Dynamic State Estimation for Large Scale Systems Based on a Parallel Proximal Algorithm

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***Abstract*—In this paper, a novel method for parallel dynamic state estimation of large scale systems is presented. Since this task requires a high amount of computational resources, a novel solution is presented based on a minimization problem, including spatial and temporal constraints solved with a parallel proximal dual approach. In order to evaluate the performance of the proposed method, experiments are carried out to dynamically estimate sparse brain activity resulting from a large scale real brain model. To this end, simulated and real signals are used in the state estimation process. Results show that the temporal and spatial constraints consider the dynamic state evolution in time and the sparseness inherent to the estimated activity, respectively. Besides, the parallel solution significantly reduces the computational burden required to perform the task. It is worth noting that, for real electroencephalographic signals of each subject, the estimated activity into the brain is located in the areas removed during the successful surgery.**

***Index Terms*—Dynamic inverse problem, Sparse, Spatio-Temporal Constraints, State Estimation.**

I. Introduction

OVER the last years, reconstruction of neural activity from non-invasive electroencephalographic (EEG) recordings has allowed reaching a better understanding of the brain functions and neural dynamics [1]. Accordingly, the spatial distribution and time courses of brain current sources estimated from scalp EEG data have been used in many applications, like analyzing the functional cognitive state of the brain [2], developing human-machine interactions [3], and even for diagnosing several disorders [4], [5].

It is well known that the above-mentioned reconstruction problem is ill-conditioned and mathematically undetermined [6], [7], [8]. Hence, both spatial and temporal prior information about brain activity is required to obtain a unique solution [9], [10]. As a result, the last attempts to solve this problem consider two main issues, i) sparsity: based on the premise that source generators of EEG are sparse compared to the large number of potential sources [11]; ii) temporal dynamics: several methods have improved source reconstruction estimating brain activity as a dynamics state space model [7], [12].

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However, dynamic state estimation of large sparse systems is a task that requires a high amount of computational resources [13]. Accordingly, the computational complexity can be reduced by using spatial operators that reduced the spatial dimension and therefore the number of states to be estimated [8]. Nevertheless, those spatial projectors tend to harm the source reconstruction quality. Another approach to reduce the amount of computational resources is to use alternative solutions to the minimization problems as described in [14], where parallel and distributed estimation methods are proposed, or in [15], where a unified framework for block structured optimization is proposed.

In this paper is presented a novel method for parallel dynamic state estimation of large sparse systems. The proposed method is based on a parallel proximal dual approach, in order to reduce the amount of computational resources and also to include simultaneously spatial and temporal constraints. The temporal constraint considers the dynamic state evolution in time while the spatial constraint considers the sparseness inherent to the estimated activity. This paper is organized as follows: In section II the theoretical framework for state estimation of sparse systems is presented. In section III the experimental setup and the performance evaluation and discussion is analyzed and in IV the conclusion and final remarks are presented.

II. Theoretical Framework

A. Dynamic estimation framework

Consider a discrete state space system described by using an output equation:

$$y_k = M x_k + \mu_k, \tag{1}$$

and the state space equation defined by

$$x_k = f(x_{k-1}, x_{k-2}) + \eta_k, \tag{2}$$

being $x_k \in \mathbb{R}^{n\times 1}$ the neural activity, $M \in \mathbb{R}^{m\times n}$ the lead-field matrix with $m \ll n$, and $y_k \in \mathbb{R}^{m\times 1}$ the electroencephalographic activity, where $f(\cdot)$ can be a nonlinear function as defined in [16], [17].

By considering the dynamic evolution in time of x_k, the following cost function can be defined:

$$\hat{x}_k = \underset{x_k}{arg\,min}(\|y_k - M x_k\|_2^2 + \lambda_k^2 \|x_k - x_k^-\|_2^2), \tag{3}$$

being x_k^- the prior estimation computed by (2), which can be solved as:

$$\hat{x}_k = (M^T M + \lambda_k^2 I_n)^{-1}(M^T y_k + \lambda_k^2 x_k^-), \tag{4}$$

where $x_k^- = f(\hat{x}_{k-1}, \hat{x}_{k-2})$. However, since the term $(M^T M + \lambda_k^2 I_n)^{-1}$ is an $n \times n$ inverse, the computational resources for solving (4) increases as long as n increases [18]. Furthermore, the estimation of (4) can be modified to obtain an $m \times m$ inverse by using the matrix inversion lemma, resulting in the following solution:

$$\hat{x}_k = x_k^- + M^T(MM^T + \lambda_k^2 I_m)^{-1}(y_k - Mx_k^-). \quad (5)$$

When considering sparse constraints, as in [12], an additional term in the cost function (3) can be included:

$$\hat{x}_k = \underset{x_k}{arg\,min}(\|y_k - Mx_k\|_2^2 + \lambda_k^2 \left\|x_k - x_k^-\right\|_2^2 + \gamma_k^2 \|x_k\|_1), \quad (6)$$

and according to [8], the solution of (6) can be obtained as follows:

$$\hat{x}_k = (WM^TM + \lambda_k^2 W + \gamma_k^2 I_n)^{-1} W (M^T y_k + \lambda_k^2 x_k^-), \quad (7)$$

being $W = \mathrm{diag}(|\hat{x}_{k-1}|), \in \mathbb{R}^{n\times n}$. It is noticeable that (7) also requires high computational resources specially for high n values, since the term $(WM^TM + \lambda_k^2 W + \gamma_k^2 I_n)^{-1}$ is an $n \times n$ inverse. Additionally, the inclusion of the l_1 term hinders the reduction of (7) using the matrix inversion lemma. However, the computational resources needed for solving this optimization problem can be reduced by including a projection matrix in the state form which reduces the number of states n into a projection of dimension s being $s << n$, as stated in [8]. However, the inclusion of this projection matrix requires detailed knowledge of the spatial behavior of the forward model and its corresponding spatial features since the resulting weighted average in the state vector might produce innaccurate estimation results.

B. Parallel estimation framework

In this work, an alternative approach for reducing the computational time of dynamic sparse estimation of brain activity is proposed. Here, we use proximal splitting operators for solving the cost functions in (3) and (7). Our approach is based on previous efforts of proximal splitting operators to obtain parallel solutions of minimization problems, as discussed in [19], [14].

Consequently, in order to apply parallel proximal operators, a column block distribution of the lead-field matrix is considered as follows:

$$Mx_k = \begin{bmatrix} M^1 & \dots & M^N \end{bmatrix} \begin{bmatrix} x_k^1 \\ \vdots \\ x_k^N \end{bmatrix}, \quad (8)$$

being N the total number of blocks, and M^j and x_k^j the corresponding $j-th$ lead-field block and state vector block, respectively. Therefore, (1) can be rewritten as:

$$y_k = \sum_{i=1}^{N} M^i x_k^i + \mu_k. \quad (9)$$

By considering (8), the following parallel proximal algorithm based on a Beck-Teboulle proximal gradient algorithm can be proposed for (3), where at each $j-th$ block the following equations are computed [19]:

$$\bar{x}_k^j = x_k^{j,p} + \frac{k-2}{k+1}\left(x_k^{j,p} - x_k^{j,p-1}\right) \quad (10)$$

$$g^j = (M^j)^T \left(\sum_{j=1}^{N} M^j \bar{x}_k^j - y_k\right) + \lambda_k^2 (x_k^{j,p} - \hat{x}_k^j) \quad (11)$$

$$x_k^{j,p+1} = \mathrm{prox}_{\delta_p, \|\|_2^2}\left(\bar{x}_k^j - \delta_p g^j\right), \quad (12)$$

being p the iteration number, $\mathrm{prox}_{\|\|_2^2}$ the proximal operator of l_2 norm, and $\hat{x}_k^j$ the $j-th$ block of x_k^-.

By considering the Parallel Dykstra-like splitting and the Beck-Teboulle proximal gradient algorithm discussed in [19], the following parallel proximal algorithm can be proposed for (6), where at each $j-th$ block the following equations are computed:

$$\bar{x}_k^j = x_k^{j,p} + \frac{k-2}{k+1}\left(x_k^{j,p} - x_k^{j,p-1}\right) \quad (13)$$

$$g_1^j = (M^j)^T \left(\sum_{j=1}^{N} M^j \bar{x}_k^j - y_k\right) + \lambda_k^2 (x_k^{j,p} - \hat{x}_k^j) \quad (14)$$

$$z_k^{j,p} = \mathrm{prox}_{\|\|_2^2}\left(\bar{x}_k^j - \delta_p g_1^j\right) \quad (15)$$

$$\bar{z}_k^j = z_k^{j,p} + \frac{k-2}{k+1}\left(z_k^{j,p} - z_k^{j,p-1}\right) \quad (16)$$

$$g_2^j = (M^j)^T \left(\sum_{j=1}^{N} M^j \bar{z}_k^j - y_k\right) \quad (17)$$

$$x_k^{j,p+1} = \mathrm{prox}_{\gamma_k, \|\|_1}\left(\bar{z}_k^j - \delta_p g_2^j\right), \quad (18)$$

being $\mathrm{prox}_{\gamma_k, \|\|_1}$ the proximal operator of l_1 norm.

III. Results and discussion

A large scale model is considered for evaluating sparse dynamic state estimation. The model considers simulated dynamic sparse state activity, and an output equation corresponding to a real head model obtained from a high resolution structural Magnetic Resonance Imaging. The model is called the New York head model and considers $m = 230$ outputs and $n = 75000$ states [20]. The structure of the model is shown in Fig. 1.

Two windowed sinusoidal time series are selected to simulate the state activity into the brain, as described in [21] and [22]. The source activity at the $i-th$ dipole describing the dynamic of the state evolution in time, represented by (2), is defined as follows:

$$x_k = e^{-\frac{(c_i - t_k)^2}{2\sigma^2}} sin(2\pi f_i t_k). \quad (19)$$

Thus, by using (1), the resulting EEG y_k is obtained. Moreover, considering a sampling rate of 200Hz, one second of simulated EEG activity is obtained. The parameters of (19) are defined as $c_1 = 0.3$ s, $c_2 = 0.8$ s, $f_1 = 4$ Hz and $f_2 = 4$ Hz, $\sigma = 0.1$. The simulated activity for the two active sources is shown in Fig. 2.

Figure 3 shows the spatial and temporal patterns corresponding to the simulated activity. Framed patches in gray scale show the squared amplitude of the simulated

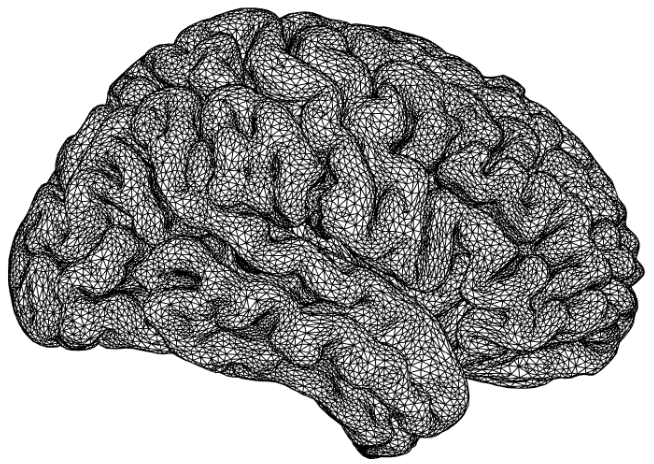

Fig. 1. Brain structure of the New York head model with $n = 75000$ states

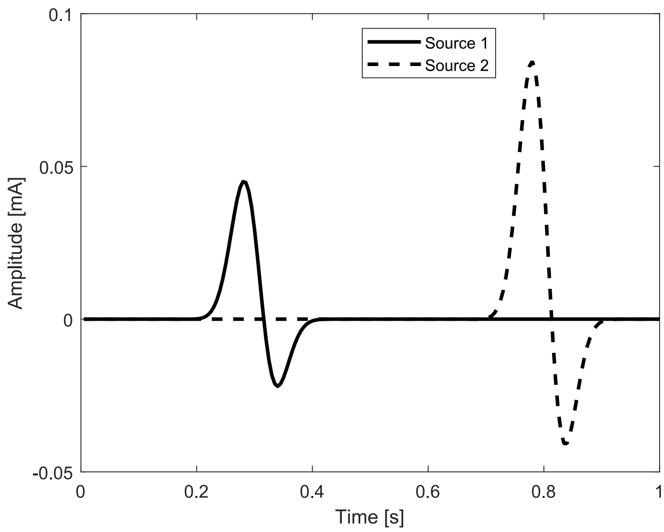

Fig. 2. Simulated activity of two sources

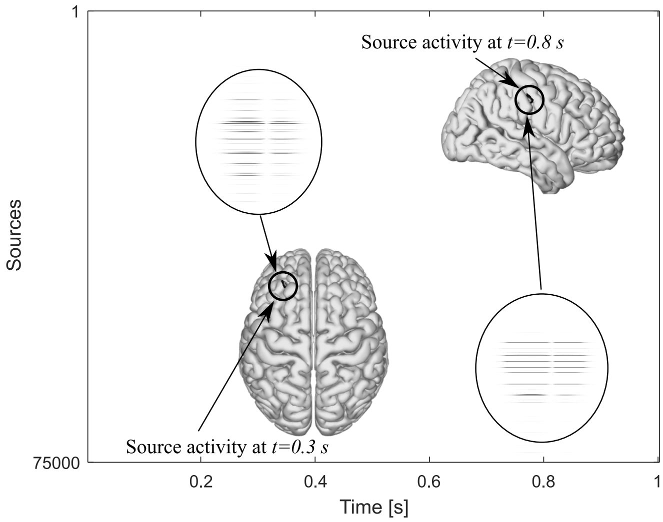

Fig. 3. Simulated activity and its corresponding spatial and temporal pattern

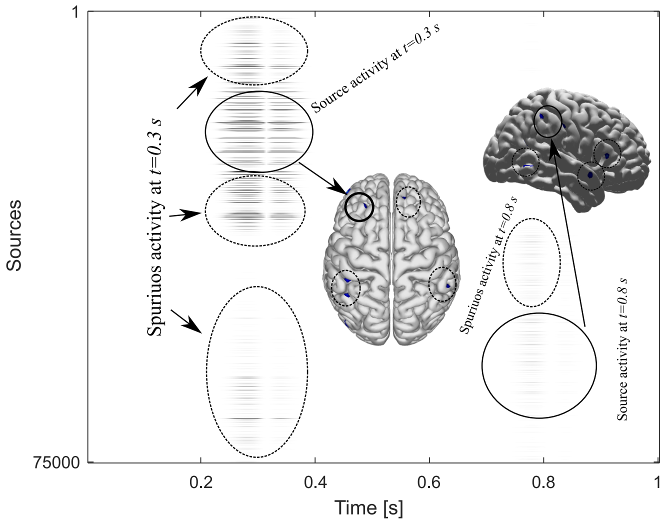

Fig. 4. Estimated activity by using the parallel estimator with l_2 norm and its corresponding spatial and temporal pattern

activity x_k, being white the zero activity and black the maximum activity of the sources. It can be seen that the activity peaks are located at times 0.3 an 0.8 respectively, corresponding to the central time of each simulated source.

Figure 4 presents the reconstructed brain activity by using the parallel l_2 norm estimator described in the cost function (3) and proposed in equations (10) to (12). It can be seen that the highest activity peaks appear around the spatial and temporal locations of the estimated sources, but surrounded by some spurious activity. This spurious activity appears due to the lack of a term in the cost function able to constraint the number of active sources.

Further, Fig. 5 shows the estimated activity by using the parallel estimator with l_1 an l_2 norms described in the cost function (6) and proposed in equations (13) to (18). It is noticeable that including the l_1 penalty term, intended to restrict the number of active sources, the spurious activity in Fig. 5 is significantly reduced in comparison with the estimated activity of Fig. 4.

Furthermore, Fig. 6 allows comparing the computational time required to estimate the dynamic solution with temporal (l_2 norm) and spatio-temporal (l_1 and l_2 norms) constraints. In this figure, the averaged time and the corresponding standard deviation required for reconstructing each time sample of a 200 samples recording is shown. The reconstruction is performed for $j = 1, 2, 4, 6, 8, 10$ blocks using an Intel Xeon Silver 4116 Processor with 12 cores, and 2.10GHZ operational frequency for each core, and 64GB of RAM Memory.

It can be seen that a reduction in the computational time is achieved when the number of parallel processes is increased. Moreover, when the number of parallel processes is low, there is a noticeable difference between using temporal and spatio-temporal constraints. However, as long as the number of parallel processes is increased, the difference between the computational time required for both methods is significantly reduced.

Additional results are evaluated for a real subject by using the proposed estimation method. The database used to this end has 30 subjects with focal epilepsy including structural magnetic resonances before and after the surgery and their corresponding electroencephalographic signals recorded before surgery. The database was recorded at the center for epilepsy treatment named “Instituto de Epilepsia y Parkinson del eje cafetero - Neurocentro”. It is worth noting that all the

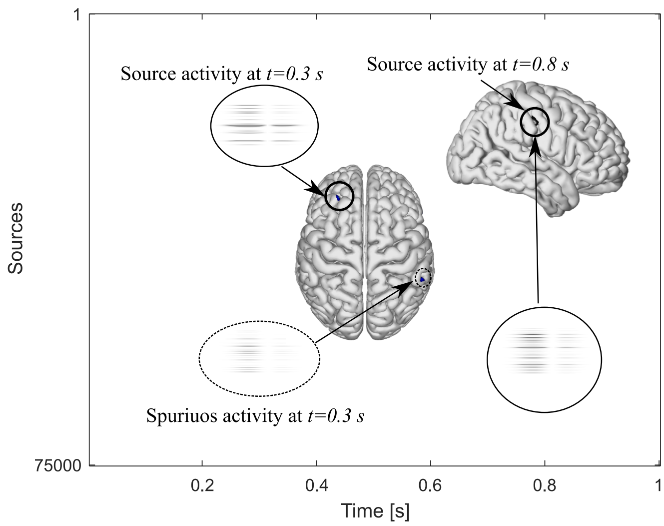

Fig. 5. Estimated activity by using the parallel estimator with l_2 and l_1 norm and its corresponding spatial and temporal pattern

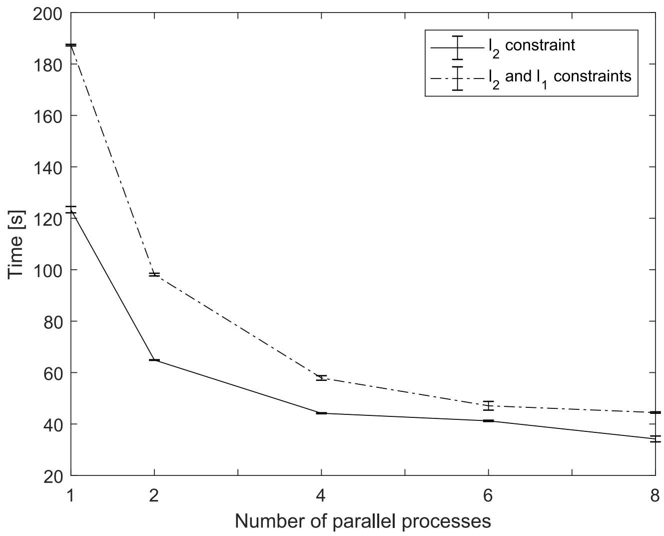

Fig. 6. Computational time of the methods with l_2 constraints (temporal) and l_1 and l_2 constraints (spatio-temporal)

subjects in the database have signed the informed consent. In this case, one subject is analyzed. The subject is labeled in the database as subject 109: male, 26 years. In Fig. 7 is shown the structural magnetic resonance of the subject before surgery. The subject is diagnosed with Focal Epilepsy with seizures starting in the parietal right lobe with rapid diffusion to the frontotemporal region.

In Fig. 8 is shown the head model of the subject which is obtained directly from a structural magnetic resonance imaging depicted in Fig. 7.

The estimated activity is presented in Fig. 9 and Fig. 10 and is overlapped with the structural magnetic resonance obtained after surgery. It can be seen that the estimated activity is shown over the magnetic resonance obtained after surgery and is located in the zone that is removed during surgery.

IV. Conclusion

In this paper a novel method for parallel dynamic state estimation of large scale systems is presented. The parallel proximal dual approach method, which is based on the solution of a minimization problem with spatial and temporal constraints, is evaluated in a framework holding sparse brain activity resulting from a large scale real brain model. Obtained results shows that the proposed method allows to obtain the dynamic state estimation by including into the solution the l_1 and l_2 norms. It can be seen that a significant improvement in the dynamic inverse problem solution is obtained when the l_1 and l_2 norms are considered simultaneously in comparison with the l_2 solution.

In addition, the inclusion of proximal operators in order to solve the minimization problems related to the dynamic inverse problem solution with spatial or spatio-temporal constraints allows to obtain a feasible solution for a high resolution model without the inclusion of a spatial projection operator. In this sense, the increase of the computational time produced when the sparse l_1 spatial constraint is included, is mitigated by using an higher number of parallel processes.

As future work, we propose to include a nonlinear

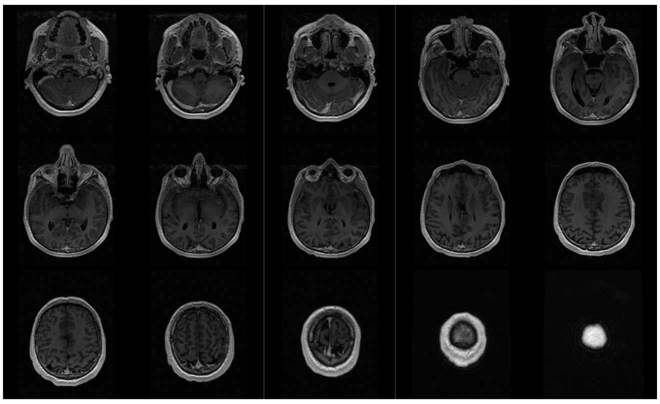

Fig. 7. Structural magnetic resonance imaging

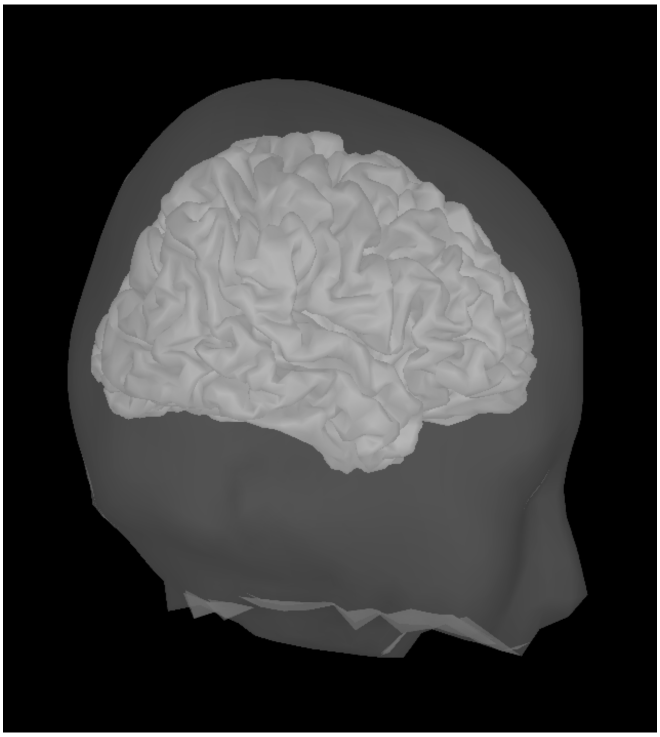

Fig. 8. Real head model of a subject

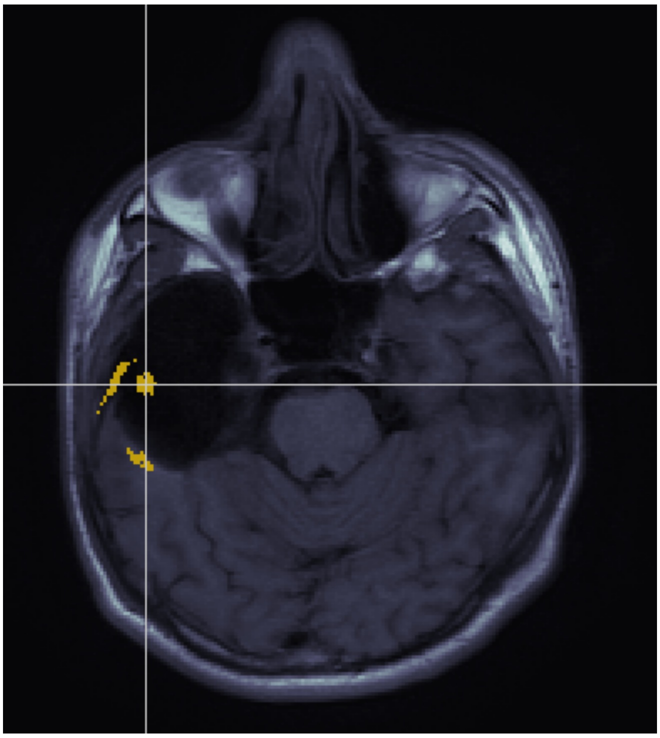

Fig. 9. Estimated activity Axial view

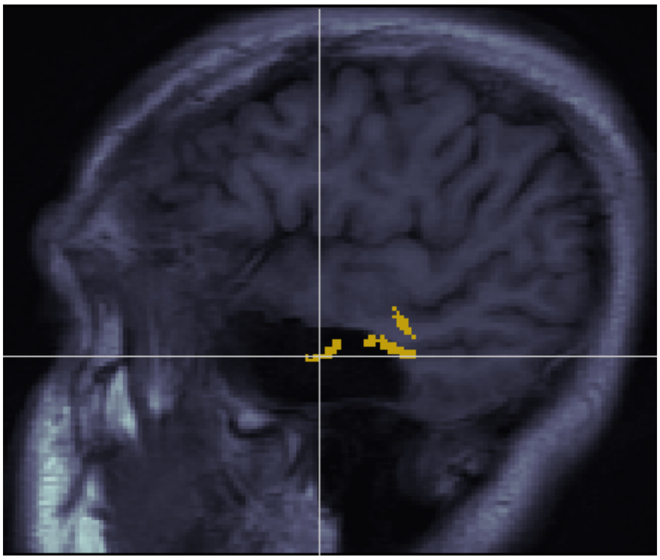

Fig. 10. Estimated activity Sagital view

dynamical model to describe the temporal evolution of the brain activity. Thus, besides the estimated source activity, we expect to obtain large scale brain networks that could be used as biomarkers of some neurological disorders. Furthermore, we would like to analyse the influence both in space and time of using a reduced set of electrodes.

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