemigroup. Then S is left fuzzy hypersimple if and only if $S \circ x = \chi_S$ for all $x \in S$.

Proof. Suppose that S is left fuzzy hypersimple. Let $x \in S$. Then by Theorem 3.6, $S \circ x = \chi_S \circ x$. By assumption, $\chi_S \circ x = \chi_S$. Hence $S \circ x = \chi_S$.

Conversely, assume that $S \circ x = \chi_S$ for all $x \in S$. Let μ be a left fuzzy hyperideal of S and $a, x \in S$. Then by Theorem 3.6,

$$\begin{aligned} (S \circ \mu)(a) &= (\chi_S \circ \mu)(a) \\ &= \bigcup_{r,s \in S} (\chi_S(r) \wedge (r \circ s)(a) \wedge \mu(s)) \\ &= \bigcup_{r,s \in S} ((r \circ s)(a) \wedge \mu(s)) \\ &= \bigcup_{r \in S} (r \circ \mu)(a) \\ &\leq \mu(a) \end{aligned}$$

and

$$\begin{aligned}(S\circ\mu)(a) &= (\chi_S\circ\mu)(a)\\ &= \bigcup_{r,s\in S}(\chi_S(r)\wedge(r\circ s)(a)\wedge\mu(s))\\ &= \bigcup_{s\in S}((\chi_S\circ s)(a)\wedge\mu(s))\\ &= \bigcup_{s\in S}((S\circ s)(a)\wedge\mu(s))\\ &= \bigcup_{s\in S}(\chi_S(a)\wedge\mu(s))\\ &= \bigcup_{s\in S}\mu(s).\end{aligned}$$

Then $\mu(a)\le\bigcup_{s\in S}\mu(s)=(S\circ\mu)(a)\le\mu(a).$ Hence $\bigcup_{s\in S}\mu(s)=\mu(a).$ By assumption,

$$\begin{aligned}(\mu\circ x)(a) &= \bigcup_{s\in S}(\mu(s)\wedge(s\circ x)(a))\\ &\le \bigcup_{s\in S}\mu(s)\wedge\bigcup_{s\in S}(s\circ x)(a))\\ &= \bigcup_{s\in S}\mu(s)\wedge(S\circ x)(a)\\ &= \bigcup_{s\in S}\mu(s)\wedge\chi_S(a)\\ &= \bigcup_{s\in S}\mu(s)\\ &= \mu(a).\end{aligned}$$

Consequently, S is left fuzzy hypersimple.

V. Fuzzy hyper bi-ideals

In Sen (2008) a new approach to the fuzzy hyper bi-ideal of fuzzy semihypergroup was introduced. In the sequel we follow Sen (2008) in order to extend this approach to fuzzy hyper bi-ideals of fuzzy LA-hypersemigroups. In order to do this, we need to give some notions, as follows.

Definition 5.1. Let S be a fuzzy LA-hypersemigroup. A fuzzy LA-subhypersemigroup μ of S is called a **fuzzy hyper bi-ideal** of S if $(\mu\circ s)\circ\mu\subseteq\mu$ for all $s\in S.$

Now the following theorem is one of the prominent characterization of the fuzzy hyper bi-ideal in fuzzy LA-hypersemigroups.

Theorem 5.2. Let μ be a fuzzy LA-subhypersemigroup of a fuzzy LA-hypersemigroup $S.$ Then μ is a fuzzy hyper bi-ideal of S if and only if $(\mu\circ S)\circ\mu\subseteq\mu.$

Proof. Suppose that μ is a fuzzy hyper bi-ideal of $S.$ Let $a\in S.$ Then by Theorem 3.6,

$$\begin{aligned}((\mu\circ S)\circ\mu)(a) &= ((\mu\circ\chi_S)\circ\mu)(a)\\ &= \bigcup_{r,s\in S}((\mu\circ\chi_S)(r)\wedge (r\circ s)(a)\wedge\mu(s))\\ &= \bigcup_{r,s,t,w\in S}(\mu(t)\wedge(t\circ w)(r)\wedge \chi_S(w)\wedge(r\circ s)(a)\wedge\mu(s))\\ &= \bigcup_{r,s,w\in S}((\mu\circ w)(r)\wedge 1 \wedge(r\circ s)(a)\wedge\mu(s))\\ &= \bigcup_{s,w\in S}(((\mu\circ w)\circ s)(a)\wedge\mu(s))\\ &= \bigcup_{s\in S}((\mu\circ w)\circ\mu)(s)\\ &\le \mu(a).\end{aligned}$$

Hence $(\mu\circ S)\circ\mu\subseteq\mu.$

Conversely, assume that $(\mu\circ S)\circ\mu\subseteq\mu.$ Since $(\mu\circ s)\circ\mu\subseteq(\mu\circ S)\circ\mu\subseteq\mu$ for all $s\in S,$ we have μ is a fuzzy hyper bi-ideal of $S.$

The next theorem establishes a similar result for fuzzy LA-hypersemigroups.

Theorem 5.3. If μ and ν are two fuzzy hyper bi-ideals of a fuzzy LA-hypersemigroup $S,$ then $\mu\cap\nu$ is a fuzzy hyper bi-ideal of $S.$

Proof. Let μ and ν be two fuzzy hyper bi-ideals of $S.$ Then by Theorem 5.2,

$$\big((\mu\cap\nu)\circ S\big)\circ(\mu\cap\nu)\subseteq\big(\mu\circ S\big)\circ\mu$$

and

$$\big((\mu\cap\nu)\circ S\big)\circ(\mu\cap\nu)\subseteq\big(\nu\circ S\big)\circ\nu.$$

This implies that $\mu\cap\nu$ is a fuzzy hyper bi-ideal of $S.$

In what follows, we consider a first connection between left fuzzy hyperideals and fuzzy hyper bi-ideal of fuzzy LA-hypersemigroups.

Theorem 5.4. Let S be a fuzzy LA-hypersemigroup with left identity, μ be a left fuzzy hyperideal of S and let ν be a fuzzy hyper bi-ideal of $S.$ Then the following statements hold:

1. $\nu\circ\mu$ is a fuzzy hyper bi-ideal of $S.$
2. $\mu^2\circ\nu$ is a fuzzy hyper bi-ideal of $S.$

Proof. 1. By assumption,

$$(\nu\circ\mu)\circ(\nu\circ\mu)=(\nu\circ\nu)\circ(\mu\circ\mu)\subseteq\nu\circ\mu,$$

which implies that $\nu\circ\mu$ is a fuzzy LA-subhypersemigroup of $S.$ By Theorem 3.6,

$$\begin{aligned}\big((\nu\circ\mu)\circ S\big)\circ(\nu\circ\mu) &= \big((\nu\circ\mu)\circ\chi_S\big)\circ(\nu\circ\mu)\\ &= \big((\nu\circ\mu)\circ\nu\big)\circ(\chi_S\circ\mu)\\ &\subseteq \big((\nu\circ\chi_S)\circ\nu\big)\circ\mu\\ &= \big((\nu\circ S)\circ\nu\big)\circ\mu\\ &\subseteq \nu\circ\mu.\end{aligned}$$

Then by Theorem 5.2, $\nu\circ\mu$ is a fuzzy hyper bi-ideal of $S.$

2. By assumption,

$$(\mu^2 \circ \nu) \circ (\mu^2 \circ \nu) = (\mu^2 \circ \mu^2) \circ (\nu \circ \nu) \subseteq \mu^2 \circ \nu,$$

which implies that $\mu^2 \circ \nu$ is a fuzzy LA-subhypersemigroup of S. By Theorem 3.6,

$$\begin{aligned}\big((\mu^2 \circ \nu) \circ S\big) \circ (\mu^2 \circ \nu) &= \big((\mu^2 \circ \nu) \circ \chi_S\big) \circ (\mu^2 \circ \nu) \\ &= \big((\chi_S \circ \nu) \circ \mu^2\big) \circ (\mu^2 \circ \nu) \\ &= \big(\mu^2 \circ (\nu \circ \chi_S)\big) \circ (\mu^2 \circ \nu) \\ &= \big(\mu^2 \circ \mu^2\big) \circ ((\nu \circ \chi_S) \circ \nu) \\ &\subseteq \mu^2 \circ ((\nu \circ S) \circ \nu) \\ &\subseteq \mu^2 \circ \nu.\end{aligned}$$

Then by Theorem 5.2, $\mu^2 \circ \nu$ is a fuzzy hyper bi-ideal of S.

As a direct consequence, we have the following result.

Theorem 5.5. Let S be a fuzzy LA-hypersemigroup with left identity. If μ and ν are two fuzzy hyper bi-ideals of S, then $\mu \circ \nu$ is a fuzzy hyper bi-ideal of S.

Proof. By assumption,

$$(\mu \circ \nu) \circ (\mu \circ \nu) = (\mu \circ \mu) \circ (\nu \circ \nu) \subseteq \mu \circ \nu,$$

which implies that $\mu \circ \nu$ is a fuzzy LA-subhypersemigroup of S. By Theorem 3.6,

$$\begin{aligned}((\mu \circ \nu) \circ S) \circ (\mu \circ \nu) &= ((\mu \circ \nu) \circ \chi_S) \circ (\mu \circ \nu) \\ &= ((\mu \circ \nu) \circ (e \circ \chi_S)) \circ (\mu \circ \nu) \\ &= ((\mu \circ e) \circ (\nu \circ \chi_S)) \circ (\mu \circ \nu) \\ &= ((\mu \circ e) \circ \mu) \circ ((\nu \circ \chi_S) \circ \nu) \\ &\subseteq ((\mu \circ S) \circ \mu) \circ ((\nu \circ S) \circ \nu) \\ &\subseteq \mu \circ \nu.\end{aligned}$$

Then by Theorem 5.2, $\mu \circ \nu$ is a fuzzy hyper bi-ideal of S.

The following theorem presents the connection between fuzzy hyper bi-ideals and uzzy hyperideals of fuzzy LA-hypersemigroups.

Theorem 5.6. Let μ be a fuzzy hyper bi-ideal of a fuzzy LA-hypersemigroup S with left identity. If $\mu^2 = \mu$, then μ is a fuzzy hyperideal of S.

Proof. By assumption,

$$\begin{aligned}\mu \circ S &= \mu \circ \chi_S \\ &= \mu^2 \circ \chi_S \\ &= (\chi_S \circ \mu) \circ \mu \\ &= (\chi_S \circ \mu) \circ \mu^2 \\ &= \mu^2 \circ (\mu \circ \chi_S) \\ &= ((\mu \circ \chi_S) \circ \mu) \circ \mu \\ &= ((\mu \circ S) \circ \mu) \circ \mu \\ &\subseteq \mu \circ \mu \\ &\subseteq \mu.\end{aligned}$$

Then by Theorem 4.6, μ is a fuzzy hyperideal of S.

Corollary 5.7. If μ is a left fuzzy hyper ideal of a fuzzy LA-hypersemigroup S with left identity, then μ^2 is a fuzzy hyper bi-ideal of S.

Proof. By assumption,

$$\begin{aligned}(\mu^2 \circ S) \circ \mu^2 &= \big(\mu^2 \circ (S \circ S)\big) \circ \mu^2 \\ &= \big((\mu \circ S) \circ (\mu \circ S)\big) \circ \mu^2 \\ &= \big((S \circ \mu) \circ (S \circ \mu)\big) \circ \mu^2 \\ &\subseteq \mu \circ \mu \\ &= \mu^2.\end{aligned}$$

Then by Theorem 5.2, μ is a fuzzy hyper bi-ideal of S.

VI. Conclusion

We extend the study initiated in Sen (2008) about fuzzy semihypergroups to the context of fuzzy LA-hypersemigroups.In this paper, we introduce and analyze a new type of fuzzy LA-hypersemigroups, as a generalization of fuzzy hypersemigroup and left almost semihypergroups.Then we discuss the relations between the fuzzy LA-hypersemigroups and the fuzzy hypersemigroups. We introduce the notion of fuzzy LA-subhypersemigroup, left fuzzy hyperideal (right fuzzy hyperideal, fuzzy hyperideal), left fuzzy hypersimple and fuzzy hyper bi-ideals of fuzzy LA-hypersemigroups and obtain their basic properties. Finally, we obtain some characterizations of left fuzzy hyperideal (right fuzzy hyperideal, fuzzy hyperideal), left fuzzy hypersimple and fuzzy hyper bi-ideals were obtained.

References

[1] V. Amjad, K. Hila and F. Yousafzai, “Generalized hyperideals in locally associative left almost semihypergroups”, *New York J. Math.*, vol. 20, pp. 1063 - 1076, 2014.

[2] R. Ameri, M.A. Larimi and N. Firouzkouhi, “Fuzzy geometric spaces associated to fuzzy hypersemigroups”, *Journal of Intelligent & Fuzzy Systems*, vol. 34, pp. 703 - 709, 2018.

[3] R. Ameri and M. Motameni, “Fuzzy hypervector spaces (redefined)”, *Italian Journal of Pure and Applied Mathematic*, vol. 29, pp.149 - 162, 2012.

[4] R. Ameri and T. Nozari, “Fuzzy hyperalgebras”, *Computers and Mathematics with Applications*, vol. 61, pp. 149 - 154, 2011.

[5] R. Ameri and R. Sadeghi, “On fuzzy gamma hypermodules”, *Ratio Mathematica*, vol. 24, pp. 11 - 30, 2013.

[6] M. Azar, M. Gulistan, N. Yaqoob and S. Kadry, “On fuzzy ordered LA-semihypergroups”, *Int. J. Anal. Appl.*, vol. 16, no. 2, pp. 276 - 289, 2018.

[7] P. Corsini, M. Shabir and T. Mahmood, “Semisimple semihypergroups in terms of hyperideals and fuzzy hyperideals”, *Iranian Journal of Fuzzy Systems*, vol. 8, no. 1, pp. 95 - 111, 2011.

[8] V.L. Fotea, “Fuzzy hypermodules”, *Computers and Mathematics with Applications*, vol. 57, pp. 466 - 475, 2009.

[9] V.L. Fotea, “Fuzzy join n-ary spaces and fuzzy canonical n-ary hypergroups”, *Fuzzy Sets and Systems*, vol. 161, pp. 3166 - 3173, 2010.

[10] V.L. Fotea and B. Davvaz, “Fuzzy hyperrings”, *Fuzzy Sets and Systems*, vol. 160, pp. 2366 - 2378, 2009.

[11] P. He and X. Xin, “Fuzzy hyperlattices”, *Computers and Mathematics with Applications*, vol. 62, pp. 4682 - 4690, 2011.

[12] K. Hila and J. Dine, “On hyperideals in left almost semihypergroups”, *ISRN Algebra*, vol. 2011, pp. Article ID953124, 8 pages, 2011.

[13] A. Khan, M. Farooq, M. Izhar and B. Davvaz, “Fuzzy hyperideals of left almost semihypergroups”, *Int. J. Anal. Appl.*, vol. 15, no. 2, pp. 155 - 171, 2017.

[14] F. Marty, “Sur Une generalization de la notion de group”, \emph{*8th Congress Math. Scan- denaves Stockholm*, pp. 45 - 49, 1934.

[15] M. Motameni, V. L. Fotea and R. Ameri, “Classes of fuzzy hyperideals”, *Filomat*, vol. 30, no. 8, pp. 2329 - 234, 2016.
[16] T. Nozari, “Commutative fundamental relation in fuzzy hypersemigroups”, *Italian Journal of Pure and Applied Mathematics*, vol. 36, pp. 455 - 464, 2016.
[17] I. Rehman, N. Yaqoob and S. Nawaz, “Hyperideals and hypersystems in LA-hyperrings”, *Songklanakarin J. Sci. Technol.*, vol. 39, no. 5, pp. 651 - 657, 2017.
[18] S. Saelee and R. Chinram, “A study on rough, fuzzy and rough fuzzy bi-ideals of ternary semigroups”, IAENG International Journal of Applied Mathematics, vol. 41, no. 3, 172 - 176, 2011.
[19] M.K. Sen, R. Ameri and G. Chowdhury, “Fuzzy hypersemigroups”, *Soft Comput*, vol. 12, pp. 891 - 900, 2008.
[20] N. Yaqoob, R. Chinram, A. Ghareeb and M. Aslam, “Left almost semigroups characterized by their interval valued fuzzy ideals”, *Afr. Mat.*, vol. 24, pp. 231 - 245, 2013.
[21] Y. Yin, J. Zhan and X. Huang, “A new view of L-fuzzy polygroups”, *Neural Comput & Appli*, vol. 20, pp. 589 - 602, 2011.
[22] Y. Yin, J. Zhan, D. Xu and J. Wang, “The *L*-fuzzy hypermodules”, *Computers and Mathematics with Applications*, vol. 59, pp. 953 - 963, 2010.