Output Feedback Stabilization via Reduced Order Observer for a Class of Feedforward Nonlinear Systems with Input Saturation

Yanling Shang, Fangzheng Gao, Guifang Qiao, Jiacai Huang and Xiaochun Zhu

Abstract—The problem of saturated output feedback stabilization for a class of nonlinear systems with upper-triangular structure is addressed in this paper. By constructing a reduced order observer to estimates the unmeasurable states and by skillfully using the homogeneous domination approach and the nested saturation technique, a saturated output feedback control scheme is successfully developed. It is prove that the proposed controller with appropriate design parameters can render the states of the closed-loop system globally asymptotically to zero without violation of the input constraint. A simulation example is provided to demonstrate the effectiveness of the proposed method.

Index Terms—Feedforward nonlinear systems, Input saturation, Homogeneous domination approach, Reduced-order observer

I. INTRODUCTION

During the past few decades, feedforward systems have received widely attention because they can be used to model many practical systems, such as the ball and beam system, the cart-pendulum system, the TORA system, and so forth. However, the design of globally stabilizing controller for a feedforward system has proven to constitute a challenging task due to the fact that such system is neither feedback linearizable nor stabilized by applying the frequently-used backstepping approach. To give this difficulty a solution, a number of intelligent approaches have been developed such as the nested-saturation method [1-6] and the forwarding technique [7, 8]. Thanks to these effective approaches, the state feedback stabilization problem has been well-studied recently. Nevertheless, when only part of state variables are measurable, the problem of global stabilization by output feedback is more challenging and has received little attention. As a matter of fact, the upper-triangular structure leads to an intrinsic obstacle that makes it difficult to achieve even semi-global output feedback stabilization of general feedforward systems [9].

In the existing literature, there are numerous valuable results in coping with the output feedback stabilization problem of feedforward systems under different growth conditions. For example, by imposed the restriction that the nonlinear term is a linear growth, the global output feedback stabilization for uncertain feedforward systems was first studied in [10]. Later, by employing the homogeneous domination approach introduced in [11], the linear growth condition was lifted in [9] where global output feedback stabilization was achieved for more general nonlinearities under a homogeneous growth condition, and stimulated a series of subsequent works [12-17]. However, the effect of the input constraint is omitted in the above-mentioned results.

As we all know, the actuator saturation is a common phenomenon in practical systems due to the inherent physical limitations of devices. Its existence often severely limits system performance, giving rise to undesirable inaccuracy or leading to instability [18]. Thus, it is of great significance to study the problem of saturated output feedback stabilization of feedforward nonlinear systems. Nevertheless, to the best of our knowledge, this issue has not been well-addressed in the literature.

Based on the above observations, in this paper we focus our attention to solve the problem of global stabilization for a class feedforward nonlinear systems by saturated output feedback. The major obstacle to tackle this problem lies in that, the presence of input constraint may lead to a system uncontrollable even if it is indeed controllable for the unconstrained case, that is, the common assumptions and output feedback control techniques mainly for unsaturated feedforward systems are infeasible here. Until now it still remains unanswered that under what conditions the feedforward nonlinear systems may exist saturated output feedback controller. To overcome the aforementioned difficulty, we first place a general homogeneous growth condition and design an unsaturated state feedback controller for the considered system by employing the homogeneous domination approach. Then, we impose a series of nested saturations to the developed controller and obtain a saturated state feedback controller. Moreover, different from the full order observers proposed in [19,20], in this paper we construct a reduced-order observer to estimate unmeasurable states, and obtain a saturated output feedback controller that renders that the states of the closed-loop system globally asymptotically convergence to zero.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we consider a class of feedforward nonlinear systems represented by

\[
\begin{align*}
\dot{x}_i &= x_{i+1} + f_i(t,x_{i+2},\ldots,x_n,u),
\quad i = 1,\ldots,n-2
\\
\dot{x}_{n-1} &= x_n + f_{n-1}(t,u)
\\
\dot{x}_n &= u
\\
y &= x_1
\end{align*}
\] (1)

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Yanling Shang, Fangzheng Gao, Guifang Qiao, Jiacai Huang and Xiaochun Zhu are School of Automation, Nanjing Institute of Technology, Nanjing 211167, P. R. China, hnnhsyl1@126.com

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where $x = (x_1, \cdots, x_n)^T \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}$ are the system state, control input and system output, respectively, and $x_2, \cdots, x_n$ are unmeasurable. The continuous functions $f_i : \mathbb{R} \times \mathbb{R}^{n-1} \to \mathbb{R}$, $i = 1, \cdots, n-1$ represent unknown nonlinear perturbations.

The objective of this paper is to present an output feedback control design strategy which globally stabilizes the system (1) under the following saturation constraint:

$$-u_{\max} \leq u \leq u_{\max}$$

(2)

where $u_{\max}$ is a priori known positive real number.

To this end, the following assumption regarding system (1) is imposed.

**Assumption 1.** For $i = 1, \cdots, n-1$, there are constants $b > 0$ and $\tau \in (-1/n, +\infty)$ such that

$$|f_i(x)| \leq b \sum_{j=i+2}^{n+1} |x_j|^{(r_i + \tau)/r_j}$$

(3)

where $x_{n+1} = u$, $r_1 = 1$, $r_{i+1} = r_i + \tau > 0$, $i = 1, \cdots, n$.

For simplicity, it is assumed that $\tau = -\delta$ with $p$ being any even integer and $q$ being any odd integer. Based on this, we know that $r_i \in (0, 1)$ is a ratio of two positive odd integers.

In what follows, we review some useful definitions and lemmas which will serve as the basis of the coming control design and performance analysis.

**Definition 1.** Consider a system

$$\dot{x} = f(x) \quad \text{with} \quad f(0) = 0, \quad x \in \mathbb{R}^n$$

(4)

Then, the origin is Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq \mathbb{R}^n$ of the origin $0$. A solution $x(t)$ of (4) is called finite-time stable if $\|x(t)\| \leq C(t)$ for all $t \geq 0$ where $C(t)$ tends to a finite limit as $t \to 0^+$.

**Lemma 1.** Consider a system

$$\dot{x} = f(x) \quad \text{with} \quad f(0) = 0, \quad x \in \mathbb{R}^n$$

(5)

Then, the origin $x = 0$ of the system $\dot{x} = f(x)$ is Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq \mathbb{R}^n$ of the origin $0$. A solution $x(t)$ of (5) is called finite-time stable if $\|x(t)\| \leq C(t)$ for all $t \geq 0$ where $C(t)$ tends to a finite limit as $t \to 0^+$.

**Lemma 2.** Consider a system

$$\dot{x} = f(x) \quad \text{with} \quad f(0) = 0, \quad x \in \mathbb{R}^n$$

(6)

Then, the origin $x = 0$ of the system $\dot{x} = f(x)$ is Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq \mathbb{R}^n$ of the origin $0$. A solution $x(t)$ of (6) is called finite-time stable if $\|x(t)\| \leq C(t)$ for all $t \geq 0$ where $C(t)$ tends to a finite limit as $t \to 0^+$.

**Lemma 3.** Consider a system

$$\dot{x} = f(x) \quad \text{with} \quad f(0) = 0, \quad x \in \mathbb{R}^n$$

(7)

Then, the origin $x = 0$ of the system $\dot{x} = f(x)$ is Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq \mathbb{R}^n$ of the origin $0$. A solution $x(t)$ of (7) is called finite-time stable if $\|x(t)\| \leq C(t)$ for all $t \geq 0$ where $C(t)$ tends to a finite limit as $t \to 0^+$.

**Lemma 4.** Consider a system

$$\dot{x} = f(x) \quad \text{with} \quad f(0) = 0, \quad x \in \mathbb{R}^n$$

(8)

Then, the origin $x = 0$ of the system $\dot{x} = f(x)$ is Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq \mathbb{R}^n$ of the origin $0$. A solution $x(t)$ of (8) is called finite-time stable if $\|x(t)\| \leq C(t)$ for all $t \geq 0$ where $C(t)$ tends to a finite limit as $t \to 0^+$.

**Lemma 5.** Consider a system

$$\dot{x} = f(x) \quad \text{with} \quad f(0) = 0, \quad x \in \mathbb{R}^n$$

(9)

Then, the origin $x = 0$ of the system $\dot{x} = f(x)$ is Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq \mathbb{R}^n$ of the origin $0$. A solution $x(t)$ of (9) is called finite-time stable if $\|x(t)\| \leq C(t)$ for all $t \geq 0$ where $C(t)$ tends to a finite limit as $t \to 0^+$.

**Lemma 6.** Consider a system

$$\dot{x} = f(x) \quad \text{with} \quad f(0) = 0, \quad x \in \mathbb{R}^n$$

(10)

Then, the origin $x = 0$ of the system $\dot{x} = f(x)$ is Lyapunov stable and finite-time convergent in a neighborhood $U \subseteq \mathbb{R}^n$ of the origin $0$. A solution $x(t)$ of (10) is called finite-time stable if $\|x(t)\| \leq C(t)$ for all $t \geq 0$ where $C(t)$ tends to a finite limit as $t \to 0^+$.
Proposition 1. Assume that at step $i - 1$, there is a $C^1$, proper and positive definite Lyapunov function $V_{i-1}$, and a set of virtual controllers $z_1^i, \ldots, z_n^i$ defined by

$$
\begin{align*}
z_1^i &= 0, \\
z_2^i &= -\beta_1^i \xi_1^i, \\
&\vdots \\
z_n^i &= -\beta_n^i \xi_n^i,
\end{align*}
$$

with $\beta_1^i > 0$, $j = 1, \ldots, i - 1$, being constants, such that

$$
V_{i-1} \leq - (n - i + 2) \sum_{j=1}^{i-1} \xi_j^2 + \sum_{j=1}^{i-1} \frac{\partial V_{i-1}}{\partial z_j} \frac{f_j}{L_k} + L \xi_i^{(2p-\tau-r_i-1)\rho}(z_i - z_i^*)
$$

Then the $i$th Lyapunov function defined by

$$
V_i = V_{i-1} + \int_{z_i^*}^{z_i} \left( s^{\rho/r_i} - z_i^{\rho/r_i} \right)^{(2p-\tau-r_i)/\rho} ds
$$

is $C^1$, proper and positive definite, and there exists the $C^0$ virtual controller $z_1^{i+1} = -\beta_n^i \xi_n^i$ such that

$$
\dot{V}_i \leq - (n - i + 2) \sum_{j=1}^{i} \xi_j^2 + \sum_{j=1}^{i} \frac{\partial V_i}{\partial z_j} \frac{f_j}{L_k} + L \xi_i^{(2p-\tau-r_i)/\rho}(z_i - z_i^*)
$$

where $\beta_1 > 0$ is a constant.

Hence at step $n$, choosing

$$
V_n = \int_{z_n^*}^{z_n} \left( s^{\rho/r_n} - z_n^{\rho/r_n} \right)^{(2p-\tau-r_n)/\rho} ds
$$

and

$$
\begin{align*}
z_{n+1}^* &= -\beta_n^n \xi_n^{r_n+1/p} \\
&= -\beta_n^n \left( z_n^{r_n/p} + \beta_n^{n-1} \left( z_{n-1}^{r_{n-1}/p} + \beta_{n-1}^{n-2} \left( \cdots + \beta_2^0 \left( z_2^{r_2/p} + \beta_1^{r_1/p} z_1^{r_1/p} \right) \right) \right) \right)^{r_{n+1}/p} \\
&= -\beta_n^* \left( z_n^{r_n/p} + \beta_n^{n-1} \left( z_{n-1}^{r_{n-1}/p} + \beta_{n-1}^{n-2} \left( \cdots + \beta_2^0 \left( z_2^{r_2/p} + \beta_1^{r_1/p} z_1^{r_1/p} \right) \right) \right) \right)^{r_{n+1}/p}
\end{align*}
$$

where

$$
\beta_n^* = \{ \beta_n^{i/r_n}, i = 1, \ldots, n \}
$$

from Proposition 1, we arrive at

$$
\dot{V}_n \leq - L \sum_{j=1}^{n} \xi_j^2 + \sum_{j=1}^{n} \frac{\partial V_n}{\partial z_j} \frac{f_j}{L_k} + L \xi_n^{(2p-\tau-r_n)/\rho}(v - z_n^*)
$$

Consequently, the following result is obtained.

Lemma 5. For the nonlinear system (5) under Assumption 1, the unsaturated state feedback controller $v = z_n^* + 1$ in (13) renders the origin of the closed-loop system is globally asymptotically stable.

B. Saturated state feedback controller design

In this subsection, a saturated state feedback controller is designed to solve the global stabilization problem for system (5). By the combined saturation technique, we impose a series of nested saturations to the controller $v = z_n^* + 1$ in (13) and obtain a saturated controller as following form

$$
v_{ssf} = v_n(Z_n) = -\beta_n \sigma_{r_n+1/p} \left( z_n^{r_n/p} - v_n^{r_n+1/p}(Z_n) \right)
$$

where $v_0 = 0$, $v_i(Z_i) = -\beta_i \sigma_{r_i+1/p} \left( z_i^{r_i/p} - v_i^{r_i+1/p}(Z_i) \right)$,

$$
\sigma(x) = \left\{ \begin{array}{ll}
\text{sign}(x), & |x| > \varepsilon \\
x, & |x| \leq \varepsilon
\end{array} \right.
$$

for a small constant $\varepsilon > 0$ to be determined later, and the gains $\beta_i$'s are selected as

$$
\beta_i = \max \left\{ \beta_1, 2^{1+i} \right\} \quad \beta_j = \max \left\{ \beta_i, 2^{i+j+1/p} \left( 4(1 + \beta_i-1) \alpha_i-1(\cdot) + 2 \right) \right\}, \quad j = 2, \ldots, n-1
$$

Remark 1. From (16) and the definition of saturation function $\sigma(\cdot)$, it can clearly be seen that the controller $v_{ssf} = v_n(Z_n)$ is bounded by a constant $\beta_n e^{r_{n+1}}$, which means that the bound of controller (16) can be arbitrarily small by choosing appropriate design constant $\varepsilon$.

We begin our the main result of this subsection by introducing an important lemma, whose similar proof can be found in [2].

Lemma 6. Consider the system (5) with saturated controller (16). For $i = 1, \ldots, n - 1$, under the condition $|z_j| \leq \varepsilon^{r_{i-1}/r_i}(1 + \beta_{i-1})$, $j = i + 1, \ldots, n + 1$, there exist a series of functions $\alpha_i(\beta_1, \ldots, \beta_i)$ defined as (18) and a constant $0 < \varepsilon_1 < 1$ such that for any $0 < \varepsilon \leq \varepsilon_1, T \geq 1$, the following inequalities hold:

$$
\left| \frac{f_i}{L_k} \right| \leq L e^{r_{i+1}/p}
$$

$$
\left| v_i^{r_{i+1}}(Z_i(T)) - v_i^{r_{i+1}/r_i}(Z_i(\tilde{t})) \right| \leq L \alpha_i(\cdot) e^{(\rho+\tau)/r_i(T - \tilde{t})}
$$

With the help of Lemmas 5 and 6, we are ready to state the main result of this subsection.

Theorem 1. For the nonlinear system (5) under Assumption 1, the saturated state feedback controller (16) renders that the origin of the closed-loop system is globally asymptotically stable.

C. Reduced order observer and main result

Since $z_2, \ldots, z_n$ are not available for feedback, the controller (16) is not implementable. To estimate the unmeasurable states, we construct a homogeneous observer

$$
\begin{align*}
\dot{\hat{z}}_i &= -L_i \hat{z}_i, \\
\hat{z}_i &= (\eta_i + L_i \hat{z}_i)^{r_i/r_i-1}, \quad i = 2, \ldots, n
\end{align*}
$$
where \( z_1 = z_1 \). Based on (16), we design a saturated output feedback controller

\[
v_{sof}(z) = -\beta_n z_1^{\sigma_{r+1}/\rho}(z_n - z_{n-1}^{\rho/r_1}(z_{n-1}^{\rho/r_1} z_{n-1}^{\sigma_{r+1}/\rho} - \beta_{r+1} z_{n-1}^{\rho/r_1} z_{n-1}^{\sigma_{r+1}/\rho}))
\]

where \( v_0 = 0, v_i(z_i) = -\beta_n z_1^{\sigma_{r+1}/\rho}(z_n - z_{n-1}^{\rho/r_1}(z_{n-1}^{\rho/r_1} z_{n-1}^{\sigma_{r+1}/\rho} - \beta_{r+1} z_{n-1}^{\rho/r_1} z_{n-1}^{\sigma_{r+1}/\rho})) \),

\( z_i = (\hat{z}_1, \cdots, \hat{z}_i), i = 1, \cdots, n \) and \( \beta_i \)'s are determined by (17).

**Remark 2.** From (22) and the definition of saturation function \( \sigma(\cdot) \), one can easily verify the following inequality holds:

\[
|v_{sof}(z)| \leq \beta_n \left( \beta_{n-1} z_1^{\rho/r_1} + \beta_{n-1} z_1^{\rho/r_1} + \cdots + \beta_1 z_1^{\rho/r_1} \right)^{1/\rho}
\]

Define the estimate errors \( e_i = (z_i^{\rho/r_1} - \hat{z}_i^{\rho/r_1}) \), \( i = 2, \cdots, n \), and choose the Lyapunov function

\[
V_i = \int_{z_{i-1}^{(2\rho - r_1)/r_1}}^{z_i^{(2\rho - r_1)/r_1}} (z_i^{(2\rho - r_1)/r_1} - \hat{z}_i^{(2\rho - r_1)/r_1})\,ds
\]

where \( \gamma_i = \eta_i + l_{i-1} z_{i-1} \). Then, for \( i = 2, \cdots, n \), it follows from (21) and (25) that

\[
\dot{V}_i = \frac{\partial V_i}{\partial z_i} (L z_i + f_i) - L \frac{\partial V_i}{\partial \hat{z}_i} \hat{z}_i - L \frac{\partial V_i}{\partial z_{i-1}} \hat{z}_{i-1} + \int_{z_{i-1}^{(2\rho - r_1)/r_1}}^{z_i^{(2\rho - r_1)/r_1}} (z_i^{(2\rho - r_1)/r_1} - \hat{z}_i^{(2\rho - r_1)/r_1}) \,ds
\]

where \( z_{n+1} = v_{sof}(\hat{z}) \). The following propositions give the proper estimations of some terms of the right-hand side of (26) whose proofs can be achieved by lemmas 3 and 4.

**Proposition 2.** There exists a positive constant \( \lambda_i \) such that

\[
-l_i e_i z_i^{\rho/r_1} \leq -l_i \lambda_i e_i^2
\]

**Proposition 3.** For \( i = 2, \cdots, n - 1 \), there holds

\[
\frac{2\rho - r_1}{z_i^{(2\rho - r_1)/r_1}} \leq \frac{1}{2} \sum_{j=i+1}^{n} \xi_j^2 + m_i e_i^2 + g_i l_{i-1} e_i^2
\]

where \( g_i \) is a continuous function of \( l_{i-1} \) and \( m_i > 0 \) is a constant.

**Proposition 4.** For the saturated output feedback controller \( v_{sof}(\hat{z}) \), we obtain

\[
\frac{2\rho - r_1}{z_n^{(2\rho - r_1)/r_1}} \leq \frac{1}{8} \sum_{j=1}^{n} \xi_j^2 + c e_i^2 + g_n (l_{n-1}) e_i^2
\]

where \( g_n \) is a continuous function of \( l_{n-1} \) and \( c > 0 \) is a constant.

**Proposition 5.** For \( i = 3, \cdots, n \), there holds

\[
-l_i e_i z_i^{\rho/r_1} \leq \frac{1}{8} \xi_i^2 - h_i (l_{i-1}) e_i^2 + h_i (l_{i-1}) e_i^2
\]

where \( h_i \) is a continuous function of \( l_{i-1} \).

**Proposition 6.** There exists a positive constant \( \mu \) such that

\[
\xi_i^{(2\rho - r_1)/r_1} \leq \frac{1}{4} \sum_{i=1}^{n} \xi_i^2 + \mu e_i^2
\]

With the help of Proposition 6, defining \( \Gamma = V_n + U \), combining (15) and (31), and recursively choosing

\[
l_{n-1} \geq \lambda_{n-1} \left( \frac{1}{4} + 1 + c + \mu \right)\\l_i \geq \lambda_i \left( \frac{1}{4} + 1 + c + \mu + g_i (l_i) + h_i (l_i) \right)\]

we obtain

\[
\dot{\Gamma} \leq -\frac{L}{4} \sum_{i=1}^{n} \xi_i^2 - \frac{L}{4} \sum_{i=2}^{n} \xi_i^2 + \frac{n-1}{2} \sum_{j=1}^{n-1} \frac{\partial V_j}{\partial z_j} ||f_j|| L z_j^r
\]
renders the system (39) globally asymptotically (finite-time) stable, with the requirement of $\|Z\|_{\Delta} = \sqrt{(\sum_{i=1}^{n} |z_i|^{2/r_1}) + (\sum_{j=1}^{m} |\eta_j|^{2/r_2})}$. Similarly, since $\sum_{i=1}^{n} \xi_i^2 + \sum_{j=1}^{m} \eta_j^2$ is homogeneous of degree $2\rho$, by Lemma 2.2, there is a constant $m_2$ such that

$$\dot{\Gamma} \leq -m_2 L \|Z\|_{\Delta}^{2\rho} + \sum_{j=1}^{m} \left( \frac{\partial \Gamma}{\partial \xi_j} \right)_{\xi_j} f_j (\xi_j)$$

(37)

Noting (23), we can find a constant $k$ such that

$$\dot{\Gamma} \leq -k \Gamma^{2\rho/(2\rho - 1)}$$

(38)

Therefore, the closed-loop system (5) with (21) and (22) is globally asymptotically stable. Furthermore, by noting that (4) is an equivalent transformation, the closed-loop system consisting of (1), $u^{p_n} = L^{p_n+1} x_0$ in (4), (21) and (22), has the same properties as the system (5) with (21) and (22). Thus, the proof is completed.

**Remark 4.** By noting the fact that $0 < L < 1$ and $\kappa_n + 1 > 0$, it is easily observed from Remark 1 that the control law $u(t)$ is bounded by a constant $\beta_n \varepsilon^{r-1}$; that is, by choosing design parameters $\varepsilon$ and $\beta_n$ as $\beta_n \varepsilon^{r-1} < u^{max}$, $|u(t)| \leq u^{max}$ can be guaranteed.

**IV. SIMULATION EXAMPLE**

Consider the following feedforward system

$$\begin{align*}
\dot{x}_1 &= x_2 + u^3 \\
\dot{x}_2 &= u \\
y &= x_1
\end{align*}$$

(39)

with the requirement of $|u| \leq u^{max} = 1$. Choosing $\tau = -\frac{\varepsilon}{\beta} \in (-1, 0)$, we have $r_1 = 1$, $r_2 = \frac{\varepsilon}{\beta}$ and $r_3 = \frac{\varepsilon}{\beta}$. It is obvious that Assumption 1 holds with $b = 1$. Therefore, by Theorem 2, we can explicitly construct a saturated output feedback controller for this example. Specifically, we can choose

$$\begin{align*}
\dot{\eta}_2 &= -L \dot{\xi}_2 \\
\dot{\xi}_2 &= (\eta_2 + l_1 y)^{3/5} \\
u &= -L^3 \beta_2 \sigma^{1/3} \left( \frac{\dot{\xi}_2}{\beta_1} + \beta_1^{1/3} \sigma(y) \right)
\end{align*}$$

(40)

with appropriate positive constants $l_1$, $\beta_1$, $\beta_2$, $\varepsilon$ and a small enough gain $L$, such that the output feedback controller (40) renders the system (39) globally asymptotically (finite-time) stable.

In the simulation, by choosing the design parameters as $\beta_1 = 1.2$, $\beta_2 = 1.4$, $l_1 = 3$, $\varepsilon = 0.6$, $L = 0.85$ and the initial condition as $(x_1(0), x_2(0), \eta_2(0)) = (1, -2, 1)$, Figs. 1-3 are obtained to exhibit the responses of the closed-loop system, from which the validity of the proposed method is demonstrated.

**V. CONCLUSION**

This paper has solved the problem of saturated stabilization by output feedback for a class of feedforward nonlinear systems. With the help of the homogeneous domination approach and the nested saturation technique, a constructive design procedure for reduced order observer-based output feedback control is given, which can guarantee that the closed-loop system states are globally asymptotically regulated to zero and the amplitude of the control signal is bounded.

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