Extinction and Stability of a Discrete Competitive System with Beddington-DeAngelis Functional Response

Jiehua Zhang, Shengbin Yu, Qingjuan Wang

Abstract—**We study A two species discrete competitive system with Beddington-DeAngelis functional response in this paper. Sufficient conditions which guarantee the extinction of a species and the global attractivity of the other one are obtained. Our results supplement some existing ones.**

Index Terms—**discrete; competitive; Beddington-DeAngelis; extinction; stability.**

I. Introduction

THROUGHOUT this paper, for any bounded sequence $\{h(n)\}$, we define $h^l = \inf_{n\in N}\{h(n)\}$,$h^u = \sup_{n\in N}\{h(n)\}$.

Motivated by Gopalsamy [1], Wang et al. [2] proposed the following Lotka-Volterra competitive system:

$$\begin{aligned}\dot{x_1}(t) &= x_1(t)\Big(r_1(t) - a_1(t)x_1(t) - \frac{b_1(t)x_2(t)}{1+x_2(t)}\Big),\\ \dot{x_2}(t) &= x_2(t)\Big(r_2(t) - a_2(t)x_2(t) - \frac{b_2(t)x_1(t)}{1+x_1(t)}\Big).\end{aligned} \tag{1}$$

By using a differential inequality, the authors obtained sufficient conditions which ensure the existence and global asymptotic stability of positive almost periodic solutions. For more works, one could refer to [3-21] and the references cited therein.

Corresponding to system (1), several scholars [3-6] investigated the dynamic behaviors of the discrete type two species competitive system with nonlinear inter-inhibition terms,

$$\begin{aligned}x_1(n+1) &= x_1(n)\exp\{r_1(n) - a_1(n)x_1(n)\\ &\quad -\frac{b_1(n)x_2(n)}{1+x_2(n)}\},\\ x_2(n+1) &= x_2(n)\exp\{r_2(n) - a_2(n)x_2(n)\\ &\quad -\frac{b_2(n)x_1(n)}{1+x_1(n)}\}.\end{aligned} \tag{2}$$

For the ecological meaning of model(2), see [3]. Qin et al. [3] and Wang et al. [4] obtained the permanence, stability, and almost periodic solutions of system (2). Yue [7] considered the partial extinction of system (2) with one toxin producing species.

Manuscript received Aug 1, 2019, revised Feb 6, 2020.

Jiehua Zhang, Shengbin Yu, Qingjuan Wang are with the Department of Basic Teaching and Research, Yango University, Fuzhou, Fujian 350015, China. (47756357@qq.com).

In [8], Ma et al. investigated the following discrete two-species competitive system:

$$\begin{aligned}x_1(n+1) &= x_1(n)\exp\{r_1(n) - a_1(n)x_1(n)\\ &\quad -\frac{b_1(n)x_2(n)}{1+\beta_1(n)x_1(n)}\},\\ x_2(n+1) &= x_2(n)\exp\{r_2(n) - a_2(n)x_2(n)\\ &\quad -\frac{b_2(n)x_1(n)}{1+\beta_2(n)x_2(n)}\},\end{aligned} \tag{3}$$

and obtained the almost periodic solutions of the system.

Based on the above papers, Chen, Chen and Huang [9] proposed the following two species non-autonomous competitive system with Beddington-DeAngelis functional response and the effect of toxic substances

$$\begin{aligned}\dot{x_1}(t) &= x_1(t)\{r_1(t) - a_1(t)x_1(t)\\ &\quad -\frac{b_1(t)x_2(t)}{\alpha_1(t)+\beta_1(t)x_1(t)+\gamma_1(t)x_2(t)}\\ &\quad -c_1(t)x_1(t)x_2(t)\},\\ \dot{x_2}(t) &= x_2(t)\{r_2(t) - a_2(t)x_2(t)\\ &\quad -\frac{b_2(t)x_1(t)}{\alpha_2(t)+\beta_2(t)x_1(t)+\gamma_2(t)x_2(t)}\\ &\quad -c_2(t)x_1(t)x_2(t)\},\end{aligned} \tag{4}$$

they obtained the partial extinction of system.

Recently, Yu ang Chen [10] investigated the dynamic behaviors of system (4) without the effect of toxic substances:

$$\begin{aligned}\dot{x_1}(t) &= x_1(t)\{r_1(t) - a_1(t)x_1(t)\\ &\quad -\frac{b_1(t)x_2(t)}{\alpha_1(t)+\beta_1(t)x_1(t)+\gamma_1(t)x_2(t)}\},\\ \dot{x_2}(t) &= x_2(t)\{r_2(t) - a_2(t)x_2(t)\\ &\quad -\frac{b_2(t)x_1(t)}{\alpha_2(t)+\beta_2(t)x_1(t)+\gamma_2(t)x_2(t)}\},\end{aligned} \tag{5}$$

Their results supplement the main results of [3, 9] and generalize [2].

It is well known that the discrete time models are more appropriate when the populations have nonoverlapping generations, and this motivated us to propose the discrete time version of system (5) as follows:

$$\begin{aligned}x_1(n+1) &= x_1(n)\exp\{r_1(n) - a_1(n)x_1(n)\\ &\quad -\frac{b_1(n)x_2(n)}{\alpha_1(n)+\beta_1(n)x_1(n)+\gamma_1(n)x_2(n)}\},\\ x_2(n+1) &= x_2(n)\exp\{r_2(n) - a_2(n)x_2(n)\\ &\quad -\frac{b_2(n)x_1(n)}{\alpha_2(n)+\beta_2(n)x_1(n)+\gamma_2(n)x_2(n)}\},\end{aligned} \tag{6}$$

where $x_1(n), x_2(n)$ are population density of species x_1 and x_2 at the nth generation, respectively. The coefficients $r_i(n)$, $a_i(n)$, $b_i(n)$,$\alpha_i(n)$, $\beta_i(n)$, $\gamma_i(n)$ $(i = 1, 2)$ are all bounded nonnegative sequences.

As regards the biological meaning, we assume (6) together with the initial conditions: $x_1(0) > 0$, $x_2(0) > 0$. It is not difficult to see that the solutions of (6) are positive.

The aim of this paper is to investigate the extinction property of the system (6). The organization is as follows. Section 2 is devoted to the results on extinction for system (6). In Section 3, we study the global stability of another species when the previous species is eventual extinction. Then, in Section 4, two examples with numerical simulations are given to illustrate the feasibility of the main results. Finally, we conclude in Section 5.

II. Extinction

In this section, we will establish sufficient conditions on the extinction of species x_1 and x_2. First, let us introduce the following lemma which will be useful for our main result.

Lemma 2.1 ([11]). *Assume that $\{x(n)\}$, satisfies $x(n) > 0$, and*

$$x(n + 1) \le x(n) \exp\{a(n) - b(n)x(n)\}$$

for $n \in N$, where $a(n)$ and $b(n)$ are nonnegative sequences bounded above and below by positive constants. Then

$$\limsup_{n\to+\infty} x(n) \le \frac{1}{b^l} \exp(a^u - 1).$$

Lemma 2.2. *Any positive solution $(x_1(n), x_2(n))^T$ of system (6) satisfies*

$$\limsup_{n\to+\infty} x_i(n) \le M_i,$$

where $M_i = \frac{1}{a_i^l} \exp(r_i^u - 1), i = 1, 2$.

Proof. Let $(x_1(n), x_2(n))^T$ be any positive solution of system (6). From the first and second equation of system (6), it follows that

$$x_i(n+1) \le x_i(n) \exp\{r_i(n) - a_i(n)x_i(n)\}, \quad i = 1, 2. \quad (7)$$

By applying Lemma 2.1 to (7), we have

$$\limsup_{n\to+\infty} x_i(n) \le \frac{1}{a_i^l} \exp(r_i^u - 1) \overset{\text{def}}{=} M_i, \quad i = 1, 2. \quad (8)$$

Lemma 2.2 shows that the positive solutions of system (6) are bounded eventually.

Theorem 2.1. *Assume*

$$(H_1) \quad \frac{r_2^u}{r_1^l} < \min\{\frac{b_2^l}{a_1^u(\alpha_2^u + \beta_2^u M_1 + \gamma_2^u M_2)}, \frac{a_2^l \alpha_1^l}{b_1^u}\}$$

holds, where $M_i, i = 1, 2$ is defined in Theorem 2.1, then the species x_2 will be driven to extinction, that is, for any positive solution $(x_1(n), x_2(n))^T$ of system (6), $\lim_{n\to+\infty} x_2(n) = 0$.

Proof. According to (H_1), one can choose a small enough positive constant ε_1 such that

$$\frac{r_2^u}{r_1^l} < \min\{\frac{b_2^l}{a_1^u(\alpha_2^u + \beta_2^u(M_1 + \varepsilon_1) + \gamma_2^u(M_2 + \varepsilon_1))}, \frac{a_2^l \alpha_1^l}{b_1^u}\} \quad (9)$$

By (9), there exist two positive constants p and q such that

$$\frac{r_2^u}{r_1^l} < \frac{p}{q} < \min\{\frac{b_2^l}{a_1^u(\alpha_2^u + \beta_2^u(M_1 + \varepsilon_1) + \gamma_2^u(M_2 + \varepsilon_1))}, \frac{a_2^l \alpha_1^l}{b_1^u}\}. \quad (10)$$

Thus,

$$\begin{aligned} qr_2^u - pr_1^l &\overset{\text{def}}{=} -\delta_1 < 0, \\ pa_1^u - \frac{qb_2^l}{\alpha_2^u + \beta_2^u(M_1 + \varepsilon_1) + \gamma_2^u(M_2 + \varepsilon_1)} &< 0, \\ \frac{pb_1^u}{\alpha_1^l} - qa_2^l &< 0. \end{aligned} \quad (11)$$

For the above ε_1, it follows from Theorem 2.1 that there exists a large enough N_1 such that for all $n \ge N_1$,

$$x_i(n) < M_i + \varepsilon_1, \quad i = 1, 2. \quad (12)$$

For any $k > N_1$, according to the equations of system (6) and (12), we can get

$$\begin{aligned} \ln\frac{x_1(k+1)}{x_1(k)} &= r_1(k) - a_1(k)x_1(k) \\ &\quad -\frac{b_1(k)x_2(k)}{\alpha_1(k) + \beta_1(k)x_1(k) + \gamma_1(k)x_2(k)}, \\ &\ge r_1^l - a_1^u x_1(k) - \frac{b_1^u x_2(k)}{a_1^l}, \\ \ln\frac{x_2(k+1)}{x_2(k)} &= r_2(k) - a_2(k)x_2(k) \\ &\quad -\frac{b_2(k)x_1(k)}{\alpha_2(k) + \beta_2(k)x_1(k) + \gamma_2(k)x_2(k)}, \\ &\le r_1^u - a_1^l x_2(k) \\ &\quad -\frac{b_2^l x_1(k)}{\alpha_2^u + \beta_2^u(M_1 + \varepsilon_1) + \gamma_2^u(M_2 + \varepsilon_1)}. \end{aligned} \quad (13)$$

Consider the following Lyapunov type extinction function, for $k > N_1$, from (11)-(13), we have

$$\begin{aligned} &q\ln\frac{x_2(k+1)}{x_2(k)} - p\ln\frac{x_1(k+1)}{x_1(k)} \\ &\le (qr_2^u - pr_1^l) \\ &\quad + [pa_1^u - \frac{qb_2^l}{\alpha_2^u + \beta_2^u(M_1 + \varepsilon_1) + \gamma_2^u(M_2 + \varepsilon_1)}]x_1(k) \\ &\quad + [\frac{pb_1^u}{\alpha_1^l} - qa_2^l]x_2(k) \\ &< -\delta_1 < 0 \end{aligned} \quad (14)$$

Summing both sides of the above inequalities from $N + 1$ to $n - 1$ leads to

$$q\ln\frac{x_2(n)}{x_2(N+1)} - p\ln\frac{x_1(n)}{x_1(N+1)} < -\delta_1(n - N - 1), \quad (15)$$

hence

$$x_2(n) < [\frac{x_1(n)}{x_1(N+1)}]^{\frac{p}{q}} x_2(N + 1) \exp(-\frac{\delta_1}{q}(n - N - 1)). \quad (16)$$

The above inequality together with the ultimate boundedness of $x_1(n)$ shows that $\lim_{n\to+\infty} x_2(n) = 0$. The proof is completed.

Theorem 2.2. *Let $(x_1(n), x_2(n))^T$ be any positive solution of system (6). Suppose*

$$(H_2) \quad \frac{r_2^l}{r_1^u} > \max\{\frac{b_2^u}{\alpha_2^l a_1^l}, \frac{a_2^u(\alpha_1^u + \beta_1^u M_1 + \gamma_1^u M_2)}{b_1^l}\}$$

holds, where $M_i, i = 1, 2$ are defined in Theorem 2.1, then the species x_1 will be driven to extinction, that is, $\lim_{n\to+\infty} x_1(n) = 0$.

Proof. Due to (H_2), one can choose a small enough positive constant ε_2 such that

$$\frac{r_2^l}{r_1^u} > \max\{\frac{b_2^u}{\alpha_2^l a_1^l}, \frac{a_2^u[\alpha_1^u + \beta_1^u(M_1+\varepsilon_2) + \gamma_1^u(M_2+\varepsilon_2)]}{b_1^l}\}. \tag{17}$$

By (17), there exist two positive constants p and q such that

$$\begin{aligned}\frac{r_2^l}{r_1^u} > \frac{p}{q} > \max\{&\frac{b_2^u}{\alpha_2^l a_1^l},\\ &\frac{a_2^u[\alpha_1^u + \beta_1^u(M_1+\varepsilon_2) + \gamma_1^u(M_2+\varepsilon_2)]}{b_1^l}\}.\end{aligned} \tag{18}$$

Thus,

$$\begin{aligned} pr_1^u - qr_2^l \overset{\text{def}}{=} -\delta_2 < 0,\\ -pa_1^l + \frac{qb_2^u}{\alpha_2^l} < 0,\\ -\frac{pb_1^l}{\alpha_1^u + \beta_1^u(M_1+\varepsilon_2) + \gamma_1^u(M_2+\varepsilon_2)} + qa_2^u < 0.\end{aligned} \tag{19}$$

For the above ε_2, it follows from Theorem 2.1 that there exists a large enough N_2 such that for all $n \geq N_2$,

$$x_i(n) < M_i + \varepsilon_2, \quad i = 1, 2. \tag{20}$$

For any $k > N_2$, according to the equations of system (6) and (20), we can get

$$\begin{aligned}\ln\frac{x_1(k+1)}{x_1(k)} &\leq r_1^u - a_1^l x_1(k)\\ &\quad - \frac{b_1^l x_2(k)}{\alpha_1^u + \beta_1^u(M_1+\varepsilon_2) + \gamma_1^u(M_2+\varepsilon_2)},\\ \ln\frac{x_2(k+1)}{x_2(k)} &\geq r_2^l - a_2^u x_2(k) - \frac{b_2^u x_1(k)}{\alpha_2^l}.\end{aligned} \tag{21}$$

Therefore, inequalities (19)-(21) show that

$$\begin{aligned}& p\ln\frac{x_1(k+1)}{x_1(k)} - q\ln\frac{x_2(k+1)}{x_2(k)}\\ &\leq (pr_1^m - qr_2^l) + (\frac{qb_2^u}{\alpha_2^l} - pa_1^l)x_1(k)\\ &\quad + [qa_2^u - \frac{pb_1^l}{\alpha_1^u + \beta_1^u(M_1+\varepsilon_2) + \gamma_1^u(M_2+\varepsilon_2)}]x_2(k)\\ &< -\delta_2 < 0, \quad k > N_2.\end{aligned} \tag{22}$$

Similarly to the analysis in Theorem 2.1, we can get $\lim_{n\to+\infty} x_1(n) = 0$.

III. Global stability

In Section 2, we get sufficient conditions which guarantee the extinction of the first or second species in system (6). Following, we investigate the stability property of the rest species. Let us first state several lemmas which will be useful in the proof of the main result of this section.

Lemma 3.1 ([12]). *Assume that $\{x(n)\}$ satisfies*

$$x(n+1) \geq x(n)\exp\{a(n) - b(n)x(n)\}, \quad n \geq N_0,$$

$\limsup_{n\to+\infty} x(n) \leq x^*$ *and $x(N_0) > 0$, where $a(n)$ and $b(n)$ are nonnegative sequences bounded above and below by positive constants and $N_0 \in N$. Then*

$$\liminf_{n\to+\infty} x(n) \geq \min\{\frac{a^l}{b^u}\exp(a^l - b^u x^*), \frac{a^l}{b^u}\}.$$

Lemma 3.2. *Suppose (H_1) holds, and $(x_1(n), x_2(n))^T$ be any positive solution of system (6) , then*

$$m_1 \leq \liminf_{n\to+\infty} x_1(n) \leq \limsup_{n\to+\infty} x_1(n) \leq M_1,$$

where $m_1 = \frac{r_1^l}{a_1^u}\exp(r_1^l - a_1^u M_1)$ and M_1 is defined in Lemma 2.1.

Proof. It follows from Lemma 2.1 and Theorem 2.1 that

$$\limsup_{n\to+\infty} x_1(n) \leq M_1, \qquad \lim_{n\to+\infty} x_2(n) = 0. \tag{23}$$

To end the proof of Lemma 3.2, we just need to show that

$$\liminf_{n\to+\infty} x_1(n) \geq m_1.$$

Since $r_1^l > 0$, there exists a small enough $\varepsilon_3 > 0$ such that

$$r_1^l - \frac{b_2^u\varepsilon_3}{\alpha_1^l} > 0. \tag{24}$$

According to (23), for the above $\varepsilon_3 > 0$, there exists a large enough $N_3 > 0$, such that, for $n \geq N_3$, it follows from Lemma 2.1 and Theorem 2.1 that

$$x_1(n) \leq M_1 + \varepsilon_3, \qquad x_2(n) \leq \varepsilon_3. \tag{25}$$

Thus, it follows from (25) and the first equation of system (6) that

$$x_1(n+1) \geq x_1(n)\exp\{r_1^l - a_1^u x_1(n) - \frac{b_2^u\varepsilon_3}{\alpha_1^l}\}. \tag{26}$$

Since $r_1^l - \frac{b_2^u\varepsilon_3}{\alpha_1^l} > 0$, by applying Lemma 3.1 to (26), it immediately follows that

$$\begin{aligned}\liminf_{n\to+\infty} x_1(n) \geq \min\{&\frac{r_1^l - \frac{b_2^u\varepsilon_3}{\alpha_1^l}}{a_1^u}\exp(r_1^l - \frac{b_2^u\varepsilon_3}{\alpha_1^l} - a_1^u M_1),\\ &\frac{r_1^l - \frac{b_2^u\varepsilon_3}{\alpha_1^l}}{a_1^u}\}.\end{aligned} \tag{27}$$

Setting $\varepsilon_3 \to 0$ in the above inequality, one can obtain

$$\liminf_{n\to+\infty} x_1(n) \geq \min\{\frac{r_1^l}{a_1^u}\exp(r_1^l - a_1^u M_1), \frac{r_1^l}{a_1^u}\}. \tag{28}$$

By calculation, one can easily get

$$\begin{aligned} r_1^l - a_1^u M_1 &= r_1^l - a_1^u\frac{\exp(r_1^u - 1)}{a_1^l}\\ &\leq r_1^l - \exp(r_1^u - 1)\\ &\leq r_1^l - r_1^u \leq 0.\end{aligned} \tag{29}$$

Inequality (28) together with (29) lead to

$$\liminf_{n\to+\infty} x_1(n) \geq \frac{r_1^l}{a_1^u}\exp(r_1^l - a_1^u M_1) \overset{\text{def}}{=} m_1. \tag{30}$$

This ends the proof of Lemma 3.2.

Lemma 3.3. *Suppose (H_2) holds, and $(x_1(n), x_2(n))^T$ be any positive solution of system (6) , then*

$$m_2 \le \liminf_{n\to+\infty} x_2(n) \le \limsup_{n\to+\infty} x_2(n) \le M_2,$$

where $m_2 = \dfrac{r_2^l}{a_2^u}\exp(r_2^l - a_2^u M_2)$ *and* M_2 *is defined in Lemma 2.1.*

Proof. The proof of Lemma 3.3 is similar to that of the proof of Lemma 3.2, we omit the details here.

Consider the following discrete logistic equation:

$$x(n+1) = x(n)\exp\{r_1(n) - a_1(n)x(n)\}, \quad n \in N, \tag{31}$$

where $r_1(n)$ and $a_1(n)$ are bounded nonnegative sequences.

Lemma 3.4 ([13]). *For any positive solution $x(n)$ of (31), we have*

$$m_1 \le \liminf_{n\to+\infty} x(n) \le \limsup_{n\to+\infty} x(n) \le M_1,$$

where m_1 and M_1 are defined by Lemma 3.2.

Consider the following discrete logistic equation:

$$x(n+1) = x(n)\exp\{r_2(n) - a_2(n)x(n)\}, \quad n \in N, \tag{32}$$

where $r_2(n)$ and $a_2(n)$ are bounded nonnegative sequences.

Lemma 3.5 ([13]). *For any positive solution $\tilde{x}(n)$ of (32), we have*

$$m_2 \le \liminf_{n\to+\infty} \tilde{x}(n) \le \limsup_{n\to+\infty} \tilde{x}(n) \le M_2,$$

where m_2 and M_2 are defined by Lemma 3.3.

Now, we come to showing the main results of this section.

Theorem 3.1. *Suppose (H_1) holds, further suppose that*

(H_3) $\dfrac{a_1^u}{a_1^l}\exp(r_1^u - 1) < 2$, *then*

$$\lim_{n\to+\infty}(x_1(n) - x(n)) = 0, \qquad \lim_{n\to+\infty} x_2(n) = 0.$$

where $(x_1(n), x_2(n))^T$ and $x(n)$ are any two positive solutions of system (6) and (31), respectively.

Proof. It follows from Theorem 2.1 that

$$\lim_{n\to+\infty} x_2(n) = 0. \tag{33}$$

Set $y(n) = \ln x_1(n) - \ln x(n)$, then it follows from the first equation of system (6) and (31) that

$$\begin{aligned}
&y(n+1)\\
&= \ln x_1(n) + \{r_1(n) - a_1(n)x_1(n)\\
&\quad - \frac{b_1(n)x_2(n)}{\alpha_1(n) + \beta_1(n)x_1(n) + \gamma_1(n)x_2(n)}\}\\
&\quad - \ln x(n) - \{r_1(n) - a_1(n)x(n)\},\\
&= y(n) - a_1(n)(x_1(n) - x(n))\\
&\quad - \frac{b_1(n)x_2(n)}{\alpha_1(n) + \beta_1(n)x_1(n) + \gamma_1(n)x_2(n)}\\
&= y(n) - a_1(n)x(n)[\exp(y(n)) - 1]\\
&\quad - \frac{b_1(n)x_2(n)}{\alpha_1(n) + \beta_1(n)x_1(n) + \gamma_1(n)x_2(n)}
\end{aligned} \tag{34}$$

Using the mean value theorem, we can obtain

$$\exp(y(n)) - 1 = y(n)\exp[\theta(n)y(n)], \quad \theta(n) \in (0,1). \tag{35}$$

Substituting (35) into the equation (34), we can get

$$\begin{aligned}
y(n+1) &= [1 - a_1(n)x(n)\exp(\theta(n)y(n))]y(n)\\
&\quad - \frac{b_1(n)x_2(n)}{\alpha_1(n) + \beta_1(n)x_1(n) + \gamma_1(n)x_2(n)}
\end{aligned} \tag{36}$$

Considering (H_3) implies that $-1 < 1 - a_1^u M_1$, there exists a small enough $\varepsilon > 0$ such that

$$-1 < 1 - a_1^u(M_1 + \varepsilon). \tag{37}$$

According to Lemma 3.2, Lemma 3.4, and (33), for the above $\varepsilon > 0$, there exists large enough $N > 0$, such that, for $n \ge N$,

$$\begin{aligned}
&m_1 + \varepsilon \le x_1(n) \le M_1 + \varepsilon,\\
&m_1 + \varepsilon \le x(n) \le M_1 + \varepsilon, \quad x_2(n) \le \varepsilon.
\end{aligned} \tag{38}$$

Note that $\theta(n) \in (0,1)$ implies that $x(n)\exp(\theta(n)y(n))$ lies between $x(n)$ and $x_1(n)$. From (36) and (38), for $n \ge N$, one can get

$$\begin{aligned}
|y(n+1)| \quad &\le \quad \max\{|1 - a_1^u(M_1 + \varepsilon)|,\\
&\qquad |1 - a_1^l(m_1 - \varepsilon)|\}|y(n)| + \frac{b_1^u}{\alpha_1^l}\varepsilon\\
&= \quad \lambda_\varepsilon|y(n)| + \frac{b_1^u}{\alpha_1^l}\varepsilon,
\end{aligned} \tag{39}$$

where $\lambda_\varepsilon = \max\{|1 - a_1^u(M_1 + \varepsilon)|, |1 - a_1^l(m_1 - \varepsilon)|\}$. This implies that

$$|y(n)| \le \lambda_\varepsilon^{n-N}|y(N)| + \frac{1 - \lambda_\varepsilon^{n-N}}{1 - \lambda_\varepsilon}\frac{b_1^u}{\alpha_1^l}\varepsilon, \quad (n \ge N). \tag{40}$$

Note that

$$-1 < 1 - a_1^u(M_1 + \varepsilon) \le 1 - a_1^l(m_1 - \varepsilon) < 1,$$

hence $0 < \lambda_\varepsilon < 1$. Thus, $\lim_{n\to+\infty} y(n) = 0$ can be immediately obtained by (40), and so $\lim_{n\to+\infty}(x_1(n) - x(n)) = 0$. This ends the proof of Theorem 3.1.

Similarly, by using Lemmas 3.3 and 3.5, we have the following theorem.

Theorem 3.2. *In addition to the conditions of Theorem 2.2,* *further suppose that* (H_4) $\dfrac{a_2^u}{a_2^l}\exp(r_2^u-1)<2$. *Then for any positive solution* $(x_1(n),x_2(n))^T$ *of system (6) and any positive solution* $\tilde{x}(n)$ *of system (32), we have*

$$\lim_{n\to+\infty}(x_2(n)-\tilde{x}(n))=0,\qquad \lim_{n\to+\infty}x_1(n)=0.$$

IV. Numeric simulations

In this section, we give the following two examples to verify the feasibilities of our results.

Example 4.1. Consider the following system:

$$\begin{aligned}
x_1(n+1) &= x_1(n)\exp\{1.5-1.5x_1(n)\\
&-\frac{(1+0.8sin(\sqrt{7}n))x_2(n)}{0.6+(0.2+sin(\sqrt{5}n))x_1(n)+0.2x_2(n)}\},\\
x_2(n+1) &= x_2(n)\exp\{0.7-0.9x_2(n)\\
&-\frac{(3+cos(\sqrt{5}n))x_1(n)}{0.9+(0.6+0.1cos(\sqrt{4}n))x_1(n)+0.2x_2(n)}\},
\end{aligned}\tag{41}$$

Take easy calculation, we have $\dfrac{r_2^u}{r_1^l}\approx 0.4667$, $M_1\approx 1.0991$, $M_2\approx 0.8231$, $\dfrac{b_2^l}{a_1^u(\alpha_2^u+\beta_2^uM_1+\gamma_2^uM_2)}\approx 0.7270$, $\dfrac{a_2^l\alpha_1^l}{b_1^u}\approx 0.63$. Thus, condition (H_1) is satisfied and it follows from Theorem 2.1 that, for any positive solution $(x_1(n),x_2(n))^T$ of system (41), we have $\lim_{n\to+\infty}(x_1(n)-x(n))=0$, $\lim_{n\to+\infty}x_2(n)=0$, where $x(n)$ is any positive solution of the system

$$x_1(n+1)=x_1(n)\exp\{1.5-1.5x_1(n)\}.$$

Figure 1 supports the conclusion.

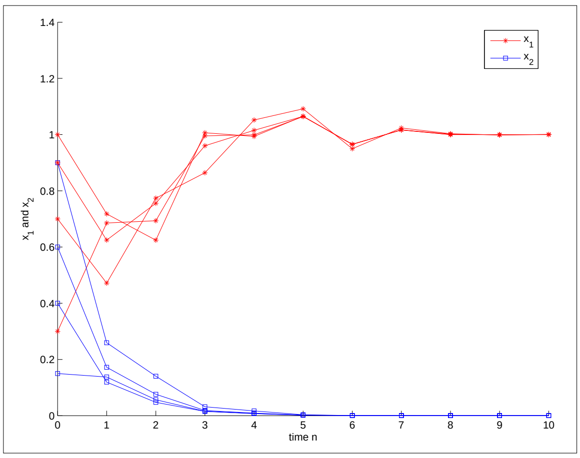

Fig. 1. Dynamic behaviors of system (41) with initial conditions $(x_1(n),x_2(n))=(1,0.4)^T$, $(0.3,0.15)^T$, $(0.7,0.9)^T$, and $(0.9,0.6)^T$, respectively.

Example 4.2. Consider the following system:

$$\begin{aligned}
x_1(n+1) &= x_1(n)\exp\{0.4-1.75x_1(n)\\
&-\frac{(3.4+0.4sin(\sqrt{3}n))x_2(n)}{2+(3.5+0.5cos(\sqrt{7}n))x_1(n)+x_2(n)}\},\\
x_2(n+1) &= x_2(n)\exp\{1.6-1.3x_2(n)\\
&-\frac{(3+cos(\sqrt{5}n))x_1(n)}{0.8+(0.6+0.4sin(\sqrt{7}n))x_1(n)+0.2x_2(n)}\},
\end{aligned}\tag{42}$$

Take easy calculation, we have $\dfrac{r_2^l}{r_1^u}=4$, $M_1\approx 0.3136$, $M_2\approx 1.4016$, $\dfrac{b_2^u}{\alpha_2^la_1^l}\approx 2.8571$, $\dfrac{a_2^u(\alpha_1^u+\beta_1^uM_1+\gamma_1^uM_2)}{b_1^l}\approx 2.0176$. It shows that (H_2) holds and according to Theorem 2.2, for system (42), species $(x_1(n)$ is driven to extinction while species $(x_2(n)$ is asymptotic to any positive solution of

$$x_2(n+1)=x_2(n)\exp\{1.6-1.3x_2(n)\}.$$

Figure 2 also supports our result.

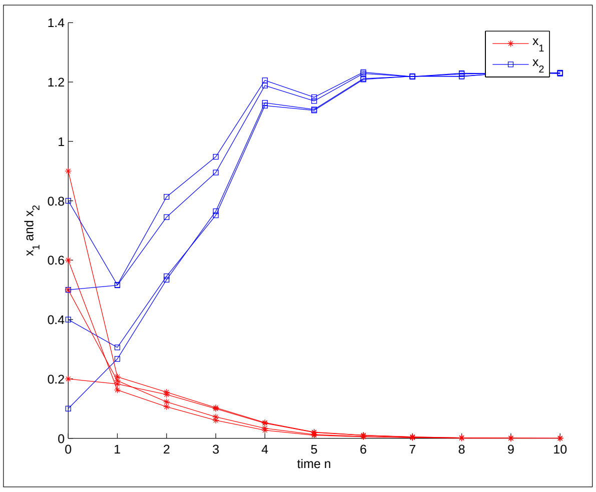

Fig. 2. Dynamic behaviors of system (42) with initial conditions $(x_1(n),x_2(n))=(0.9,0.4)^T$, $(0.2,0.1)^T$, $(0.6,0.8)^T$, and $(0.5,0.5)^T$, respectively.

V. Conclusion

In this paper, we study a discrete competitive system with Beddington-DeAngelis functional response and obtain sufficient conditions on partial extinction and stability property of the other species. When $\beta_i(n)=0$, $\alpha_i(n)=\gamma_i(n)=1$ $(i=1,2)$, (6) becomes (2) which was investigated by Qin et al. [3] and Wang et al. [4]. Moreover, when $\alpha_i(n)=1$, $\gamma_i(n)=0$ $(i=1,2)$, system (6) reduces to system (3) studied by Ma et al. [8]. So our results generalize [3,5,8].

Acknowledgment

The work is supported by the Youth science and technology Program of Fujian Provincial Education Department (JAT170771), the Natural Science Foundation of Fujian Province (2019J01089) and Program for New Century Excellent Talents in Fujian Province University (2018).

REFERENCES

[1] K. Gopalsamy, "Stability and Oscillations in Delay Differential Equation of Population Dynamics," *Mathematics and Its ApplicationsKluwer Academic Publishers, Dordrecht, The Netherlands*, vol. 74, 1992.

[2] Q. Wang, Z. Liu, Z. Li, and R. A. Cheke, "Existence and global asymptotic stability of positive almost periodic solutions of a two-species competitive system," *Int. J. Biomath.*, vol. 7, no. 4, Article ID 1450040, 18 pages, 2014.

[3] W. Qin, Z. Liu and Y. Chen, "Permanence and global stability of positive periodic solutions of a discrete competitive system," *Discrete Dyn. Nat. Soc.*, vol. 2009, Article ID Art. ID 830537,13 pages, 2009.

[4] Q. Wang, Z. Liu, "Uniformly asymptotic stability of positive almost periodic solutions for a discrete competitive system," *J. Appl. Math.*, vol. 2013, Article ID 182158, 9 pages, 2013.

[5] Q. Wang, Z. Liu and Z. Li, "Positive almost periodic solutions for a discrete competitive system subject to feedback controls," *J. Appl. Math.*, vol. 2013, Article ID 429163, 14 pages, 2013.

[6] S. Yu, "Permanence for a discrete competitive system with feedback controls," *Commun. Math. Biol. Neurosci.*, vol. 2015, Article ID 16, 2015.

[7] Q. Yue, "Extinction for a discrete competition system with the effect of toxic substances," *Adv. Differ. Equ.*, Article ID 1, 15 pages, 2016.

[8] H. Ma, J. Gao and L. Xie, "Global stability of positive periodic solutions and almost periodic solutions for a discrete competitive system," *Discrete Dyn. Nat. Soc.*, vol. 2015, Article ID 658758, 13 pages, 2015.

[9] F. Chen, X. Chen, and S. Huang, "Extinction of a two species non-autonomous competitive system with Beddington-DeAngelis functional response and the effect of toxic substances," *Open Math.*, vol. 14, pp. 1157-1173, 2016.

[10] S. Yu and F. Chen, "Dynamic behaviors of a competitive system with Beddington-DeAngelis functional response," *Discrete Dyn. Nat. Soc.*, vol. 2019, Article ID 4592054, 12 pages, 2019.

[11] L. Chen, L. Chen and Z. Li, "Permanence of a delayed discrete mutualism model with feedback controls," *Math. Comput. Model.*, vol. 50, pp. 1083-1089, 2009.

[12] F. Chen, "Permanence for the discrete mutualism model with time delays," *Math. Comput. Model.*, vol. 47, pp. 431-435, 2008.

[13] F. Chen, X. Gong and W. Chen, "Extinction in two dimensional discrete Lotka-Volterra competitive system with the effect of toxic substances." *Dyn. Contin. Discrete Impuls. Syst. Ser. B, Appl. Algorithms*, vol. 20, pp. 449-461, 2013.

[14] S. Yu, J. Zhang, "Permanence of a discrete modified Leslie-Gower Predator-prey system with delay and feedback controls." *Acta Anal. Funct.*, Appl. vol. 16, no.3, pp. 244-249, 2014.

[15] S. Yu, "Effect of Predator Mutual Interference on an Autonomous Leslie-Gower Predator-prey Model. " *IAENG International Journal of Applied Mathematics*, vol. 49, no.2, pp. 229-233, 2019.

[16] L. Pang, Y. Liao and T. Zhang, "Almost periodic solutions for Lasota-Wazewska model with multiple delays, " *IAENG International Journal of Applied Mathematics*, vol. 46, no.3, pp. 311-316, 2016.

[17] Y. Li, L. Xu and T. Zhang, "Dynamics of almost periodic mutualism model with time delays," *IAENG International Journal of Applied Mathematics*, vol. 48, no.2, pp. 168-176, 2018.

[18] Q. Yue, "Permanence of a Delayed Biological System with Stage Structure and Density-dependent Juvenile Birth Rate," *Engineering Letters*, vol. 27, no.2, pp. 263-268, 2019.

[19] B. Chen, "The Influence of Density Dependent Birth Rate to a Commensal Symbiosis Model with Holling Type Functional Response," *Engineering Letters*, vol. 27, no.2, pp. 295-302, 2019.

[20] L. Wang, "Dynamic Analysis on an Almost Periodic Predator-Prey Model with Impulses Effects," *Engineering Letters*, vol. 26, no.3, pp. 333-339, 2018.

[21] Y. Liao, "Dynamics of Two-species Harvesting Model of Almost Periodic Facultative Mutualism with Discrete and Distributed Delays," *Engineering Letters*, vol. 26, no.1, pp. 7-13, 2018.