Stochastic Green Production Planning with Lot Sizing

X. J. Wang, S. H. Choi

***Abstract*—This paper studies carbon management for optimisation of lot sizing in manufacturing under the stochastic make-to-order production planning environment, with an aim to maximise operational profit. It covers not only about the business benefits, but also about the ecological impingements involved with production planning. Our main concerns are focused on two primary green factors—carbon tax and public green awareness—to address their respective impacts on production planning, because carbon tax inevitably increases cost while public green awareness affects customers' inclination towards green products and hence market demand. The result underscores the critical roles played by carbon management in manufacturing for achieving both ecological and economic benefits. It also provides managerial insight into operations optimisation in production planning to help firms abate ecological deterioration resulting from carbon emissions when achieving their economic performance.**

***Index Terms*—production planning, carbon emissions, carbon tax, public green awareness, lot sizing**

I. Introduction

EXCESSIVE emissions by human activities of heat-trapping greenhouse gases (GHGs), especially carbon dioxide (CO_2), have been defacing the vulnerable earth with abnormal climatic disasters and impingements, which are expected to aggravate in the coming decades. Facing far-reaching impingements of global warming and climate change, there are exigent needs to take concerted effort on a global scale to substantially reduce emissions of GHGs [1]. This paper investigates two relatively more practical and proliferated green measures, namely carbon tax, and the implicit green factor of public green awareness.

Carbon tax is relatively simple yet effective and less costly to achieve long-term carbon mitigation [2], and it has two significant advantages in comparison with other regulations [3]. Indeed, it has been receiving greater scrutiny due to its practicability and effectiveness in reducing anthropogenic global climate change [4]. Green awareness of the public may affect or translate into consumer behaviour, ending up with customers' preference for low-carbon products. This shift of purchasing behaviour towards green products increases market demands for products of companies with a better green brand image. This paper studies the impacts of these two factors by incorporating them into a manufacturing model, with our primary concern on the impacts of green policies on both businesses and the environment.

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II. Lot Sizing Production Planning

A. Problem Description

We adopt an extensive uncertain production planning model, as Fig. 1. Orders arrive randomly. When they accumulate to a batch, they are gathered and transferred to the setup stage for initial setup on a batch-by-batch basis. Subsequently, these partially completed orders are released to the processing stage to be processed and then completed one by one. The completed orders will be individually delivered to end customers without waiting for the whole batch to complete.

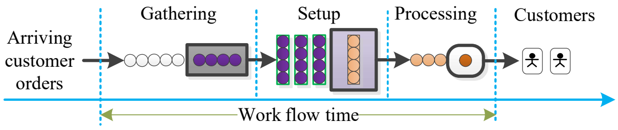

Fig. 1. Uncertain lot sizing production planning

Notations are defined as below to clarify the model derivation.

NOTATION	DESCRIPTION
Q	Lot size
$\Re$	Operational profit
D	Interarrival rate of individual orders without carbon tax
$\tilde{D}$	Interarrival rate of individual orders after considering carbon tax and public green awareness
γ	Unit sales price of products
C_V	Total variable cost expenses
C_F	Total fixed cost expenses
s	Unit batch setup cost expense
h	Unit inventory cost of holding WIPs per unit time
ω	Unit variable cost irrelevant to the lot size and work flow time
W	Lead time
λ	Expected interarrival time of individual customer orders ($\lambda D = 1$)
σ_X^2	Variance of interarrival of individual customer orders
τ	Expected batch setup time
σ_Y^2	Variance of batch setup time
μ	Expected processing rate of individual orders
σ_Z^2	Variance of processing time
ρ	Traffic intensity
e	Total emission of GHGs into the atmosphere
e_m	Carbon emission from processing procedures
e_{WIP}	Carbon emission from holding WIPs
κ_0	Fixed emission factor for production
κ_1	Variable emission factor for production

g_0	Fixed emission factor for WIPs inventory
g_1	Variable emission factor for WIPs inventory
η	Unit carbon tax
Q^*	Optimal lot size for BenchMark Model (BEM)
$Q^\#$	Optimal lot size for Carbon Tax Model (CTM)
a	Intermediate parameter without practical meaning
b	Intermediate parameter without practical meaning
c	Intermediate parameter without practical meaning
$E()$	Expected value function
$\chi()$	Sensitivity function of market demands to carbon emissions

B. Benchmark Model Derivation

For the problem illustrated in Fig. 1, the operational profit of the firm can be formulated as:

$$\Re = D\gamma - C_V - C_F \tag{1}$$

where

$$C_V = D\left(s\frac{1}{Q} + E(W)h + \omega\right) \tag{2}$$

$$E(W) = \frac{Q-1}{2\lambda} + \frac{Q(\sigma_X^2+\sigma_Z^2)+\sigma_Y^2}{2\left[Q\left(\frac{1}{\lambda}-\frac{1}{\mu}\right)-\tau\right]} + \frac{Q+1}{2\mu} + \tau \tag{3}$$

C. Production Planning with Carbon Regulations

The manufacturing process in question consists of two carbon emissions sources — machinery operations and WIPs stocks. Carbon emissions may be quantified as:

$$e_m = \kappa_0 + D\kappa_1 . \tag{4}$$

Carbon emissions incurred from the WIP holding inventory is given by:

$$e_{WIP} = g_0 + g_1 DE(W) \tag{5}$$

From these two emission sources, it follows that the total carbon emissions may be summarized as:

$$e = e_m + e_{WIP} \tag{6}$$

D. Carbon Tax Model

The production planning model in Fig. 1 needs to be adjusted as below to consider impacts of carbon emissions:

$$\Re = D\gamma - C_V - C_F - \eta e . \tag{7}$$

E. Public Green Awareness

Impacts of public green awareness on product demands may be expressed as:

$$\tilde{D} = \chi(e) . \tag{8}$$

Therefore, we adjust (1) as follows:

$$\Re = \chi(e)\gamma - C_V - C_F \tag{9}$$

F. Operations Constraints

The three production planning models formulated above share two common operations constraints summarized as follows:

$$\rho < 100\%, \quad Q \geq 1 \tag{10}$$

where

$$\rho = \frac{\lambda(\mu\tau + Q)}{\mu Q} \tag{11}$$

III. Model Analysis

The first task in the analysis is to derive some optimality properties pertinent to the benchmark model without considering any green regulations. Then, the effects of carbon taxes and public green awareness on production planning, operational profit and carbon emissions are explored.

A. Benchmark Model (BEM)

Proposition 1. For (1), $\Re$ is concave in its domain.

[PROOF] The first-order convexity condition [5] proves that the negative $\Re$ is convex as to Q.

Proposition 2. The optimal lot size Q^* for maximisation of $\Re$ can be obtained by solving the following equation:

$$ab^2hQ^4 - 2ab\tau hQ^3 + \left[ah\tau^2 - 2sb^2 - ch\right]Q^2 + 4bs\tau Q - 2s\tau^2 = 0 \tag{12}$$

where $a = \frac{1}{D} + \frac{1}{\mu}$, $b = \frac{1}{D} - \frac{1}{\mu}$, $c = (\sigma_X^2 + \sigma_Z^2)\tau + b\sigma_Y^2$.

[PROOF] (1) implies continuity and differentiability of $\Re$ as to Q. Based on these characteristics, it follows that

$$\Re'\big|_{Q^*} = \frac{d\Re}{dQ}\bigg|_{Q^*} = -\frac{dC_V}{dQ}\bigg|_{Q^*} = 0 . \tag{13}$$

Reorganizing this equality completes the proof.

Proposition 3. Solving (12) gives a unique globally optimal lot sizing policy for maximisation of $\Re$.

[PROOF] Concavity of $\Re$ suggests that there should be only one optimal lot size that can maximise $\Re$. Therefore, this unique optimal lot size can be obtained by solving the necessary condition stated in Proposition 2.

B. Carbon Tax Model (CTM)

To study the effects of carbon tax, we explore (7).

Proposition 4. For (1), $\Re$ stays concave in the case of carbon tax considered.

Proposition 5. After carbon tax is introduced, the optimal lot size $Q^\#$ for maximising $\Re$ can be obtained by solving:

$$\begin{aligned} &ab^2(h+\eta g_1)Q^4 - 2ab\tau(h+\eta g_1)Q^3 \\ &+\left[a(h+\eta g_1)\tau^2 - 2sb^2 - c(h+\eta g_1)\right]Q^2 + 4bs\tau Q - 2s\tau^2 = 0 \end{aligned} \tag{14}$$

where a, b, and c hold identical definitions as Proposition 2.

Proposition 6. Through solving (14), we can obtain the unique globally optimal lot sizing policy for maximisation of the operational profit under carbon tax regulation.

C. Green Awareness Model (GAM)

In practice, a customer's desire to purchase a product would fade if he or she is aware that its production emitted relatively more carbon into the atmosphere, giving a negative rate of change of demand with respect to emission, that is,

$$\tilde{D}' = \frac{d\chi(e)}{de} < 0 \tag{15}$$

As such, we can substitute the decreased market demand into (12) to obtain the optimal lot sizing solution that takes into account of the customers' green awareness.

IV. Numerical Experiments

To test our model, we use the following operational data taken from a real manufacturing environment [6]: $1/\lambda$ =1.0000 minutes, σ_X^2 =0.5000, τ =10.0000 minutes, σ_Y^2 =10.0000, $1/\mu$ =0.5000 minutes, σ_Z^2 =0.0625, k_0 =2 ton, k_1 = 0.02 kg, g_0 = 3 ton, g_1 = 0.01 kg, ω = \$2.5, s = \$1200, h = \$1.5, CF = \$2.0 million, L = 3 years, γ = \$150, and r = 30%, η = \$30 per ton.

A. Global Optimisation Demonstration

The first case to explore is the BEM model. Solving (12) we note that Q = 34.9502 is the only feasible optimal lot size that leads to the global maximal operational profit of \$5.0195 million, as stated in Proposition 3. Accordingly, 0.1221 million tons of carbon emissions are pumped into the atmosphere, in order to achieve this maximal profit.

Similarly, using Proposition 5 and Proposition 6, we know that Q = 30.7208 is only one global optimal solution to the CTM model, corresponding to the maximal operational profit of \$1.4985 million and the total carbon emissions of 0.1140 million tons.

B. Impacts of CTM on Profit and Carbon Emissions

TABLE I summarises the theoretical solutions to both the BEM and CTM models. In comparison with BEM, CTM can really reduce carbon emissions by 6.63%, yet at the cost of dramatically sacrificing corporate profit, that is, a sharp plunge of 70.15% in this case from \$5.0195 million to \$1.4985 million.

TABLE I
COMPARISON BETWEEN BEM AND CTM MODELS

Scenario	BEM (No Carbon Tax)	CTM (With carbon Tax)	Change
Lot size	34.9502	30.7208	-12.10%
Carbon emission (million tons)	0.1221	0.1140	-6.63%
Profit (\$ million)	5.0195	1.4985	-70.15%

C. Impacts of Carbon Tax on Production Planning

TABLE II lists the in-depth effects of carbon tax rates on the lot sizing policy, the carbon emissions, and the operational profit, as it gradually increases from \$0 through \$44 per ton.

TABLE II
IMPACTS OF CARBON TAX ON LOT SIZE, CARBON EMISSIONS, AND PROFIT

Carbon Tax (\$/ton)	Lot Size	Carbon Emission (million tons)	Profit (\$ million)
0	34.9502	0.1221	5.0195
10	33.0947	0.1184	3.8177
20	31.7362	0.1158	2.6472
30	30.7208	0.1140	1.4985
40	29.9469	0.1127	0.3656
42	29.8146	0.1124	0.1405
44	29.6888	0.1122	-0.0842

It can be observed that when a higher carbon tax is levied, a firm would immediately respond by trimming down its lot size so as to earn a maximal profit, even if it is a bit less than in the case of a relatively lower carbon tax rate. Along with it is the decrease of carbon emissions.

When the carbon tax rate increases from \$30 to \$40 per ton, for example, carbon emissions reduce slightly by about 1%, from 0.1140 to 0.1127 million tons. However, the operational profit dramatically drops by 75.60% to a mere amount of \$0.3656 million. Moreover, any further increase in carbon tax even makes the firm unprofitable, but imposes virtually no effect on carbon emissions, as shown in the last row of TABLE II.

Two points in TABLE II are worth noticing. First, the first row with a zero carbon tax indicates a special case where there is no tax levied on carbon emissions, which is in fact the BEM model. Second, there is a critical carbon tax rate (CCTR) beyond which a firm's profit become negative, as demonstrated by the last row in TABLE II.

D. Critical Carbon Tax Rate (CCTR)

CCTR results in \$0 operational profit. Accordingly, we can set (7) to zero to reflect this merit of CCTR, as in

$$\Re = D\gamma - C_V - C_F - \eta e = 0 . \tag{16}$$

The second characteristic of CCTR is a bit implicit. We know that with CCTR a firm can earn not more than \$0 operational profit. In fact, things can become far worse with CCTR, because when the carbon tax rate is set to CCTR, a firm can only break even at best, and any other operations will incur losses.

Considering its continuity and differentiability it follows

$$\Re' = \frac{d\Re}{dQ} = 0 . \tag{17}$$

Solving (16) and (17), we get a CCTR of \$43.25 per ton of carbon emissions.

E. Impacts of Public Green Awareness

As stated earlier, (15) reflects general negativity of market aggregate ecological attitude against carbon emissions. Fig. 2 illustrates a hypothetical case where customers are insensitive to carbon emissions associated with their purchased products. The firm can earn a maximal profit at the equilibrium point A with a yield of Q(A) and a price of P(A).

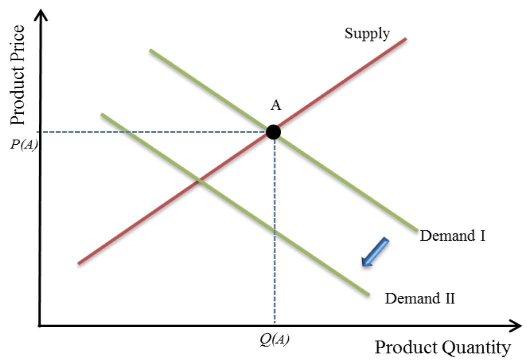

Fig. 2. Impingements of rising public green awareness on demand

In reality, however, rising green awareness may result in a decreasing demand for a product with relatively more carbon emissions, causing the demand curve to move downward from Demand I to Demand II.

The first possible choice is to decrease its production while keeping its unit sales price stable (Point B in Fig. 3). To achieve this target, the firm has to cut down its yield from Q(A) to Q(B). The reduction in demand leads to a decrease of a firm's profit, as illustrated in the black area in Fig. 3. The other loss in profit is mainly attributed to a firm's current decision making deviating from its market equilibrium — Point E. Such quantitative impingements is shown in TABLE

III.

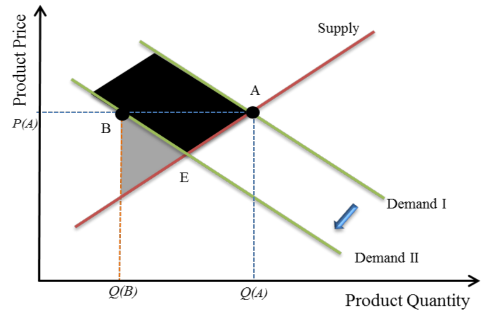

Fig. 3. Responding to the demand decrease by cutting down production, but keeping the unit sales price unchanged.

TABLE III
IMPINGEMENTS OF REDUCED DEMAND DUE TO INCREASED PUBLIC GREEN AWARENESS

Demand Change	Lot Size	Carbon Emissions (million tons)	Profit ($ millions)
0%	34.9502	0.1221	5.0195
-2%	34.8541	0.1197	4.6450
-4%	34.7294	0.1173	4.2704
-6%	34.8543	0.1148	3.8962
-8%	35.0212	0.1124	3.5218
-10%	34.8541	0.1099	3.1474

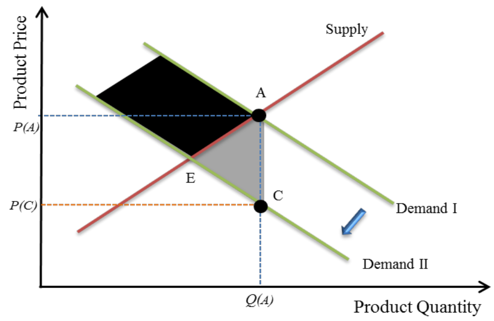

Fig. 4. Responding to the reduced demand by only cutting price

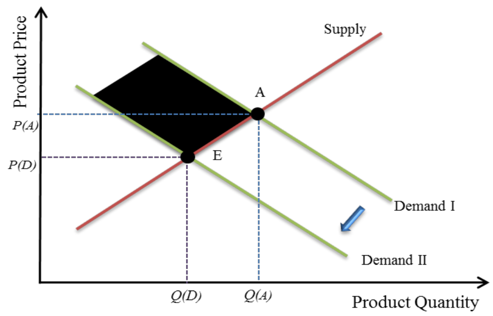

Fig. 5. Profit loss arising from public green awareness

The second choice that a firm can make is to keep its production yield unchanged but cut down its sales price to attract more customers to offset the impacts of increased public green awareness on the product demand. Similarly, the profit loss is two-fold: one from increased public green awareness (the dark area) and the other due to shifting of market equilibrium from E to C (the gray area) in Fig. 4.

Such loss in profit is ascribed to two factors. The first factor is increased public green awareness. The other factor (the deviation away from the market equilibrium), however, can be avoided by means of optimised operations.

Fig. 5 gives a demonstration of the third alternative case, where a firm needs to respond to public green awareness by both reducing its production yield and cutting down its sales price at the same time. The market equilibrium point moves from Point A to Point E.

V. CONCLUSION

The impacts of carbon tax and the increasing green awareness (and even actions) on the lot sizing policy, carbon emissions, and operational profit are explored. A few significant numerical illustrations are carried out to validate the proposed production planning model, and to underscore the practical significance of carbon emissions in both production optimisation and environmental preservation. Several propositions are proved to demonstrate the theoretical effectiveness of the proposed model in solving for globally optimal solutions.

The numerical studies reveal that incorporation of carbon management by means of carbon tax can indeed help reduce carbon emission dramatically, despite at a cost of reduced operational profit. This highlights the importance of carbon management to both business interests and environmental protection. It is therefore beneficial for manufacturers to gain insight into possible interactions between carbon tax and its operational decisions to minimise impacts on business profit when carbon tax is strictly enforced. More importantly, authorities should set carbon tax at practicable levels that can motivate manufacturing management to cut down their carbon emissions in production planning. There is a CCTR over which both the resulting marginal carbon reduction and business profitability become negative.

This paper also explores the increasing public green awareness. It is spontaneously shaped with the strengthening green education and quality on a global scale, and completely not controlled or supervised by any green laws and regulations. Such spontaneity makes this green factor difficult to perform quantitative analysis, but its impacts on carbon emissions are never negligible. We apply microeconomic illustrations to demonstrate that public green awareness helps a lot in reducing carbon emissions, and thus can motivate firms to assume more green responsibilities to attract more customers. As a prompt operational response, a firm needs to simultaneously cut down both its yield and sales price to minimise negative impingements of this factor on their profits.

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