An Optimization Model with Concerning Product Deterioration for Parameters Decision

Chien-Ping Chung

Abstract—**Product parameters decision plays an important role in enhancing product competitiveness, which can effectively improve quality and reduce cost. As we know, there is a dependency between the process mean and process tolerance of the product that they need to be decided at the same time to get the best solution under quality and cost considerations. However, when the product is deteriorating over time, it is necessary to concern the initial setting and the using time to avoid a high loss due to product failure. Thus, this study establishes an optimization model of quality and cost to make parameters decision with concerning product deterioration. For global solutions, sufficient and necessary optimality conditions for parameters are also proposed. Finally, numerical example and sensitivity analysis are used to fully explain and relevant steps for product parameters decision are also provided to help researchers apply the results of this study.**

Index Terms—**Parameters decision, Quality loss, Production cost, Product deterioration**

I. Introduction

FANCING an increasingly competitive environment, manufacturing needs to consider quality and cost factors in the development of new products. At the product and process design stage, there is a two-stage method for reducing variations in product quality [1–2]. The first stage is parameter design, which aims to reduce the sensitivity of product quality to variation, by setting the process mean. The second stage is tolerance design, which reduces process tolerance by controlling the variability of product parameters. In general, reducing process tolerance can result in increased costs; therefore process mean and process tolerance need to be considered at the same time in the product and process design stage [3–4].

Taking into account product deterioration factors, failure rates may increase at later stages of product life [5–6]. Product designers need to focus on how to operate the product correctly and safely over its expected life under consumer conditions. Deterioration may result in failure of the product function and even cause serious damage; therefore, it is necessary to design a product for product life application in consideration of the deterioration of the product.

It is well known that product parameters may change continuously and product applications may terminate before their life expectancy expires [7–9]. Products are usually assembled from many components based on their functional requirements. Thus, the life of a product is a function of the life of its components, i.e., when the assembled product undergoes deterioration, it may be necessary to replace one or more components. Simply stated, all product parameters must meet the design target before the product is used. However, during the use of the product, the product parameters of the components may change, which may gradually reduce the functionality of the product [10]. As the quality of product performance declines, it may increase the risk of product failure over time. That is, product quality may change gradually, and product components may deteriorate before their expiration. Therefore, considering the initial setting of the process mean of quality compensation and product life based on possible deterioration becomes an important factor in design activities [11]. In addition, the determination of process tolerance that affects product parameters during application is also a key factor.

Changes in process tolerance will affect both quality loss and production cost, and the initial setting of process mean will result in various quality loss; quality loss and production cost must be considered to appropriately reflect the total cost in the proposed model. Due to the dependencies between process mean and tolerance, some researchers synchronize the product parameters of product design and process planning [12]. However, product deterioration may occur and result in continuous changes in its product parameters. In this regard, the present study aims to develop a model in which the initial setting of process mean, process tolerance and using time are simultaneously determined during the deterioration process.

This study has five sections. Section I is the introduction. Section II describes the background information needed in this study. Section III presents the problem formulation. Section IV provides a numerical example for applications, and a conclusion is given in Section V.

II. Relevant Background

A. Process Mean and Tolerance

In the course of product manufacturing and processing, tolerance configuration plays an important role. In [13], Irani et al. built the relationship between design tolerance and process tolerance using a dimension chain for each fixed dimension, thereby enlarging process tolerance as much as possible within a reasonable range. In [14], Ji proposed the solution objective equation of maximum process tolerance,

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meaning enlarging the process tolerance as much as possible, while meeting the restrictions of design tolerance and process capability. In [15], Ngoi and Fang included cost factors and considered different costs due to varied tolerances. They thus converted the process tolerance of each processing operation by a specific economic evaluation method into different weights, in order to determine the maximum process tolerance allocation amount. However, related studies did not consider quality loss when products leave the factory and are purchased by consumers. Therefore, regarding the productivity of products, manufacturers only require the quality characteristic value of products to fall within specification limits, and they broaden tolerance levels as much as possible in order to reduce manufacturing costs. Thus, with the current increasingly strict quality requirements, many problems will occur. In [16], Wei indicates that the broadening of tolerance levels implies opportunities for producing products of poorer quality. Therefore, a nonlinear mathematical model, which simultaneously considers quality loss, manufacturing costs, and process capability, was developed in order to optimize process tolerance. On the other hand, some scholars have expounded the importance of quality, and combined the quality loss function, as proposed by Dr. Genichi Taguchi, with tolerance allocation. The aim is to deduce a nonlinear mathematical model, which minimizes quality loss and manufacturing costs, while appropriately balancing quality and cost, in order to minimize total cost and optimize process tolerance [17–20]. Literature in this respect seems not to consider the quality loss function. If the process mean is not set as the design target value, the process mean is assumed as the design target value for related research. In [21], the authors integrated quality loss with manufacturing costs to design the optimal process tolerance. The case of reworking failed workpieces and quality characteristic value entails particular asymmetric loss; thus, the new concept of a process mean unequal to the design target value was proposed, where process mean and process tolerance are regarded as product parameters, respectively, in order to determine the optimal process mean and process tolerance.

B. Quality Loss and Production Cost

Quality should meet consumer demand, and under economic considerations, customers require that products are produced the most economically. In the view of customers, quality means satisfactory product functions or good after-sales service during the usage period of products. The quantification of quality loss can be discussed from the concept of Taguchi secondary quality loss [22–24]. Although the quality characteristic value of products has met specification limits, as it deviates from the design target value to different extents, there are different quality losses. According to the concept of secondary quality loss function, as proposed by Taguchi, it is obvious that there is no loss only when the quality characteristic value completely meets the design target value. Once the quality characteristic value of products deviates from the design target value, even if the quality characteristic value still meets specification limits, there remains nonlinear quality loss according to quality loss coefficient K. The K value depends on consumer demand; namely, the quality loss function reflects consumers' requirements for quality. If S denotes the offset of quality characteristic value y from the set target value T, the loss to consumers is A, then $K = A / S^2$. This phenomenon describes that although the products reaching customers are qualified, the product specifications or functions fail to completely meet the design target value; thus, there will be social cost loss. Mathematical expression in Eq. (1) and Fig. 1 are presented for description.

$$L(x) = K\,(x - T)^2 \tag{1}$$

where x is the quality value, K is quality loss coefficient, and T is the design target.

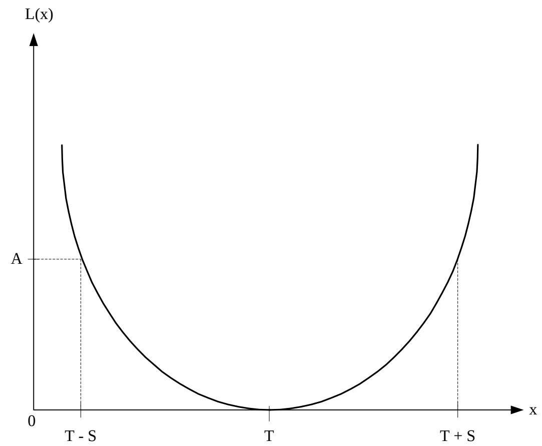

Fig. 1 Quality loss expression

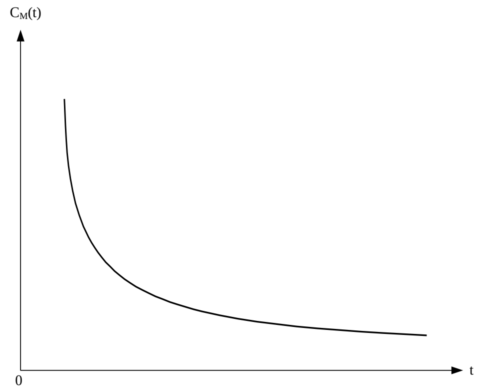

Fig. 2 Tolerance cost expression

In the product manufacturing process, different tolerances result in different costs, as given in Fig. 2. Precise tolerance will result in higher cost, and loose tolerance will result in lower cost. However, for the measurement of product cost, this study directly uses the tolerance-cost function [25], as proposed:

$$C_M(t) = a + b \cdot \exp(-c \cdot t) \tag{2}$$

where a, b, and c are coefficients and t is the process tolerance.

Coefficient a is part of the fixed costs, representing setup and preparation costs before processing; $b \cdot \exp(-c \cdot t)$ is part of the variable costs, representing the costs of effort and time put into processing, i.e., variable cost in direct relation to process tolerance. The coefficients a, b, and c are regarded as tolerance-cost coefficients determined using a statistical regression analysis equation to calculate previous data or actual data of a working field.

III. Problem formulation

All costs incurred in a product life cycle include quality loss and production cost. Quality loss occurs during use by the consumer. Production cost refers to the costs incurred before the product is sold to consumers. Therefore, the total cost can be obtained through the sum of quality loss and production cost.

When the initial setting of process mean is time-dependent due to deterioration, it can be expressed as a function of time s:

$$U(s) = a_0 + (B + W \cdot s) \quad (3)$$

Here, a_0 is the initial setting of process mean, B is a constant, and W is the deterioration rate. Suppose that the dimension value x is a random variable and is in accordance with a normal distribution f(x) of the mean U(s) and the standard deviation σ. The variance is a function of the process tolerance [21]:

$$\sigma^2 = \left(\frac{t}{3 \times C_P}\right)^2 \quad (4)$$

where C_P is the process capability index.

Let x_1, x_2, …, x_n be the quality values appearing in different situations. The average quality loss in its symmetric quality loss function is as follows:

$$E[L(x)] = K\,[(U(s) - T)^2 + \sigma^2] \quad (5)$$

Let Q be the using time that needs to be determined in the proposed model; the expected total cost TC per unit time for the duration Q can then be expressed by:

$$TC = \frac{1}{Q}\{\int_0^Q E[L(x)]ds + C_M(t)]\} \quad (6)$$

Decision variables include initial setting of process mean a_0, process tolerance t, and using time Q. For global solutions [26], sufficient optimality conditions for parameters a_0^*, t^*, and Q^* are shown in Appendix A, and necessary optimality conditions are given in Appendix B.

IV. Numerical example and sensitivity analysis

Consider the following numerical example: T = 30 mm, S = 0.12 mm, K = \$7200, W = 0.05 mm/month, B = 0.0000001, C_P = 1, a = 50.1135, b = 119.3737, and c = 31.5878. By inserting these values in Eq. (6), the total cost function represents our objective function. GAMS software [27] was used to solve the mathematical model. Because the objective function contains integral expressions, they first have to be converted into polynomial equations to be readable by GAMS. The optimal solutions are shown in Table 1.

A sensitivity analysis was performed using the deterioration rate W values. The results are shown in Table 2, and we have the following conclusion. When the deterioration rate increases, it drives TC to increase. An increase in the deterioration rate results in a reduction of the Q^* value, and vice versa. This may be explained as follows: when the deterioration rate increases, the possibility that the upper specification limit will be exceeded increases as well in time. Hence, we would rather keep the product in a short using time to ensure that the product performs in a normal function. Moreover, when the deterioration rate increases, the probability for the product parameters to fall in an acceptable range is reduced.

Table 1 Optimal solutions of the problem formulation

Variables	Values
Initial setting of process mean a_0^*	29.9279 mm
Process tolerance t^*	0.0638 mm
Using time Q^*	2.5644 months
Total cost TC	$39.2300

Table 2 The values of a_0^*, t^*, Q^* and TC vs. W

W	a_0^*	t^*	Q^*	TC
0.03	29.9406	0.0648	3.8186	$28.4007
0.04	29.9340	0.0640	3.0487	$33.9572
0.05	29.9279	0.0638	2.5644	$39.2300
0.06	29.9218	0.0641	2.2335	$44.2606
0.07	29.9149	0.0649	2.0019	$49.0575

From the above results, it can be known that when there is deterioration in the components of the product, the initial setting of process mean, process tolerance, and using time should be considered. Optimal product parameters can be easily solved using optimization software. In other words, the optimization model constructed in this study can be used as a method for parameters decision when product components have deteriorated. At the same time, using the proposed solution can significantly reduce cost and improve quality.

Robust design delivers high quality products at a lower total cost, which increases the competitiveness of the manufacturer; however, during product use, the product's component functionality may be shorter than expected due to deterioration. Avoiding the high cost of troubleshooting and ensuring quality requirements when making parameter decisions, including the initial setting of process mean, process tolerance, and using time is critical. The constructed optimization model considers minimizing costs, including quality loss and production cost, while optimizing the above parameters. To help the researcher apply the results of this study, the relevant steps are as follows:

Step 1: Define the functionality of the product and components, and determine the product parameters of interest. This also provides design target values, design tolerances, and quality loss coefficient for the product. If there is a deterioration process in the components of the product, a deterioration rate should be provided.

Step 2: Construct an optimization model taking into account the total cost with quality loss and production cost.

Step 3: Regarding whether the found product parameters are global optimal solutions, it is necessary to verify the sufficient optimality conditions.

Step 4: Simultaneously optimize the product parameters by finding necessary optimality conditions.

Step 5: Finally, sensitivity analysis and model discussion are performed on some decision variables for product design.

V. Conclusion

The results of this study show that when the product is deteriorating over time, the product parameters, including the initial setting of process mean, process tolerance, and using time, should be optimized simultaneously. Therefore, the product can operate in quality function, and product failure result in expensive and expensive payments that can be avoided. For optimal solutions with the proposed optimization model can be obtained from package software such as GAMS. In other words, the problem formulation can be easily applied by most product designers. Namely, product parameters decision can significantly improve product quality and reduce cost. These achievements can improve the ability of today's manufacturing industry to face fierce competition.

APPENDIX

A. Sufficient Optimality Conditions for Parameters

The Hessian matrix [28–29] is:

$$\mathbf{H} = \begin{bmatrix} \frac{\partial^2 TC}{\partial a_0^2} & \frac{\partial^2 TC}{\partial a_0 \partial t} & \frac{\partial^2 TC}{\partial a_0 \partial Q} \\ \frac{\partial^2 TC}{\partial t \partial a_0} & \frac{\partial^2 TC}{\partial t^2} & \frac{\partial^2 TC}{\partial t \partial Q} \\ \frac{\partial^2 TC}{\partial Q \partial a_0} & \frac{\partial^2 TC}{\partial Q \partial t} & \frac{\partial^2 TC}{\partial Q^2} \end{bmatrix}$$

The sufficient optimality conditions for parameters a_0^*, t^*, and Q^* are that M_1, M_2, and M_3 are positive. They are:

$$M_1 = \frac{\partial^2 TC}{\partial a_0^2} \geq 0 \tag{7}$$

$$M_2 = \frac{\partial^2 TC}{\partial a_0^2} \frac{\partial^2 TC}{\partial t^2} - \frac{\partial^2 TC}{\partial a_0 \partial t} \frac{\partial^2 TC}{\partial t \partial a_0} \geq 0 \tag{8}$$

$$M_3 = \frac{\partial^2 TC}{\partial a_0^2} \frac{\partial^2 TC}{\partial t^2} \frac{\partial^2 TC}{\partial Q^2} + \frac{\partial^2 TC}{\partial t \partial a_0} \frac{\partial^2 TC}{\partial Q \partial t} \frac{\partial^2 TC}{\partial t \partial a_0} + \frac{\partial^2 TC}{\partial Q \partial a_0} \frac{\partial^2 TC}{\partial t \partial Q} \frac{\partial^2 TC}{\partial a_0 \partial t} - \frac{\partial^2 TC}{\partial a_0 \partial Q} \frac{\partial^2 TC}{\partial t^2} \frac{\partial^2 TC}{\partial Q \partial a_0} - \frac{\partial^2 TC}{\partial t \partial Q} \frac{\partial^2 TC}{\partial Q \partial t} \frac{\partial^2 TC}{\partial a_0^2} - \frac{\partial^2 TC}{\partial Q^2} \frac{\partial^2 TC}{\partial t \partial a_0} \frac{\partial^2 TC}{\partial a_0 \partial t} \geq 0 \tag{9}$$

To prove $M_1 \geq 0$ is true.

$$M_1 = 2K \geq 0 \tag{10}$$

To prove $M_2 \geq 0$ is true.

$$M_2 = 2K(bc^2 e^{-ct})/Q + \left(\frac{2K}{3C_P}\right)^2 \geq 0 \tag{11}$$

To prove $M_3 \geq 0$ is true.

$$M_3 = \frac{e^{-ct} K[54b^2c^2C_P^2 + 2e^{ct}KQ(12a + B^2KQ^3W^2)]}{27C_P^2Q^4} + \frac{e^{-ct}K\{3be^{ct}[36ac^2C_P^2 + KQ(8 + 3B^2c^2C_P^2Q^2W^2)]\}}{27C_P^2Q^4} \geq 0 \tag{12}$$

B. Necessary Optimality Conditions for Parameters

To find necessary optimality conditions for parameters, take the first derivative of Eq. (6) with respect parameters and equate them to zero, respectively. The derivative of Eq.(6) with respect parameter a_0 and equate to zero.

$$K(2a_0 - 2T - BQW) = 0 \tag{13}$$

The derivative of Eq.(6) with respect parameter t and equate to zero.

$$\frac{bce^{-ct}}{Q} + \frac{2Kt}{9QC_p^2} g\left(-bce^{-ct} + \frac{2KQt}{9C_p^2}\right) = 0 \tag{14}$$

The derivative of Eq.(6) with respect parameter Q and equate to zero.

$$\frac{-3a - 3be^{-ct} + BKQ^2W(-3a_0 + 3T + 2BQW)}{3Q^2} = 0 \tag{15}$$

By solving the simultaneous equations with Eq.(13), Eq.(14), and Eq.(15), the optimal solution for parameters a_0^*, t^*, and Q^* can be found.

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