Research on Risk-averse Retailer's Spot Procurement Decision under Fuzzy Demand

Yanchun Wan, Qiucen Chen

Abstract—**In the operation of supply chains, market demand uncertainty often leads to supply-demand imbalance. In the research of supply chain contracts, scholars often consider the market to be deterministic or random and presume that supply chain participants are rational. However, market demand is not only random but also fuzzy, especially for short life-cycle products with high risk, for which supply chain participants often have certain risk preferences when ordering these products. This paper considers a two-echelon supply chain system consisting of one supplier and one retailer under fuzzy demand and uses fuzzy mathematics to study the optimal decision of the risk-averse retailer when he/she takes spot contracts. First, regarding market demand as a triangular fuzzy variable, this paper provides a spot purchasing model of the risk-neutral retailer based on the newsboy model. Second, this paper uses CVaR theory to study the risk-averse retailer's decision-making behavior and derives the optimal decision by fuzzy set theory. Finally, a numerical study is provided to verify the relationships among the parameters, optimal decision and retailer's profit and shows that the optimal spot order quantity fluctuates between a fuzzy low value and fuzzy high value of the fuzzy market demand represented by the triangular fuzzy number. It is necessary to consider the fuzziness of market demand and retailer's risk preference when studying some short life-cycle products.**

Index Terms—**fuzzy demand, procurement decision, risk-averse, spot contract**

I. INTRODUCTION

IN recent years, fierce competition, price volatility, and uncertain supply and demand in supply chain coordination have attracted increasing attention from industries and researchers. In the process of purchasing and supply chain management, appropriate procurement decisions are particularly important.

At present, retailers normally use supply chain contracts to purchase products from suppliers. In the research field, the study of supply chain contracts has been quite mature, and the research results are fairly rich. The application of supply chain contracts, such as wholesale price contracts, buy-back contracts, revenue sharing contracts, quantity discount contracts and price discount contracts, effectively coordinates the supply chain [1]. When coping with risks in terms of market demand, the above contracts reduce risk by adjusting the order volume and price. However, when studying market demand uncertainty, most spot contract models assume that market demand is subject to a certain probability distribution. In fact, it is difficult for decision makers to describe the fluctuation rules of market demand with exact data or probability theory due to the lack of historical data and sufficient information, especially for short life-cycle products, such as fresh agriproducts, high-end clothing and electronics. With the development of mathematical theory, scholars find that market demand uncertainty is mostly fuzzy. Fuzzy mathematics is a powerful tool for quantifying market demand, the attitudes of players and so on. Therefore, it is of great practical significance to study the spot contracts used by supply chains under fuzzy demands.

Fuzzy theory, first proposed by Zadeh in the international academic journal *Control and Information* in 1965, is mainly used to study uncertainty [2]. In 1976, fuzzy mathematics received widespread attention from scholars in China. In 1978, Zadeh [3] proposes the concept of a possibility measure, which is used to measure fuzzy sets. However, such a possibility measure has the limitation of lacking self-duality. To make up for this defect, B. Liu and K. Liu [4] put forward the concept of a credibility measure in 2002. Then, B. Liu [5] establishes a perfect axiomatic system of credibility theory in 2004 that lays a solid mathematical foundation for later research on fuzzy environment decisions. At present, fuzzy mathematics is widely used in computer simulations, portfolio evaluation, asset evaluation, quality evaluation, disaster forecasting and other fields and has achieved increasingly good results.

In a supply chain system, the demand market is a complex system with multiple types of uncertainty, and there is fuzziness in the fluctuation of the demand market. Using fuzzy theory to study supply chain contracts has increasingly attracted the attention of scholars. Wang et al. [6][7] study a revenue-sharing supply chain contract, single-term transaction contract and long-term transaction contract with the supplier as the leader and the retailer as the follower under a fuzzy demand environment. Analyzing a two-echelon supply chain system composed of a single supplier and retailer in a fuzzy demand environment, Sang et al. [8] assume market demand is a fuzzy variable, construct an integrated supply chain model and revenue sharing contract model based on credibility distribution and identify the optimal strategy.

Ryu and Yücesan [9] assume that market demand, the retail price and the sales price are fuzzy variables and study the supply chain coordination problem of quantity discount contracts, revenue sharing contracts and buy-back contracts under fuzzy demand. Considering fuzzy demand and cost structure, Sang [10] designs a benefit sharing contract based

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on fuzzy demand and cost to coordinate the supply chain by analyzing the influence of centralized decision-making and distributed decision-making on the supply chain participants. Sang [11] regards market demand as a positive trapezoidal fuzzy number and constructs models of centralized decision-making systems and spanning revenue sharing to study the behavior of a three-stage supply chain. Hong [12] considers the market demand function, manufacturing costs, and retail operating costs as triangular fuzzy variables and uses a Stackelberg game to solve the problem between the retailer and the manufacturer and identify the optimal wholesale price and order quantity. However, most of the above studies assume that the supply chain participants are risk neutral and seldom take their risk preferences into consideration. To study the supply chain management concepts of benefit and risk sharing and participants' risk preferences, Zhao et al. [13] use fuzzy random numbers to express demand information updating, construct a coordination model for a buy-back contract and propose pessimistic, optimistic and risk-neutral decision criteria.

Existing research on using fuzzy mathematics in supply chain spot contracts has the following characteristics:

1) Most of the literature considers only randomness and ignores the fuzziness of the external environment of the supply chain when studying the demand market.
2) As an important branch of mathematics, fuzzy mathematics is widely used in economics, science and technology and other fields. In the study of supply chain contracts, fuzzy mathematics can effectively depict the fuzziness of the demand market, which is closer to reality.
3) In the study of spot contracts under fuzzy mathematics, most of the literature assumes that the participants are risk neutral and pays less attention to their risk preferences.

This paper makes the following contributions. First, we regard market demand as a triangular fuzzy variable for short life-cycle products. Second, we use CVaR theory to study the retailer's decision-making behavior and describe the process used to identify the optimal decisions. This paper is organized as follows. Section 2 introduces some definitions and propositions about fuzzy set theory and the typical procurement model. Section 3 describes the problem and proposes a procurement model under fuzzy demand. A numerical example and some analyses are provided in Section 4. Section 5 summarizes the work and provides directions for future studies.

II. Preliminaries

A. Definitions and Propositions for the Fuzzy Sets

Definition 1. Let $\tilde{A}$ be a fuzzy set of domain U ; for any $x \in U$, there exits $\mu_{\tilde{A}} \in [0,1]$ to indicate that the extent of x that belongs to $\tilde{A}$. Then, we use $\mu_{\tilde{A}} : U \to [0,1]$ to show the membership functions of $\tilde{A}$, and $\mu_{\tilde{A}}$ refers to the membership of the element in U in fuzzy set $\tilde{A}$.

Definition 2. A fuzzy variable $\tilde{A} = (\underline{a},\ a,\ \overline{a})$ is called a triangular fuzzy variable when it has the following membership function [14]

$$\mu_{\tilde{A}(x)} = \begin{cases} \dfrac{x-\underline{a}}{a-\underline{a}}, & if\ \underline{a} \le x \le a \\ \dfrac{\overline{a}-x}{\overline{a}-a}, & if\ a < x \le \overline{a} \\ 0, & if\ x < \underline{a}\ or\ x > \overline{a} \end{cases}$$

where $\underline{a},\ a,\ \overline{a}$ are real numbers and $\underline{a} < a < \overline{a}$. The support set θ of $\mu_{\tilde{A}(x)}$ is $[\underline{a}, \overline{a}]$.

Proposition 1. Let $\tilde{A}$ be a triangular fuzzy variable that has the credibility density function $\varphi(x)$ and credibility distribution function $\phi(x)$ [5]

$$\varphi(x) = \begin{cases} \dfrac{1}{2(a-\underline{a})}, & if\ \underline{a} \le x \le a \\ \dfrac{1}{2(\overline{a}-a)}, & if\ a < x \le \overline{a} \\ 0, & if\ x < \underline{a}\ or\ x > \overline{a} \end{cases}$$

where $\underline{a},\ a,\ \overline{a}$ are real numbers and $\underline{a} < a < \overline{a}$.

$$\phi(x) = \begin{cases} 0, & if\ x < \underline{a} \\ \dfrac{x-\underline{a}}{2(a-\underline{a})}, & if\ \underline{a} \le x \le a \\ \dfrac{x+\overline{a}-2a}{2(\overline{a}-a)}, & if\ a < x \le \overline{a} \\ 1, & if\ x > \overline{a} \end{cases}$$

where $\underline{a},\ a,\ \overline{a}$ are real numbers and $\underline{a} < a < \overline{a}$.

Proposition 2. Let ε be a fuzzy variable with support set $\theta = (u,v)$ that has a credibility density function $\varphi(x)$ and credibility density function; then, we have [8]

$$E[\min(z,\varepsilon)] = z - \int_u^z (z-x)\varphi(x)dx$$

where $0 \le u \le z \le v$.

B. Newsboy Model

This paper considers a two-echelon supply chain consisting of one retailer and one supplier who provides a short life-cycle product. The retailer faces a newsboy problem. The supply chain participants are risk neutral, and the information is symmetrical.

The following notations are used in the newsboy model.

Parameters

D : the random market demand of product and the actual value of x ;

$f(x)$: the probability density function of random market demand;

$F(x)$: the distribution function of random market demand; this parameter is nonnegative, derivable and strictly increasing;

p : the unit price of the product;

w : the unit wholesale cost of the product;

s : the unit shortage cost of the product; and

v : the unit salvage value of the product, where $0 < v < p$.

Decision variables

q : the order quantity;

π_r : the profit of the retailer.

The retailer's expected profit function is formulated as:

$$E[\pi_r(q)] = \int_0^q [px + v(q-x) - wq] f(x)dx + \int_q^\infty [pq - s(x-q) - wq] f(x)dx$$

Therefore, the problem can be summarized as follows: when $f(x)$, p, w, s, and v are known, the optimal q is found to maximize $E[\pi_r(q)]$; therefore, this is a typical newsboy model.

III. Problem Analysis and Mode Formulation

A. Problem Description

This paper considers a two-echelon supply chain consisting of one risk-averse retailer and one risk-neutral supplier who provides a short life-cycle product, and the retailer faces uncertain market demand that is hard to describe by any random numbers.

Before the sales season, the supplier provides a wholesale price contract to the retailer that determines the order quantity based on the market demand information obtained. During the sales season, the retailer is responsible for selling the products; after the sales season, the retailer has to dispose of the unsold products and bears all risks of market demand uncertainty and inventory costs, while the supplier's profits are fixed [15].

As the most commonly used supply chain contract, the wholesale price contract, characterized by simple operation, easy implementation and low implementation cost, is often used in a decentralized decision-making supply chain with a low unit price and high procurement cost. In reality, supply chain participants prefer to use a wholesale price contract, which is the simplest form of contract, and the additional burden of using other coordination contracts exceeds the potential benefits [16].

B. Assumptions and Symbol Definitions

The assumptions related to the market environment are as follows:

Assumption 1. The supplier is risk neutral, while the retailer is risk averse with the purpose of achieving the maximum conditional value for a specific level of risk.

Assumption 2. Before the sales season, the supplier can deliver all orders to the retailer on time.

Assumption 3. The two supply chain participants are symmetrical, which means that they know all the costs, contract parameters and market rules.

Assumption 4. The external requirements can be roughly predicted and represented by a triangular fuzzy number.

The following notations are used in the spot procurement model under fuzzy demand.

Parameters

$\tilde{D}$: the fuzzy market demand of the product; $\tilde{D}$ is a triangular fuzzy number;

p : the unit price of the product;

w : the unit wholesale cost of the product, where $0 < w < p$;

s : the unit shortage cost of the product;

v : the unit salvage value of the product, where $s < p - v$ and $0 < v < w$.

Decision variables

q : the order quantity;

$\tilde{\pi}_r(q)$: the fuzzy profit of the retailer.

C. Construction of a Spot Purchasing Model for a Risk-neutral Retailer under Fuzzy Demand

Giannoccaro et al. suggested that the decision-making methods used for supply chain coordination can be divided into centralized decision-making and distributed decision-making [17]. Centralized decision-making is generally applied to a supply chain system with only one decision maker, which is infrequent in reality; distributed decision-making is generally applied to a supply chain system with different decision makers in different supply chain links, which frequently occurs in reality. This paper applies distributed decision-making to study the supply chain system under fuzzy demand.

In a distributed decision-making supply chain, the main factors affecting the retailer's profit are sales income, salvage value income, shortage cost and procurement cost, where the retailer's fuzzy sales volume, fuzzy inventory volume, and fuzzy shortage volume are $\min(\tilde{D}, q)$, $\max(q - \tilde{D}, 0)$, and $\max(\tilde{D} - q, 0)$, respectively.

This paper denotes the expected fuzzy sales volume, expected fuzzy inventory volume, and expected fuzzy shortage volume as $S(q)$, $I(q)$, and $O(q)$, respectively. Therefore, the expected fuzzy profit function of the risk-neutral retailer can be described as follows:

$$E[\tilde{\pi}_r(q)] = p \times S(q) + v \times I(q) - s \times O(q) - wq \quad (1)$$

In a distributed decision-making supply chain, the retailer's goal is maximizing the retailer's fuzzy expected revenue $\tilde{\pi}_r(q)$ by setting the optimal order quantity q. Therefore, two issues need to be addressed: first, how can fuzzy market demand be solved? Second, how can fuzzy functions be solved?

1) Fuzzy market demand representation

When one or more retailers cannot influence the market price, which is determined by the external market, the market demand $\tilde{D}$ can be expressed as a single fuzzy number and most commonly is expressed as a triangular fuzzy number [12][15][18][19]. It is more intuitive and realistic to use triangular fuzzy numbers to represent uncertain information regarding market demand. The triangular fuzzy number and related fuzzy algorithms used in this paper are mainly based on the perfect axiomatic system of credibility theory developed by Liu [5].

This paper assumes that market demand is a triangular fuzzy number $\tilde{D} = (\underline{a}, a, \overline{a})$, where $\underline{a} < a < \overline{a}$ and $\underline{a} < q < \overline{a}$. Market demand has a credibility density function $\varphi(x)$ and credibility distributions function $\phi(x)$.

2) Solving the fuzzy functions

To maximize the fuzzy expected profit $\tilde{\pi}_r$, the optimal order quantity q must be determined. Therefore, the problem can be described as

$$\max_q E[\tilde{\pi}_r(q)] = p \times S(q) + v \times I(q) - s \times O(q) - wq \quad (2)$$

where $\underline{a} < q < \overline{a}$.

In addition, $S(q)$, $I(q)$, $O(q)$ can be calculated as follows:

$$S(q)=E\left[\min\left(\tilde{D},q\right)\right]=q-\int_{\underline{a}}^{q}(q-x)\varphi(x)dx$$

$$I(q)=E[\max(q-\tilde{D},0)]=-E[\min(q,\tilde{D})-q]$$
$$=-E[\min(\tilde{D},q)]+q=q-S(q)$$
$$=\int_{\underline{a}}^{q}(q-x)\varphi(x)dx$$

$$O(q)=E[\max(\tilde{D}-q,0)]=E[\tilde{D}]-E[\min(\tilde{D},q)]$$
$$=E[\tilde{D}]-S(q)=\int_{\underline{a}}^{\overline{a}}x\varphi(x)dx-S(q)$$
$$=\int_{q}^{\overline{a}}(x-q)\varphi(x)Udx$$

According to the fuzzy number algorithm, equation (2) can be written as

$$\max_q E\left[\tilde{\pi}_r(q)\right]=p\times\left(q-\int_{\underline{a}}^{q}(q-x)\varphi(x)dx\right)$$
$$+v\times\int_{\underline{a}}^{q}(q-x)\varphi(x)dx \qquad (3)$$
$$-s\times\int_{q}^{\overline{a}}(x-q)\varphi(x)dx-wq$$

where $\underline{a}<q<\overline{a}$.

According to equation (3), the first derivative $\frac{dE\left[\tilde{\pi}_r(q)\right]}{dq}$ and the second derivative $\frac{d^2E\left[\tilde{\pi}_r(q)\right]}{dq^2}$ of $E\left[\tilde{\pi}_r(q)\right]$ can be obtained as

$$\frac{dE[\tilde{\pi}_r(q)]}{dq}=p-p\phi(q)+v\phi(q)+s-s\phi(q)-w$$

$$\frac{d^2E[\tilde{\pi}_r(q)]}{dq^2}=-p\varphi(q)+v\varphi(q)-s\varphi(q)$$

Since $s<p-v$, $\frac{d^2E\left[\tilde{\pi}_r(q)\right]}{dq^2}$ is negative, $E\left[\tilde{\pi}_r(q)\right]$ is concave in q, and the optimal order quantity of a risk-neutral retailer can be obtained by solving the condition as follows:

$$p-p\phi(q)+v\phi(q)+s-s\phi(q)-w=0 \quad (4)$$

By solving equation (4), the optimal order quantity of the risk-neutral retailer under fuzzy demand can be expressed as follows:

$$q^*=\phi^{-1}\left(\frac{p+s-w}{p+s-v}\right) \quad (5)$$

One thing to note is that $\phi^{-1}(x)$ is the inverse function of $\phi(x)$. This paper sets the function $\varphi'(x)$ as the derivative function of $\phi^{-1}(x)$. Therefore, $\phi^{-1}(x)$ and $\varphi'(x)$ can be represented as

$$\phi^{-1}(x)=\begin{cases}\underline{a}, & if\ x=0\\ 2\left(a-\underline{a}\right)x+\underline{a}, & if\ 0<x\le 0.5\\ 2\left(\overline{a}-a\right)x+2a-\overline{a}, & if\ 0.5<x\le 1\\ \overline{a}, & if\ x=1\end{cases}$$

$$\varphi'(x)=\begin{cases}2(a-\underline{a}), & if\ 0\le x\le 0.5\\ 2(\overline{a}-a), & if\ 0.5<x\le 1\\ 0, & if\ x<0\ or\ x>1\end{cases}$$

Therefore, the optimal fuzzy profit of the retailer is

$$\max_q E\left[\tilde{\pi}_r(q)\right]=\left(p+s-w\right)\phi^{-1}\left(\frac{p+s-w}{p+s-v}\right)-s\overline{a}$$
$$-\left(p-v\right)\int_{\underline{a}}^{\phi^{-1}\left(\frac{p+s-w}{p+s-v}\right)}\phi(x)dx$$
$$+s\int_{\phi^{-1}\left(\frac{p+s-w}{p+s-v}\right)}^{\overline{a}}\phi(x)dx$$

Thus, we can make the following conclusions:

Conclusion 1. Since

$$\frac{\partial q^*}{\partial p}=\frac{\partial q^*}{\partial s}=\varphi'\left(\frac{p+s-w}{p+s-v}\right)\times\frac{w-v}{\left(p+s-v\right)^2}>0,$$

$$\frac{\partial q^*}{\partial v}=\varphi'\left(\frac{p+s-w}{p+s-v}\right)\times\frac{p+s-w}{\left(p+s-v\right)^2}>0,$$

$$\frac{\partial q^*}{\partial w}=-\varphi'\left(\frac{p+s-w}{p+s-v}\right)\times\frac{1}{p+s-v}<0,$$

The optimal order quantity of retailer q^* increases with an increasing unit price p, shortage cost s and salvage value v of the product and decreases with an increasing unit wholesale cost w of the product.

Conclusion 2. Since $\frac{p+s-w}{p+s-v}\in(0,1)$, $q\in\left(\underline{a},\overline{a}\right)$; that is, regardless of how the parameters change, the optimal order quantity q^* takes a value in the interval $\left(\underline{a},\overline{a}\right)$ under fuzzy demand.

Conclusion 3. When there is a change in the contract parameters, the change trend of the risk-neutral retailer's order quantity under fuzzy demand is consistent with the risk-neutral retailer's under random demand. When less market information is known, regarding market demand as a fuzzy variable can effectively improve the accuracy and reduce the risks of decision-making, and the retailer can avoid ordering too many products.

D. Construction of a Spot Purchasing Model for a Risk-averse Retailer under Fuzzy Demand

The risk preference of the retailer will have an impact on the optimal purchasing decision. The retailer sells short life-cycle products and thus tends to avoid market risk when making purchasing decisions. At present, there are three main methods for measuring risk factors: mean-variance (MV), value at risk (VaR) and conditional value at risk (CVaR) [20].

In this paper, CVaR theory is introduced to analyze the risk-averse behavior of retailers. According to the generalization definition of CVaR [21][22], the objective function of the risk-averse retailer's decision can be adjusted to

$$CVaR\left(\tilde{\pi}_r(q)\right)=\max_{u\in R}\left\{u+\frac{1}{\eta}E\left[\min\left(\tilde{\pi}_r(q)-u,0\right)\right]\right\} \quad (6)$$

Parameters

u: the value at risk at the confidence level η, which represents the possible upper income limit;

E: the CVaR criterion measures the average profit falling below the η-quantile level, which represents the expected value of a decision variable;

η: the degree of risk aversion, with a confidence level of $\eta\in(0,1]$. When $\eta=1$, the retailer is risk neutral. As η decreases, the retailer becomes more risk averse.

To simplify the computation, we set the function as

$$G(q,u)=u+\frac{1}{\eta}E\left[\min\left(\tilde{\pi}_r(q)-u,0\right)\right] \quad (7)$$

In addition, equation (3) can be written as

$$\max_q E\left[\tilde{\pi}_r(q)\right]=\int_{\underline{a}}^{q}\left((p-w)q-(p-v)(q-x)\right)\varphi(x)dx+\int_{q}^{\bar{a}}\left((p-w)q-s(x-q)\right)\varphi(x)dx \quad (8)$$

According to equation (8), equation (7) can be written as

$$\begin{aligned}G(q,u)&=u+\frac{1}{\eta}E\left[\min\left(\tilde{\pi}_r(q)-u,0\right)\right]\\&=u-\frac{1}{\eta}\int_{\underline{a}}^{q}\left(u-(v-w)q-(p-v)x\right)^{+}\varphi(x)dx\\&\quad-\frac{1}{\eta}\int_{q}^{\bar{a}}\left(u-(p-w)q+s(x-q)\right)^{+}\varphi(x)dx\end{aligned} \quad (9)$$

Let us calculate the optimal q and u that will maximize $G(q,u)$.

We set the notation as follows:

$$q_1=\frac{u-(v-w)q}{p-v}$$

$$q_2=\frac{(p-w+s)q-u}{s}$$

$$y_1=(p-v)\underline{a}+(v-w)q$$

$$y_2=(p-w)q-s(\bar{a}-q)$$

$$y_3=(p-w)q$$

The maximum problem will be discussed by considering two situations: $y_1\le y_2$ and $y_1>y_2$.

Situation 1. If $y_1\le y_2$, then $q\ge\frac{(p-v)\underline{a}+s\bar{a}}{p+s-v}$.

In this situation, this paper discusses the problem in four cases: $u\le y_1\le y_2\le y_3$, $y_1\le u\le y_2\le y_3$, $y_1\le y_2\le u\le y_3$ and $y_1\le y_2\le y_3\le u$.

Case 1. When $u\le y_1\le y_2\le y_3$, equation (9) can be written as

$$G(q,u)=u$$

Then, it is easy to obtain

$$\frac{\partial G(q,u)}{\partial u}=1>0$$

$$\frac{\partial^2 G(q,u)}{\partial u^2}=0$$

Thus, there is no optimal u^* that maximizes $G(q,u)$.

Case 2. When $y_1\le u\le y_2\le y_3$, equation (9) can be written as

$$\begin{aligned}G(q,u)&=u\\&\quad-\frac{1}{\eta}\int_{\underline{a}}^{q_1}\left(u-(v-w)\frac{u-q_1(p-v)}{v-w}-(p-v)x\right)\varphi(x)dx\\&=u-\frac{p-v}{\eta}\left(\int_{\underline{a}}^{q_1}\phi(x)dx-(q_1-\underline{a})\phi(\underline{a})\right)\\&=u-\frac{p-v}{\eta}\int_{\underline{a}}^{q_1}\phi(x)dx\end{aligned}$$

Then, it is easy to obtain

$$\frac{\partial G(q,u)}{\partial u}=1-\frac{1}{\eta}\left(\phi(q_1)-\phi(\underline{a})\right)=1-\frac{1}{\eta}\phi(q_1)$$

$$\frac{\partial^2 G(q,u)}{\partial u^2}=-\frac{1}{\eta}\varphi(q_1)\frac{1}{p-v}<0$$

By solving $\frac{\partial G(q,u)}{\partial u}=0$, it is easy to obtain

$$u_1^*=(p-v)\phi^{-1}(\eta)+(v-w)q$$

Obviously, $u_1^*\ge y_1$.

Therefore, if $u_1^*<y_2$, that is, if $q<\frac{(p-v)\phi^{-1}(\eta)+s\bar{a}}{p+s-v}$, there is an optimal $u^*=u_1^*$ that maximizes $G(q,u)$.

Then, is easy to obtain

$$\frac{\partial G(q,u)}{\partial u}=\frac{v-w}{\eta}\phi(q_1)=v-w<0$$

Therefore, $G(q,u)$ is a strictly decreasing function of q, and the optimal order quantity q^* equals $\frac{(p-v)\underline{a}+s\bar{a}}{p+s-v}$ where $\frac{(p-v)\underline{a}+s\bar{a}}{p+s-v}\le q<\frac{(p-v)\phi^{-1}(\eta)+s\bar{a}}{p+s-v}$.

If $u_1^*\ge y_2$, that is, if $q\ge\frac{(p-v)\phi^{-1}(\eta)+s\bar{a}}{p+s-v}$, there is no optimal u^* that maximizes $G(q,u)$. This is discussed in Case 3.

Case 3. When $y_1\le y_2\le u\le y_3$, equation (9) can be written as

$$\begin{aligned}G(q,u)&=u-\frac{p-v}{\eta}\left(\int_{\underline{a}}^{q_1}\phi(x)dx-(q_1-\underline{a})\phi(\underline{a})\right)\\&\quad-\frac{s}{\eta}\left((\bar{a}-q_2)\phi(\bar{a})-\int_{q_2}^{\bar{a}}\phi(x)dx\right)\\&=u-\frac{p-v}{\eta}\int_{\underline{a}}^{\frac{u-(v-w)q}{p-v}}\phi(x)dx\\&\quad-\frac{s}{\eta}\left(\bar{a}-\frac{(p-w+s)q-u}{s}\right)\phi(\bar{a})\\&\quad+\frac{s}{\eta}\int_{\frac{(p-w+s)q-u}{s}}^{\bar{a}}\phi(x)dx\end{aligned}$$

Then, it is easy to obtain

$$\frac{\partial G(q,u)}{\partial u}=1-\frac{1}{\eta}\left(1+\phi(q_1)-\phi(q_2)\right)$$

$$\frac{\partial^2 G(q,u)}{\partial u^2}=-\frac{1}{\eta}\left(\varphi(q_1)\frac{1}{p-v}+\varphi(q_2)\frac{1}{s}\right)<0$$

Therefore, $G(q,u)$ is concave in u. Let $u_2^*(q)$ be the solution of equation

$$1-\frac{1}{\eta}\left(1+\phi(q_1)-\phi(q_2)\right)=0$$

If $u_1^*\ge y_2$ that is, if $q\ge\frac{(p-v)\phi^{-1}(\eta)+s\bar{a}}{p+s-v}$, there is an optimal $u^*=u_2^*(q)$ that maximizes $G(q,u)$.

By solving the following equation composition

$$\frac{\partial G(q,u)}{\partial u}=0$$

$$\frac{\partial G(q,u)}{\partial q}=0,$$

it is easy to obtain

$$u_2^*=\frac{s(v-w)\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}{p+s-v}+\frac{(p-w+s)(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$$

$$q_1^*=\frac{s\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)+(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$$

Since $q\ge\frac{(p-v)\phi^{-1}(\eta)+s\bar{a}}{p+s-v}$, and

$$q_1^*=\frac{s\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)+(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v},$$
$$<\frac{(p-v)\phi^{-1}(\eta)+s\bar{a}}{p+s-v}$$

the optimal q and u that maximize $G(q,u)$ are

$$u^*=\frac{s(v-w)\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}{p+s-v}+\frac{(p-w+s)(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$$

$$q^*=\frac{(p-v)\phi^{-1}(\eta)+s\bar{a}}{p+s-v}$$

Case 4. When $y_1\le y_2\le y_3\le u$, equation (9) can be written as

$$G(q,u)=u-\frac{1}{\eta}\int_{\underline{a}}^{q}\big(u-(v-w)q-(p-v)x\big)\varphi(x)dx-\frac{1}{\eta}\int_{q}^{\bar{a}}\big(u-(p-w)q+s(x-q)\big)\varphi(x)dx$$

Then, it is easy to obtain

$$\frac{\partial G(q,u)}{\partial u}=1-\frac{1}{\eta}\int_{\underline{a}}^{q}\varphi(x)dx-\frac{1}{\eta}\int_{q}^{\bar{a}}\varphi(x)dx=1-\frac{1}{\eta}\le 0$$

$$\frac{\partial^2 G(q,u)}{\partial u^2}=0$$

Thus, there is no optimal u^* that maximizes $G(q,u)$.

Situation 1 is summarized as follows:

In Case 2, the optimal solutions are

$$u^*=(p-v)\phi^{-1}(\eta)+(v-w)q$$

$$q^*=\frac{(p-v)\underline{a}+s\bar{a}}{p+s-v}$$

Here, we can infer that

$$y_1=y_2<(p-v)\phi^{-1}(\eta)+(v-w)q\le y_3,$$

which is discussed in Case 3.

Therefore, the solutions in Case 3 are the optimal solutions of Situation 1. The optimal solutions are

$$u^*=\frac{s(v-w)\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}{p+s-v}+\frac{(p-w+s)(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$$

$$q^*=\frac{(p-v)\phi^{-1}(\eta)+s\bar{a}}{p+s-v}$$

At this moment, we set the maximum of $G(q,u)$ as g_1.

Situation 2. If $y_1>y_2$, then $q<\frac{(p-v)\underline{a}+s\bar{a}}{p+s-v}$.

The method used in Situation 2 is also used in Situation 1. Therefore, we can obtain the optimal solutions of Situation 2

$$u^*=\frac{s(v-w)\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}{p+s-v}+\frac{(p-w+s)(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$$

$$q^*=\frac{s\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)+(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$$

At this moment, we set the maximum of $G(q,u)$ as g_2.

By combining Situation 1 and Situation 2, we can prove this theorem:

$$g_1\le g_2$$

We set the function as

$$h(q)=u-\frac{p-v}{\eta}\int_{\underline{a}}^{\frac{u-(v-w)q}{p-v}}\phi(x)dx-\frac{s}{\eta}\left(\bar{a}-\frac{(p-w+s)q-u}{s}\right)\phi(\bar{a})+\frac{s}{\eta}\int_{\frac{(p-w+s)q-u}{s}}^{\bar{a}}\phi(x)dx$$

while $q\in R$ and

$$u=\frac{s(v-w)\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}{p+s-v}+\frac{(p-w+s)(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$$

Obviously, when $h(q)$ reaches the maximum value, q equals $\frac{s\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)+(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$.

Therefore, $g_1\le g_2$.

In conclusion, the optimal solutions that maximize $G(q,u)$ are

$$u^{*}=\frac{s(v-w)\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}{p+s-v}+\frac{(p-w+s)(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$$

$$q^{*}=\frac{s\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)+(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$$

Therefore, it is easy to verify that

$$\begin{aligned}CVaR(\tilde{\pi}_r(q))&=\max G(q,u)\\&=\frac{s(v-w)\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}{p+s-v}\\&\quad+\frac{(p-w+s)(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}\\&\quad-\frac{p-v}{\eta}\int_{\underline{a}}^{\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}\phi(x)dx\\&\quad-\frac{s}{\eta}\left(\overline{a}-\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)\right)\\&\quad+\frac{s}{\eta}\int_{\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}^{\overline{a}}\phi(x)dx\end{aligned}$$

Therefore, we can make the following conclusions:

Conclusion 1. The optimal order quantity of retailer q^* increases with an increasing unit price p, shortage cost s and salvage value v of the product and decreases with an increasing unit wholesale cost w of the product.

Conclusion 2. $q^*\in(\underline{a},\overline{a})$. Regardless of how the parameters change, the optimal order quantity q^* will take a value in the interval $(\underline{a},\overline{a})$.

Conclusion 3. The optimal order quantity q^* is monotonically increasing according to the degree of risk aversion η; the optimal order quantity of the risk-neutral retailer with $\eta=1$ is higher than that of the risk-averse retailer with $\eta\in(0,1)$ when other parameters are the same. That is, the risk-averse retailer's decision is conservative.

Conclusion 4. When the contract parameters change, the change trend of the risk-averse retailer's order quantity is consistent with that of the risk-neutral retailer under fuzzy demand.

IV. Numerical Study

A. Basic Test

To verify the validity of the proposed fuzzy model and analyze the relationships among the parameters, optimal order quantity and retailer's profit, a numerical study is provided to investigate the impact of the degree of risk aversion and other contract parameters on optimal decision and risk profit. In particular, a short life-cycle fresh agriproduct is used as an example. Since the 21st century, with the continuous advancement of genetically modified (GM) technology, the market has been increasingly receptive to GM agriproducts. The 13th National Five-Year Plan on Science, Technology and Innovation Planning of China clearly proposes strengthening the research and development of GM cotton, corn and soybean and promotes the commercialization process of GM agriproducts. Considering that a new GM agriproduct has gained great popularity among consumers abroad, a domestic GM agriproduct retail company (hereinafter referred to as company A) decided to import this GM agriproduct to increase product diversity and enhance market competitiveness. Due to insufficient historical data on the supply chain of this GM agriproduct, it is difficult for company A and its supplier to infer the random distribution of its domestic market demand. Therefore, experts are invited to infer the fuzzy distribution of this GM agriproduct. This GM fresh agriproduct is susceptible to decay, damage, moisture and nutritional losses during production and distribution, which affect the profit of the retailer and may lead to greater externality losses if spoiled products flow to the consumers. In addition, among the supply chain members of this GM agriproduct, the supplier is a foreign supplier with a relatively wide and stable foreign market for this product. However, company A is a domestic retailer, and the domestic market reception of this GM agriproduct is fuzzy. This company will face considerable uncertainty regarding selling this GM agriproduct in the domestic market. Therefore, this paper assumes that the supplier is risk neutral and company A is risk averse. Fresh agriproducts can be used for deep processing after the sales period, so a unit salvage value is considered.

This paper assumes that the market demand of the GM agriproduct is a triangular fuzzy number $\tilde{D}=(2000,5000,6500)$. The values of other contract parameters are given as $p=10$, $w=4$, $s=0.8$, $v=1.8$, and $\eta=0.8$.

For the spot purchasing model of a risk-averse retailer under fuzzy demand, solved by MATLAB 2016b, the optimal order quantity q of company A is

$$q=\frac{s\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)+(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}\approx 5366.67$$

The value of risk u at the confidence level η is

$$u=\frac{s(v-w)\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)+(p-w+s)(p-v)\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$$

$$\approx 31762.67$$

The expected fuzzy profit G of company A is

$$G(q,u)=\frac{s(v-w)\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}{p+s-v}$$
$$+\frac{(p-w+s)(p-v)\phi^{-}\left(\frac{p-w+s}{p+s-v}\eta\right)}{p+s-v}$$
$$-\frac{p-v}{\eta}\int_{\underline{a}}^{\phi^{-1}\left(\frac{p-w+s}{p+s-v}\eta\right)}\phi(x)dx$$
$$-\frac{s}{\eta}\left(\overline{a}-\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)\right)$$
$$+\frac{s}{\eta}\int_{\phi^{-1}\left(1+\frac{v-w}{p+s-v}\eta\right)}^{\overline{a}}\phi(x)dx$$
$$\approx 22244.25$$

B. Sensitivity Analysis

In this section, to gain further insights, this paper will perform a sensitivity analysis of the key parameters η, $\underline{a}$, $\overline{a}$, p, w and v of a spot purchasing model involving a risk-averse retailer under fuzzy demand. Here, this paper considers that the market demand is a triangular fuzzy variable $\tilde{D}=\left(\underline{a},\ a,\ \overline{a}\right)$, where $\underline{a}<a<\overline{a}$, and the range of η is in a numerical interval $(0,1]$. Another thing to note is that when $\eta=1.0$, the retailer is risk neutral. The unit shortage cost and the unit salvage value of the product are $s=0.8$ and $v=1.8$, respectively. The other contract parameters should follow these volume relationships:

$$s<p-v$$
$$0<v<w<p$$

To simplify the representation, the optimal order quantity and the expected fuzzy profit are indicated by q and G, respectively.

Here, this paper investigates the impact of key parameters on the optimal decision by a numerical study. The numerical results are presented in Fig. 1 to Fig. 11, and the results are discussed along with our qualitative analysis results.

1) The impact of the degree of risk aversion on the optimal decision

$\tilde{D}=(2000,5000,6500)$, $p=10$, $w=4$, $s=0.8$, and $v=1.8$, the impact of the degree of risk aversion η on the optimal spot order quantity q, and the expected fuzzy profit G is shown in Fig. 1. Note that the degree of risk aversion increases with a decreasing value of η.

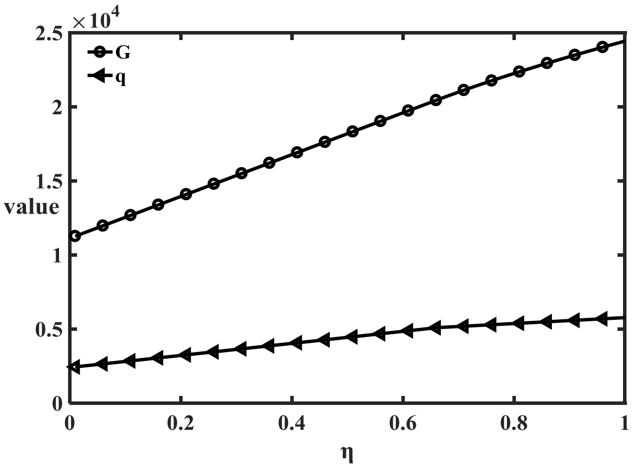

Fig. 1. The impact of η on q and G

From Fig. 1, we can see that

a) The optimal order quantity q and the expected fuzzy profit G of retailer increases with an increasing η; that is, the higher the degree of risk aversion of the retailer is, the lower the optimal order quantity of the retailer.

b) When η reaches zero, that is, the degree of risk aversion of the retailer is infinitesimal, the optimal order quantity is approaching 2400. The derivative process is

$$q^*=\frac{0.8\times 6500+(10-1.8)\times 2000}{10+0.8-1.8}$$
$$=2400$$

c) When $\eta=1.0$, the order quantity reaches a fuzzy high value of 5366.67; that is, the optimal order quantity of a risk-neutral retailer is higher than that of a risk-averse retailer when other parameters are the same and the risk-averse retailer's procurement decision is conservative. The derivative process is

$$q^*=\frac{0.8\phi^{-1}\left(1+\frac{1.8-4}{10+0.8-1.8}\right)+(10-1.8)\phi^{-1}\left(\frac{10-4+0.8}{10+0.8-1.8}\right)}{10+0.8-1.8}$$
$$\approx 5366.67$$

d) $q\in(2000,6500)$. Regardless of how the degree of risk aversion and other parameter combinations change, the optimal order quantity q takes a value in the interval $(2000,6500)$. Furthermore, considering that $p=10$, $w=4$, $s=0.8$, $v=1.8$, and $\eta=0.8$, the value range of q can be narrowed to $(2400,5366.67)$.

2) The impact of a fuzzy low value of market demand on the optimal decision

For the degree of risk aversion η, this paper considers three different values $\eta=1.0,0.8,0.6$. $a=5000$, $\overline{a}=6500$, $p=10$, $w=4$, $s=0.8$, and $v=1.8$, the impact of fuzzy low value $\underline{a}$ on the optimal order quantity q, and the expected fuzzy profit G is shown in Fig. 2 and Fig. 3. Note that the larger the difference between the fuzzy low value $\underline{a}$ and the fuzzy high value $\overline{a}$ of the triangular fuzzy number is, the higher the fuzzy degree of market demand is, and the greater the market demand uncertainty will be.

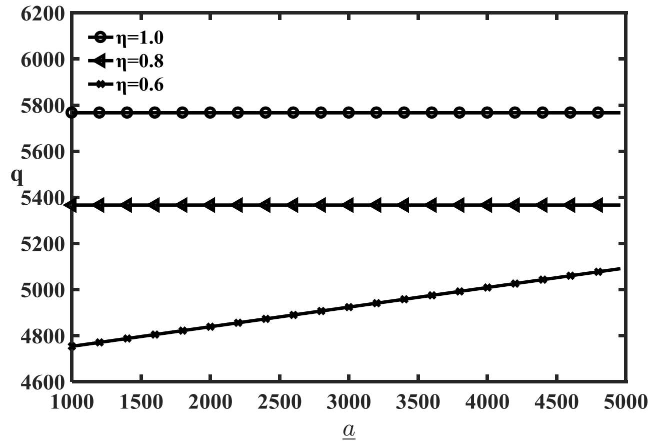

Fig. 2. The impact of $\underline{a}$ on q

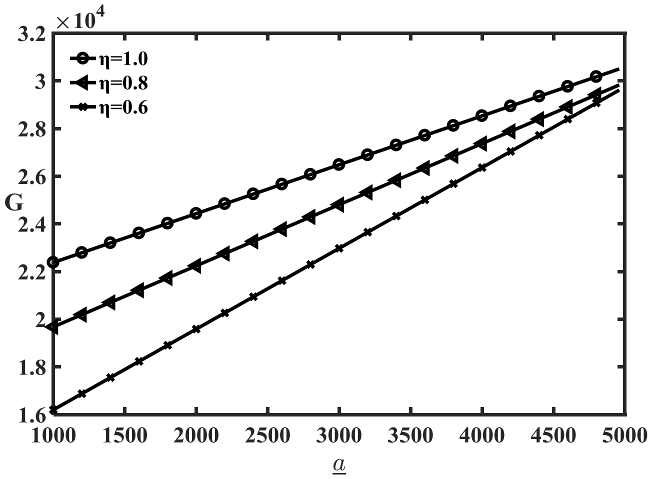

Fig. 3. The impact of $\underline{a}$ on G

From Fig. 4 and Fig. 5, we can see that

a) As $\underline{a}$ increases, the optimal order quantity represented by the curve with $\eta = 0.6$ increases, while the curve with $\eta = 1.0$ and the curve with $\eta = 0.8$ keep a certain value because the credibility distribution function of fuzzy market demand is a piecewise function. When $0.5 < 1 + \frac{v-w}{p+s-v}\eta \le 1$ and $0.5 < \frac{p-w+s}{p+s-v}\eta \le 1$, $\frac{\partial q^*}{\partial \underline{a}} = 0$ and the change in $\underline{a}$ cannot affect the value of the optimal order quantity. When $0 \le 1 + \frac{v-w}{p+s-v}\eta \le 0.5$ and $0 \le \frac{p-w+s}{p+s-v}\eta \le 0.5$, $\frac{\partial q^*}{\partial \underline{a}} > 0$ and the optimal order quantity increases with an increasing $\underline{a}$. In addition, as $\underline{a}$ decreases, the optimal order quantity of the risk-neutral retailer is not sensitive to a reduction and is always maintained at a high level, so the retailer may be exposed to great risk caused by uncertain market demand.

b) As $\underline{a}$ increases, the expected fuzzy profit with different degree of risk aversion increases because when it is an increasing fuzzy low value, the uncertainty of fuzzy market demand decreases and the expected market demand increases, while the optimal order quantity either remains the same or increases, so the retailer's expected fuzzy profit increases. In addition, the higher the degree of risk aversion is, the faster the profit increases, and the profit difference between a risk-neutral retailer and a risk-averse retailer gradually decreases as $\underline{a}$ increases. This occurs because the risk-neutral retailer's procurement strategy and expected fuzzy profit are less susceptible than that of a risk-averse retailer to change in market demand uncertainty in this condition.

3) The impact of the fuzzy high value of market demand on optimal decision

For the degree of risk aversion η, this paper considers three different values $\eta = 1.0, 0.8, 0.6$. When $\underline{a} = 2000$, $a = 5000$, $p = 10$, $w = 4$, $s = 0.8$, and $v = 1.8$, the impact of the fuzzy high value c of market demand on the optimal order quantity q. The expected fuzzy profit G is shown in Fig. 4 and Fig. 5.

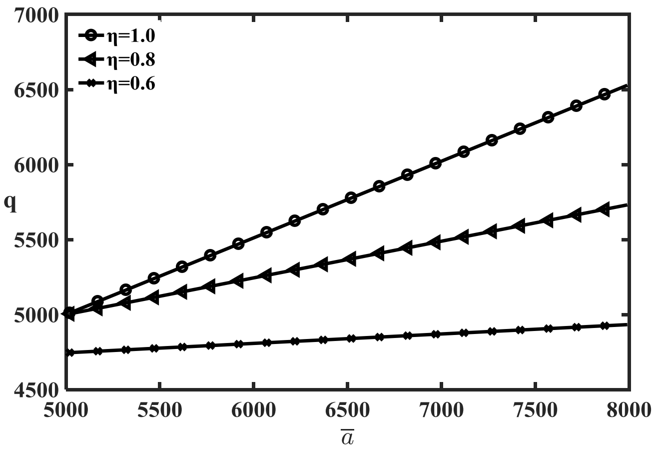

Fig. 4. The impact of $\overline{a}$ on q

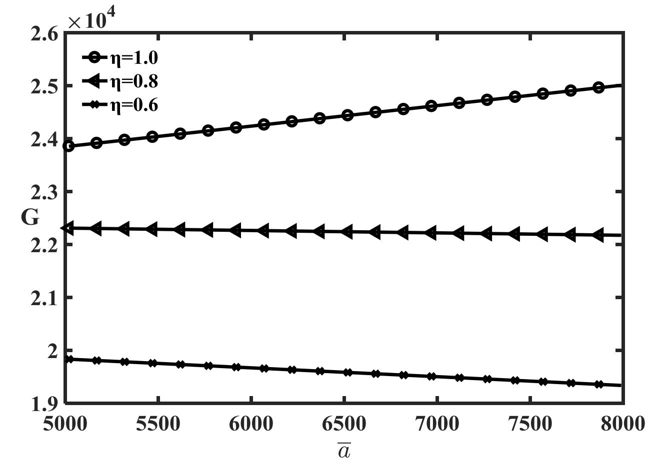

Fig. 5. The impact of $\overline{a}$ on G

From Fig. 4 and Fig. 5, we can see that

a) The optimal order quantity q and the expected fuzzy profit G of the retailer increase with increasing fuzzy high value c.

b) Regardless of how the fuzzy high value $\overline{a}$ and other combinations of parameters change, the optimal order quantity q takes the value between the interval $(2000, \overline{a})$. Furthermore, considering that $p = 10$, $w = 4$, $s = 0.8$, $v = 1.8$, $\eta = 1.0, 0.8, 0.6$, the value range of q can be narrowed to $(4746.15, 6528.22)$.

c) When the fuzzy high value $\bar{a}$ is a certain value, the slope of the curves in Fig. 4 increase with increasing η; that is, compared to the risk-neutral retailer, the risk-averse retailer's procurement strategy is less susceptible to change by uncertain market demand.

d) When the fuzzy high value $\bar{a}$ is a certain value, the slope of the curves in Fig. 5 increase with increasing η. In particular, when η equals 1.0 or 0.8, the slope is positive. However, the slope changes into negative when η equals 0.6, that is, as the fuzzy degree increases, the higher the degree of risk aversion of the retailer, the more focus on loss caused by the market demand uncertainty is, and the lower marginal profit of expected profit is.

e) It can be seen from Fig. 4 and Fig. 5 that when η equals 1.0 or 0.8, q and G both increase with increasing $\bar{a}$. When η equals 0.6, as $\bar{a}$ increases, q increases slowly, but G decreases; that is, when the retailer is too conservative, as $\bar{a}$ increases, the retailer's order quantity increases slightly, but the retailer's estimated fuzzy loss exceeds the fuzzy profit, which leads to a decrease in the expected fuzzy profit. In practice, when η equals 0.6 or less, the overconservative retailer will not increase order quantity substantially with increasing fuzzy high value of the fuzzy market demand. In the meantime, the overconservative retailer overestimates the possible loss, which results in an overconservative procurement strategy and missing market opportunity further.

4) The impact of the unit price of the product on the optimal decision

For the degree of risk aversion η, this paper considers three different values $\eta = 1.0, 0.8, 0.6$. When $\tilde{D} = (2000, 5000, 6500)$, $w = 4$, $s = 0.8$, and $v = 1.8$, the impact of the unit price p on the optimal order quantity q, The expected fuzzy profit G is shown in Fig. 6 and Fig. 7.

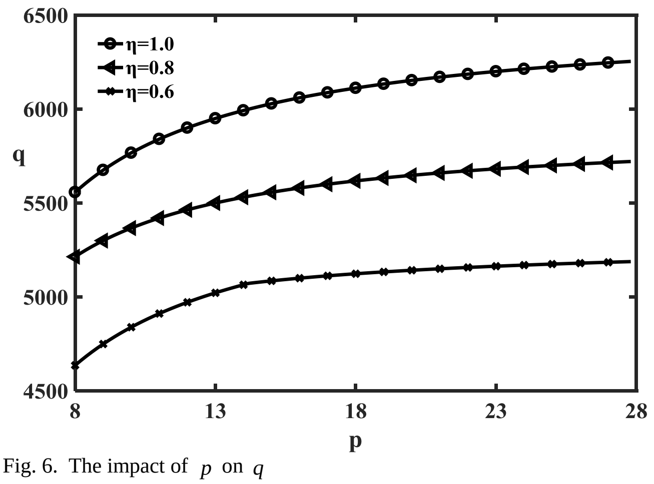

Fig. 6. The impact of p on q

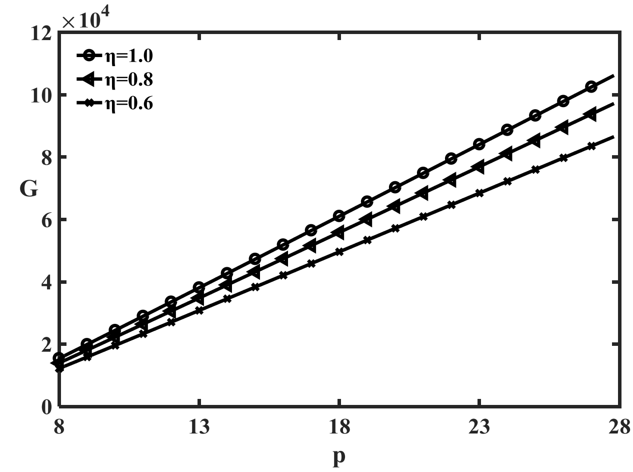

Fig. 7. The impact of p on G

From Fig. 6 and Fig. 7, we can see that

a) The optimal order quantity q and the expected fuzzy profit G increase with an increasing unit price p of the product.

b) The trend of the three curves is consistent. All three curves increase as p increases. When p equals 10, η equals 0.6, 0.8 or 1.0, and the optimal order quantity is 4839.11, 5366.67 and 5766.67, respectively. When the price p reaches infinity, the optimal order quantity of a risk-neutral retailer with $\eta = 1.0$ equals the fuzzy high value $c = 6500$; the optimal order quantity of a risk-averse retailer with $\eta = 0.8$ or $\eta = 0.6$ is approaching 5900 or 5600. The derivative process is

$$\lim_{p\to\infty} \frac{s\phi^{-1}\left(1+\dfrac{v-w}{p+s-v}\eta\right)+(p-v)\phi^{-1}\left(\dfrac{p-w+s}{p+s-v}\eta\right)}{p-v+s}$$
$$=\lim_{p\to\infty} \frac{0.1\phi^{-1}\left(1+\dfrac{1}{p-0.4}\eta\right)+(p-v)\phi^{-1}\left(\dfrac{p-1.4}{p-0.4}\eta\right)}{p-0.4}$$
$$=\phi^{-1}(\eta)$$

where $\phi^{-1}(0.6) = 5600$, $\phi^{-1}(0.8) = 5900$ and $\phi^{-1}(1.0) = 6500$.

c) The slopes of the curves in Fig. 6 show that a risk-averse retailer's procurement strategy is less susceptible to change in the unit price p. Therefore, the difference between a risk-neutral retailer's order quantity and that of a risk-averse retailer increases with increasing unit price p.

d) The slopes of the curves in Fig. 7 show that as the unit price p increases, the marginal fuzzy profit of a risk-neutral retailer is higher than that of a risk-averse retailer.

5) The impact of the unit wholesale cost of the product on the optimal decision

For the degree of risk aversion η, this paper considers three different values $\eta = 1.0, 0.8, 0.6$. When $\tilde{D} = (2000, 5000, 6500)$, $p = 10$, $s = 0.8$,and $v = 1.8$, the impact of the unit wholesale cost w on the optimal order quantity q. The expected fuzzy profit G is shown in Fig. 8 and Fig. 9.

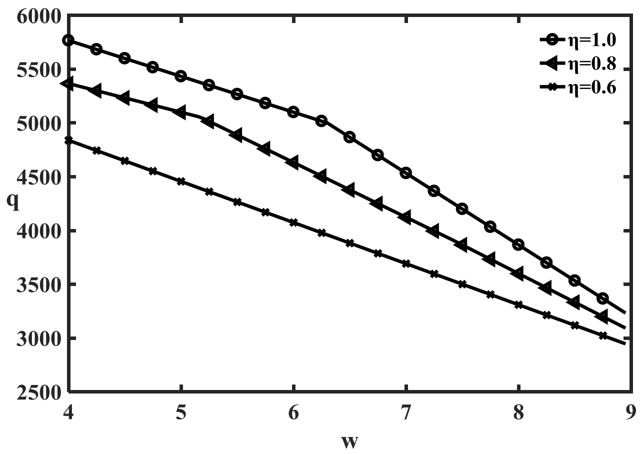

Fig. 8. The impact of w on q

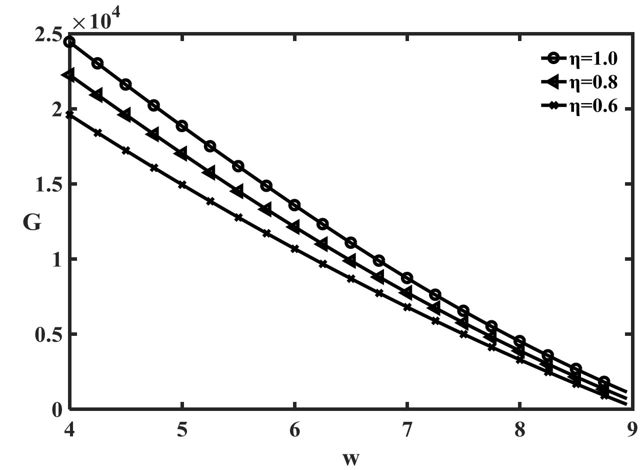

Fig. 9. The impact of w on G

From Fig. 8 and Fig. 9, we can see that

a) The optimal order quantity q and the expected fuzzy profit G decrease with increasing unit wholesale cost w of the product.

b) The slopes of the curves in Fig. 8 show that a risk-averse retailer's procurement strategy is less susceptible to change by the unit wholesale cost w. As w increases, the difference in the optimal order quantity with different risk-averse degrees gradually decreases. When w approximates p, the optimal order quantities with different risk-averse degrees are similar.

c) When other parameters are the same, the trend of the curve with different degree of risk aversions is consistent. In addition, each curve in Fig. 8 has a turning point, which is influenced by the membership function of the triangular fuzzy variable that is a piecewise function.

d) The slopes of the curves in Fig. 9 show that when w is a fixed value, the slopes of the curves decrease with increasing risk aversion degree. In other words, the risk-averse retailer's procurement strategy is less susceptible to changes in w than that of the risk-neutral retailer. When w approximates p, the expected fuzzy profits with the same degree of risk aversion are similar.

6) The impact of the unit salvage value of the product on the optimal decision

For the degree of risk aversion η, this paper considers three different values $\eta = 1.0, 0.8, 0.6$. When $\tilde{D} = (2000, 5000, 6500)$, $p = 10$, $w = 4$, $s = 0.8$, the impact of the unit salvage value v on the optimal order quantity q. The expected fuzzy profit G is shown in Fig. 10 and Fig. 11.

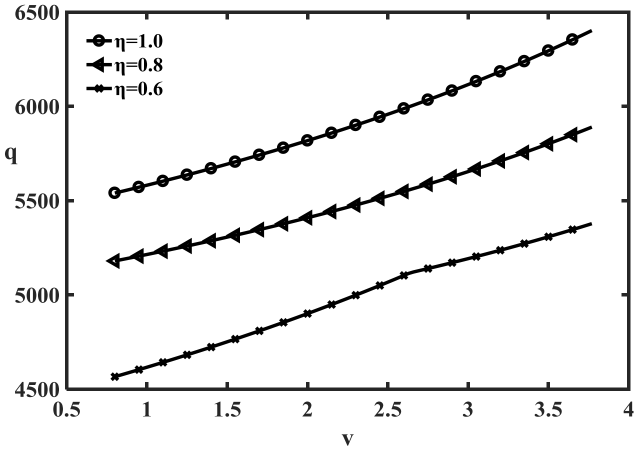

Fig. 10. The impact of v on G

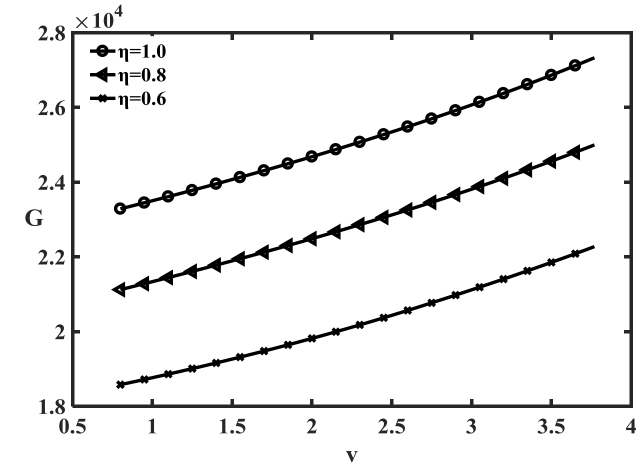

Fig. 11. The impact of v on G

From Fig. 10 and Fig. 11, we can see that

a) The optimal order quantity q and the expected fuzzy profit G increase with an increasing unit salvage value v of the product. Therefore, the retailer can increase its expected profits by deep processing and special promotions in practice.

b) The slopes of the curves in Fig. 10 show that the risk-averse retailer's procurement strategy is less susceptible to changes in the unit salvage value v than that of the risk-neutral retailer.

c) The slopes of the curves in Fig. 11 show that as the unit salvage value increases, the marginal profit of the risk-neutral retailer is higher than that of the risk-averse retailer.

V. Conclusion

To solve the uncertainty problem regarding the market demand of short life-cycle products, this paper establishes a spot purchasing model of a risk-averse retailer under fuzzy demand in a two-echelon supply chain system consisting of one supplier and one retailer. This paper uses CVaR theory to characterize the risk-averse retailer and risk profit to analyze the optimal procurement strategy of the retailer.

The conclusions of this paper are as follows. First, the optimal order quantity and expected fuzzy profit of the retailer increase with an increasing fuzzy low value for market demand, the unit price of the product and the unit salvage value of the product. The optimal order quantity and expected fuzzy profit of the retailer decrease with an increasing degree of risk aversion and the unit wholesale cost. However, as the fuzzy high value of market demand increases, the expected fuzzy profit can be either positive or negative, depending on the degree of risk aversion. One thing to note is that a risk-averse retailer's behavior is less susceptible to change using these parameters except the fuzzy low value. The fuzzy low value has more influence on the procurement strategy of the risk-averse retailer than for the risk-neutral retailer. Therefore, risk-averse behavior is more conservative and safe when the retailer is faced with great uncertainty regarding the low value of fuzzy demand insufficient market information. Second, regardless of how other contract parameters change, the optimal order quantity of the risk-neutral retailer or the risk-averse retailer takes a value between the fuzzy low value and the fuzzy high value of fuzzy market demand; that is, the order quantity will be not too high or too low, which effectively reduces the risks of uncertain market demand and increases the prediction accuracy.

Based on this research, future research can consider the following aspects:

1) When market demand is fuzzy, it can be represented by other fuzzy number forms based on different conditions, such as trapezoidal fuzzy numbers and fuzzy random numbers.
2) When studying an oligopolistic or monopolistic market, it is necessary to consider the relationship between the retail price and market demand and take the parameters of market size and consumer price sensitivity into consideration.
3) This paper assumes that the retailer is risk averse and that the supplier is risk neutral. However, many suppliers are risk seeking in reality, so studying the risk preferences of more supply chain members could have merits.
4) Future studies can consider a competition model with multiple retailers and multiple suppliers and study the supply chain decision-making problems of multiple supply chain members with different risk preferences.
5) When the market demand of agriproducts increases, to reduce the degree of dependence on the spot market and reduce the market risks caused by demand and price uncertainty, the retailer's procurement can shift from using a single strategy (the spot market) to using a portfolio strategy combining the spot market and option market, which could be analyzed in future studies.
6) The research subject of this paper is fresh agriproducts, which are short life-cycle products. Other short life-cycle products, such as high-end clothing, electronics and innovative products, can be considered.

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