Bio-inspired Control Based on a Cerebellum Model Applied to a Multivariable Nonlinear System

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Abstract—**In this paper, a novel bio-inspired control system is applied over a nonlinear multivariable system in real time. The control system is based on a bi-hemispheric neural network and a generalized polynomial controller. A ball and plate system is selected to perform the experiments since it adequately describes the brain interaction performed through the cerebellum that implies the sensor (human eyes or artificial vision system) and the actuator (human motor system or servo-motors related with the angles of the plate). The proposed method is evaluated by analyzing the settling time and steady-state error in comparison with a nonlinear controller obtained by using state feedback control law which is computed through extended linearization. The proposed approach is evaluated in a simulated and real environment, where obtained results show that the proposed method reduces the steady-state error of the nonlinear multivariable system.**

Index Terms—**Bio-inspired control, cerebellum, non-linear, multivariable.**

I. Introduction

Estimation and control of non-linear systems are highly complex tasks, even more when they are performed simultaneously. These tasks are usually solved through classical methods such as exact linearization, sliding mode control, fuzzy and neural networks based control [1], [2], [3]. But also, these tasks are solved by using a combination of neural networks or intelligent controllers and classical controllers [4], even under uncertainties conditions [5].

A particular structure of neural networks combined with classical controllers is proposed in [6], which considers a bio-inspired control based on a bi-hemispheric neural network. It can be seen that the model successfully controls an unstable robot. Besides, in [7] and [8], it is shown that a bi-hemispherical neural network of the cerebellum with realistic climbing fiber can reproduce the asymmetrical motor learning process adequately during a robot control task. That results show that a neural network with a cerebellum based structure is a feasible design for non-linear control systems. However, the methods mentioned above must be simplified to perform a real-time control [9], [10].

The ball and plate system is a widely used nonlinear model that consists of a plate and a ball. The plated is pivoted in its center in such a way that the slope of the plate can be manipulated in two perpendicular directions by using servos, and the ball can move freely over the plate, where the measurement of x-y position is made through a coupled vision system [11],[12]. Under this construction, the ball and plate is a dynamic system with two inputs and two outputs which describes adequately the brain interaction performed through the cerebellum. That interaction implies the sensor (human eyes or artificial vision system) and the actuator (human motor system or servo-motors related to the angles of the plate) where several control schemes linear and nonlinear have been applied [13], [14]. The structure mentioned above of the ball and plate system is a crucial factor to be used as a referential framework to evaluate the cerebellum-based controller, as stated by [15].

In this work, a novel bio-inspired control system based on a bi-hemispheric neural network is proposed. The model is simplified in order to obtain a feasible implementation in both simulation and real-time. The performance of the proposed controller is evaluated over a ball and plate system, which is a nonlinear multivariable system that is suitable for a cerebellum-based controller. The performance of the methods is evaluated by using the settling time and steady-state error in comparison with state of the art methods. The paper is organized as follows: In section II, the model of the cerebellum is proposed, in section III the results and discussions are presented and in IV the final remarks and future works are presented.

II. Theoretical Framework

A. Cerebellum Neural Network Model

The controller consists of a Bi-hemispheric neural network and a nonlinear controller, where the nonlinear controller helps the brain controller while it is in the learning process. Therefore, the parameters of the nonlinear controller are designed in such a way that the controller can only initially balance the system [6].

The structure of the brain network is configured according to the microcircuit of the cerebellum. In Fig. 1 is shown the neural circuit of the cerebellum where BA: basket cell, CF: climbing fiber, DCN: deep cerebellar nuclei, GO: Golgi cell, GC: granule cell, GPe: globus pallidus extern, GPi: globus pallidus intern, IO: inferior olive, LU: Lugaro cell, MFs: mossy fibers, PFs: parallel fibers, PN: pontine nucleus, SR: striatum, SNr: substantia nigra pars reticulata, ST: stellate cell, STN: subthalamic nucleus, TAL: thalamus, UBC: unipolar brush cell [16].

It can be seen that the cerebellum inputs are transported through moss fibers MFs in the brain follicle involved in engine learning. MFs are postulated to provide desired signals, copy efficiency of engine commands, and error

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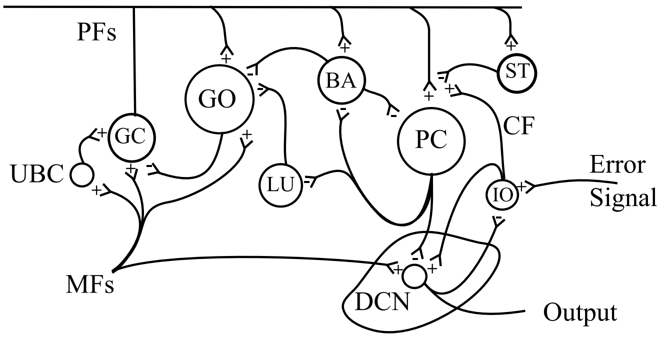

Fig. 1. Neural Circuit of the Cerebellum

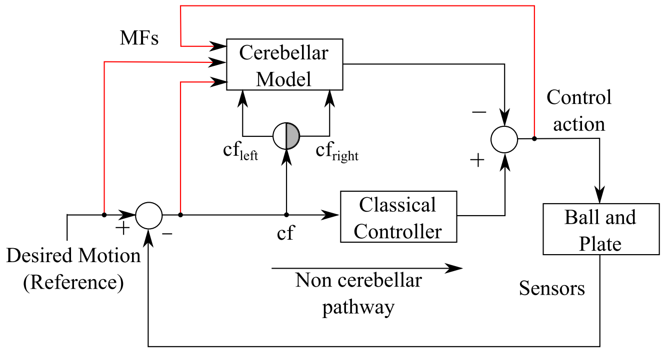

Fig. 2. Control diagram based on a bi-hemispheric cerebellum structure

signals (desired trajectory - actual trajectory) [7]. There are eight types of MFs, each of which carries the following signal:

1) Desired angle of inclination of the body (rad).
2) Desired body inclination angular velocity (rad/s),
3) Desired plate angle (rad),
4) Desired angle (rad),
5) Error of angular control of the inclination of the body (rad),
6) Error of control of the angle of inclination of the body (rad),
7) Control of the angular speed of the inclination of the Error (rad),
8) Engine command efficiency.

All this emulating the cell types found so far in the cerebral cortex [8] (cells (Gr), Golgi cells (Go), cells (Ba/St), and cells (Pk)) and these are interconnected through moss fibers mf, parallel fiber (pf) olive nucleus (IO) and climbing fibers (cf). Each cell model is described as follows:

$$X_{Gr} = \sum_{mf=1}^{7} W_{Grmf} \cdot Y_m f + \sum_{go=1}^{5} W_{GrGo} \cdot Y_{GO} \quad (1)$$

$$Y_{GO} = \frac{2}{1 + e^{-X_{Go}/u}} - 1 \quad (2)$$

$$X_{Ba/St} = \sum_{Gr=1}^{755} W_{Ba/(StGr)} \cdot Y_{Gr} \quad (3)$$

$$X_{Pk} = \sum_{Gr=1}^{755} W_{PkGr} \cdot Y_{Gr} \quad (4)$$

$$Y_{Ba/St} = \frac{2}{1 + e^{-X_{(Ba/st)}/u}} - 1 \quad (5)$$

$$Y_{Pk} = \sum_{Gr=1}^{755} W_{PkGr} \cdot Y_{Gr} + \sum_{Ba/st=1}^{5} W_{PkBa} \cdot Y_{Ba/St} \quad (6)$$

where Y is the nonlinear output of each cell, W represents the synaptic step between the x cell and the y cell. The subscripts $Gr, Go, mf, Pk, Ba/St$ are parameters for $Gr, Go, Pk, Ba/St$ respectively.

Therefore, by considering the aforementioned structure of the cerebellum, the following bi-hemispheric controller can be proposed, as described in Fig. 2.

B. Non-linear least squares

Nonlinear least-squares are the form of least squares analysis used to fit a set of m observations with a model that is nonlinear in unknown parameters $(m > n)$. It is used in some forms of nonlinear regression. The basis of the method is to approximate a system iteratively by a linear model [17].

Considering a set of m observations $(x_1, y_1), (x_2, y_2), ..., (x_m, y_m)$, and a curve (model function) $y = f(x, \beta)$, which besides the variable x also depends on n parameters $\beta = \beta_1, \beta_2, ...\beta_n$ with $m \geq n$. You want to find the vector β, parameters such that the curve fits better to the data given, that is, the sum of squares:

$$S = \sum_{i=1}^{m} r_i^2 \quad (7)$$

This is minimized when r_i errors are given by:

$$r_i = y_i - f(x_i, \beta) \ (i = 1, 2, ..., m) \quad (8)$$

The minimum value of S occurs when the gradient is zero. Since the model contains n parameters, there are n gradient equations:

$$\frac{\partial S}{\partial \beta_i} = \sum_i r_i \frac{\partial r_i}{\partial \beta_j} = 0 \ (j = 1, ..., n) \quad (9)$$

In a nonlinear system, the derivatives $\frac{\partial r_i}{\partial \beta_j}$ are functions of both the independent variable and the parameters, so these gradient equations don't have a final solution. Instead, the initial values should be chosen for the parameters [18]. Then, the parameters are refined iteratively, that is, the values are obtained by successive approximation:

$$\beta \approx \beta_j^{k+1} + \Delta\beta_j \quad (10)$$

Here, k is an iteration number, and the increment vector, $\Delta\beta_i$ is known as the displacement vector. In each iteration of the model is linearized by approximation to first order, in Taylor series the expansion over β^k:

$$f(x_i, \beta) \approx f(x_i, \beta^k) + \sum_j, \frac{\partial f(x_i, \beta_k)}{\partial \beta_j}(\beta_j - \beta_j^j) \quad (11)$$

$$\approx f(x_i, \beta^k) + \sum_j J_i j \delta\beta_j \quad (12)$$

The Jacobian J, is a function of the constants, the independent variable and the parameters, so it changes from one iteration to the next. So, in terms of the linear model, $\frac{\partial r_i}{\partial \beta_j} = -J_i j$ and the waste is given by:

$$r_i = \Delta y_i y_i - \sum_{s=1}^{n} \Delta\beta_s;\ \Delta y_i = y_i - f(x_i, \beta) \tag{13}$$

By substituting these expressions into the gradient equations, they become:

$$\sum_{i=1}^{m}\sum_{s=1}^{n} J_i j J_i s \delta\beta_s = \sum_{i=1}^{m} J_i j \Delta y_i \ (j = 1, ..., n) \tag{14}$$

C. Extended linearization

The extended linearization method initially proposes to obtain a linear controller by the feedback of the state vector, or a combination of observer-controller or even a classical controller obtained by means inherent to the description by transfer functions. In this way, the linear controller derived is defined in terms of the operational point and therefore stabilize a family of linear models obtained in the initial stage of the design [19].

The nonlinear control signal by state feedback is defined as

$$u = k(x) \tag{15}$$

Suppose the analytical system is derived into the following theorem of existence of the function $\frac{dx}{dt} = f(x, u)$ is such that $A(U), B(U)$ is a controllable pair. Then, there is a nonlinear gain or feedback function $k(cdot)\Re^n \to \Re$, such that the self-values of the closed-loop linear system of the $[A(U) - B(U)K(U)]$ matrix have previously specified values, which are locally invariant with respect to U.

The test is essentially divided in two steps: the first step is $\frac{dX(U)}{dU} dU \neq 0$. This quantity is useful in the second step where a function $k(\cdot)$ is defined in in order to obtain a linearization of $u = -k(x)$ as $\delta u = -k(U)\delta x$.

Without loss of generality, it can be assumed that $A(U)$ is reversible. However, if it is not, we can do using previous feedback of the state vector that all the auto-values are non-zero and therefore, $A(U) = \frac{\partial f}{\partial x} \mid_{U, X(U)}$ would be reversible.

$$\frac{\partial f}{\partial x} = \frac{\partial f(x, u)}{\partial x} + \left[\frac{\partial k}{\partial u}\right] - \left(\frac{\partial k}{\partial x}\right) \tag{16}$$

The nonlinear gain to be obtained must satisfy:

$$\frac{\partial k}{\partial x} \mid_{X(U)} = K(U) = [K_1(U), ..., K_n(U)] \tag{17}$$

$$K(X(U)) = -U \tag{18}$$

By differentiating $k(x)$ with respect to $x_i, i = 1, 2, 3, ..., n$ and evaluating in $X(U)$, the following expression is obtained:

$$\frac{\partial k}{\partial x_i}(X(U), U) = K_i(X_1^{-1}(X_1(U))) = K_i(U) \tag{19}$$

By proceeding in a similar way with respect to x_1 and evaluating at point $X_1^{-1}(x_1)$, the following equation is obtained:

$$\frac{\partial k}{\partial x_1} \mid x_1^{-1}(x_1) = K_1(X_1^{-1}(x_1) - \frac{dX_1^{-1}(x_1)}{dX_1} \tag{20}$$

Finally, by particularizing the last expression with respect to $U = X_1^{-1}(x_1)$ the following equation is obtained:

$$\frac{\partial k}{\partial x_1} = (X(U)) = K_1(U) \tag{21}$$

III. Results and discussion

The results are shown in simulation and a real environment by using a two-dimensional prototype called ball and plate. The ball and plate is an unstable, non-linear, multivariable, open-loop system. The system consists of a plate pivoted in its center in such a way that the slope of the plate can be manipulated in two perpendicular directions, where the measurement is made through a coupled vision system. The ball and plate is a dynamic system with two inputs and two outputs, so the controller must be accurate to keep the ball at the desired point. In this system, there are two control problems: the stabilization of the system around equilibrium points and the tracking of trajectories. The first problem is to take the ball to a specific position and keep it there. For the second problem, the objective is to make the ball follow a predefined trajectory (linear, square, circle, or Lissajous curves) [11].

Two controllers are tested over the system: The bi-hemispheric controller based on the cerebellum with a generalized polynomial controller and a linear state feedback method based on an extended linearization method, which results in a nonlinear controller designed in terms of the operational point.

In order to test the performance of the controllers, two measures are used: the settling time and steady-state error. Additional analysis under external disturbances is also applied at several operational points.

For the two-dimensional system of the ball and plate, two models are obtained that determine the estimated output for each axis of the plate. The models are represented in a third-order transfer function, where the parameters of the numerator and denominator are estimated in an experimental way with a sampling time of $100ms$, and at the same time, the controller will be carried out under the same sampling time.

A. System identification

In order to obtain a model that describes the dynamical of the nonlinear system around the operational point (small input angles), a randomly generated output is obtained experimentally to obtain movements throughout the operating range of the system an therefore identify the system. In order to perform this task, 120 seconds are obtained using a sampling time of 100 ms. The input data for each angle is generated and applied simultaneously, but the identification is performed decoupled. As a result, only two discrete transfer functions are obtained, one for each input-output relationship associated to each axis.

As shown in Fig. 3 and Fig. 4, the estimated outputs for each axis (x axis and y axis) adequately track the nonlinear behaviour of the ball and plate system.

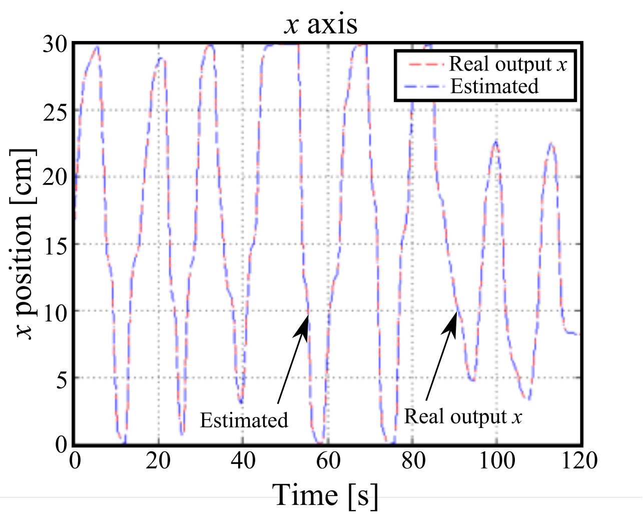

Fig. 3. Estimated and real output for x axis.

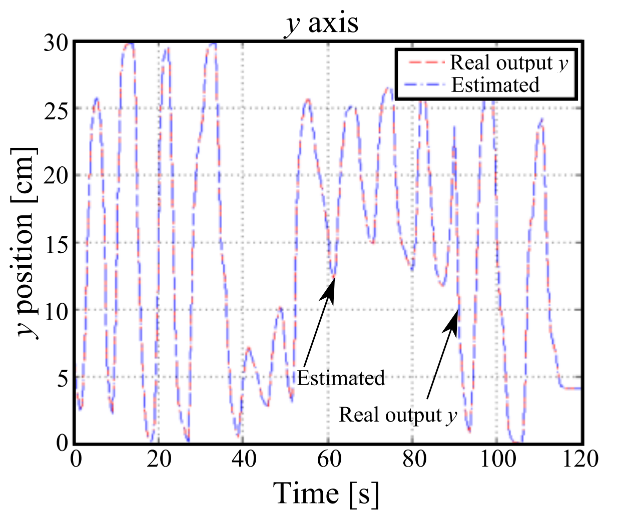

Fig. 4. Estimated and real output for y axis.

It can be seen from Fig. 3 and Fig. 4 that the nonlinear dynamics can be approximated around the operational point by two equations, where (22) is the estimated transfer function for the x axis and (23) is the estimated transfer function for the y axis.

$$H(z) = \frac{0.0340482z^2 + 0.0975331z - 0.238661}{z^3 + 1.12029z^2 - 0.09114312} \quad (22)$$

$$H(z) = \frac{-0.0115444z^2 + 0.0977724z - 0.668972}{z^3 + 4.1573z^2 - 3.89773} \quad (23)$$

B. Simulation of the bi-hemispherical controller based on the estimated system

By considering (22) and (23) a generalized polynomial controller can be designed with transfer function defined by:

$$C(z) = \frac{l_0z^2 + l_1z + l_2}{z^3 + p_1z^2 + p_2z + p_3} \quad (24)$$

As a result of the design process the parameters presented in Table I for both axis (x axis and y axis) are obtained.

TABLE I
POLYNOMIAL CONTROLLER PARAMETERS CONSTANTS FOR THE BALL AND PLATE SYSTEM

Controller parameters	x-axis	y-axis
$p1$	2.3608	-0.426
$p2$	-11.3577	2.78
$p3$	31.9423	5.6466
$l0$	75.4195	28.6904
$l1$	33.0461	-10.90975
$l2$	17.8976	-4.1212

The values of Table I of the polynomial controller are included into the structure of a bi-hemispherical controller based on the cerebellum model in such a way that the system is able to train the neural network and achieve a new control action, thus improving the response of the system. It is noticeable that the polynomial controller is included in the structure presented in Fig. 2. As a result, the tracking response of the controlled system is presented in Fig. 5 and Fig. 6 for x and y axis, respectively.

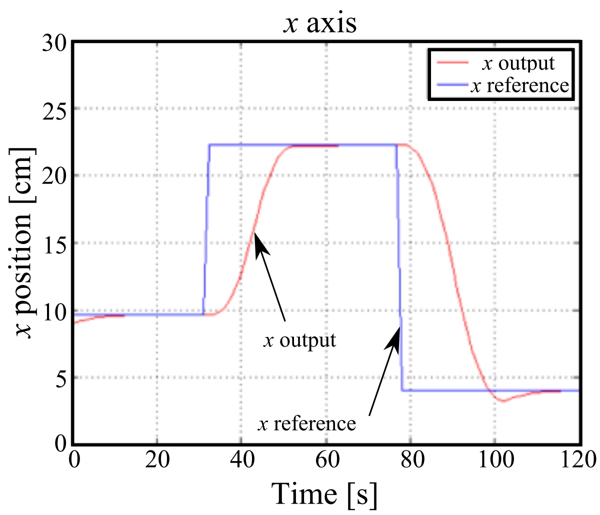

Fig. 5. Tracking response for a bi-hemispherical controller based on the cerebellum with the polynomial controller for the x-axis.

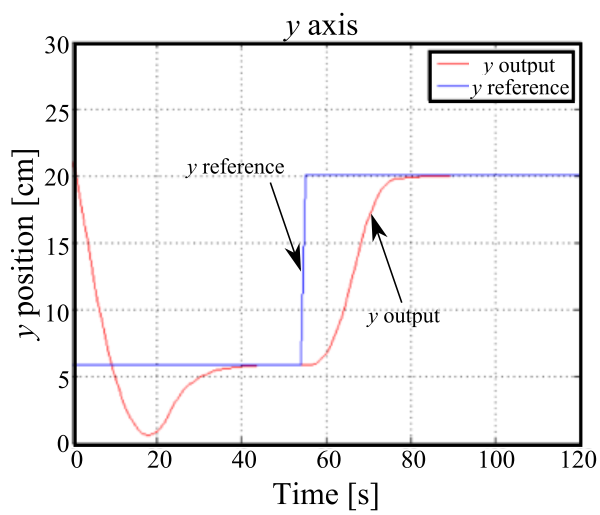

Fig. 6. Tracking response for a bi-hemispherical controller based on the cerebellum with the polynomial controller for the y-axis.

From Fig. 5 and Fig. 6, it can be seen that the controller manages to bring the system outputs to a desired reference value. The output has no oscillations before the reference change for both axes and it can also be seen that the error in steady state is zero, so it is determined that the control system operates efficiently for the parameters estimated on the ball and plate system.

C. *State feedback by extended linearization*

In order to compare the performance of the proposed method, a comparison with the state space feedback by using an extended linearization method is performed, as shown in Fig. 7 and Fig. 8.

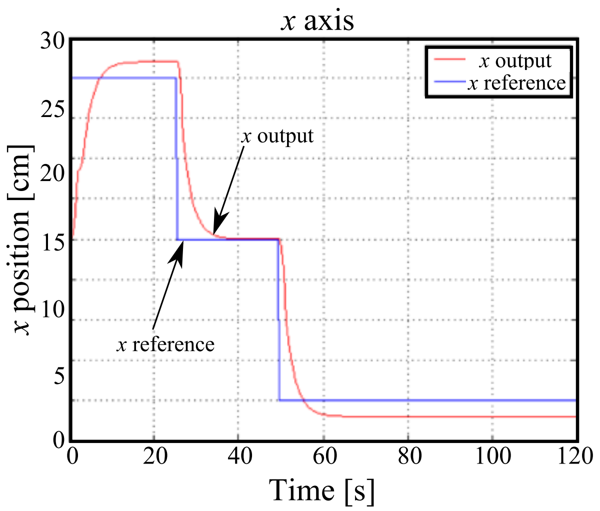

Fig. 7. Tracking response by using state feedback controller based on an extended feedback linearization method in the x-axis.

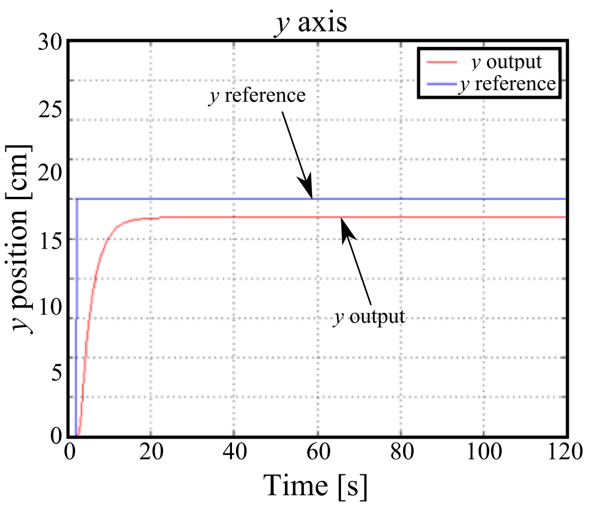

Fig. 8. Tracking response by using state feedback controller based on an extended feedback linearization method in the y-axis.

It can be seen that for both axis the closed loop response has steady error greater than zero, since the state space feedback based on an extended linearization method considers an approximated model. Therefore, as presented in Fig. 7 and Fig. 8 these differences between the nonlinear model an the approximated model result in a steady error greater than zero for references non equal to zero.

In Fig. 9 a tracking response of a simulated ball and plate system for a x-y trajectory generated with two sinusoidal signals of 0.4Hz and 0.2Hz are presented by using the bi-hemispheric cerebellum based controller. It can be seen that the steady-state error is near to zero and also that the system tracks x-y trajectory adequately.

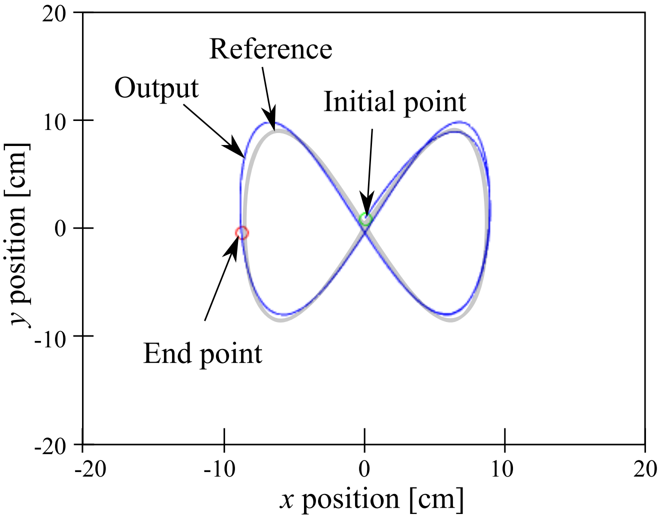

Fig. 9. System response for a trajectory tracking in simulation for x-axis and y-axis considering a bi-hemispheric cerebellum based controller.

D. *Real-time MIMO controller*

The tracking results of a real time implementation of a bi-hemispheric neural network controller with a generalized polynomial controller are shown in Fig. 10 and Fig. 11. In this case, only a set point is considered for both axis, and external impulse type disturbances are added a times $t = 10$s, $t = 40$s, $t = 70$s.

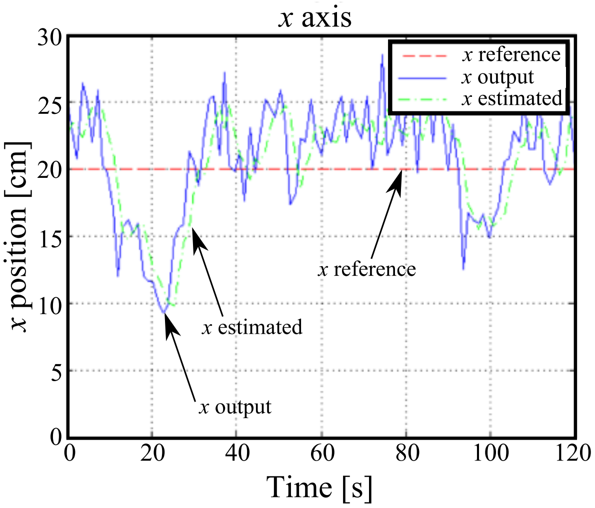

Fig. 10. System response in real time for x-axis considering a bi-hemispheric cerebellum based controller.

From Fig. 10 and Fig. 11, it is observed that the reference tracking used by the system tries to reduce the error. However, it can be seen that both axes have a steady-state error. That behavior can be justified considering the inherent dynamic of the system where it can be seen that a reduction of the steady error is not achieved effectively. In the y axis a smaller steady-state error is observed where the learning of the neural network is better in terms of convergence.

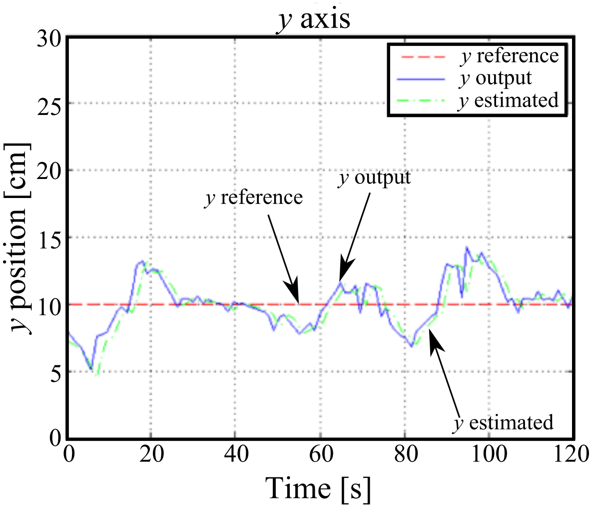

Fig. 11. System response in real time for y-axis considering a bi-hemispheric cerebellum based controller.

It is also noticeable that the identification of the system is performed on-line in order to adjust the generalized polynomial controller adequately. On the other hand, it can be seen that the controller tries to reduce the effect of disturbances with the corresponding reference without diminishing the estimation of the outputs.

IV. Conclusions

In this work, a novel bio-inspired control system applied to a multivariable nonlinear system is presented. It can be seen that by using a control-based in a Bi-hemispheric neural network and a nonlinear controller, the steady-state error of the nonlinear multivariable system is reduced. That is proved over a ball and plate system and is evaluated in terms of the settling time and steady-state error in comparison with the extended linearization method, where it can be seen that the proposed method outperforms the state of the art method. The technique is successfully proved over a simulated and a real environment. As a result, the proposed method based on a bi-hemispheric neural network is feasible for implementation in real-time, at least for systems with visual-motor structures. As future works, several tests over nonlinear multivariable systems are proposed involving different structures and inputs-outputs relations. In addition, future performance analysis must be made by considering computational complexity and system requirements.

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