Robust Proportional-Derivative Control on SO(3) to Compensate The Unknown Center of Gravity of Quadrotor

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***Abstract*—In this paper, a robust controller for hovering stabilization of a quadrotor is proposed. The proposed controller is based upon the compensation of the position of center of gravity *CoG* by a bounded of control torque. To avoid complexities and ambiguities associated with other attitude representations such as Euler angles or quaternions, both of the attitude dynamics and the proposed control system are globally expressed on the special orthogonal-3 (*SO(3)*) group. The robust compensator is introduced to restrain the influence of uncertain parameters of quadrotor such as the center of gravity. The stability of the designed controller is verified by the Lyapunov stability theorem, and the validity of the proposed controller is demonstrated by simulations under different simulation scenarios. Simulation results show that the proposed controller is robust to stabilize hovering condition, given a deviated *CoG*. Moreover, this controller gives a better performance compared to the nominal proportional derivative controller.**

***Index Terms*—Proportional-Derivative, Center of Gravity, Quadrotor, Stabilization, Special Orthogonal-3.**

I. Introduction

THE unmanned aerial vehicles (*UAV*) are increasingly being considered as a means of performing complex functions or assisting humans in carrying out dangerous missions within dynamic environments. Quadrotor is one of many types of *UAV* which consists of two pairs of counter-rotating rotors. Quadrotor has been a popular *UAV* platform, due to its unique abilities such as hovering, *VTOL* (Vertical Take Off and Landing), and maneuvering in tight spaces. The usage area of a quadrotor can be separated into three major parts such as: military operations (security, intelligence), public applications (*SAR*), and civil applications (photography).

In practical applications, the position in space of the UAV is generally controlled by an operator through a remote-control system using a visual feedback from an on-board camera, while the attitude is automatically stabilized via an on-board controller. The attitude controller is an important feature since it allows the vehicle to maintain a desired orientation [1]. There are lots of developed control algorithms to control the attitude of a quadrotor [2].

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Attitude of a quadrotor is an orientation of the body fixed frame with respect to the inertial frame. It depends on the moment and the thrust of the two pairs of its rotor. By varying the rotor speed, one can change the lift force to create motion [3]. Since the quadrotor requires the stable hovering control to work in many areas, attitude and altitude control are essentially required [4].

In order to control the attitude of a quadrotor, one needs to represent the attitude properly. Most of the prior work on the attitude control is based on minimal representations of an attitude like euler angles [5] and quaternions [6]. With euler angles representation, the system will suffer from the problem of singularities for a large angle rotational maneuvers. Some works use a quaternion to represent the attitude of quadrotor to avoid singularity. Although the quaternions do not increase to singularities, they have double cover of the set of attitudes *SO(3)* (no unique representation called ambiguities) in the sense that each attitude corresponds to two different quaternion vectors [7]. One method to avoid from singularities and ambiguities is to use the geometric control. The geometric control technique depends on the rotation matrix representation. This representation is global and unique. In geometric control method, the rotation matrix can be developed with an exponential matrix of exponential coordinate of an attitude which map a vector $\Re^3$ into a matrix *SO(3)* (Special Orthogonal-3). The exponential coordinate is one of the intrinsic properties of Lie group *SO(3)* [8].

Quadrotor has wide applications in a wide spectrum of scenarios and also has a lot of kind of disturbances such as wind, blade flapping, and unmodeled motor and propeller dynamics. Robustness of the flight controller performance is a fundamental feature for any micro aerial vehicle (*MAV*) application. In particular, the trajectory control law is made adaptive with respect to the presence of external forces and moments (e.g., due to the wind) [9] and the uncertainty of parameters of the dynamic model [10]. Integral-based actions can be used to counteract external disturbances, such as wind and presence of small loads. Nevertheless, an adaptive or integral action may result in an additional disturbance when the nonlinearities of the model are not properly taken into account. This nonlinearities can be caused by an uncertainties parameter model. Thus, the design of a flight controller, which is able to maintain attitude and track a special target accurately and robustly in the presence of uncertainties parameter model is an important step [11], [12], [13], [14].

All of the reviewed literature above assumes a balanced quadrotor model, i.e., the center of gravity (*CoG*) coincides with the geometric center (*CG*) of quadrotor. In this condition, the *CoG* is assumed to be static and known. In fact, the *CoG* of a quadrotor may not be located on its geometric center and can not be easily measured. For example, the placement of the on-board electronic component, battery and the payloads, may not be fixed symmetrically with the geometric center of a quadrotor. This payload can affect the stability of a quadrotor such as inertia perturbation [15] and unbalanced gravitational forces of a quadrotor [16], [17], [18].

Based to the unbalanced gravitational force, due to the dynamic change in the quadrotor's center of gravity, Palunko in [16] uses an input-output feedback linearization to design an adaptive controller to compensates the dynamic changes of *CoG*. Another adaptive control scheme was proposed in [17] to compensate the presence of unknown dynamic parameters, e.g., the position of *CoG*. In these two previous work, the position of the *CoG* is directly considered as an internal part of the rigid body, which causes a totally remodeling of the quadrotor *UAV*. This leads to a very complicated expression of the dynamic and also increases the difficulty of controller design.

Xian in [18] using another approach to consider the CoG deviation as an external payload, which reduces the complexity of the dynamic model. In this work, a new nonlinear adaptive control based on immersion and invariance (*I&I*) approach was proposed to address the effect of unknown *CoG* deviation of the external payload, which causes the unbalanced gravitational force.

This paper present a new control scheme to cope with unbalanced gravitational force, due to the presence of unknown position of the *CoG* of a quadrotor. In this paper, the *CoG* deviation is considered as an external payload, just like in [18], where the position of *CoG* can change. This can be seen when a quadrotor is used for spraying agricultural pest using liquid pesticides, where the liquid pesticides is always move. This fluid movement causes the *CoG* position to change, which can disturb the quadrotor stability. Since an adaptive control is too sophisticated and consumes relatively high cost for computational, an upper-bound controller based on Proportional-Derivative method is proposed to compensate the effect of *CoG* position. Like in [19], a designed robust compensator is added to the nominal control law, to compensate the effect of *CoG* position. Instead of the Euler angles or unit quaternions, the attitude of quadrotor is represented by an exponential coordinate on *SO(3)* group.

This paper is organized as follows. The basic mathematics for operation in SO(3) group is explained in section II. The modeling of a quadrotor and the effect of the gravitational force will be addressed in section III. The designed of this controller and the analysis of the stability will be explained in section IV. The numerical simulation and discussion of this paper is presented in section V and the conclusion of this paper is presented in section VI.

II. Mathematics of Rotation in *SO(3)*

In this section, the basic mathematics for operation in *SO(3)* group is described, mainly based on [20] and [21]. Some mathematical tools will be given for rotation parametrization and some basic knowledge of Lie Group and Lie Algebra.

The position of a point at the rigid body as a function of time is denoted by *q(t)*. If the body is rotated at constant unit velocity about the axis ω, the velocity of the point can be written as

$$\dot{q}(t) = \omega \times q(t) = \hat{\omega}\, q(t). \tag{1}$$

This equation in (1) can be integrated to give

$$q(t) = e^{\hat{\omega} t} q(0), \tag{2}$$

where *q(0)* is the initial position of the point (*t=0*) and $e^{\hat{\omega} t}$ is a matrix exponential as

$$e^{\hat{\omega} t} = I + \hat{\omega} t + \frac{(\hat{\omega} t)^2}{2!} + \frac{(\hat{\omega} t)^3}{3!} + \dots, \tag{3}$$

where $I \in \Re^{3\times 3}$ is an identity matrix.

The hat operator $\wedge$ maps a vector in $\Re^3$ to a skew-symmetric matrix, defined as

$$\hat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}. \tag{4}$$

The matrix $\hat{\omega}$ is a skew-symmetric matrix, which satisfies $\hat{\omega}^T = -\hat{\omega}$. The vector space of all 3×3 skew-symmetric matrices is denoted *so(3)*. The *so(3)* group, which contains a skew-symmetric matrices, is the Lie algebra of *SO(3)*, and the space of 3×3 skew-symmetric matrices is

$$so(3) = \left\{ S \in \Re^{3\times 3} \ : S^T = -S \right\}. \tag{5}$$

According to the euler's theorem for rotates body, any 3-dimensional rotation of a rigid body can be represented by a rotation of a given axis $\omega \in \Re^3$ by an angle $\theta \in [0, 2\pi]$. Let $\omega \in \Re^3$ is a unit vector which specifies the direction of a rigid body rotation (rotation axis), and $\theta \in \Re$ is an angle of rotation in radians. If the body is rotated at unit velocity for θ units of time, then (3) can be re-written as

$$\begin{aligned} R(\omega, \theta) &= e^{\hat{\omega}\theta} \\ &= I + \hat{\omega}\theta + \frac{(\hat{\omega}\theta)^2}{2!} + \frac{(\hat{\omega}\theta)^3}{3!} + \dots, \end{aligned} \tag{6}$$

where $R \in SO(3)$ is a rotation matrix, $\omega \in \Re^3$ is a vector of rotation axis, and $\theta \in \Re$ is an angle of rotation.

Equation (6) is an infinite series and hence not useful from a computational viewpoint. To obtain a closed-form expression of $e^{\hat{\omega}\theta}$, (6) commonly written in a Rodrigues' formula as

$$e^{\hat{\omega}\theta} = I + \frac{\hat{\omega}}{\|\omega\|} sin(\|\omega\|\, \theta) + \frac{\hat{\omega}^2}{\|\omega\|^2}(1 - cos(\|\omega\|\, \theta)), \tag{7}$$

and for$\|\omega\| = 1$, (7) becomes

$$e^{\hat{\omega}\theta} = I + \hat{\omega} sin(\theta) + \hat{\omega}^2(1 - cos(\theta)). \tag{8}$$

An illustration of a rotation with an axis and an angle of rotation can be seen in Fig. 1, where the object will be rotated by an angle θ with a rotation axis *r*. This is the distinctive of *SO(3)* representation, while euler angles representation uses the composition of three consecutive elementary rotations [22].

Rotation matrix $R \in SO(3)$ is a Lie group which has an algebraic group structure based on a matrix multiplication as the group operation, which satisfies

$$SO(3) = \left\{ R \in \Re^{3\times 3},\ R^T R = I_{3\times 3},\ detR = +1 \right\}. \tag{9}$$

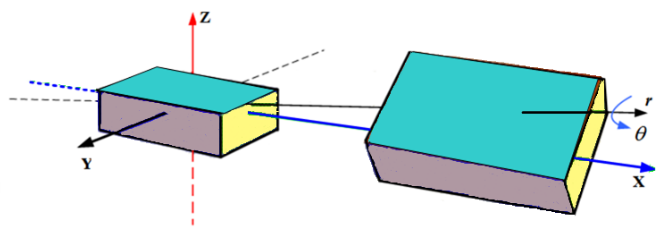

Fig. 1. Illustration of a rotation with an axis-angle

The inverse mapping from an orthogonal matrix *SO(3)* to a skew-symmetric matrix *so(3)* is done by a logarithmic map which is defined as

$$\log(R) = \frac{\theta}{2\sin\theta}(R - R^T) \in so(3), \tag{10}$$

where θ is $\cos^{-1}((tr(R) - 1)/2)$ with $|\theta| < \pi$, and *tr(R)* is the sum of the elements on the main diagonal of a rotation matrix R. Moreover, if a rotation matrix $R = I$, the rotation axis ω can be chosen arbitrarily.

The result of (10) is still in a skew-symmetric matrix of *so(3)* group. This result can be written in terms of a vector $\Re^3$ by using the vee operator $^\vee$ as

$$\zeta = \log(R)^\vee \ \in \Re^3. \tag{11}$$

where vee operator $^\vee$ is an inverse map from a skew-symmetric matrix *so(3)* to a 3-dimension vector $\Re^3$. This vector ζ is called an exponential coordinate ζ of rotation matrix, and can be calculated using *MATLAB* software. This exponential coordinate ζ is a minimal representation of the rotation matrix, and is one of the intrinsic properties of Lie group *SO(3)* [8].

III. Modeling of Quadrotor

In this section, two main issues will be discussed. First, the model of quadrotor is presented. And the second subsection, the model of quadrotor due to the effect of position of *CoG* is explained.

A. Quadrotor System Model

Consider a quadrotor *UAV* model illustrated in Fig. 2. This is a system of four identical rotors and propellers located at the vertices of a square, which generate a thrust and torque normal to the plane of this square. Two coordinate frames are defined for analyzing the motion of a quadrotor. The first frame is the moving coordinate frame which denoted by $B = [\vec{x}_B \; \vec{y}_B \; \vec{z}_B]^T$. This coordinate frame is fixed to the quadrotor's body, which is called the body fixed frame. The second coordinate frame is the earth-fixed reference frame, denoted by $E = [\vec{x}_E \; \vec{y}_E \; \vec{z}_E]^T$, called inertial coordinate frame.

Quadrotor's system inputs are the squared angular speed of its four rotor, denoted by $w = [w_1^2 \; w_2^2 \; w_3^2 \; w_4^2]^T$. Assumed that the thrust of each propeller is directly controlled, and the direction of the thrust of each propeller is normal to the quadrotor plane. The first and third propellers are assumed to generate a thrust along the direction of $-\vec{z_B}$ when rotating clockwise. The second and fourth propellers are assumed to generate a thrust along the same direction of $-\vec{z_B}$ when rotating counterclockwise. Thus, the thrust magnitude generated by the propellers are denoted by $F = \sum_{i=1}^{4} f_i$, and it is positive when the total thrust vector acts along $-\vec{z_B}$, and negative when the total thrust vector acts along $\vec{z_B}$.

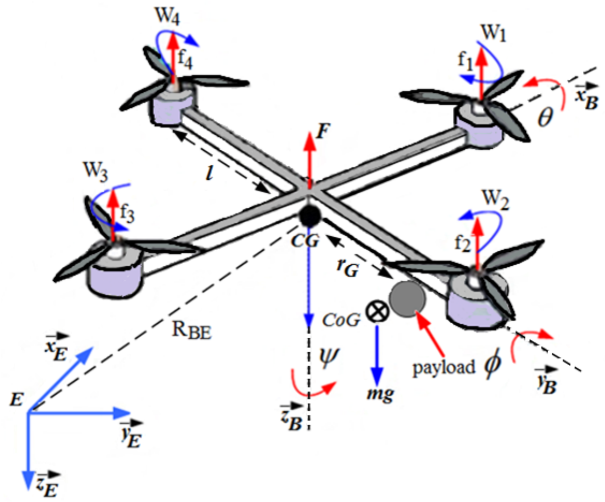

Fig. 2. Model of a quadrotor

With this assumption, the thrust of each propeller f_1, f_2, f_3, f_4 is directly converted into the thrust F and the moment τ, or vice versa. The moment $\tau = [\tau_x \; \tau_y \; \tau_z]^T$ and thrust F are generated by manipulating the angular speed as

$$\begin{aligned} \tau_x &= lb(w_4^2 - w_2^2) \\ \tau_y &= lb(w_1^2 - w_3^2) \\ \tau_z &= b(w_1^2 - w_2^2 + w_3^2 - w_4^2) \\ F &= d(w_1^2 + w_2^2 + w_3^2 + w_4^2), \end{aligned} \tag{12}$$

where l, b, and d are respectively the length of the quadrotor's arm from rotor to geometric center of a quadrotor (*CG*), propeller thrust coefficient, and drag coefficient. Using this equation, the thrust magnitude $F \in \Re$ and the moment vector $\tau = [\tau_x \; \tau_y \; \tau_z]^T \ \in \Re^3$ are viewed as control inputs in this paper.

The configuration of this quadrotor *UAV* is defined by the location of the origin and the attitude with respect to the inertial frame. The body frame orientation in space is given by a rotation matrix R from body frame B to inertial frame E ($R_{BE} \in SO(3)$). Based on this model of a quadrotor, the corresponding equation of motion of quadrotor can be written as

$$\dot{R} = R\hat{\Omega} \tag{13}$$

$$J\dot{\Omega} + \Omega \times (J\Omega) = \tau, \tag{14}$$

where $R \in SO(3)$ is a rotation matrix from body frame to the inertial frame; $\Omega \in \Re^3$ is angular velocity in body frame; $J \in \Re^{3x3}$ is a moment of inertia; and $\tau \in \Re^3$ is a moment vector;

The inertia matrix of a quadrotor is as follows:

$$J = \begin{bmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{xy} & J_{yy} & J_{yz} \\ J_{xz} & J_{yz} & J_{zz} \end{bmatrix}, \tag{15}$$

where the indices x, y, and z in (15) denote x-, y-, and z-axis in the body frame, respectively. Since the quadrotor is symmetric to its x- and y- axis, the axis coupling in quadrotor system to be eliminated. Therefore, the off-diagonal elements of the inertia matrix are zero, and (15) can be written as

$$J = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix}. \quad (16)$$

The kinematics equation in (13) is a rotation matrix, which has nine parameters. Thus it is complex for the real time application. Based on the [23], the differential equation of kinematics equation in (13) can be rewritten in terms of angle ζ as

$$\dot{\zeta} = \left(I + \frac{1}{2}\hat{\zeta} + (1 - \alpha(\|\zeta\|)) \frac{\hat{\zeta}^2}{\|\zeta\|^2} \right) \Omega, \quad (17)$$

where $\alpha(\zeta) \equiv (\zeta/2)cot(\zeta/2)$, $\Omega \in \Re^3$ is an angular velocity in body frame, and $\zeta \in \Re^3$ is exponential coordinate of a rotation matrix.

Finally, with (14) and (17), the second order system of a quadrotor in *SO(3)* is defined as

$$\begin{cases} \dot{\zeta} = \left(I + \frac{1}{2}\hat{\zeta} + (1 - \alpha(\|\zeta\|)) \frac{\hat{\zeta}^2}{\|\zeta\|^2} \right) \Omega, \\ J\dot{\Omega} = \tau - \Omega \times (J\Omega). \end{cases} \quad (18)$$

B. *The effect of* CoG *position*

Ideally, the *CoG* position of a quadrotor coincides with the quadrotor's *CG*. But in practice, even though the origin of a quadrotor is placed at the quadrotor's center of geometry (*CG*), the center of gravity (*CoG*) does not guarantee to coincide with the *CG* of quadrotor. For example, there is an additional unbalanced payload placed in the position as far as r_G as shown in Fig. 2. In this situation, the *CoG* of a quadrotor will deviate from the quadrotor's *CG*. Thus, the thrust on an axis of quadrotor become unbalance, due to the gravity force which acting at the center of mass. This gravity force can be calculated by

$$g_G(R) = r_G \times (R_{EB}\, m_p G), \quad (19)$$

where $r_G = [r_{Gx}\ r_{Gy}\ r_{Gz}]^T \in \Re^3$ is the position of *CoG* along the axis of a quadrotor, $R_{EB} \in SO(3)$ is a rotation matrix from inertial frame to the body frame, m_p is the mass of the equivalent payload, and $G = [0\ 0\ g]^T \in \Re^3$ denotes the acceleration vector of gravity with $g = 9.81\ m/s^2$.

As shown in (19), it appears that the magnitude of the gravity force will be even greater when the position of *CoG* is further away from the *CG* of quadrotor. Therefore, when the *CoG* position of a quadrotor does not coincide with the quadrotor's geometric center, the dynamic equation of quadrotor in (18) become

$$J\dot{\Omega} = \tau - \Omega \times (J\Omega) - g_G(R). \quad (20)$$

The gravity force term $g_G(R)$ in (20) represent the uncertain parameter of a quadrotor, where the value of the payload mass m_p and *CoG* position r_G is typically unknown. These effect cannot be compensated in the controller as they would require the knowledge of payload mass m_p and *CoG* position r_G. Therefore, in the next section, a robust controller is designed to compensate these uncertain parameters of a quadrotor.

IV. Robust Control for Quadrotor in *SO(3)*

In this section, a robust controller based on Proportional-Derivative (*PD*) method will be designed. This controller is designed to compensate the deviated *CoG* effect. This section is divided into two parts. In the first part, the *PD* control in nominal condition is explained. The second part will explain the design of robust *PD* controller, to guarantee the boundedness of moment (τ), using the knowledge of Lyapunov function control and a function bound ($\Delta(x)$) of the unbalanced gravitational force.

A. *PD Control in Nominal Condition*

Assumption 1. The position of *CoG* coincides with the *CG* of a quadrotor. Therefore the gravitational force on quadrotor axis is balanced.

Theorem 1: (*PD* plus feed-forward control on *SO(3)*). Consider a quadrotor system in (18) and suppose that Assumption 1 holds for the system. This quadrotor system can be asymptotically stabilized by the following *PD* control law:

$$\tau = -K_p\zeta - K_d\Omega + \Omega \times J\Omega, \quad (21)$$

where K_p and K_d be a symmetric, positive definite gain matrices, and $\zeta = log(R)^\vee$ is an attitude represented by an exponential coordinate of a rotation matrix.

Proof: By substituting the control law in (21) into (18) with Assumption 1 holds, then the closed-loop system satisfies

$$\begin{cases} \dot{\zeta} = \left(I + \frac{1}{2}\hat{\zeta} + (1 - \alpha(\|\zeta\|)) \frac{\hat{\zeta}^2}{\|\zeta\|^2} \right) \Omega, \\ \dot{\Omega} = -J^{-1}(K_p\zeta + K_d\Omega). \end{cases} \quad (22)$$

Consider the following Lyapunov candidate function

$$W = \frac{1}{2}\zeta^T K_p \zeta + \frac{1}{2}\Omega^T J \Omega + \epsilon, \quad (23)$$

where *W* is a Lyapunov candidate function, and ϵ is a small enough value. This Lyapunov function can be rewritten in matrix form as follow

$$W = \frac{1}{2}\left\langle \begin{bmatrix} \zeta \\ \Omega \end{bmatrix}, \begin{bmatrix} K_p & \epsilon I_{3\times3} \\ \epsilon I_{3\times3} & J \end{bmatrix} \begin{bmatrix} \zeta \\ \Omega \end{bmatrix} \right\rangle, \quad (24)$$

where $I_{3\times3}$ is an identity matrix. This equation can be simplified to

$$W = \frac{1}{2}\xi^T S_\epsilon \xi, \quad (25)$$

with

$$\xi = \begin{bmatrix} \zeta \\ \Omega \end{bmatrix} \quad (26)$$

and

$$S_\epsilon = \begin{bmatrix} K_p & \epsilon I_{3\times3} \\ \epsilon I_{3\times3} & J \end{bmatrix}, \quad (27)$$

where the matrix S_ϵ is positive definite for small ϵ.

The time derivative of the Lyapunov function W in (24) along the trajectories system of (22) gives

$$\begin{aligned}\dot{W} &= \left\langle \dot{\zeta}, K_p\zeta \right\rangle + \epsilon \left\langle \dot{\zeta}, \Omega \right\rangle + \epsilon \left\langle \zeta, \dot{\Omega} \right\rangle + \left\langle \Omega, J\dot{\Omega} \right\rangle \\ &= \left\langle \dot{\zeta}, K_p\zeta \right\rangle + \left\langle \Omega, J(-J^{-1}K_p\zeta - J^{-1}K_d\Omega) \right\rangle \\ &\quad + \epsilon \left\langle \dot{\zeta}, \Omega \right\rangle + \epsilon \left\langle \zeta, -J^{-1}K_p\zeta - J^{-1}K_d\Omega \right\rangle \\ &= \left\langle \dot{\zeta}, K_p\zeta \right\rangle + \left\langle \Omega, -K_p\zeta - K_d\Omega \right\rangle + \epsilon \left\langle \dot{\zeta}, \Omega \right\rangle \\ &\quad + \epsilon \left\langle \zeta, -J^{-1}K_p\zeta - J^{-1}K_d\Omega \right\rangle.\end{aligned} \tag{28}$$

Conduct to [23], the derivative of the exponential coordinate $\dot{\zeta}$ can be replaced by a body angular velocity Ω, where

$$\dot{\zeta} \leq \Omega, \tag{29}$$

then, (28) can be rewritten as

$$\begin{aligned}\dot{W} &\leq \left\langle \Omega, K_p\zeta \right\rangle + \epsilon \left\langle \Omega, \Omega \right\rangle - \epsilon \left\langle \zeta, J^{-1}K_p\zeta \right\rangle \\ &\quad - \epsilon \left\langle \zeta, J^{-1}K_d\Omega \right\rangle - \left\langle \Omega, K_p\zeta \right\rangle - \left\langle \Omega, K_d\Omega \right\rangle \\ &\leq \epsilon \left\langle \Omega, \Omega \right\rangle - \epsilon \left\langle \zeta, J^{-1}K_p\zeta \right\rangle - \epsilon \left\langle \zeta, J^{-1}K_d\Omega \right\rangle \\ &\quad - \left\langle \Omega, K_d\Omega \right\rangle.\end{aligned} \tag{30}$$

Equation (30) can be rewritten in a quadratic form as

$$\dot{W} = -\xi^T Q_\epsilon \xi, \tag{31}$$

where

$$Q_\epsilon = \begin{bmatrix} \epsilon J^1 K_p & \frac{\epsilon}{2} J^{-1} K_d \\ \frac{\epsilon}{2} J^{-1} K_d & K_d - \epsilon I_{3\times 3} \end{bmatrix}, \tag{32}$$

is positive definite matrix for small ϵ.

In order to make the closed-loop system being asymptotically stable, S_ϵ and Q_ϵ have to be guaranteed to be positive definite. Assuming that K_p, K_d and J are diagonal matrix. To guarantee matrix S_ϵ and Q_ϵ are positive definite matrix, the upper-bound condition for ϵ can be calculated by calculating the leading principal minor of S_ϵ and Q_ϵ, i.e.,

$$\begin{aligned}\epsilon < min &\left\{ \sqrt{J_{xx}K_{p1}}, \sqrt{J_{yy}K_{p2}}, \sqrt{J_{zz}K_{p3}}, \right. \\ &\frac{K_{d1}\lambda_1}{1+K_{d1}^2}, \frac{K_{d2}\lambda_2}{1+K_{d2}^2}, \frac{K_{d3}\lambda_3}{1+K_{d3}^2},\end{aligned} \tag{33}$$

where

$$\begin{aligned}\lambda_1 &= 4J_{xx}K_{p1} \\ \lambda_2 &= 4J_{yy}K_{p2} \\ \lambda_3 &= 4J_{zz}K_{p3}.\end{aligned} \tag{34}$$

Remark 1. The control law in (21) using Proportional and Derivative (*PD*) Feedback plus feed-forward compensation. The proportional feedback is in term of the exponential coordinate of the relative attitude vector (ζ), and the derivative feedback is in terms of angular velocity vector (Ω). Usually, a PD control uses proportional and derivative error term. In this work, the desired attitude is constant ($\zeta_d = 0$) since this work only consider stabilization. Therefore, there is no ζ_d in (21).

Remark 2. (Feed-forward compensation) The feed-forward term is used to enhance the performance by compensating for the plant dynamic. The coriolis term ($\Omega \times J\Omega$) in control law (21) is compensated. As a result, the coriolis term (cross-term) vanishes in the closed-loop systems dynamics in (22).

B. PD Control with uncertainties parameter

In this section, the uncertainties parameter of the gravity force $g_G(R)$ is added to the dynamic system of a quadrotor. The value of gravity force $g_G(R)$ is depend to the mass of payload (m_p) and the position of payload relative to the origin (center of geometry) r_G, where in most cases is unknown. Basically, the position of the *CoG* is not fixed in one position, it may change with the time (dynamic). For example, the payload is a container filled with liquid, where the liquid in the container is always moving. This effect needs to be properly compensated. Thus, a robust control is required to compensate this effect, in order to achieve null error at steady state.

Conduct to the dynamic system in (20), to design a robust controller, an additional feedback control u_r which is a virtual force have to be designed to counter act the effect of the uncertain parameter of the gravity force $g_G(R)$. So that, the overall control

$$\tau = \tau_0 + u_r, \tag{35}$$

stabilizes the actual system in (20) in the presence of the uncertainties parameter of the gravity force $g_G(R)$, where τ_0 is a control law in nominal condition, as shown in (21). By substituting the control in (35) into (20) leads to the closed-loop system as

$$\begin{aligned}\dot{\Omega} &= J^{-1}(\tau_0 + u_r - \Omega \times (J\Omega) - g_G(R)) \\ &= -J^{-1}(\Omega \times J\Omega) + J^{-1}u_r + J^{-1}g_G(R) \\ &\quad + J^{-1}(-K_p\zeta - K_d\Omega + \Omega \times J\Omega) \\ &= -J^{-1}K_p\zeta - J^{-1}K_d\Omega + J^{-1}u_r + J^{-1}g_G(R) \\ &= J^{-1}(-K_p\zeta - K_d\Omega + u_r + g_G(R)).\end{aligned} \tag{36}$$

By using the Lyapunov function in (24), the time derivative of Lyapunov function W along the trajectory of (36) gives

$$\begin{aligned}\dot{W} &= \left\langle \dot{\zeta}, K_p\zeta \right\rangle + \epsilon \left\langle \dot{\zeta}, \Omega \right\rangle + \epsilon \left\langle \zeta, \dot{\Omega} \right\rangle + \left\langle \Omega, J\dot{\Omega} \right\rangle \\ &= \left\langle \dot{\zeta}, K_p\zeta \right\rangle + \epsilon \left\langle \dot{\zeta}, \Omega \right\rangle - \epsilon \left\langle \zeta, J^{-1}K_p\zeta \right\rangle - \\ &\quad \epsilon \left\langle \zeta, J^{-1}K_d\Omega \right\rangle + \epsilon \left\langle \zeta, u_r \right\rangle + \epsilon \left\langle \zeta, g_G(R) \right\rangle - \\ &\quad \left\langle \Omega, K_p\zeta \right\rangle - \left\langle \Omega, K_d\Omega \right\rangle + \left\langle \Omega, u_r \right\rangle + \left\langle \Omega, g_G(R) \right\rangle.\end{aligned} \tag{37}$$

By using (29) and (31), equation (37) can be simplified as

$$\begin{aligned}\dot{W} &\leq -\xi^T Q_\epsilon \xi + (\epsilon \left\langle \zeta, J^{-1}u_r \right\rangle + \epsilon \left\langle \zeta, J^{-1}g_G(R) \right\rangle + \\ &\quad \left\langle \Omega, u_r \right\rangle + \left\langle \Omega, g_G(R) \right\rangle) \\ &\leq \dot{W}_n + \dot{W}_r.\end{aligned} \tag{38}$$

The first term on the right-hand side in (38) is due to the nominal closed-loop system (31), and the second term represents the effect of the control u_r and the uncertain term of the gravity force $g_G(R)$ on nominal system, where

$$\dot{W}_n = -\xi^T Q_\epsilon \xi \tag{39}$$

is the derivative of Lyapunov function in nominal condition, and

$$\begin{aligned}\dot{W}_r &= \epsilon \left\langle \zeta, J^{-1}u_r \right\rangle + \epsilon \left\langle \zeta, J^{-1}g_G(R) \right\rangle + \left\langle \Omega, u_r \right\rangle \\ &\quad + \left\langle \Omega, g_G(R) \right\rangle\end{aligned} \tag{40}$$

is the derivative of the Lyapunov function with the robust control u_r and the uncertain term of the gravity force $g_G(R)$

in the system.

Lemma 1. Let $\dot{W}_r$ as

$$\frac{\partial W_0}{\partial x} f(x)(ur(x) + \delta(x)). \tag{41}$$

When the uncertainty $\delta(x)$ can assume any value or function with the size of the given bounding function $\Delta(x)$, the robust control

$$u_r(x) = -\Delta(x)\, sgn(\frac{\partial W_0}{\partial x} g(x)) \tag{42}$$

guarantees $\dot{W} \leq \dot{W}_n$ and the asymptotic stability of the closed-loop system.

With Lemma 1 in mind, the robust controller can be designed from the Lyapunov function in (38). The system can be said to be stable if the time derivative of the Lyapunov function is less than or equal to zero ($\dot{W} \leq 0$). From (38), it can be seen that the first term is less than zero, or a negative definite function. For stabilizing the system, the second term must be a negative semi-definite function.

Due to the matching condition, the uncertain term of the gravity force $g_G(R)$ appears on the right-hand side exactly at the same point where the control u_r appears. It is possible to redesign the control u_r to cancel the effect of the gravity force $g_G(R)$ on nominal system such that,

$$\begin{aligned} 0 \geq \epsilon \left\langle \zeta, J^{-1} u_r \right\rangle + \epsilon \left\langle \zeta, J^{-1} g_G(R) \right\rangle + \left\langle \Omega, u_r \right\rangle \\ + \left\langle \Omega, g_G(R) \right\rangle . \end{aligned} \tag{43}$$

From (42), (43) and with given a bounding function $\Delta G(R) \geq |g_G(R)|$, a robust control u_r can be defined as

$$u_r = -\Delta G(R) \frac{\zeta^T J^{-1} + \Omega^T}{\|\zeta^T J^{-1} + \Omega^T\| + k} \tag{44}$$

where $u_r \in \Re^3$ is the robust moment vector, $\Delta G(R) \geq |g_G(R)| \in \Re$ is a bounding function of the uncertainties of the gravitational force, $\zeta \in \Re^3$ is the exponential coordinate vector, $\Omega \in \Re^3$ is an angular velocity of body frame, and $\epsilon, k \in \Re$ is constant with enough value. The constant k is added to the denominator of robust control u_r to avoid discontinuity which can cause the chattering when the value of angle ζ and angular velocity Ω is zero.

Theorem 2: (Robust Controller). Consider a quadrotor system in (20), this quadrotor system can be asymptotically stabilized by the robust control in (44).

Proof: Consider the derivative of Lyapunov Function in (38), with $\dot{W}_n$ and $\dot{W}_r$ are in (39) and (40). The system can be said to be stable if the time derivative of Lyapunov function in (38) is less than or equal to zero. The first term $\dot{W}_n$ is a negative definite function. Therefore, the second term $\dot{W}_r$ must be zero or a negative function, and can be rewritten as follows

$$\begin{aligned} \dot{W}_r &= \epsilon \left\langle \zeta, J^{-1} u_r \right\rangle + \epsilon \left\langle \zeta, J^{-1} g_G(R) \right\rangle + \left\langle \Omega, u_r \right\rangle \\ &\quad + \left\langle \Omega, g_G(R) \right\rangle \\ &= u_r(\epsilon \zeta^T J^{-1} + \Omega^T) + g_G(R)(\epsilon \zeta^T J^{-1} + \Omega^T). \end{aligned} \tag{45}$$

By substituting the compensator in (44) into (45), then

$$\begin{aligned} \dot{W}_r = -\Delta G(R) \frac{\zeta^T J^{-1} + \Omega^T}{\|\zeta^T J^{-1} + \Omega^T\| + k} (\epsilon \zeta^T J^{-1} + \Omega^T) \\ + g_G(R)(\epsilon \zeta^T J^{-1} + \Omega^T). \end{aligned} \tag{46}$$

The vector $(\zeta^T J^{-1} + \Omega^T)/\|\zeta^T J^{-1} + \Omega^T\|$ in (44) is the unit vector that denotes the direction of $\zeta^T J^{-1} + \Omega^T$. With constant k in compensator u_r is made small (near to zero), the magnitude of compensator almost equal to the bounding function ($\|u_r\| \approx -\Delta G(R)$), then (46) becomes

$$\begin{aligned} \dot{W}_r = -\Delta G(R)(\epsilon \zeta^T J^{-1} + \Omega^T) + \\ g_G(R)(\epsilon \zeta^T J^{-1} + \Omega^T), \end{aligned} \tag{47}$$

and since $\Delta G(R) \geq g_G(R)$, $\dot{W}_r$ is always a negative function. Therefore, the robust control u_r in (44) guarantees the derivative of Lyapunov function in (38) is negative semi-definite function, i.e. $\dot{W} \leq 0$. This mean that the system is stable to compensate the uncertain parameter by adding the compensator in (44) into the nominal controller in (21).

Finally, to conclude this section, the final form of the controller with compensation can be written as

$$\begin{aligned} \tau = -K_p \zeta - K_d \Omega + \Omega \times J\Omega - \\ \Delta G(R) \frac{\zeta^T J^{-1} + \Omega^T}{\|\zeta^T J^{-1} + \Omega^T\| + k} \end{aligned} \tag{48}$$

A schematic diagram of the proposed attitude stabilization control strategy is shown in Fig. 3. From Fig. 3, the error of attitude (rotation) can be obtained by multiplying the transpose of rotation target matrix R_d with a rotation current matrix R_c. By using an inverse mapping in (11), the exponential coordinate ζ of this rotation error can be calculated. A compensator u_r (44) is added to the nominal controller τ_0 (21) to obtain a robust controller τ in (48). In this control strategy, the uncertainty of the position of *CoG* is added to the physical model of a quadrotor, and a small disturbance is added to the dynamic of a quadrotor.

In this work, the quadrotor model parameters are adopted from [24] to perjoin the simulation. The parameters are shown in Table I.

TABLE I
QUADROTOR MODEL PARAMETERS

Parameter	Description	Values	Units
m	Mass	6.5×10^{-1}	kg
I_{xx}	Roll Inertia	7.5×10^{-3}	kg m^2
I_{yy}	Pitch Inertia	7.5×10^{-3}	kg m^2
I_{zz}	Yaw Inertia	1.3×10^{-2}	kg m^2
Jr	Rotor inertia	6.5×10^{-5}	kg/m^2
b	Thrust Factor	3.3×10^{-5}	N
d	Drag Factor	7.5×10^{-7}	N
l	Distance to *CG*	3×10^{-1}	m

V. Numerical Simulation and Discussion

The model of a quadrotor plant and the controller in this work are simulated in the MATLAB/SIMULINK. The initial attitude is a rotation matrix $R = [0.9956\ 0.6545\ -0.06767; -0.06767\ 0.9972\ -0.03105; 0.06545\ \ 0.0355\ \ 0.9972]$, and the controlling target is to stabilize a quadrotor in hovering condition $R = [1\ 0\ 0; 0\ 1\ 0; 0\ 0\ 1]$. This simulation using *PD* controller with gain of K_p and K_d are $K_p = diag(14,\ 14,\ 14)$ and $K_d = diag(4,\ 4,\ 4)$. And the stability of system in this work is indicated by the value of error of angle ζ. The

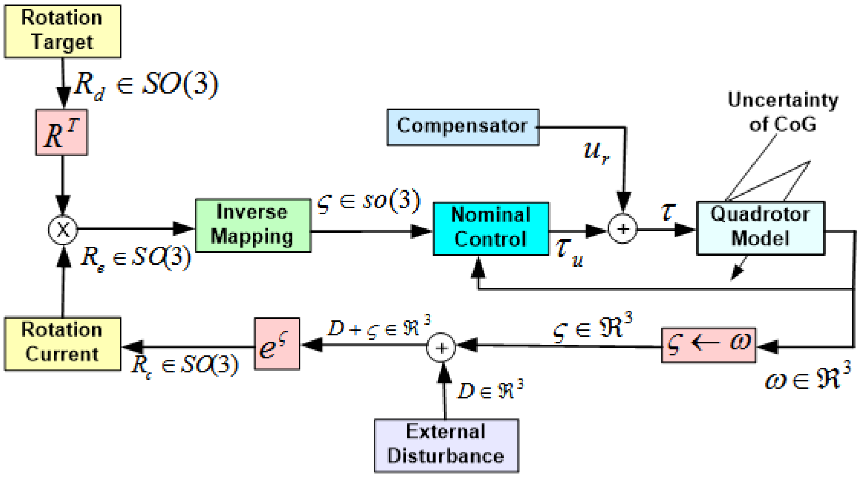

Fig. 3. The schematic diagram of the proposed control strategy

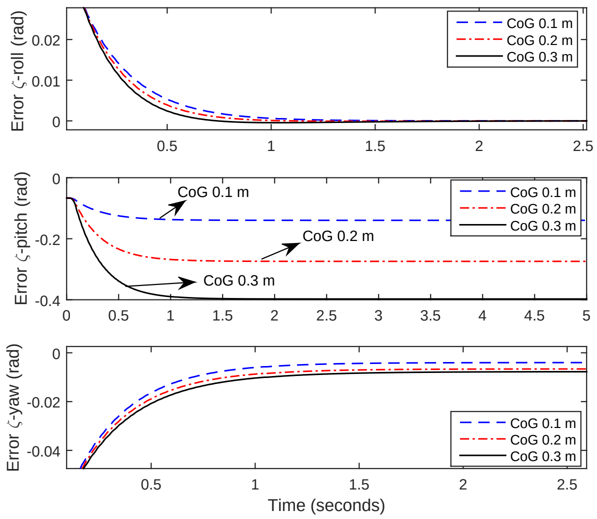

Fig. 4. The effect of *CoG* position to a quadrotor's stability

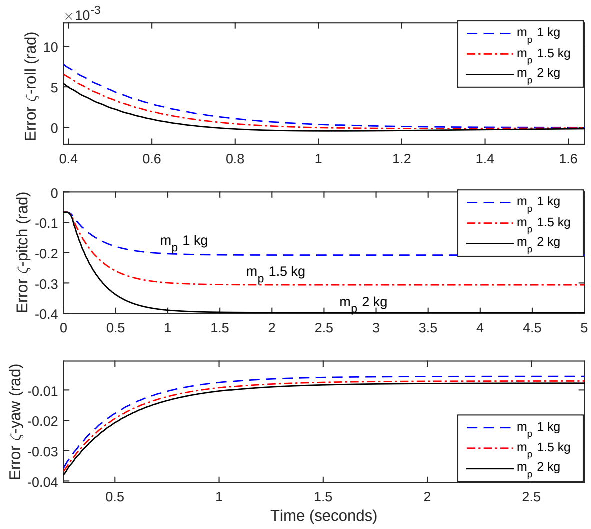

Fig. 5. The effect of mass of payload m_p to a quadrotor's stability

smaller of the error of angle ζ, the greater stability of the system.

Based on (19), the gravity force which acting in the center of mass depends to the *CoG* position r_G and the mass of payload m_p. To show the stability of the system due to the change of mass and the *CoG* position, the simulation is done as shown in Fig. 4 and 5.

Fig. 4 shows the stability of system with a constant mass of payload $m_p = 2\ kg$ and the *CoG* position is changed on the *x-axes*. This shows that the further *CoG* position from the quadrotor center of geometry, the greater stability error of the system becomes. Fig. 4 also shows that when the *CoG* position is on the *x-axes*, the distortion occurs in the pitch motion ζ_y, and the errors from the roll ζ_x and yaw ζ_z motion tend to be zero, and vice in versa.

Showing the effect of the mass of payload m_p to the system stability, the simulation is done with a constant *CoG* position is placed $0.3\ m$ far from the quadrotor center of geometry on *x-axes* as shown in Fig. 5. This shows that the greater the mass of payload, the greater distortion in the system stability.

Compensating the effect of *CoG* position r_G and the mass of payload m_p, a robust controller was designed in (48), with a bounding function $\Delta G(R) \geq |g_G(R)|$. To show the effect of a bounding function on the system stability, the first simulation is done with a small gravity force ($m_p = 0.5kg$ and $r_G = [0\ 0.1\ 0]^T m$). With this parameters, the magnitude of the gravity force can be calculated i.e. $|g_G(R)| = 0.49$, then the bounding function $\Delta G(R) = 0.5$ can be used to compensate this gravity force effect as shown in Fig. 6.

Fig. 6 shows that the system's stability is achieved when the magnitude of the gravity force on system is less than the bounding function. On the other hand, the system will be unstable if the bounding function is less than the magnitude of the gravity force ($g_G(R) = 0.98$) as shown in Fig. 6. This can be proved by using the Lyapunov function in (47). By substitute $\Delta G(R) = 0.5$ and $g_G(R) = 0.98$ into (47), the derivative of Lyapunov function $\dot{W}_r$ will be a positive definite function, which mean that the system is unstable.

Actually, the magnitude of the gravity force cannot be calculated exactly, due to the unknown parameter of mass of

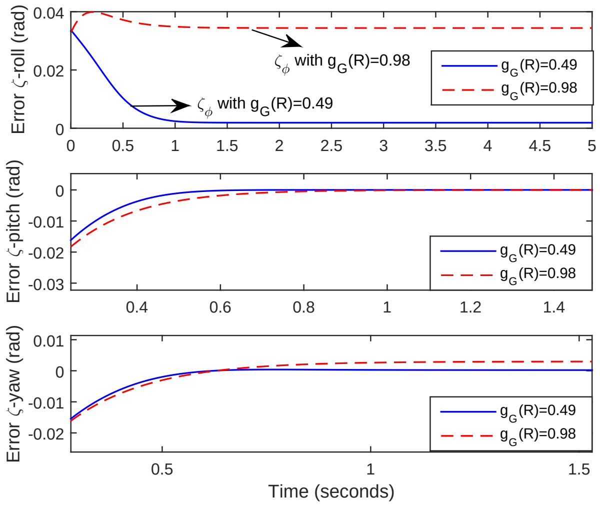

Fig. 6. The effect of bounding function to a quadrotor's stability

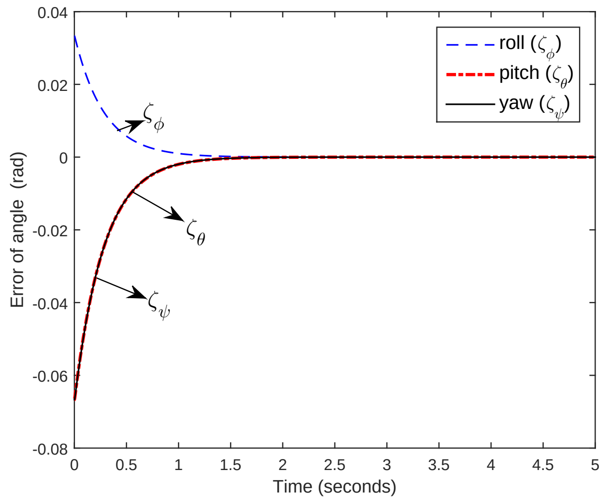

Fig. 8. Stabilization in nominal condition without compensator

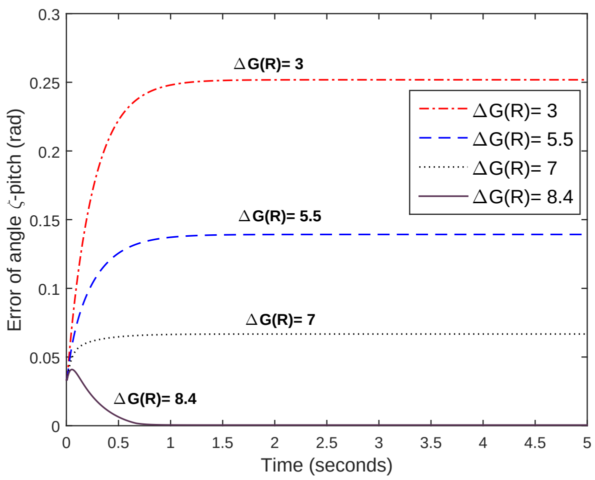

Fig. 7. The effect of bounding function $\Delta G(R)$ to a quadrotor's stability

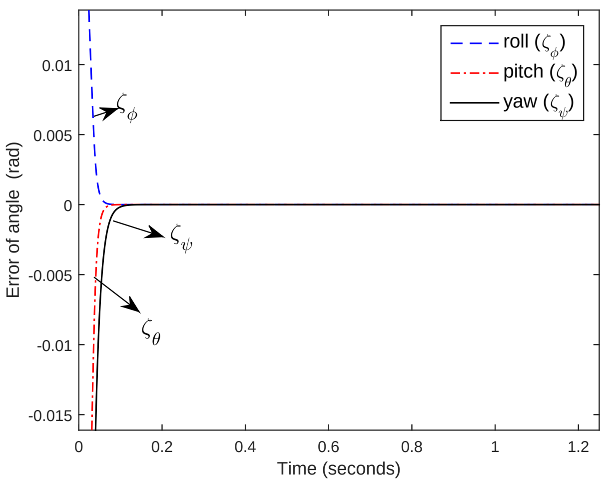

Fig. 9. Stabilization in nominal condition with compensator

payload m_p and the *CoG* position r_G. As explain before that if the bounding function is less than the magnitude of the gravity force, the system will be unstable. On the other hand, the greater bounding function, more computation cost is required. Therefore, to choose a suitable bounding function $\Delta G(R)$, the maximum magnitude of gravity force that might be occured have to be calculated first. In this work, the maximum allowable of mass of payload m_p and *CoG* position r_G are 2 kg and 30 cm respectively. By using equation in (19), the maximum magnitude of gravity force is $|g_G(R)| = 8.32$. Therefore, the bounding function $\Delta G(R)$ must be greater than 8.32. To show the effect of the choice of a bounding function to the stability of system with this maximum parameter, simulation is done as shown in Fig. 7.

As shown in Fig. 7, the system stability is not achieved when the bounding function $\Delta G(R)$ is smaller then the magnitude of the gravity force maximum $|g_G(R)|$, which indicated by the error of angle in *y-axis* (ζ_θ). Based to the simulation as shown in Fig. 7, $\Delta G(R) = 8.4$ will be used as the bounding function in the designed compensator u_r. And to show the robustness of this designed controller, the simulation is done in four different cases as shown in Table II.

TABLE II
CASES OF SIMULATION

Case	Simulation
1	Attitude stabilization in nominal condition
2	Attitude stabilization with presence of unknown *CoG* position
3	Attitude stabilization with presence of disturbances
4	Attitude stabilization with presence of disturbances and unknown *CoG* position

A. Case 1

In this simulation test, attitude stabilization only using a *PD*-Feedforward controller where proposed by Bullo in [23]. This first simulation is done in nominal condition, where there is no disturbance and the *CoG* position coincides with the *CG* of quadrotor. The performance of the closed-loop system with the nominal controller in (21) is shown in Fig. 8. This controller is able to stabilize the attitude of a quadrotor, to make the system is asymptotically stable,

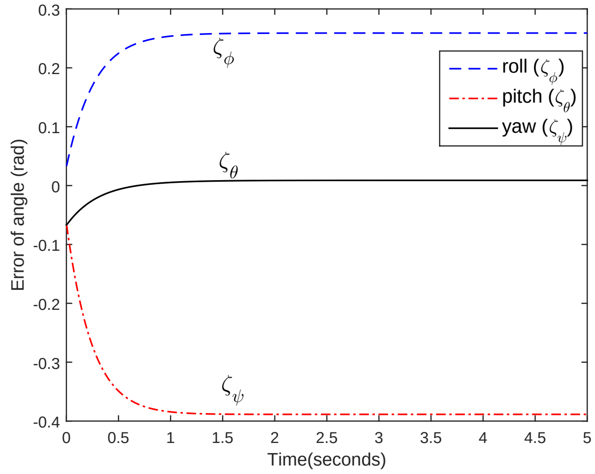

Fig. 10. Uncertainty parameter stabilization without compensator

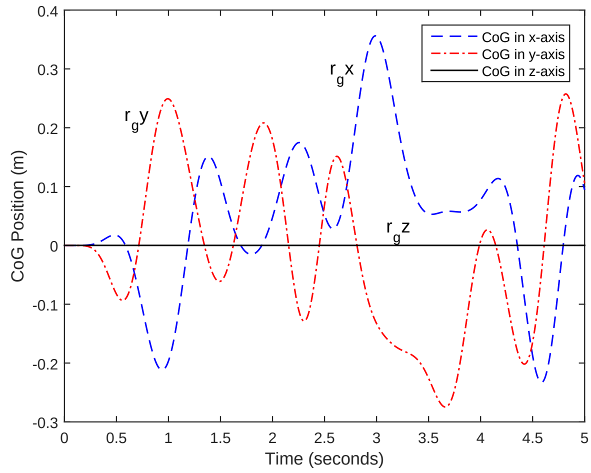

Fig. 12. Random position of *CoG*

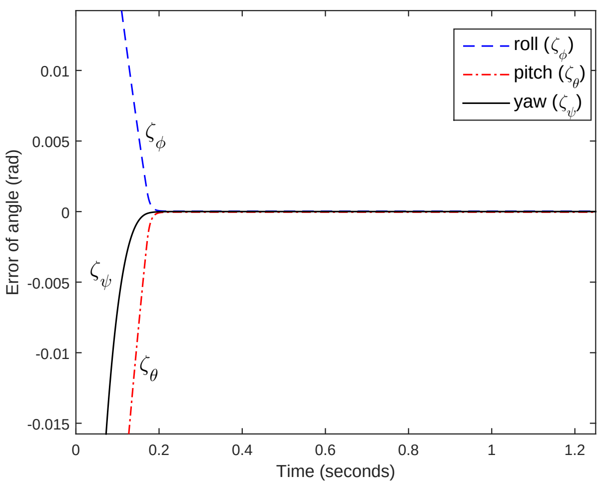

Fig. 11. Uncertainty parameter stabilization with compensator

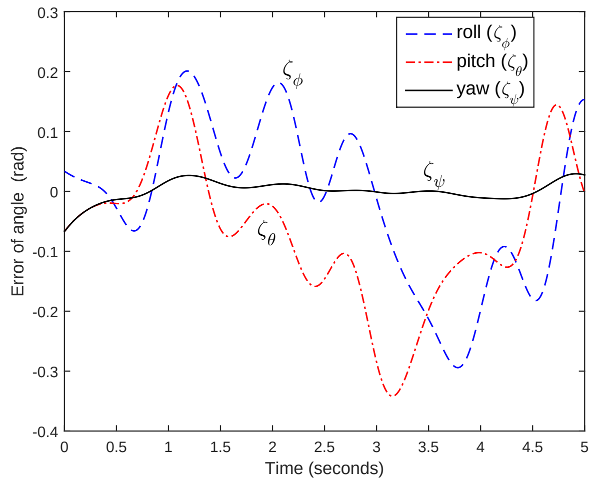

Fig. 13. Random *CoG* position stabilization without compensator

which is represented by an error of angle ζ in zero value. By adding a designed compensator to the controller, the stabilization looks better, as shown in Fig. 9. Compared to the result before in Fig. 8, faster settling time is obtained when using a compensator in the controller. By using this compensator, settling time is reached within 0.13 sec, while in the controller without a compensator, the settling time is reached in 1.62 sec.

B. Case 2

This section simulates the effect of gravitational force to the stabilization of a quadrotor. The magnitude of the gravitational force is unknown, due to the unknown position of *CoG* and the mass of payload. First simulation is about stabilization with nominal *PD* controller in (21), with a constant uncertain parameter. The uncertain parameters that used in this first simulation are: $g = 9.81\ m/s^2$, $m_p = 1\ kg$, and $r_G = [0.3\ 0.3\ 0]^T\ m$.

Conducted to this parameter, the stabilization without compensation is shown in Fig. 10, and the result stabilization with compensation is shown in Fig. 11. From Fig. 10, it seems that a nominal controller fails to stabilize the system, because they would require the knowledge of this parameter to compensate the gravitational force in the dynamic of system. Therefore, a compensator is needed to be added to the nominal *PD* controller, so the stabilization is obtained as seen in Fig. 11.

To demonstrate the robustness of this compensator, the simulation is done with a random uncertainties parameter. The random position of *CoG* is given as shown in Fig. 12, while the mass of payload and acceleration of gravitation is constant. This random *CoG* is use to indicate the dynamic *CoG* position of quadrotor.

The nominal control was fails to compensate this dynamic change of the *CoG* position. So, the stabilization is not achieved as shown in Fig. 13. By using the designed compensator, the stability was achieved as shown in Fig. 14.

Based this simulation, it can be concluded that, the designed compensator is robust to compensate the gravitational force of moment, due to the deviated *CoG*, where the position of *CoG* is typically unknown. This confirms that a *PD* controller is unable to compensate the effect of the uncertainty parameter, because they would require the knowledge of this parameter. Therefore, a compensator is needed to be added to the nominal *PD* controller.

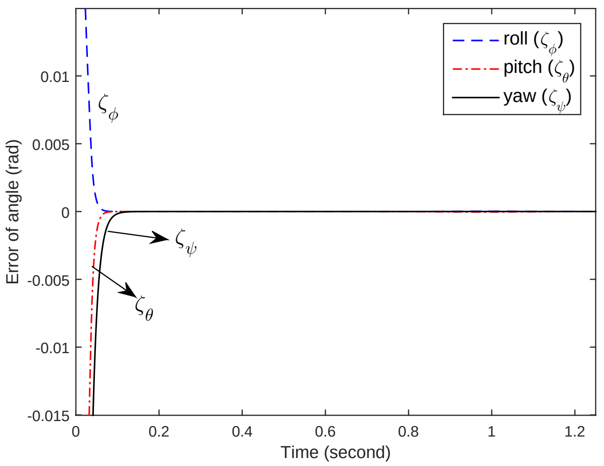

Fig. 14. Random *CoG* position stabilization with compensator

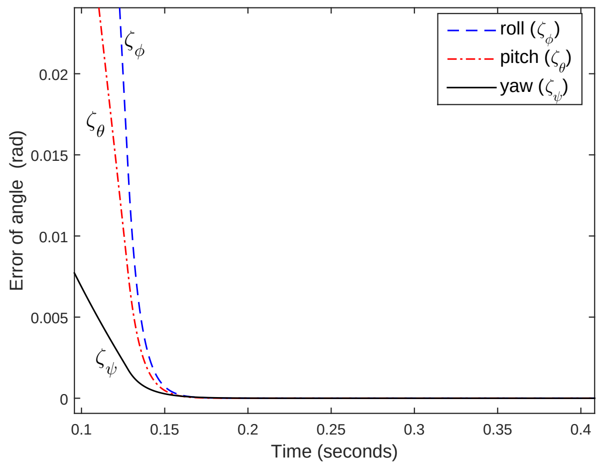

Fig. 16. Constant disturbance stabilization with compensator

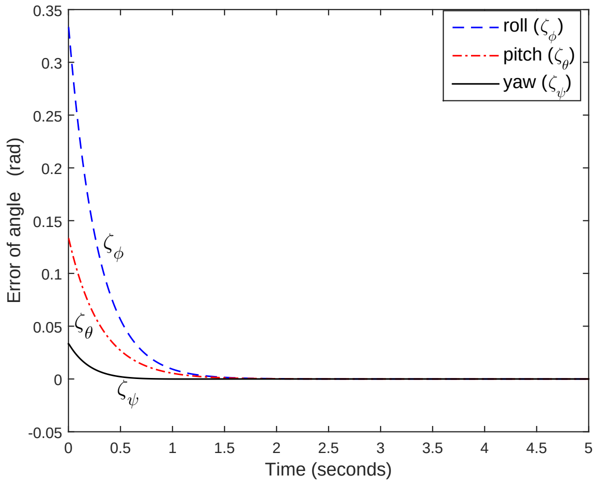

Fig. 15. Constant disturbance stabilization without compensator

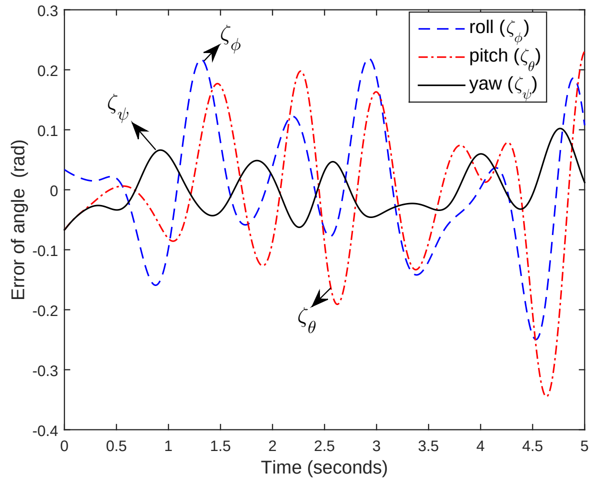

Fig. 17. Random disturbance stabilization without compensator

C. Case 3

In the third case, the stabilization is tested with the presence of a small disturbance in the system. In this case, the *CoG* position of quadrotor is assumed coincides with the *CG* of quadrotor. First simulation is done to test the performance of the nominal control in (21) to handle a constant disturbance in the system as shown in Fig. 15.

Fig. 15 shows that the nominal control able to compensate the constant disturbance in the system, so that the stability can be achieved. Furthermore, simulation is conducted to find out how reliable a compensator is when dealing with a constant disturbance. From Fig. 16, it appears that by using a compensator in a nominal control law, the better stability was achieved. Faster settling time is reached in 0.16 sec, while the nominal control needs 1.5 sec to reach the steady state.

Another scenario is tested, where a random disturbance is given into the system. First, the simulation is done using nominal *PD*- Feedforward controller in (21) without designed compensator. This controller fails to stabilize the quadrotor's attitude, when a random disturbance is presence, as shown in Fig. 17, where the error of angle ζ value is about -0.35 to 0.2 rad. A better result is given by adding a compensator to the controller. By using this designed compensator, the system become more stable as seen in Fig. 18. The error of angle ζ can be reduced to around ± 0.06 rad. It shows that the compensation reduce the bound of state variable and improve the system stability.

D. Case 4

In this section, the stability of a quadrotor will be tested, due to the presence of a disturbance and the deviated position of *CoG*. A constant disturbance signal $\left[0.3\ 0.2\ 0.1\right]^T$ is given when the system stability with deviated *CoG* is achieved. The result is shown in Fig. 19. It appears that the system's stability is disturbed when a disturbance is given. The error of angle ζ is rising up for a while, and the controller attracts the state of error back to the zero value within 0.2 sec. The stability of the system is maintained during the disturbance is added to the system.

Based on the simulation in this work, comparing to the nominal controller based on Bullo in [23], this controller has better performance in stabilization as shown in Table III.

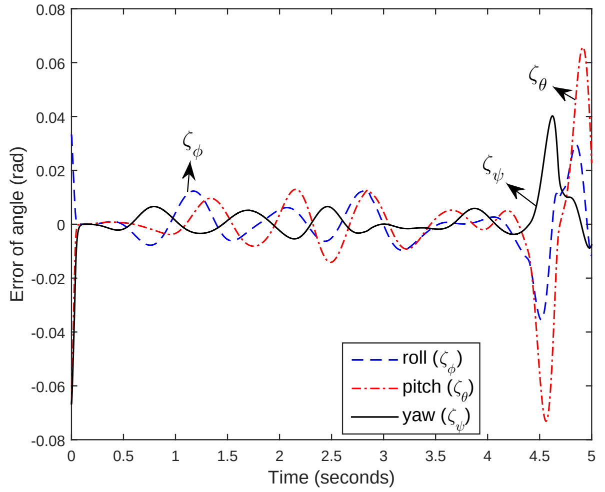

Fig. 18. Random disturbance stabilization with compensator

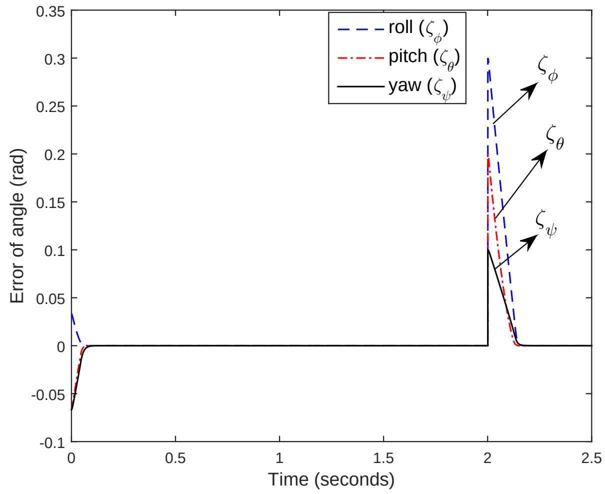

Fig. 19. Stabilization with presence of disturbance and deviated *CoG* position

TABLE III
THE COMPARISON OF NOMINAL CONTROLLER AND ROBUST CONTROLLER

Case	Nominal Controller	Robust Controller
1. Nominal Condition Stabilization Settling Time	 Stable ± 1.62 sec	 Stable ± 0.13 sec
2. With Disturbance Constant disturbance Recovery Time Random disturbance	 Stable ± 1.4 sec Not Stable	 Stable ± 0.1 sec Stable
3. With uncertain parameters Constant uncertain parameter Random uncertain parameter	 Not Stable Not Stable	 Stable Stable
4. Deviated *CoG* and disturbance	Not Stable	Stable

VI. CONCLUSION

In this paper, a robust attitude stabilization controller was proposed consisting a nominal Proportional-Derivative feed-forward and a robust compensator for quadrotor systems under the effect of the unknown position of the center of gravity. The robust compensator was introduced to achieve the robust stability against the effect of the uncertainties parameter, which is caused by unknown position of center of gravity. It is proven that the stability of quadrotor's attitude can be achieved without the knowledge of the position of *CoG*. This robust compensator also increases the performance of nominal controller by increasing the settling time of the system.

Future effort, this controller will be expanded to compensate the moment of inertia due to the unbalance thrust, and tested in real quadrotor.

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