Cost Optimization of Two-dimensional Warranty under Incomplete Preventive Maintenance Condition

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***Abstract*—In order to reduce the cost of two-dimensional warranty service, preventive maintenance of some expensive equipment is a very popular strategy for the manufacturer to provide after-sales warranty service. Preventive maintenance is further studied in this paper, and the strategy of combining two different degrees of incomplete preventive maintenance is put forward and the corresponding cost model is established. The validity of the model is proved by an example, and the sensitivity analysis is carried out. The results can offer an important reference for the manufacturer to design more competitive and lower cost warranty strategy.**

***Index Terms*—preventive maintenance, incomplete maintenance, two-dimensional maintenance, cost**

I. Introduction

AT present, in the fierce market competition, in addition to excellent performance, reasonable price and excellent quality, after-sale warranty service plays an increasingly important role in product marketing. During the warranty period, good after-sales service has become an important factor in competition among enterprises. According to the deadline of the warranty service, the warranty service can be divided into one-dimensional warranty, two-dimensional warranty and multi-dimensional maintenance strategy[1]. At present, two-dimensional warranty strategy has been widely used in many military and civil equipment maintenance services. Usually, the two-dimensional warranty period is a rectangular area. As shown in Fig. 1, the time is represented by the horizontal axis and the usage by the vertical axis. T_W and U_W are the warranty deadlines of the product in terms of time and usage respectively. Whichever deadline arrives first, the warranty service will end.

In the current warranty service, in order to improve customer satisfaction, attract users and increase sales, the competitive warranty service is mostly provided by manufacturers. In other words, when the product breaks down, the manufacturer will repair it. In order to reduce the cost of warranty and improve availability, preventive maintenance is necessary after a certain period of product operation, because for the same degree of maintenance, the cost and time of product maintenance after failure is usually greater than the cost and time of maintenance before failure[2][3]. At present, for many warranty products, the warranty service has gradually changed from a single repair-based maintenance strategy to a preventive maintenance strategy. Many domestic and foreign scholars have proved the superiority of this strategy through research.

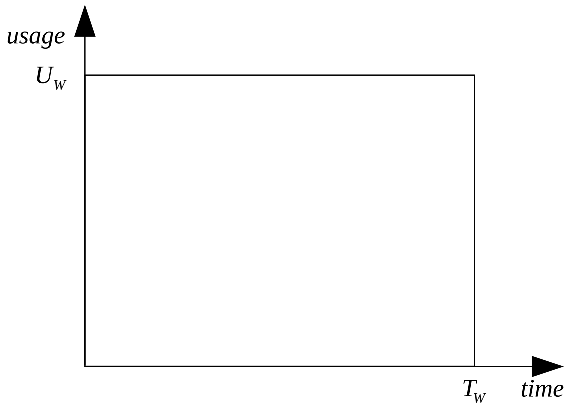

Fig.1 Two-dimensional warranty area

Huang established a cost model under preventive maintenance strategy, and obtained a maintenance plan to maximize the profit of the manufacturer[4]. Shang introduced maintenance cost threshold to decide whether to replace products after failure, and determined preventive maintenance interval under minimum maintenance cost[5]. Considering the optimization of warranty cost from the manufacturer's point of view, Alqahtani adopted a periodical preventive maintenance warranty strategy for recycling products, and developed a cost model[7]. Wang put forward two maintenance strategies: periodic preventive maintenance and sequential preventive maintenance, in which the degree of preventive maintenance is defined as incomplete maintenance. In these maintenance strategies, they were assumed that the product would be replaced after several preventive maintenance, and cost models were established. Finally, the advantages and disadvantages of the two strategies were compared[7]. Wang analyzed from the perspective of the maximum benefit obtained by users and the minimum warranty cost borne by manufacturers, respectively. It was assumed that the preventive maintenance cost was borne by both parties in proportion during the

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initial warranty period, and all the preventive maintenance cost was borne by users during the extended warranty period. The warranty strategy of periodic preventive maintenance was implemented for components, and the decision-making model of preventive maintenance cycle was established[8]]. Su proposed a warranty strategy of carrying out incomplete preventive maintenance within the warranty period, and established multi-objective maintenance optimization model considering both cost and availability. By a case study, it was proved this strategy can better satisfy the customer’s preference than the single-objective maintenance optimization[9]. Chien utilized the Non-Homogeneous Pure Birth Process (NHPBP) to model the process of general failures for durable products during warranty period, and proposed an optimal PM schedule for a NHPBP repairable system and develop the expected cost function. Taking the lowest warranty cost as target, it was proved the optimal PM schedule can offer an important analytical tool for marketing managers[10]. Tong assumed that the usage rate of product during warranty period is changed. During warranty period, the product was subjected to several maintenance and cost model was established. Through example, it was proved that the model can improve the accuracy of the calculation result of the warranty cost[11]. Wu studied the problem of preventive maintenance optimization considering aging loss during the warranty period[12]. Huang analyzed the two-dimensional warranty cost under the periodic preventive maintenance strategy[13]. From the manufacturer's point of view, Park proposed to perform preventive maintenance during the warranty period. Minimum maintenance or replacement maintenance depended on whether the repair time exceeded the threshold. Finally, the optimal value of preventive maintenance interval was determined to minimize the cost during the warranty period[14]. Huang established an aperiodic preventive maintenance model based on reliability threshold. The time for incomplete preventive maintenance was optimized with the goal of maximizing the profit of the manufacturer. Furthermore, the effects of warranty period elasticity and price elasticity on the manufacturer's earnings were considered[15]. However, in warranty strategies of the existing literature, only a single degree of preventive maintenance is provided during the warranty period.

According to the degree of product recovery after maintenance, maintenance is divided into three categories. The first is called minimum maintenance (ABAO), in which case, each repair restores the product to the level before the failure; the second is called complete maintenance (AGAN), in which case, each repair restores the product to the same level as a new product; and the third is called incomplete maintenance, with the degree of repair between the two. Hypothesis ξ ($\xi \in [0,1]$) represents repair factor. In the case of ABAO ($\xi = 0$), each repair restores the project to the level before the failure, while in the case of AGAN ($\xi = 1$), each repair restores the project to a new project. In case of incomplete maintenance[16], the value range of ξ is $0 < \xi < 1$. In order to reduce the cost of warranty, preventive maintenance usually adopts incomplete maintenance or complete maintenance. Three different maintenance degrees have their own characteristics. Comparing the maintenance time and cost expended by different degrees of maintenance activity, it is easy to know that minimum maintenance, incomplete maintenance and complete maintenance increase in turn. The research shows that preventive maintenance activities can reduce the virtual age of the system and improve the reliability of the system. Modeling with incomplete maintenance can assume not only the time during the preceding preventive maintenance interval can be reduced by incomplete maintenance, but also the time before the preventive maintenance can be reduced. The second hypothesis is used in this paper. Proportional Age Setback model[17] is used in this hypothesis, and the same hypothesis model is used in most literatures[18][19]. In the process of incomplete preventive maintenance, the individual degree of repair corresponds to different repair factors, which reduces the virtual age of products at different degrees. Generally, the higher the maintenance degree, the higher the maintenance cost. This rule is consistent with the actual application of maintenance.

In the existing literature, majority of warranty strategies provide only a single preventive maintenance strategy during the warranty period. Only one paper mentions a combination of preventive maintenance strategies. Liao proposes that preventive maintenance during the warranty period includes complete maintenance, incomplete maintenance and predictive maintenance[20]. However, at present, no literature has proposed two different degrees of incomplete maintenance combination. So, the combination will have great application space in the actual warranty service.

Because maintenance requires expensive maintenance equipment, users often outsource maintenance tasks to contractors or manufacturers. Maintenance tasks are usually performed by the contractor or manufacturer, but users with certain maintenance ability can participate in preventive maintenance. Such as large industrial and mining enterprises, due to the needs of production and work tasks, they must establish their own maintenance force. However, in the initial stage of equipment use or during the warranty period, the user's self-repairing ability is usually lower than that of the manufacturer. The maintenance task of incomplete preventive maintenance can be jointly undertaken by the manufacturer and the user. Now, this warranty mode has been done in some areas. However, due to the lack of relevant research, the implementation of most preventive maintenance relies only on experience. There is no scientific model as a guide.

How to choose the maintenance degree of incomplete preventive maintenance is a problem that needs to be considered. It is understood that in the existing literature, the degree of incomplete maintenance is the same, and the cost assumption of each incomplete maintenance is also the same. In this paper, it is pointed out that if incomplete preventive maintenance is carried out with different degree of maintenance, the corresponding maintenance cost will be different. This is a new method proposed for the first time based on the existing literature. In this paper, the corresponding model is established, and the applicability and validity of the model are proved by an example.

The rest of the article is structured as follows. In the second part, a two-dimensional maintenance cost optimization model under incomplete preventive maintenance is proposed. The third part explains the calculation of the model. In order to verify the validity of the model, an example is given in the fourth part. In the fifth part, the results are discussed. The last part gives a summary and development direction.

II. ESTABLISHING A COST MODEL

A. Symbols and Hypothesis

(T_W, U_W) : Two-dimensional warranty period for products

(T_0, U_0) : Two-dimensional preventive maintenance interval for warranty period

C_{p1}: Cost of preventive maintenance performed by the user

C_{p2} : Cost of preventive maintenance performed by the manufacturer

C_f : Maintenance cost after product failure

ξ_1 : Repair factor after preventive maintenance performed by the user

ξ_2 : Repair factor after preventive maintenance performed by the manufacturer

$C(T_0, U_0)$: The expected cost of the warranty during the warranty period is considered (T_0, U_0) as the interval of preventive maintenance

$\lambda(t \mid r)$: Initial failure rate of products

During the two-dimensional warranty period (T_w, U_w) , the design usage of the product is r_w , $r_w = U_w / T_w$. We use (T_0, U_0) as an interval for periodic incomplete preventive maintenance, and T_0 is the time interval, while U_0 is the usage interval, $r_0 = U_0 / T_0$. No matter which deadline arrives first, preventive maintenance is carried out. After preventive maintenance, the failure rate of products can be reduced. During the warranty period, a total of N_1 preventive maintenance are carried out, and it is assumed only the N_2 th ($N_2 = \text{int}(N_1/2)+1$, according to the difference of the total number of preventive maintenance between odd and even numbers, it is stipulated that the preventive maintenance should be carried out by the manufacturer once in the middle) preventive maintenance is carried out by the manufacturer, and the remaining $N_1 - 1$ times preventive maintenance are carried out by the user. The initial failure rate of the product is $\lambda(\mathrm{t} \mid \mathrm{r})$, and the failure rate during the interval between the n th($n \in [1, N_1\text{-}1]$) and $n+1$ th preventive maintenance is $\lambda_n(\mathrm{t} \mid \mathrm{r})$. The product failure rate before the first preventive maintenance is $\lambda_0(\mathrm{t} \mid \mathrm{r})$. During the warranty period, the manufacturer is responsible for the maintenance after product failure, and the maintenance degree is ABAO, that is, the failure rate of the product remains unchanged before and after maintenance. In the existing research, there are three main ways to express the two-dimensional failure rate: the two-factor variable method, the composite scale method and the utilization rate method. And usage rate method is widely used. Assuming that the usage rate r of a single product remains unchanged during the warranty period, this hypothesis has also been proved[21][22], but different users have different usage rates. For batch products, usage rate is a random variable. Assuming the manufacturer can obtain the distribution through the feedback information and survey of previous product use, the distribution function of usage and probability density function are $G(r)$ and $g(r)$, respectively. Assuming that product usage u is a linear function of usage time t , and for a given usage r is a fixed value, then there is $u = r \times t$. The failure rate function is $\lambda(t \mid r)$ when the product usage rate is r . Expression is $\lambda(t \mid r) = \theta_0 + \theta_1 r + \theta_2 t^2 + \theta_3 r t^2$, the specific value of θ_0, θ_1, θ_2, θ_3 can be estimated according to the historical failure rate of the product.

B. Modeling

According to the relationship between r_w and r_0 , the product warranty cost is divided into the following two cases.

a) If $r_0 \le r_w$, as shown in Fig. 2, there are three cases.

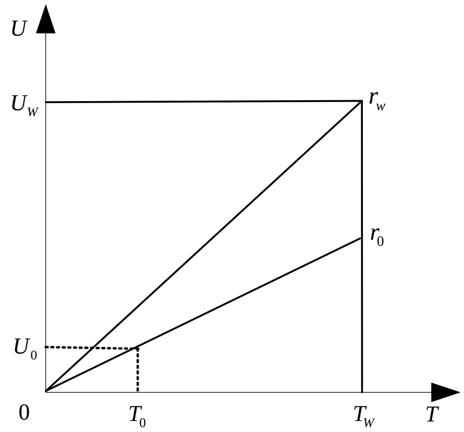

Fig.2 The relationship between and r_w （$r_0 \le r_w$）

(1)When $r \le r_0$, preventive maintenance in interval is T_0 , and warranty deadline is T_w , $N_1 = \text{int}[T_w / T_0]$.

The expression of $\lambda_n(\mathrm{t} \mid \mathrm{r})$ is

$$\lambda_n(\mathrm{t} \mid \mathrm{r}) = \begin{cases} \lambda(\mathrm{t} \mid \mathrm{r}), \mathrm{n} = 0 \\ \lambda(\mathrm{t} - \sum_{i=1}^{n} \xi_1^{\,i} T_0 \mid \mathrm{r}), 1 \le n \le N_2 - 1 \\ \lambda(\mathrm{t} - \xi_2 \sum_{i=0}^{n\text{-}1} \xi_1^{\,i} T_0 \mid \mathrm{r}), n = N_2 \\ \lambda(\mathrm{t} - \sum_{i=1}^{n-N_2} \xi_1^{\,i} T_0 - \xi_2 \sum_{j=n-N_2}^{n-1} \xi_1^{\,j} T_0 \mid r), N_2 + 1 \le n \le N_1 \end{cases}$$

In this case, the cost during the warranty period is as follows:

$$\begin{aligned} C_1(T_0, U_0) &= C_{p1} * (N_1 - 1) + C_{p2} \\ &+ C_f * (\int_0^{T_0} \lambda_0(\mathrm{t}|\mathrm{r}) dt + \int_{T_0}^{2T_0} \lambda_1(\mathrm{t}|\mathrm{r}) dt \\ &+ \ldots + \int_{(N_1-1)T_0}^{N_1 T_0} \lambda_{N_1-1}(\mathrm{t}|\mathrm{r}) dt + \int_{N_1 T_0}^{T_W} \lambda_{N_1}(\mathrm{t}|\mathrm{r}) dt) \\ &= C_{p1} * (N_1 - 1) + C_{p2} \\ &+ C_f * (\sum_{i=0}^{N_1-1} \int_{iT_0}^{(i+1)T_0} \lambda_i(\mathrm{t}|\mathrm{r}) dt + \int_{N_1 T_0}^{T_W} \lambda_{N_1}(\mathrm{t}|\mathrm{r}) dt) \end{aligned} \quad (1)$$

(2)When $r_0 < r \le r_w$, preventive maintenance interval is U_0 / r, and warranty deadline is T_w, $N_1 = \text{int}[T_w r / U_0]$.

The expression of $\lambda_n(\text{t}\,|\,\text{r})$ is

$$\lambda_n(\text{t}\,|\,\text{r}) = \begin{cases} \lambda(\text{t}\,|\,\text{r}), \text{n} = 0 \\ \lambda(\text{t} - \sum_{i=1}^{n} \xi_1^i U_0 / r|\text{r}), 1 \le n \le N_2 - 1 \\ \lambda(\text{t} - \xi_2 \sum_{i=0}^{n\text{-}1} \xi_1^i U_0 / r|\text{r}), n = N_2 \\ \lambda(\text{t} - \sum_{i=1}^{n-N_2} \xi_1^i U_0 / r - \xi_2 \sum_{j=n-N_2}^{n-1} \xi_1^j U_0 / r|\text{r}), N_2 + 1 \le n \le N_1 \end{cases}$$

In this case, the cost during the warranty period is as follows:

$$\begin{aligned} C_2(T_0, U_0) &= C_{p1} * (N_1 - 1) + C_{p2} \\ &+ C_f * (\int_0^{U_0/r} \lambda_0(\text{t}|\text{r}) dt + \int_{U_0/r}^{2U_0/r} \lambda_1(\text{t}|\text{r}) dt \\ &+ \ldots + \int_{(N_1-1)U_0/r}^{N_1 U_0/r} \lambda_{N_1-1}(\text{t}|\text{r}) dt + \int_{N_1 U_0/r}^{T_W} \lambda_{N_1}(\text{t}|\text{r}) dt) \\ &= C_{p1} * (N_1 - 1) + C_{p2} \\ &+ C_f * (\sum_{i=0}^{N_1-1} \int_{iU_0/r}^{(i+1)U_0/r} \lambda_i(\text{t}|\text{r}) dt + \int_{N_1 U_0/r}^{T_W} \lambda_{N_1}(\text{t}|\text{r}) dt) \end{aligned} \quad (2)$$

(3)When $r > r_w$, preventive maintenance interval is U_0 / r, and warranty deadline is U_w / r, $N_1 = \text{int}[U_w / U_0]$.

The expression of $\lambda_n(\text{t}\,|\,\text{r})$ is

$$\lambda_n(\text{t}\,|\,\text{r}) = \begin{cases} \lambda(\text{t}\,|\,\text{r}), \text{n} = 0 \\ \lambda(\text{t} - \sum_{i=1}^{n} \xi_1^i U_0 / r|\text{r}), 1 \le n \le N_2 - 1 \\ \lambda(\text{t} - \xi_2 \sum_{i=0}^{n\text{-}1} \xi_1^i U_0 / r|\text{r}), n = N_2 \\ \lambda(\text{t} - \sum_{i=1}^{n-N_2} \xi_1^i U_0 / r - \xi_2 \sum_{j=n-N_2}^{n-1} \xi_1^j U_0 / r|\text{r}), N_2 + 1 \le n \le N_1 \end{cases}$$

In this case, the cost during the warranty period is as follows:

$$\begin{aligned} C_3(T_0, U_0) &= C_{p1} * (N_1 - 1) + C_{p2} \\ &+ C_f * (\int_0^{U_0/r} \lambda_0(\text{t}|\text{r}) dt + \int_{U_0/r}^{2U_0/r} \lambda_1(\text{t}|\text{r}) dt \\ &+ \ldots + \int_{(N_1-1)U_0/r}^{N_1 U_0/r} \lambda_{N_1-1}(\text{t}|\text{r}) dt + \int_{N_1 U_0/r}^{U_w/r} \lambda_{N_1}(\text{t}|\text{r}) dt) \\ &= C_{p1} * (N_1 - 1) + C_{p2} \\ &+ C_f * (\sum_{i=0}^{N_1-1} \int_{iU_0/r}^{(i+1)U_0/r} \lambda_i(\text{t}|\text{r}) dt + \int_{N_1 U_0/r}^{U_w/r} \lambda_{N_1}(\text{t}|\text{r}) dt) \end{aligned} \quad (3)$$

According to Eqs. (1) - (3), in case of $r_0 \le r_w$, the expected value of maintenance cost during the warranty period is

$$\begin{aligned} C(T_0, U_0) &= \int_0^{r_0} C_1(T_0, U_0) dG(r) + \int_{r_0}^{r_w} C_2(T_0, U_0) dG(r) \\ &+ \int_{r_w}^{+\infty} C_3(T_0, U_0) dG(r) \end{aligned} \quad (4)$$

b) If $r_0 > r_w$, as shown in Fig. 3, there are three cases.

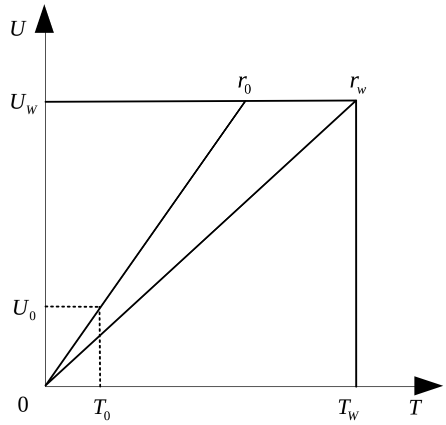

Fig.3 The relationship between r_0 and r_w （$r_0 > r_w$）

(1)When $r \le r_w$, preventive maintenance interval is T_0, and warranty deadline is T_w, $N_1 = \text{int}[T_w / T_0]$.

The expression of $\lambda_n(\text{t}\,|\,\text{r})$ is

$$\lambda_n(\text{t}\,|\,\text{r}) = \begin{cases} \lambda(\text{t}\,|\,\text{r}), \text{n} = 0 \\ \lambda(\text{t} - \sum_{i=1}^{n} \xi_1^i T_0\,|\,\text{r}), 1 \le n \le N_2 - 1 \\ \lambda(\text{t} - \xi_2 \sum_{i=0}^{n\text{-}1} \xi_1^i T_0|\text{r}), n = N_2 \\ \lambda(\text{t} - \sum_{i=1}^{n-N_2} \xi_1^i T_0 - \xi_2 \sum_{j=n-N_2}^{n-1} \xi_1^j T_0\,|\,r), N_2 + 1 \le n \le N_1 \end{cases}$$

In this case, the cost during the warranty period is as follows:

$$\begin{aligned} C_4(T_0, U_0) &= C_{p1} * (N_1 - 1) + C_{p2} \\ &+ C_f * (\int_0^{T_0} \lambda_0(\text{t}|\text{r}) dt + \int_{T_0}^{2T_0} \lambda_1(\text{t}|\text{r}) dt \\ &+ \ldots + \int_{(N_1-1)T_0}^{N_1 T_0} \lambda_{N_1-1}(\text{t}|\text{r}) dt + \int_{N_1 T_0}^{T_W} \lambda_{N_1}(\text{t}|\text{r}) dt) \\ &= C_{p1} * (N_1 - 1) + C_{p2} \\ &+ C_f * (\sum_{i=0}^{N_1-1} \int_{iT_0}^{(i+1)T_0} \lambda_i(\text{t}|\text{r}) dt + \int_{N_1 T_0}^{T_W} \lambda_{N_1}(\text{t}|\text{r}) dt) \end{aligned} \quad (5)$$

(2)When $r_w < r \le r_0$, preventive maintenance interval is T_0, and warranty deadline is U_w / r, $N_1 = \text{int}[U_w / T_0 r]$.

The expression of $\lambda_n(\text{t}\,|\,\text{r})$ is

$$\lambda_n(\text{t}\,|\,\text{r}) = \begin{cases} \lambda(\text{t}\,|\,\text{r}), \text{n} = 0 \\ \lambda(\text{t} - \sum_{i=1}^{n} \xi_1^i T_0\,|\,\text{r}), 1 \le n \le N_2 - 1 \\ \lambda(\text{t} - \xi_2 \sum_{i=0}^{n\text{-}1} \xi_1^i T_0|\text{r}), n = N_2 \\ \lambda(\text{t} - \sum_{i=1}^{n-N_2} \xi_1^i T_0 - \xi_2 \sum_{j=n-N_2}^{n-1} \xi_1^j T_0\,|\,r), N_2 + 1 \le n \le N_1 \end{cases}$$

In this case, the cost during the warranty period is as follows:

$$\begin{aligned} C_5(T_0,U_0) &= C_{p1}*(N_1-1)+C_{p2} \\ &+C_f*(\int_0^{T_0}\lambda_0(\mathrm{t}|\mathrm{r})dt+\int_{T_0}^{2T_0}\lambda_1(\mathrm{t}|\mathrm{r})dt \\ &+...+\int_{(N_1-1)T_0}^{N_1T_0}\lambda_{N_1-1}(\mathrm{t}|\mathrm{r})dt+\int_{N_1T_0}^{U_w/r}\lambda_{N_1}(\mathrm{t}|\mathrm{r})dt) \\ &= C_{p1}*(N_1-1)+C_{p2} \\ &+C_f*(\sum_{i=0}^{N_1-1}\int_{iT_0}^{(i+1)T_0}\lambda_i(\mathrm{t}|\mathrm{r})dt+\int_{N_1T_0}^{U_w/r}\lambda_{N_1}(\mathrm{t}|\mathrm{r})dt) \end{aligned} \quad (6)$$

(3)When $r>r_0$, preventive maintenance interval is U_0/r, and warranty deadline is U_w/r, $N_1=\mathrm{int}[U_w/U_0]$.

The expression of $\lambda_n(\mathrm{t}|\mathrm{r})$ is

$$\lambda_n(\mathrm{t}|\mathrm{r})=\begin{cases}\lambda(\mathrm{t}|\mathrm{r}),\mathrm{n}=0\\ \lambda(\mathrm{t}-\sum_{i=1}^{n}\xi_1^{\,i}U_0/r|\mathrm{r}),1\le n\le N_2-1\\ \lambda(\mathrm{t}-\xi_2\sum_{i=0}^{n\text{-}1}\xi_1^{\,i}U_0/r|\mathrm{r}),n=N_2\\ \lambda(\mathrm{t}-\sum_{i=1}^{n-N_2}\xi_1^{\,i}U_0/r-\xi_2\sum_{j=n-N_2}^{n-1}\xi_1^{\,j}U_0/r|\mathrm{r}),N_2+1\le n\le N_1\end{cases}$$

In this case, the cost during the warranty period is as follows:

$$\begin{aligned} C_6(T_0,U_0) &= C_{p1}*(N_1-1)+C_{p2} \\ &+C_f*(\int_0^{U_0/r}\lambda_0(\mathrm{t}|\mathrm{r})dt+\int_{U_0/r}^{2U_0/r}\lambda_1(\mathrm{t}|\mathrm{r})dt \\ &+...+\int_{(N_1-1)U_0/r}^{N_1U_0/r}\lambda_{N_1-1}(\mathrm{t}|\mathrm{r})dt+\int_{N_1U_0/r}^{U_w/r}\lambda_{N_1}(\mathrm{t}|\mathrm{r})dt) \\ &= C_{p1}*(N_1-1)+C_{p2} \\ &+C_f*(\sum_{i=0}^{N_1-1}\int_{iU_0/r}^{(i+1)U_0/r}\lambda_i(\mathrm{t}|\mathrm{r})dt+\int_{N_1U_0/r}^{U_w/r}\lambda_{N_1}(\mathrm{t}|\mathrm{r})dt) \end{aligned} \quad (7)$$

According to Eqs. (5) - (7), in case of $r_0\le r_w$, the expected value of maintenance cost during the warranty period is

$$\begin{aligned} C(T_0,U_0) &= \int_0^{r_w}C_4(T_0,U_0)dG(r)+\int_{r_w}^{r_0}C_5(T_0,U_0)dG(r) \\ &+\int_{r_0}^{+\infty}C_6(T_0,U_0)dG(r) \end{aligned} \quad (8)$$

III. MODEL ANALYSIS

From the expression of cost model, we can see that it is difficult to solve it directly by the method of function because of the complexity of the model. With the help of MATLAB tools, the model is solved by numerical algorithm. Generating finite groups with fixed step length, each group of numerical values is substituted into the model for solving, and the cost is obtained. We can see that the 3D chart of the cost is a concave surface. By comparison, the minimum cost can be obtained. At the same time, the best preventive maintenance interval can be got easily.

IV. EXAMPLE ANALYSIS

It is known that after the sale of a new type of large-scale engineering vehicle, in order to reduce the number of failures and reduce the cost during the warranty period, preventive maintenance is required. Through years of use of the same type of products, users have a certain degree of maintenance. In the initial stage of product use, compared with the manufacturer, the user's maintenance capabilities is still lacking. Thus, two degrees preventive maintenance performed by the manufacturer and the user are more viable. The two-dimensional warranty period for the agreed equipment in the warranty contract is $T_w=5$ a, $U_w=15\times10^4$ km. The remaining parameters are shown in Table I (the relevant data can be obtained from the statistics of the industrial sector).

TABLE I
PARAMETER SETTINGS

Parameter	Value
ξ_1	0.8
ξ_2	0.6
C_{p1}	5000 yuan
C_{p2}	11000 yuan
C_f	1000 yuan
$\theta_0, \theta_1, \theta_2, \theta_3$	0.8，0.7，0.9，1.1

T_0 takes the value within $[0,5]$, the step length is 0.05; U_0 takes the value within $[0,15]$, the step length is 0.15. Thus it can generates a total of 10000 groups (T_0,U_0). Solve the established model and draw a three-dimensional diagram of the corresponding expected cost values $C(T_0,U_0)$ under different conditions (T_0,U_0), as shown in Fig. 4. By calculation, minimize cost of joint preventive maintenance during the warranty period is 7.7906×10^4 yuan, under the condition of $T_0=0.85$ a, U_0=5.25×10^4 km. Compared with the traditional single preventive maintenance, if the user only performs preventive maintenance, the three-dimensional diagram of the cost model is shown in Fig. 5. The minimum cost is 8.4995×10^4 yuan. If the manufacturer only performs preventive maintenance, the three-dimensional m diagram of the cost model is shown in Fig. 6. The minimum cost is 8.042×10^4 yuan. Joint preventive maintenance cost 9.11% less than preventive maintenance by the user alone; 3.21% less than preventive maintenance by the manufacturer alone. It can be seen from the calculation results that the model is effective.

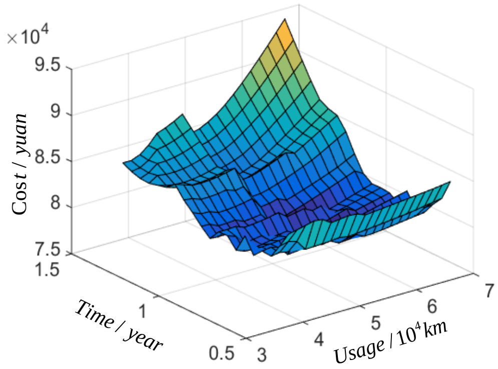

Fig.4 Joint preventive maintenance cost

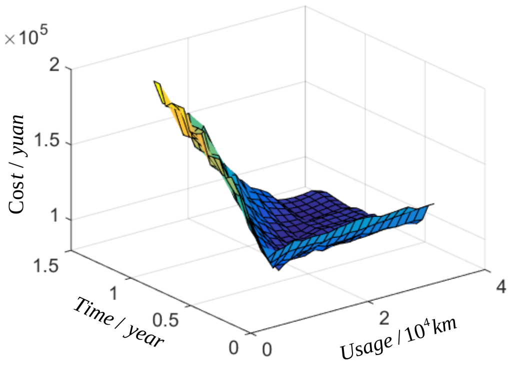

Fig.5 User independent preventive maintenance cost

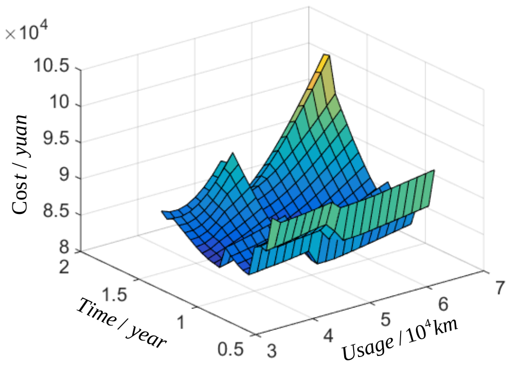

Fig.6 Manufacturer independent preventive maintenance cost

V. SENSITIVITY ANALYSIS

The warranty strategy is divided into three categories: A, B, and C. Strategy A is for the user to perform preventive maintenance independently. Strategy B is for the manufacturer to perform preventive maintenance independently during the warranty period. Strategy C is for the user and manufacturer proposed in this paper to jointly implement preventive maintenance. service.

1) Taking C_{p2} as a variable, the remaining parameters remain unchanged, and 10 sets of values are calculated. The results are shown in Table II. We can see that as C_{p2} increases, the optimal strategy changes from policy B to policy C from the 3th group. The obtained data is plotted to see the trend of change. It can be seen from Fig. 7 that as C_{p2} increases, the advantages of Strategy B and Strategy C gradually decrease. The comparison results show that with the change of the preventive cost of the manufacturer, the optimal strategy may be converted between the three strategies. In a considerable scope, the strategy C is the optimal strategy.

TABLE II
COST COMPARISON UNDER DIFFERENT C_{p2}

Warranty cost/10^4 yuan					
C_{p1}	C_{p2}	Policy A	Policy B	Policy C	Optimal Policy
	9000	8.4995	7.5398	7.6954	B
	9500	8.4995	7.6653	7.7192	B
	10000	8.4995	7.7909	7.7430	C
	10500	8.4995	7.9165	7.7668	C
5000	11000	8.4995	8.0420	7.7906	C
	11500	8.4995	8.1676	7.8144	C
	12000	8.4995	8.2932	7.8382	C
	12500	8.4995	8.4187	7.8620	C
	13000	8.4995	8.5443	7.8858	C

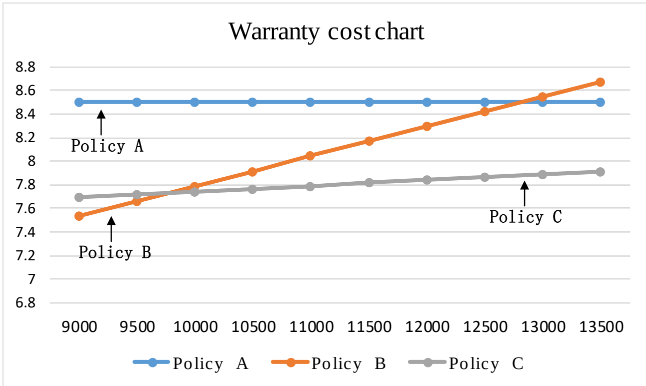

Fig.7 Warranty cost chart under different C_{p2}

2) Taking ξ_1 as a variable, the remaining parameters remain unchanged, and 7 sets of values are calculated. The results are shown in Table III. It can be seen from Fig. 8 that with the increase of ξ_1, under the conditions of Strategy A and Strategy C, the warranty fee is gradually increased, but the growth rate of Strategy C is lower than that of Strategy A. The optimal strategy is transformed between A, B, and C. It can be seen that with the change of ξ_1 and maintenance cost in two kinds of incomplete preventive maintenance, the optimal maintenance strategy will change. Under certain conditions, strategy C is the optimal strategy.

TABLE III
COST COMPARISON UNDER DIFFERENT CONDITIONS

Warranty cost/10^4 yuan				
ξ_1	Policy A	Policy B	Policy C	Optimal Policy
0.65	6.4761	8.0420	6.7631	A
0.7	7.0150	8.0420	7.0665	A
0.75	7.6790	8.0420	7.4075	C
0.8	8.4995	8.0420	7.7906	C
0.85	9.5157	8.0420	8.2208	B
0.9	10.777	8.0420	8.7041	B
0.95	12.344	8.0420	9.2469	B

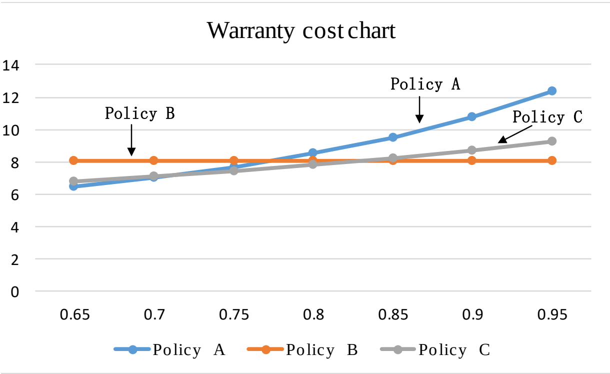

Fig.8 Warranty cost chart under different ξ_1

In this paper, the cost of the two-dimensional warranty product in warranty period is studied, the maintenance strategy is proposed, and corresponding cost model is established. Through the calculation of example, the proposed strategy can reduce cost during the warranty period, and provide a scientific basis for the manufacturer and the user to select a reasonable preventive maintenance interval and select the corresponding degree of incomplete preventive maintenance. However, this paper only considers the manufacturer to perform preventive maintenance once, and in the following research, it should be studied to perform multiple preventive maintenance. As we all know, for some uses, the availability of the product during the warranty period is just as important as the maintenance cost of the product, and sometimes it may be more important than the maintenance cost. Therefore, in the next step, we should also study the availability during the warranty period.

REFERENCES

[1] M. Park and H. Pham, "A new warranty policy with failure times and warranty servicing times," *IEEE Transactions on Reliability*, vol. 61, no. 3, pp. 822-831, 2012.

[2] M. S. Kadyan and J. Kumar, "Stochastic Modeling of a Single-Unit Repairable System with Preventive Maintenance under Warranty," *International Journal of Computer Applications*, vol. 75, no. 14, pp. 36-41, 2014.

[3] C. Su and X. Wang, "A two-stage preventive maintenance optimization model incorporating two-dimensional extended warranty," *Reliability Engineering and System Safety*, vol. 155, pp. 169–178, 2016.

[4] Y. S. Huang and C. Yen, "A study of two-dimensional warranty policies with preventive maintenance," *IIE Transactions*, vol. 41, no. 4, pp. 299-308, 2009.

[5] L. Shang and Z. Cai, "Optimal replacement policy of products with repair-cost threshold after the extended warranty," *Journal of Systems Engineering and Electronics*, vol. 28, no. 4, pp. 725-731, 2017.

[6] A. Y. Alqahtani and S. M. Gupta, "Money-back guarantee warranty policy with preventive maintenance strategy for sensor-embedded remanufactured products," *Journal of Industrial Engineering International*, vol. 14, no. 4, pp. 767-782, 2018.

[7] G. Wang, Y. Zhang and R. C. M. Yam, "Preventive maintenance models based on the generalized geometric process," *IEEE Transactions on Reliability*, vol. 66, no. 4, pp. 1380-1388, 2017.

[8] J. Wang, Z. Zhou and H. Peng, "Flexible decision models for a two-dimensional warranty policy with periodic preventive maintenance," *Reliability Engineering and System Safety*, vol. 162, pp. 14-27, 2017.

[9] C. Su and Y. Liu, "Multi-objective imperfect preventive maintenance optimisation with NSGA-II," *International Journal of Production Research*, vol. 5, pp. 1-17, 2019.

[10] Y. H. Chien, "The optimal preventive-maintenance policy for a NHPBP repairable system under free-repair warranty," *Reliability Engineering and System Safety*, vol. 188, pp. 444-453, 2019.

[11] P. Tong, X. Song and Z. Liu, "A maintenance strategy for two-dimensional extended warranty based on dynamic usage rate," *International Journal of Production Research*, vol. 55, no. 19, pp. 1-17, 2017.

[12] J. Wu, M. Xie and Ng. T. S. Adam, "On a general periodic preventive maintenance policy incorporating warranty contracts and system ageing losses," *International Journal of Production Economics*, vol. 129, no. 1, pp. 102-110, 2011.

[13] Y. S. Huang, W. Y. Gau and J. W. Ho, "Cost analysis of two-dimensional warranty for products with periodic preventive maintenance," *Reliability Engineering and System Safety*, vol. 134, pp. 51-58, 2015.

[14] M. Park, K. M. Jung and D. H. Park, "Optimal maintenance strategy under renewable warranty with repair time threshold," *Applied Mathematical Modelling*, vol. 43, pp. 498-508, 2017.

[15] Y. S. Huang, "Two-Dimensional Warranty With Reliability-Based Preventive Maintenance," *IEEE Transactions on Reliability*, vol. 62, no. 4, pp. 898-907, 2013.

[16] H. Pham and H. Wang, "Imperfect maintenance," *European Jpurnal of Operational Research*, vol. 94, no. 3, pp. 425-438, 1996.

[17] S. Martorell, A. Sanchez and V. Serradell, "Age-dependent reliability model considering effects of maintenance and working conditions," *Reliability Engineering and System Safety*, vol. 64, no. 1, pp. 19-31, 1999.

[18] Z. Lin, Y. S. Huang and C. Fang "Non-periodic preventive maintenance with reliability thresholds for complex repairable systems," *Reliability Engineering and System Safety*, vol. 136, pp. 145-156, 2015.

[19] E. F. Sami and B. D. Mohamed, "Age-based hybrid model for imperfect preventive maintenance," *IIE Transactions*, vol. 38, no. 4, pp. 365-375, 2006.

[20] G. L. Liao. "Optimum Maintenance Strategy of a Repairable System Under Long-Term Free Preventive Maintenance Warranty with Predicted Maintenance," *International Journal of Reliability, Quality and Safety Engineering*, vol. 23, no. 4, pp. 1-7, 2016.

[21] J. Lawless and J. Hu, J. Cao, "Methods for the estimation of failure distributions and rates from automobile warranty data," *Lifetime Data Analysis*, vol. 1, no. 3, pp. 227-240, 1995.

[22] S. K. Gupta, S. De and A. Chatterjee, "Warranty forecasting from incomplete two-dimensional warranty data," *Reliability Engineering and System Safety*, vol. 126, pp. 1-13, 2014.

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