Optimization of Overall Equipment Effectiveness with Integrated Modeling of Maintenance and Quality

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Abstract— **The Overall Equipment Effectiveness (OEE) significantly improves with the integrated optimization of preventive maintenance and quality control. This paper presents a new mixed-integer non-linear programming model. This model determines the optimal preventive maintenance interval as well as the optimal parameters of the $\bar{X}$ control chart taking into account the OEE. The optimization performs by considering the production system as a continuous-time homogeneous Markov chain, to minimize the costs per time unit and maximize OEE. A numerical example is used to evaluate this new model. Sensitivity analysis performs to determine the effect of the model parameters on optimal decisions. This analysis further illustrates the relationship between preventive maintenance and statistical quality control and also their effect on OEE.**

Keyword— **Chart control, Maintenance, Markov processes, Overall Equipment Effectiveness.**

I. INTRODUCTION

The increase in productivity leads to the survival of manufacturing organizations in today's competitive environment. One of the best opportunities for increasing productivity is the scheduling of preventive maintenance and the use of control charts.

Overall Equipment Effectiveness (OEE) is a powerful index to assess the performance of repairs. The OEE affects repairs by eliminating six big losses in equipment. These six big losses classify into three categories: availability rate (equipment failure, setup, and adjustment), performance rate (idling, minor stoppage, and reduced speed), and quality rate (defects in the process and reduced yield). Therefore, production planning, quality, and maintenance have a direct effect on OEE.

Generally, there are two failure modes in the production systems. In the first failure mode, the production immediately stops due to the machine breaks down. The second failure mode reduces the quality of the process by changing the mean of the process.

It is obvious that the first failure mode reduces the availability of the machine, and the second failure mode increases the production rate of non-conforming products. Therefore, both of modes reduce OEE. The control charts are an appropriate tool to determine the optimal time of preventive maintenance. Timely preventive maintenance prevents the production of non-conforming products and also machine failure. Therefore, the optimization of preventive maintenance, the economic design of the control chart, and OEE must be performed simultaneously in one model.

Reference [1] provides a review of the papers in the field of overall equipment effectiveness. Reference [2] reviews the papers on maintenance performance measurement. This paper also studies OEE as an evaluation indicator for repair improvement. In the field of calculation of OEE, many articles have presented, but these papers are case studies in various fields, and OEE is calculated based on empirical data resulted from case studies of the various industries. Table I lists a summary of the papers and their application areas.

TABLE I
SUMMARY OF PAPERS IN THE FIELD OF OEE

References	Case study	Area of application
[3]	√	A tile manufacturing company
[4]	√	A spinning plant
[5]	√	On broaching machine from one of manufacturing enterprise
[6]	√	In the printing industry
[7]	√	The croissant production line
[8]	√	Empirical OEE data gathered from 98 Swedish companies in the manufacturing industry
[9]	√	In a Beverage plant

Despite the direct effect of repair decisions and control charts parameters on the OEE, to our knowledge, there is no mathematical modeling for the integrated optimization of repairs, quality, and OEE. Reference [10] has provided a paper for the integrated optimization of preventive maintenance decisions (preventive maintenance interval) and control chart parameters (sampling interval, sample size, and control limit) in the form of a continuous-time Markov chain. The present paper is an extension of [10]. In the present paper, control chart parameters and the preventive maintenance intervals are optimized to minimize the cost per time unit and maximize the OEE in the form of a homogeneous continuous-time Markov chain.

The process of a machine can be a stochastic process. The parameter of this process is time. Various states of the process, including in-control state and out of control states, and the state of failure (the state of corrective maintenance), the sampling state, the states of various levels of preventive maintenance, and the state of false alarm can be the status space of this stochastic process. Each state can be the status of the stochastic process at the time of t. In this stochastic process, the conditional probability distribution of each

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future state, is independent of the previous situation, and only it depends on its current status then this stochastic process can be considered as a Markov chain. In this Markov chain, the possibility of transition from one state to another is independent of time (or stage); therefore, this Markov chain can be homogeneous.

In this Markov chain, the process is moving from one state to another state with a probability. The time distribution of each state is exponential. The processing time in different states is independent of each other, and the duration of each state depends on its state. Therefore, the different states of a machine process can be a continuous-time homogeneous Markov chain.

The following assumptions consider in this model:

1- The time length is not negligible for sampling, search for false alarms, performing preventive maintenance, and corrective maintenance.
2- The distribution of the qualitative key characteristic of the machine (process) is normal.
3- The repairs include both preventive maintenance and corrective maintenance.
4- Integration concepts include both maintenance and quality.
5- The control chart is the $\bar{X}$ control chart.
6- The type of system is a job-shop system.
7- The assignable cause of the process is one cause.
8- The type of design is statistical economics.
9- The repair is either perfect (Based on Renewal theory) or imperfect.
10- The machine deterioration process is based on a continuous-time homogeneous Markov chain.
11- The distribution of process time (machine) until the shift is exponential.
12- The sample size, the width of the control limits, and the sampling interval are fixed.
13- When the process is out of control, the mean of the process changes, but the variance remains constant.
14- The number of process states includes one in-control state, several out of control states, and one failure state.
15- The performance rate of the machines is 100%.

The most important contribution of this paper is the presentation of a new mathematical model that, while optimizes the preventive maintenance interval and control chart parameters, minimizes the cost and maximizes the OEE.

In this model, all the stages of a machine, including different machine states, sampling state, false alarm state, corrective maintenance state, and also different levels of preventive maintenance, are considered in the form of a continuous-time homogeneous Markov chain. The time lengths of the various levels of preventive maintenance up to the entry to the operational states are a hyper exponential random variable of type $f-1$. Also, the length of time before entering the various levels of preventive maintenance is a hyper exponential random variable of type L'. To our knowledge, this distribution is not in any of the papers that optimize the maintenance and quality, simultaneously. So, it can be another contribution.

Also, the time lengths of different states of the machine up to entering the sampling mode are a hyper exponential random variable of type $f-1$. After sampling, the process is detected with probability 0.5 either in-control or out of control. When the process is in-control, it detects either in-control with probability $1-\alpha$, or out of control with probability α. Also, when the process is out of control, it detects either in-control with probability β, or out of control with probability $1-\beta$. The probability of type I error and the probability of type II error are α and β, respectively. Therefore, the time length until the exit from the sampling state is a hyper exponential random variable of type 4.

II. MODEL DESCRIPTION

This model is presented for a job-shop system. When the machine is out of control, it produces non-conforming products. The percentage of non-conforming products produced in out-of-control states varies depending on the out of control state . However, the worse the out of control state produces, the higher the percentage of non-conforming products. The production rate of each machine for each product is known. A component of each machine is considered as a one-piece that preventive maintenance must perform on it.

In the following, the integrated model is described. First, indices, parameters, and variables are listed, and then objective function and constraints are introduced. It should be noted that the l is the level of the preventive maintenance and i, j are different states of the machine that the state i is worse than the state j and the state s is the sampling state.

Sets of indices

M : Set of machines $\{ m = 1,2,...,M \}$.

P : Set of products $\{ p = 1,2,...,P \}$.

I : Set of machine states $\{ i = 1,2,..., f \}$.

L : Set of preventive maintenance levels $\{ l = 1,2,...,L \}$.

Parameters

ais_{imp} : Transmission probability of the state i to the state s of the machine m while producing the product p.

asi_{imp} : Transmission probability of the state s to the state of i the machine m while producing the product p.

asl_{lmp} : Transmission probability of the state s to the state l of the machine m while producing the product p.

ali_{limp} : Transmission probability of the state l to state i of the machine m while producing the product p.

λij_{ijmp} : Arrival rate of the state i to the state j of the machine m while producing the product p.

λli_{limp} : Arrival rate of the level l to the state i of the machine m while producing the product p.

λins_{mp} : Arrival rate of the inspection state of false alarm to in control state of the machine m while producing the product p.

λf_{mp} : Arrival rate of the failure state to in control state of the machine m while producing the product p.

δ_{mp} : The magnitude of the process mean shift for the machine m while producing the product p.

θs_{mp} : Expected length of time to check a sample taken of the machine m while producing the product p.

T_{mp} : Duration of the planning period of the machine m while producing the product p.

Pr_{imp} : Production rate of non-conforming products in the state i of the machine m while producing the product p.

R_{mp} : Production rate of the machine m while producing the product p (both conforming and non-conforming products) .

r_{mp} : Revenue from the machine m for producing the product p according to index o_{mp}.

ci_{imp} : Cost of each time unit in the state i of the machine m while producing the product p.

L' : The number of preventive maintenance levels.

cl_{lmp} : Cost of each time unit at the level l of the machine m while producing the product p.

ccm_{mp} : Cost of each time unit in the corrective maintenance state of the machine m while producing the product p.

$cstop_{mp}$: Cost of each time unit of stopping of the machine m while producing the product p.

cf_{mp} : Fixed cost of each time unit in the state s of the machine m while producing the product p.

cv_{mp} : Variable cost of each sample unit of the machine m while producing the product p.

$cins_{mp}$: Cost of each time unit for inspection of the machine m while producing the product p for the false alarm.

Decision variables

z : The objective function.

α_{mp} : Type I error of machine m while producing the product p.

πi_{imp} : Percentage of time in the state i of the machine m while producing the product p.

πl_{lmp} : Percentage of time in the level l of the machine m while producing the product p.

πf_{mp} : Percentage of time in the failure state of the machine m while producing the product p.

πins_{mp} : Percentage of time in the inspection state of the machine m while producing the product p.

πs_{mp} : Percentage of time in the sampling state of the machine m while producing the product p.

λIS_{mp} : Arrival rate from the operational states to the sampling state of the machine m while producing the product p.

n_{mp} : Number of samples taken at each sampling time of the machine m while producing the product p.

k_{mp} : Amount of standard deviations of the machine m while producing the product p.

β_{mp} : Type II error of the machine m while producing the product p.

o_{mp} : Overall equipment effectiveness of machine m while producing the product p.

h_{mp} : Sampling interval of the machine m while producing the product p.

Av_{mp} : Availability rate of the machine m while producing the product p.

Qu_{mp} : Quality rate of the machine m while producing the product p.

τ_{mp} : Preventive maintenance interval of the machine m while producing the product p.

Objective function

The objective function contains a sum of nine functions. Each function derives by multiplying the cost per time unit of that state in the length of time the process remains in that state in the long term. The limiting probability of each state in the Markov chain calculates the time length of each state.

The production system is job-shop. The calculations must perform for all products and machines.

Eight functions are of the cost type and must minimize. The ninth function is of the income type; therefore, with a negative sign enters in the objective function.

$$\begin{aligned} Min\, z = & \sum_{i=1}^{f-1}\sum_{m=1}^{M}\sum_{p=1}^{P} ci_{imp}\,\pi i_{imp} \\ & + \sum_{m=1}^{M}\sum_{p=1}^{P} ccm_{mp}\,\pi f_{mp} + \sum_{m=1}^{M}\sum_{p=1}^{P} cstop_{mp}\,\pi f_{mp} \\ & + \sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{p=1}^{P} cl_{lmp}\,\pi l_{lmp} + \sum_{l=1}^{L}\sum_{m=1}^{M}\sum_{p=1}^{P} cstop_{mp}\,\pi l_{lmp} \\ & + \sum_{m=1}^{M}\sum_{p=1}^{P} cv_{mp}\, n_{mp}\,\pi s_{mp} + \sum_{m=1}^{M}\sum_{p=1}^{P} cins_{mp}\,\pi ins_{mp} \\ & + \sum_{m=1}^{M}\sum_{p=1}^{P} cf_{mp}\,\pi s_{mp} - \sum_{m=1}^{M}\sum_{p=1}^{P} r_{mp}\, o_{mp} \end{aligned} \quad (1)$$

Constraints

Equations of equilibrium are written for each node (state). In this Markov chain, the nodes include in-control state, out of control states, failure state, sampling state, preventive maintenance states, and inspection state for the false alarm. In equations, the inputs of each node are indicated by the positive sign and outputs by the negative sign.

Equation (2) is the equilibrium equation of the in-control node (node one). Inputs to this node are from the inspection node for the false alarm, the failure node (corrective maintenance), the sampling node, and the nodes of the various levels of preventive maintenance. Outputs are to other machine nodes (both operational nodes and failure node) and the sampling node. Equation (3) is equilibrium equations for out of control nodes other than failure state. Inputs to these nodes are from the sampling node, the in-control node, the out of control nodes that have better conditions than this node (the out of control nodes before this node), and the nodes of the various levels of preventive maintenance. Outputs are to the sampling node, the out of control nodes that are worse than this node (out of control nodes after this node), and failure node. Equation (4) is the equilibrium equation for the failure node. Inputs to this node are from the operational nodes (both in control and the out of control nodes). The output of this node is only to the in-control node. The machine returns to in control state after the corrective maintenance.

Equation (5) is the equilibrium equation for the sampling node. Inputs to this node are from different states of the machine (other than failure node), and outputs are to the in-control node, the out of control nodes (other than the node of failure), nodes of preventive maintenance levels, and false alarm node.

Equation (6) is the equilibrium equation for various levels of preventive maintenance. Input is from the sampling node, and outputs are to the in-control node and out of control nodes of the machine (other than the failure node). Equation (7) is the equilibrium equation for the inspection node (false alarm). The node's input is from the sampling node, and the output of this node is to the in-control node.

$$\begin{aligned}&\sum_{l=1}^{L}(f-1)ali_{limp}\,\lambda li_{limp}\,\pi l_{lmp}+\lambda ins_{mp}\,\pi ins_{mp}\\&+2(1-\alpha_{mp})(1/(\theta s_{mp}\,n_{mp}))\pi s_{mp}+\lambda f_{mp}\,\pi f_{mp}\\&-\sum_{j=2}^{f}\lambda ij_{ijmp}\,\pi i_{imp}-(f-1)ais_{imp}\,\lambda IS_{mp}\,\pi i_{imp}=0\\&i=1,\forall m\in M,\forall p\in P\end{aligned}\tag{2}$$

$$\begin{aligned}&\sum_{l=1}^{L}(f-1)ali_{limp}\,\lambda li_{limp}\,\pi l_{lmp}-\sum_{j=i+1}^{f}\lambda ij_{ijmp}\,\pi i_{imp}\\&+2asi_{imp}(\beta_{mp})(1/(\theta s_{mp}\,n_{mp}))\pi s_{mp}\\&+\sum_{j=1}^{i-1}\lambda ij_{jimp}\,\pi i_{jmp}-(f-1)ais_{imp}\,\lambda IS_{mp}\,\pi i_{imp}=0\\&i=2,3,\ldots,f-1,\forall m\in M,\forall p\in P\end{aligned}\tag{3}$$

$$\begin{aligned}&\sum_{i=1}^{f-1}\lambda ij_{ijmp}\,\pi i_{imp}-\lambda f_{mp}\,\pi f_{mp}=0\\&j=f,\forall m\in M,\forall p\in P\end{aligned}\tag{4}$$

$$\begin{aligned}&(f-1)\sum_{i=1}^{f-1}ais_{imp}\,\lambda IS_{mp}\,\pi i_{imp}\\&-2(1-\alpha_{mp})(1/(\theta s_{mp}\,n_{mp}))\pi s_{mp}\\&-2\sum_{i=2}^{f-1}asi_{imp}\,\beta_{mp}(1/(\theta s_{mp}\,n_{mp}))\pi s_{mp}\\&-2\alpha_{mp}(1/(\theta s_{mp}\,n_{mp}))\pi s_{mp}\\&-2L'\sum_{l=1}^{L}asl_{lmp}(1-\beta_{mp})(1/(\theta s_{mp}\,n_{mp}))\pi s_{mp}=0\\&\forall m\in M,\forall p\in P\end{aligned}\tag{5}$$

$$\begin{aligned}&2L'(asl_{lmp})(1-\beta_{mp})(1/(\theta s_{mp}\,n_{mp}))\pi s_{mp}\\&-\sum_{i=1}^{f-1}(f-1)ali_{limp}\,\lambda li_{limp}\,\pi l_{lmp}=0\\&\forall m\in M,\forall p\in P,\forall l\in L\end{aligned}\tag{6}$$

$$\begin{aligned}&2(\alpha_{mp})(1/(\theta s_{mp}n_{mp}))\pi s_{mp}-\lambda ins_{mp}\,\pi ins_{mp}=0\\&\forall m\in M,\forall p\in P\end{aligned}\tag{7}$$

$$h_{mp}=1/(\lambda IS_{mp})\qquad\forall m\in M,\forall p\in P\tag{8}$$

$$\begin{aligned}&\tau_{mp}=[1/((1-\beta_{mp})\lambda IS_{mp})]+\theta s_{mp}\,n_{mp}\\&\forall m\in M,\forall p\in P\end{aligned}\tag{9}$$

$$\begin{aligned}&Av_{mp}=(T_{mp}-T_{mp}\,\pi l_{mp}-T_{mp}\,\pi f_{mp})/(T_{mp}-T_{mp}\,\pi l_{mp})\\&\forall m\in M,\forall p\in P\end{aligned}\tag{10}$$

$$\begin{aligned}Qu_{mp}=&\Big(\sum_{i=1}^{f-1}R_{mp}\pi i_{imp}T_{mp}-\sum_{i=1}^{f-1}Pr_{imp}\pi i_{imp}T_{mp}\Big)\\&/\Big(\sum_{i=1}^{f-1}R_{mp}\pi i_{imp}T_{mp}\Big)\qquad\forall m\in M,\forall p\in P\end{aligned}\tag{11}$$

$$o_{mp}=Av_{mp}\,Qu_{mp}\qquad\forall m\in M,\forall p\in P\tag{12}$$

$$\begin{aligned}&\sum_{i=1}^{f-1}\pi i_{imp}+\sum_{l=1}^{L}\pi l_{lmp}+\pi f_{mp}+\pi ins_{mp}+\pi s_{mp}=1\\&\forall m\in M,\forall p\in P\end{aligned}\tag{13}$$

$$\alpha_{mp}=2\Phi(-k_{mp})\qquad\forall m\in M,\forall p\in P\tag{14}$$

$$\begin{aligned}&\beta_{mp}=\Phi(k_{mp}-\delta_{mp}\sqrt{n_{mp}})-\Phi(-k_{mp}-\delta_{mp}\sqrt{n_{mp}})\\&\forall m\in M,\forall p\in P\end{aligned}\tag{15}$$

Equation (8) calculates the sampling interval. It derives by the inverse of the entry rate from the operational nodes to the sampling node. Equation (9) calculates the preventive maintenance interval. It is equal to inverse multiplication of the entry rate from the operational nodes to the sampling node in the probability of entering from the sampling node to the preventive maintenance node plus the inverse multiplication of the sample number in the time length of checking a sample.

Equation (10) calculates the ratio of the planned production time of the machine minus the failure time (stop time) to the planned production time. It is the availability rate. Equation (11) is the ratio of the total production products minus the non-conforming products to the total production products (both conforming and non-conforming products). This equation calculates the quality rate. Equation (12) calculates the OEE. Equation (13) is necessary to obtain the percentage of process time remaining in each of the nodes.

Equation (14) calculates the probability of type I error. Equation (15) calculates the probability of type II error where $\Phi(x)$ denotes the standard normal cumulative distribution function.

III. RESULTS AND DISCUSSION

In the following, a numerical example presents to evaluate the proposed model, and then sensitivity analysis examines the effect of model parameters on the optimal solution. Consider a production system consisting of a machine and a product $(m=1,\ p=1)$. The process of controlling the machine performs by the $\bar{X}$ control chart. The machine has four states: in-control state $(i=1)$, out of control states $(i=2,3)$, and failure state $(i=4)$.

The process of the machine, without repair, only changes to worse conditions. One-level is considered for preventive maintenance $(l=1)$. For this numerical example, the values of the parameters introduced in the model description section are presented in Tables II-VII.

TABLE II
EXIT RATES BETWEEN MACHINE STATES.

λij_{1211}	λij_{1311}	λij_{1411}	λij_{2311}	λij_{2411}	λij_{3411}	λf_{11}
0.00025	0.00016	0.000125	0.0002	0.00011	0.0002	0.005

TABLE III
EXIT RATES BETWEEN REPAIRS AND MACHINE STATES.

λli_{1111}	λli_{1211}	λli_{1311}
0.0222	0.033	0.05

TABLE IV
THE TRANSFER PROBABILITY BETWEEN MACHINE STATES AND SAMPLING STATE.

ais_{111}	ais_{211}	ais_{311}	asi_{211}	asi_{311}
0.0001	0.2999	0.7	0.6	0.4

TABLE V
THE TRANSFER PROBABILITY BETWEEN REPAIRS AND MACHINE STATES.

ali_{1111}	ali_{1211}	ali_{1311}
0.6	0.3	0.1

TABLE VI
COSTS.

ci_{111}	ci_{211}	ci_{311}	ccm_{11}	cl_{111}	cf_{11}	cv_{11}	$cins_{11}$	$cstop_{11}$
0	100	200	2000	200	20	5	50	1000

TABLE VII
OTHER PARAMETERS.

Pr_{111}	Pr_{211}	Pr_{311}	R_{mp}	δ_{11}	r_{11}	θs_{11}	L'	T_{11}	λins_{11}
0	0.2	0.4	5	1.5	200	20	1	480	0.05

A program to solve this model is written in GAMS (version 24.9.1).

The optimal solution obtained by using the BARON solver is $z^*=76.31, h^*_{11}=118, \tau^*_{11}=1576, n^*_{11}=4$, $o^*_{11}=88\%$.

Sensitivity analysis is performed to observe the effect of model parameters on the optimal solution. These parameters include the cost of corrective maintenance (ccm_{11}), the cost of preventive maintenance (cl_{111}), the magnitude of the quality shift of the process (δ_{11}) and the production rate of non-conforming products in out of control states (Pr_{211}, Pr_{311}).

The variations of parameters and results of the sensitivity analysis are presented in Table VIII. In this table, the values of parameters, either negative or positive, are considered, where negative numbers mean the decreased value and positive numbers indicate the increased value of the number defined for this parameter in the numerical example as much as that number.

It should be noted that when changing one parameter for sensitivity analysis, the values of other parameters are the same as numerical example values (Tables II-VII). In other words, in each problem solved for sensitivity analysis, only one parameter of the numerical example parameters above is changed. Therefore, for the sensitivity analysis, eight problems are solved. The values of the objective function and the variables of each problem are presented in Table VIII.

TABLE VIII
VARIATIONS OF PARAMETERS AND SENSITIVITY ANALYSIS RESULTS.

Parameter	Variation	z^*	h^*_{11}	τ^*_{11}	n^*_{11}	k^*_{11}	o^*_{11}
ccm_{11}	-1000	69.18	132	1835	3	1.84	76%
	+2000	91.34	101	1439	5	2.1	93%
cl_{111}	-100	78.45	112	1547	3	1.87	91%
	+200	83.73	129	1763	5	2.21	84%
δ_{11}	-0.5	101.48	108	1427	5	1.81	91%
	+1.5	74.61	156	1786	3	2.3	73%
Pr_{211},	-0.1, -0.2	93.71	131	1821	3	1.85	79%
Pr_{311}	+0.2, +0.4	106.28	101	1486	5	2.11	89%

As seen in Table VIII, the followings can be noted:

Variations of the cost of corrective maintenance: As the cost of corrective maintenance (ccm_{11}) increases (from 1000 to 4000), both the sampling interval (h_{11}) and the preventive maintenance interval (τ_{11}) decrease, as can be seen in Table VIII, the sampling interval is decreased from 132 to 101, and the preventive maintenance interval is reduced from 1835 to 1439. But the OEE, the sample size (n_{11}) and the magnitude of amount of standard deviations of machine (k_{11}) increase (from 76% to 93%, from 3 to 5, and from 1.84 to 2.1, respectively) The OEE has increased due to the reduction of non-conforming products and increased availability of the machine due to the decrease in preventive maintenance interval and sampling interval. The overall cost (z) increase due to increased sample size, sampling, and preventive maintenance (from 69.18 to 91.34).

Variations of preventive maintenance cost: The cost of preventive maintenance (cl_{11}) affects both the sampling interval and the preventive maintenance interval. As the cost of preventive maintenance increases (from 100 to 400), the magnitude of standard deviations of the machine (k_{11}), the sample size (n_{11}), the sampling interval, and the preventive maintenance interval increase (from 1.87 to 2.21, from 3 to 5, from 112 to 129 and from 1547 to 1763, respectively). The OEE is decreased by the increase of non-conforming products and the reduction of availability of the machine.

Variations of the magnitude of the process mean shift: The change in the magnitude of the process mean shift (δ_{11}) affects the value of k_{11}. As δ_{11} increases (from 1 to 3), k_{11} also increases (from 1.81 to 2.3). This change is logical that larger process changes necessarily require a larger control limit. Also, as δ_{11} increases the sampling interval, and the preventive maintenance interval increase, but both the OEE and the sample size decrease. As the sampling interval and preventive maintenance interval increase, the probability of both machine failure, and producing the non-conforming product will increase due to delay in sampling and late detection of out of control state.

For this reason, the OEE reduces.

Variations of the production rate of non-conforming products in the out of control state: As the production rate of non-conforming products (Pr_{211}, Pr_{311}) increases in out-of-control states (from 0.1 to 0.4 in state 2 and from 0.2 to 0.8 in state 3), the preventive maintenance interval, and the sampling interval reduce. By decreasing the sampling interval, the probability of producing a non-conforming product decreases because the out-of-control state detects earlier. As the preventive maintenance interval reduces, the probability of the machine failure reduces, and therefore the availability of the machine increases. With the reduction of non-conforming products and increased availability, OEE increases. Also, both the sample size and the value of standard deviations of machine increase, which are quite logical.

The results of the sensitivity analysis show that the model responds logically to variations of the parameters of the problem. The change in input parameters affects both the preventive maintenance and the statistical process control policies and also the OEE.

This model is a combinatorial optimization problem. These problems are known as NP-hard problems due to the complexity and difficulty of the solution. In these problems, as the dimensions of the problem increase, the solution time expands exponentially. A numerical example is presented with a product and a machine here. This example is solved within a reasonable time in GAMS by using the BARON solver due to its small size. For problems with larger dimensions, the computational time needed to solve these problems is not acceptable. Therefore, there is a need for the development of heuristic optimization approaches due to the large production environments with a large number of products and machines.

IV. Conclusions and Future Studies

This paper presents an integrated model of preventive maintenance, quality control, and OEE in the form of a continuous-time homogeneous Markov chain. The proposed model can be applied to manufacturing processes, which shift to an out-of-control state over time.

The most significant contribution of the paper is the development of an integrated model that optimizes both the control chart parameters and the preventive maintenance interval by maximizing the OEE and minimizing the cost. Both the time length before entering to various levels of preventive maintenance and the time length at different levels of preventive maintenance until entering to different machine states are considered as a hyper exponential random variable. This distribution is notably another contribution of this paper.

A numerical example is presented to validate the model. The sensitivity analysis performed shows the dependence between preventive maintenance, statistical process control, and the OEE. This model is a combinational optimization problem; in the future research suggests the use of meta-heuristic algorithms to solve this model. Simultaneous optimization of production planning, preventive maintenance, and statistical quality control considering OEE is an interesting topic for future research.

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