Numerical Solutions of the Moisture Transport in Rough Rice

Bubpha Jitsom, Surattana Sungnul and Ekkachai Kunnawuttipreechachan

Abstract—**In this work, we investigate numerical solutions of the moisture transport in rough rice. In Thailand, moisture content of rough rice is one of the most important factors for setting price of rough rice. In computational method, the moisture transport in rough rice can be considered as an infinite cylinder. The model can be discribed by the diffusion equation with a convective boundary condition. The explicit and implicit schemes of finite difference methods are applied to solve such the problem. The numerical results show that the moisture content is distributed from centre to boundary surface of the rough rice. The numerical simulations of the moisture content at various radius are also illustrated.**

Index Terms—**moisture content, drying, diffusion model, infinite cylinder, finite difference method.**

I. Introduction

ROUGH rice is one of the most consumed crops as it provides staple food for more than half of world's population. Moisture content is one of the most important factors influencing the quality of rough rice during storage and it remains at a high level during the harvest, namely 14-15% of moisture content [13], and then it is reduced with appropriate drying process.

Storage of rough rice requires moisture control to be at an appropriate level because newly harvested rough rice still has high humidity. Therefore, research has to be done to find ways to reduce the humidity of rough rice and store it as efficiently as possible to maintain the rice's quality. In general, if the moisture of rough rice is too high, then it will spoil the grains of rice. On the other hand, if the humidity is too low, it may cause commercial weight loss and may cause the kernels of the rough rice to be breaken and deteriorate nutritional values.

The moisture content in rough rice is defined as the water in liquid state. To measure the moisture content, there are both direct methods and indirect methods [14]. Loss of drying on moisture is a direct method that can be used as a reference method [15]. The direct method provides high accuracy and high precision of moisture content values. However, this method is time-consuming. Hence, indirect methods such as resistance-type and capacitance-type instruments are developed to reduce time-consuming for drying measurement [16].

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The humidity in rough rice is important to extend the storage. After harvesting the rough rice, the moisture content of the rough rice should be reduced as soon as possible. Dehumidification in rough rice can be done in two ways:

1. The natural drying is the use of heat from sunlight. The air flow causes heat and mass transfers between rough rice and air, causing the moisture content of the rough rice to decrease.

2. For artificial drying, the dryer has an advantage of being able to dry in all weather conditions, even when it rains or has little sunlight. This method does not waste space for drying and is able to control the humidity reduction to the desired level time spent in reducing less moisture.

At present, there are few previous works related to numerical solutions of moisture transport in rough rice. There are some studies which conduct to study the moisture transport from rough rice. For examples, O. Hacıhafızog, et al. [3] show that the suitability of several drying models available in literature in defining thin layer drying behaviour of long-grain rough rice has been examined by using statistical analysis. In 2009, R. Dong, et al. [4] study the moisture distribution in rough rice kernel during temperature drying process. The results show that the model is capable of accurately simulating drying and tempering processes of grain.

Z. Naghavi, et al. [5] study a non-equilibrium model of the grain fixed deep-bed drying for rough rice. The accumulation terms are kept in both energy and mass balance equations and the set of coupled partial differential equations derived from the model is solved simultaneously by using the backward implicit method. W. P. da Silva, et al. [6] investigate the inverse method, which is used to fit analytical solutions of the diffusion equation with convective boundary conditions to the experimental data of the thin-layer drying kinetics of products with cylindrical shape. In 2013, J. Shanthilal and C. Anandharamakrishnan [7] use the finite element method to solve numerical solutions for rice hydration and dehydration models. This review clearly indicates that the model with computational and numerical methods are valuable tools for prediction of temperature profile and moisture content distribution in rough rice during hydration and dehydration. In 2017, P. Klomklao, et al. [8] present that the moisture content can be measured by gravimetric method which is a direct method. It is found that the measurement results have a standard uncertainty of 1.23% of the moisture content in the range of 14% to 20%.

In this work, we investigate numerical solutions of the moisture transport in rough rice, which is one of the most important factors for setting prices of rough rice. The moisture transport in rough rice is considered as an infinite cylinder and can be discribed by diffusion equations with a convective boundary condition. The explicit and implicit finite difference schemes are applied to evaluate numerical solutions of such the problem.

II. Mathematical Formulation

A. The diffusion equation

The diffusion equation for a property Φ can be written in a general form as [1]-[2], namely

$$\frac{\partial}{\partial t}(\lambda\Phi) = \nabla\cdot\left(\Gamma^{\Phi}\nabla\Phi\right) + S, \tag{1}$$

where λ and Γ^{Φ} are parameters of the diffusion process and S is a source term. For a cylinder with length L, which is much larger than its radius R, a one-dimensional diffusion equation can be applied by

$$\frac{\partial}{\partial t}(\lambda\Phi) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\Gamma^{\Phi}\frac{\partial\Phi}{\partial r}\right) + S, \tag{2}$$

where r is the distance of a point from the cylinder axis.

Setting $\lambda = 1$, $\Gamma^{\Phi} = D_f$ (effective diffusivity), $\Phi = M$ (dry basis moisture content) and $S = 0$, then Eq. (2) can be rewritten for the moisture transport considered as an infinite cylinder

$$\frac{\partial M}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(rD_f\frac{\partial M}{\partial r}\right). \tag{3}$$

The diffusion equation for the description of water transport in solid can be analytically solved under the following assumsions:

- the solid is homogeneous and isotropic,
- the initial moisture distribution is uniform,
- the dimensions of the solid do not vary during the diffusion,
- the effective diffusivity do not vary during the diffusion,
- the convective mass transfer coefficient is constant during the diffusion.

B. Boundary condition

The initial moisture content M_0 at the centre of grain is defined by the dirichlet boundary condition as $M(0,t) = 1$. For the initial time, its boundary is defined as $M(r,0) = 1$.

The convective boundary condition, also called boundary condition of the third kind or Cauchy boundary condition [6], is expressed by imposing equaly internal diffusive flux at the boundary of the infinite cylinder and external convective flux near the boundary, i.e.

$$-D_f\frac{\partial}{\partial r}M(r,t)|_{r=R} = h\left(M(r,t)\,|_{r=R} - M_{eq}\right). \tag{4}$$

Here, h is the convective mass transfer coefficient, $M(r,t)$ is the moisture content at radial distance r and time t, M_{eq} is the equilibrium moisture content in a solid of given drying matter, and R is the radius of the infinite cylinder.

III. Numerical Methods

This section presents the finite difference methods by using the FTCS (forward-time central-space) explicit scheme and the FTCS implicit scheme for infinite cylinder to evaluate numerical solutions of Eq. (3).

A. The FTCS explicit scheme

Setting the space step-size $\Delta r = \frac{R}{m}$ and the time step-size $\Delta t = \frac{T_{max}}{n}$, where T_{max} is the time-length of the problem. Eq. (3) can be written into the form of the FTCS explicit scheme:

$$\frac{M_i^{n+1} - M_i^n}{\Delta t} = D_f\left(\frac{M_{i+1}^n - 2M_i^n + M_{i-1}^n}{(\Delta r)^2}\right) + \frac{D_f}{r_i}\left(\frac{M_{i+1}^n - M_{i-1}^n}{2\Delta r}\right), \tag{5}$$

where $M_i^n = M(i\Delta x, n\Delta t)$, $i = 0,1,2,...,m$ and $n = 1,2,...,T_{max}$.

Simplify Eq.(5) to the explicit form as

$$M_i^{n+1} = \left(A + \frac{B}{r_i}\right)M_{i+1}^n - 2AM_i^n + \left(A - \frac{B}{r_i}\right)M_{i-1}^n + M_i^n, \tag{6}$$

where $A = \frac{D_f\Delta t}{(\Delta r)^2}$ and $B = \frac{D_f\Delta t}{2\Delta r}$.

The convergency condition of the FTCS explicit scheme is that $\Delta t \le \frac{(\Delta r)^2}{4D_f}$. If Δt does not satisfy with this condition, then the solutions may converge or diverge.

B. The FTCS implicit scheme

Using the same notations for the step-sizes, the FTCS implicit scheme of Eq.(3) can be written as

$$\frac{M_i^{n+1} - M_i^n}{\Delta t} = D_f\left(\frac{M_{i+1}^{n+1} - 2M_i^{n+1} + M_{i-1}^{n+1}}{(\Delta r)^2}\right) + \frac{D_f}{r_i}\left(\frac{M_{i+1}^{n+1} - M_{i-1}^{n+1}}{2\Delta r}\right). \tag{7}$$

It can be rearranged into a simple form as

$$a_iM_{i-1}^{n+1} + b_iM_i^{n+1} + c_iM_{i+1}^{n+1} = M_i^n, \tag{8}$$

where

$$\begin{aligned} a_i &= -\frac{D_f\Delta t}{(\Delta r)^2} + \frac{D_f\Delta t}{2r_i\Delta r}, \\ b_i &= 1 + 2\frac{D_f\Delta t}{(\Delta r)^2}, \\ c_i &= -\frac{D_f\Delta t}{(\Delta r)^2} - \frac{D_f\Delta t}{2r_i\Delta r}, \end{aligned}$$

for $i = 2,...,m-1$.

The system (8) can be written into the matrix equation with tridiagonal coefficient matrix. The sweep method can be applied to solve this problem. In addition, the FTCS implicit scheme is unconditionally stable.

Algorithm of the sweep method

The system of linear equations (8) can be written in matrix form $A\mathbf{M}^{n+1} = \mathbf{B}$, ie.

$$\begin{bmatrix} b_2 & c_2 & 0 & 0 & \cdots & 0 \\ a_3 & b_3 & c_3 & 0 & \cdots & 0 \\ 0 & a_4 & b_4 & c_4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & a_{m-2} & b_{m-2} & c_{m-2} \\ 0 & 0 & 0 & \cdots & a_{m-1} & b_{m-1} \end{bmatrix} \begin{bmatrix} M_2 \\ M_3 \\ M_4 \\ \vdots \\ M_{m-2} \\ M_{m-1} \end{bmatrix}^{n+1}$$

$$= \begin{bmatrix} f_2 \\ f_3 \\ f_4 \\ \vdots \\ f_{m-2} \\ f_{m-1} \end{bmatrix},$$

where

$$\begin{bmatrix} f_2 \\ f_3 \\ f_4 \\ \vdots \\ f_{m-2} \\ f_{m-1} \end{bmatrix} = \begin{bmatrix} M_2^n - a_2 M_1^{n+1} \\ M_3^n \\ M_4^n \\ \vdots \\ M_{m-2}^n \\ M_{m-1}^n - c_{m-1} M_m^{n+1} \end{bmatrix}.$$

In component form, we have

$$\begin{aligned} b_2 M_2^{n+1} + c_2 M_3^{n+1} &= f_2, \\ a_i M_{i-1}^{n+1} + b_i M_i^{n+1} + c_i M_{i+1}^{n+1} &= f_i, i = 3, 4, ..., m-2; \\ & \qquad n = 1, ..., [T/\tau] - 1, \\ a_{m-1} M_{m-2}^{n+1} + b_{m-1} M_{m-1}^{n+1} &= f_{m-1}. \end{aligned} \tag{9}$$

We assume that
$M_i^{n+1} = \alpha_i M_{i+1}^{n+1} + \beta_i$,

$$i = 3, 4, ..., m-2; \ n = 1, ..., [T/\tau] - 1. \tag{10}$$

Note that $i = 2, \ M_2^{n+1} = \alpha_2 M_3^{n+1} + \beta_2$ and compare with the first equation of (9) we have

$$\alpha_2 = -\frac{c_2}{b_2}, \ \beta_2 = \frac{f_2}{b_2}. \tag{11}$$

Substituting $M_{i-1}^{n+1} = \alpha_{i-1} M_i^{n+1} + \beta_{i-1}$ into the second equation of (9) gives that
$a_i(\alpha_{i-1} M_i^{n+1} + \beta_{i-1}) + b_i M_i^{n+1} + c_i M_{i+1}^{n+1} = f_i$,
where $i = 3, 4, ..., m-2$. Above equation can be simplified as

$$M_i^{n+1} = -\frac{c_i}{a_i \alpha_{i-1} + b_i} M_{i+1}^{n+1} + \frac{f_i - a_i \beta_{i-1}}{a_i \alpha_{i-1} + b_i}.$$

Comparing with (10), we have

$$\alpha_i = -\frac{c_i}{a_i \alpha_{i-1} + b_i}, \quad \beta_i = \frac{f_i - a_i \beta_{i-1}}{a_i \alpha_{i-1} + b_i}, \ i = 3, ..., m-2. \tag{12}$$

For $i = m-2$, substitute $M_{m-2}^{n+1} = \alpha_{m-2} M_{m-1}^{n+1} + \beta_{m-2}$ into the last equation of (9), we have

$$\begin{aligned} a_{m-1} \left(\alpha_{m-2} M_{m-1}^{n+1} + \beta_{m-2} \right) + b_{m-1} M_{m-1}^{n+1} \\ = M_{m-1}^n - c_{m-1} M_m^{n+1}. \end{aligned} \tag{13}$$

From the convective boundary condition in Eq.(4), we get

$$M_m^{n+1} = \frac{1}{1+F} \left[F \cdot M_{eq} + M_{m-1}^{n+1} \right], \text{where } F = \frac{h \Delta r}{D_f}.$$

Eq.(13) can be written as

$$\begin{aligned} & a_{m-1} \left(\alpha_{m-2} M_{m-1}^{n+1} + \beta_{m-2} \right) + b_{m-1} M_{m-1}^{n+1} + \\ & \qquad + c_{m-1} \left\{ \tfrac{1}{1+F} \left[F \cdot M_{eq} + M_{m-1}^{n+1} \right] \right\} = M_{m-1}^n. \end{aligned}$$

It can be rearranged as

$$M_{m-1}^{n+1} = \frac{M_{m-1}^n - a_{m-1} \beta_{m-2} - \frac{c_{m-1} F \cdot M_{eq}}{1+F}}{a_{m-1} \alpha_{m-2} + b_{m-1} + \frac{c_{m-1}}{1+F}}. \tag{14}$$

We can present the algorithm of the sweep method as follows

Step 1. $\alpha_2 = -\dfrac{c_2}{b_2}, \ \beta_2 = \dfrac{f_2}{b_2}$.

Step 2. $\alpha_i = -\dfrac{c_i}{a_i \alpha_{i-1} + b_i}, \quad \beta_i = \dfrac{f_i - a_i \beta_{i-1}}{a_i \alpha_{i-1} + b_i}, \quad i = 3, ..., m-2.$

Step 3. $M_{m-1}^{n+1} = \dfrac{M_{m-1}^n - a_{m-1} \beta_{m-2} - \frac{c_{m-1} F \cdot M_{eq}}{1+F}}{a_{m-1} \alpha_{m-2} + b_{m-1} + \frac{c_{m-1}}{1+F}}, \quad n = 1, ..., [T/\tau] - 1.$

Step 4. $M_i^{n+1} = \alpha_i M_{i+1}^{n+1} + \beta_i, \ i = m-2, m-3, ..., 2; \quad n = 1, ..., [T/\tau] - 1.$

The time evolutions of the moisture contents at $r = 0$ and $r = R$, as well as the average moisture content and the radial distributions of the local moisture content obtained by the FTCS explicit scheme and the FTCS implicit scheme are presented in the next section.

IV. Results

Once the parameters of the drying kinetic have been determined, the moisture content at distance r from the axis of the infinite cylinder can be calculated by Eq.(3).

The numerical results have been calculated for various values for $0 \leq r \leq R$ and $0 \leq t \leq 1200$. Then, using the FTCS explicit scheme and the FTCS implicit scheme applied to solve Eq. (3). Let us consider the infinite cylinder of Eq.(3) and setting the parameters from the experimental data [6]: $D_f = 1.997 \times 10^{-5} (mm)^2 s^{-1}, h = 3.798 \times 10^{-4} (mm) s^{-1}, R = 1.2\,(mm)$. According to [6], the moisture values suitable for purchasing rough rice in Thailand is 15%, i.e. $M_{eq} = 0.15$.

A. The numerical solutions of the FTCS explicit scheme

Let us consider the explicit form with $\Delta r = 0.05$ and $\Delta t = 31.29$ in Eq.(6). The graphs of time evolution of the moisture content are shown in Fig. 1. We can see that as time goes by, the moisture of rough rice is decreased from the centre to the surface. This means that the surface is drier than the inside of the grain. Likewise, the graphs of radial moisture distribution in Fig. 2 show that the moisture of the rough rice is decreased from the centre to the surface over time. Moreover, if Δt is not satisfied the convergence condition for the FTCS explicit scheme, i.e. $\Delta t > \frac{(\Delta r)^2}{4 D_f}$ then the numerical solutions may not converge as shown in Fig. 3 and Fig. 4 for $\Delta r = 0.05$ and $\Delta t = 70$.

The graphs of time evolution of the moisture content at the centre, the average value, and the boundary in Fig. 1 are similar to the results in previous experimental works [6].

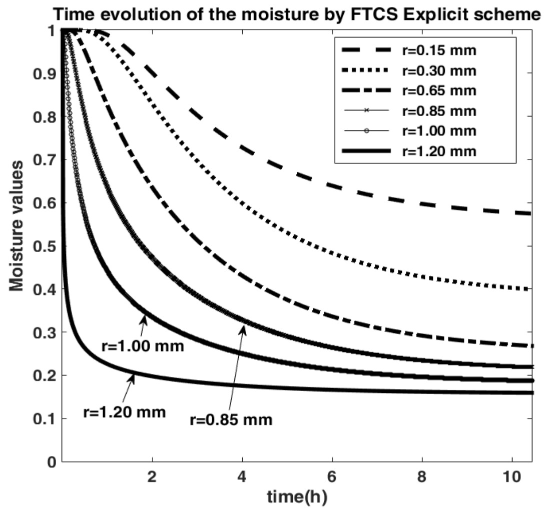

Fig. 1. Time evolution of the moisture content with $0 < r < 1.2\,mm$ by explicit scheme at $\Delta r = 0.05, \Delta t = 31.29$ with $D_f = 1.997 \times 10^{-5}(mm)^2 s^{-1}$

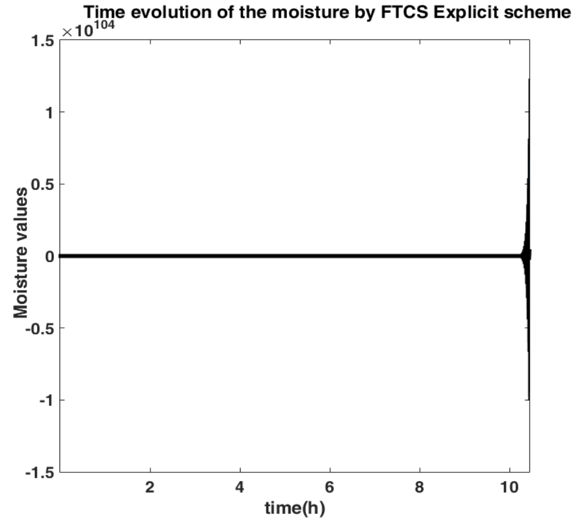

Fig. 3. Time evolution of the moisture content with $0 < r < 1.2\,mm$ by explicit scheme (divergent) at $\Delta r = 0.05, \Delta t = 70$ with $D_f = 1.997 \times 10^{-5}(mm)^2 s^{-1}$

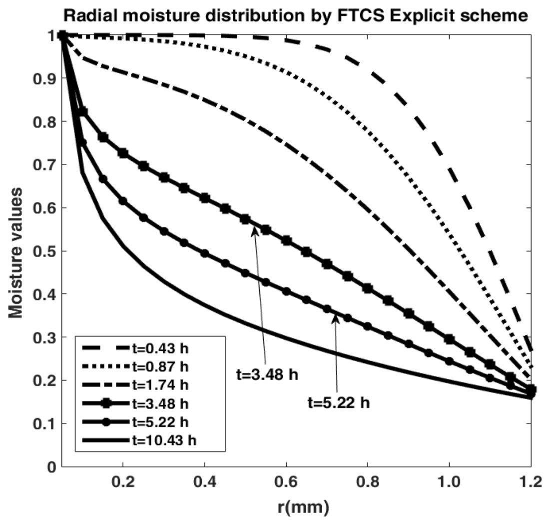

Fig. 2. Radial moisture distribution by explicit scheme at $\Delta r = 0.05, \Delta t = 31.29$ with $D_f = 1.997 \times 10^{-5}(mm)^2 s^{-1}$

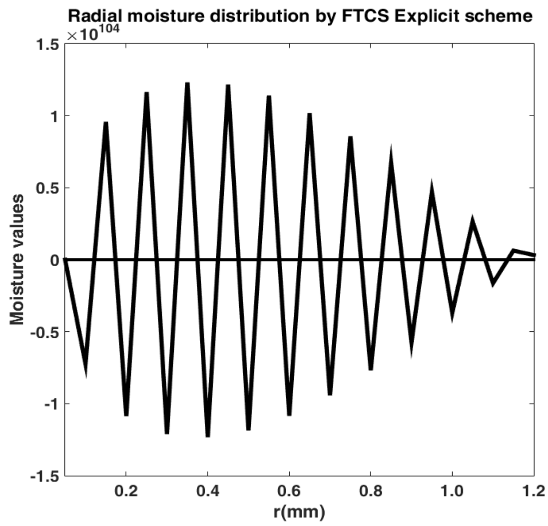

Fig. 4. Radial moisture distribution by explicit scheme (divergent) at $\Delta r = 0.05, \Delta t = 70$ with $D_f = 1.997 \times 10^{-5}(mm)^2 s^{-1}$

B. The numerical solutions of the FTCS implicit scheme

Using the system of the implicit scheme with $\Delta r = 0.05$ and $\Delta t = 31.29$ in Eq. (8). The graphs of time evolution of the moisture content are shown in Fig. 5. We can see that as time goes by, the moisture content of rough rice is decreased from the centre to the surface, which means that the surface is drier than the inside of the grain. Likewise, the graphs of radial moisture distribution in Fig. 6 show that the moisture of the rough rice is decreased from the centre to the surface over time. In addition, the FTCS implicit scheme is unconditionally stable, which means that convergence of solution does not depend on the step size. For example, when $\Delta r = 0.05$ and $\Delta t = 70$ which is not satisfied the condition $\Delta t \leq \frac{(\Delta r)^2}{4D_f}$ but we have found that the numerical solutions still converge, Fig. 7 and Fig. 8 are illustrated numerical simulations for this case.

C. The numerical simulations of the FTCS implicit scheme with $0.5 \leq D_f \leq 10$

In this section, numerical simulations of the FTCS implicit scheme with $0.5 \leq D_f \leq 10$ are illustrated. We found that the moisture content is rapidly distributed from the centre to the surface of the rough rice as the diffusivity D_f is increased. Moreover we have found that the moisture content near the centre is decreased as time t increases. On the contrary, the moisture content near the surface of rough rice is increased as time t increase as shown in Fig. 9-12.

According to Fig. 13-16, we can see that if the time t is fixed as $t = 10$ hr., the moisture content at the surface of rough rice is increased when D_f is increased.

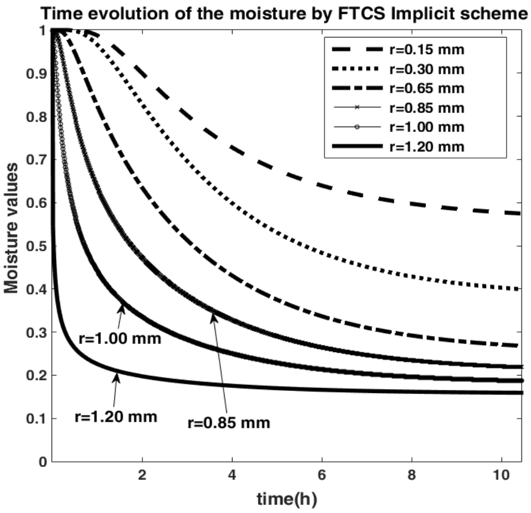

Fig. 5. Time evolution of the moisture content with $0 < r < 1.2\,mm$ by implicit scheme at $\Delta r = 0.05, \Delta t = 31.29$ with $D_f = 1.997 \times 10^{-5}(mm)^2 s^{-1}$

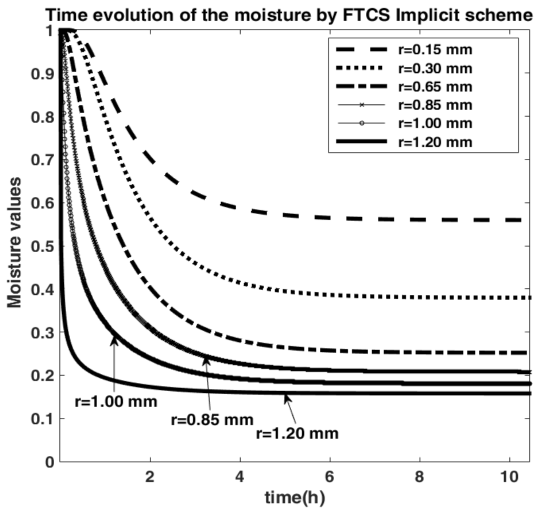

Fig. 7. Time evolution of the moisture content with $0 < r < 1.2\,mm$ by implicit scheme at $\Delta r = 0.05, \Delta t = 70$ with $D_f = 1.997 \times 10^{-5}(mm)^2 s^{-1}$

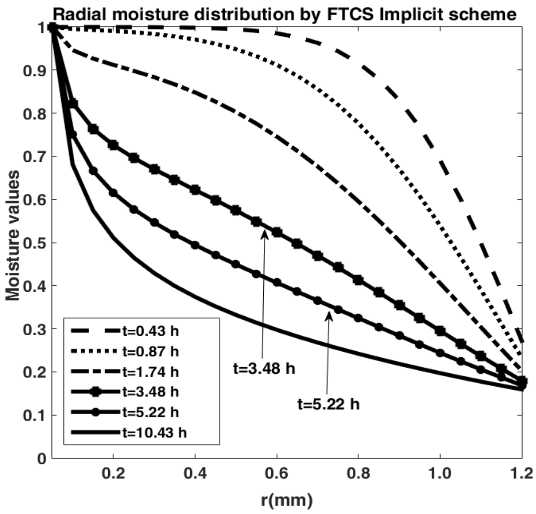

Fig. 6. Radial moisture distribution by implicit scheme at $\Delta r = 0.05, \Delta t = 31.29$ with $D_f = 1.997 \times 10^{-5}(mm)^2 s^{-1}$

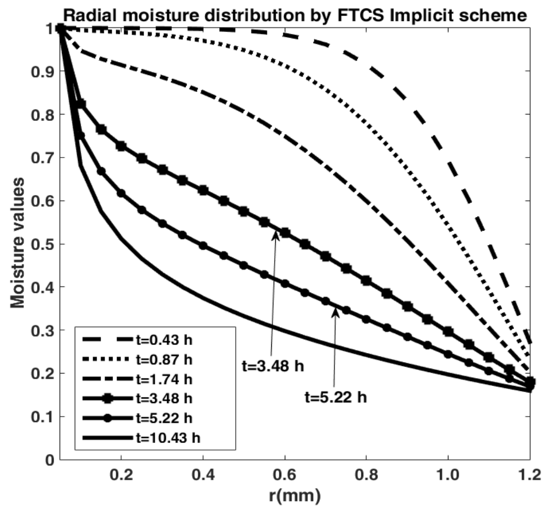

Fig. 8. Radial moisture distribution by implicit scheme at $\Delta r = 0.05, \Delta t = 70$ with $D_f = 1.997 \times 10^{-5}(mm)^2 s^{-1}$

D. The numerical simulations of the FTCS implicit scheme with $0.05 \leq M_{eq} \leq 0.5$

In general, moisture content affects the weight and quality of rough rice. From [13], the results show that, during husking process, if the humidity is too high, rough rice is broken easily. The moisture content of suitable rough rice is 14-15%. Moreover, when the moisture content of rough rice is not satisfied the standard values, the rice price will be reduced.

According to Fig. 17-20, the numerical simulations are illustrated for the cases that the moisture content is fixed $D_f = 1.997 \times 10^{-5}\,(mm)\,s^{-1}$ with various equilibrium moisture contents, $0.05 \leq M_{eq} \leq 0.5$. It can be seen that the moisture content of rough rice surface is converged to M_{eq} when time increases. Moreover, the moisture content from centre to surface of rough rice is also converged to M_{eq} as shown in Fig. 21-24.

V. Conclusion

The moisture content is one of the most importanct parameters for setting prices of rough rice. In this work, the moisture transport in rough rice is considered as an infinite cylinder described by the diffusion equation with the convective boundary condition. The explicit and implicit schemes of finite difference method are used to solve the problem.

The numerical solutions which obtained by the explicit scheme is converged if $\Delta t \leq \frac{(\Delta r)^2}{4D_f}$, but the implicit scheme is unconditionally stable for all step size. So, the implicit scheme is suitable to solve the infinite cylinder equation of moisture transport of rough rice.

We can see that as time increasing, then the moisture of rough rice is decreasing. According to the grain radius from the centre to the surface, we can explain that the surface is drier than the inside of the grain. Likewise, the graphs of radial moisture distribution show that the moisture of the rough rice is decreased from the centre to the surface over

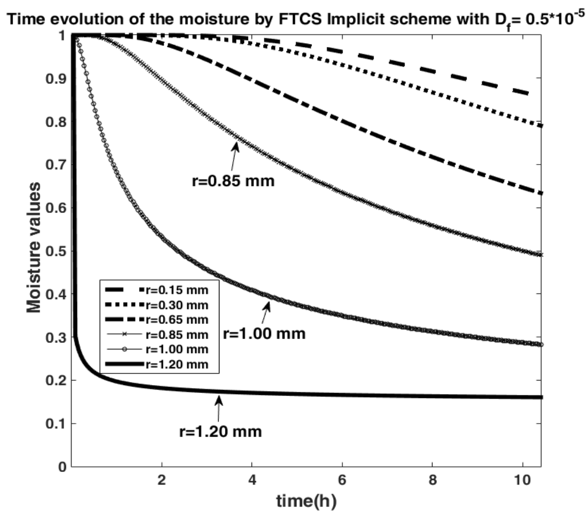

Fig. 9. Time evolution of the moisture content with $0 < r < 1.2\,mm$ by implicit scheme with $D_f = 0.5 \times 10^{-5}\,(mm)^2\,s^{-1}$

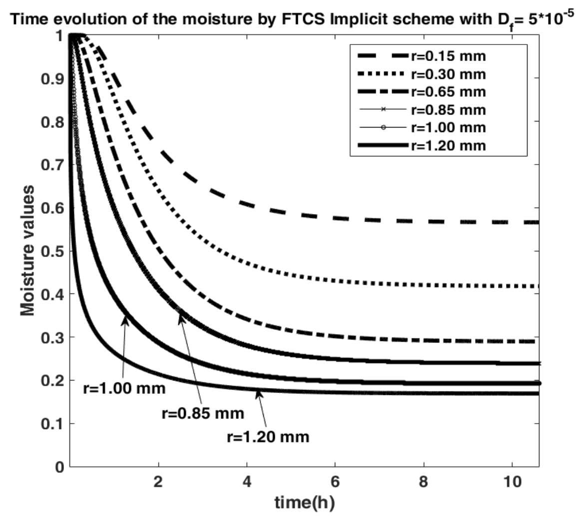

Fig. 11. Time evolution of the moisture content with $0 < r < 1.2\,mm$ by implicit scheme with $D_f = 5 \times 10^{-5}\,(mm)^2\,s^{-1}$

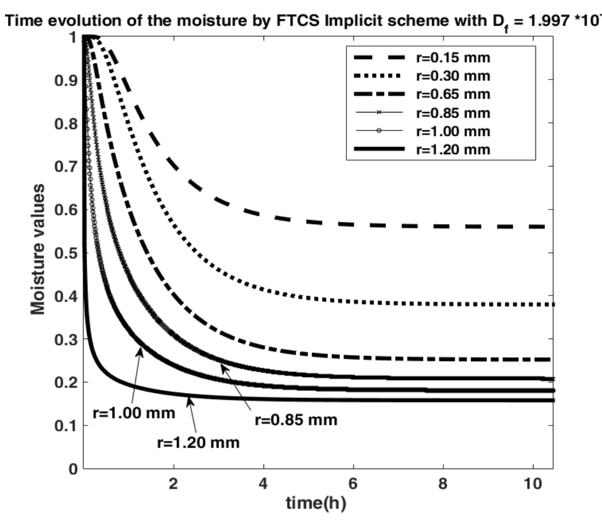

Fig. 10. Time evolution of the moisture content with $0 < r < 1.2\,mm$ by implicit scheme with $D_f = 1.997 \times 10^{-5}\,(mm)\,s^{-1}$

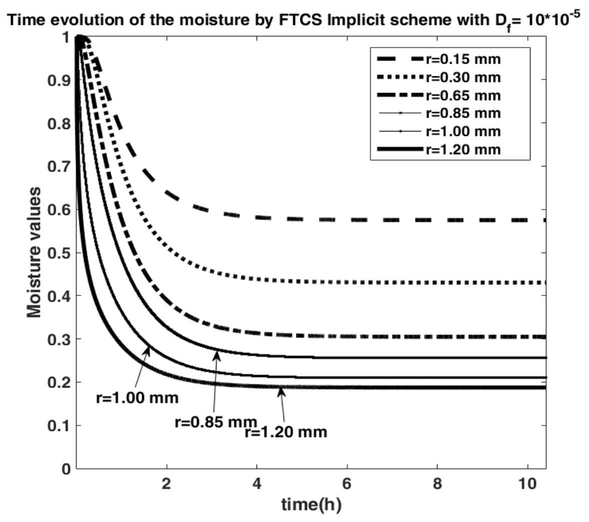

Fig. 12. Time evolution of the moisture content with $0 < r < 1.2\,mm$ by implicit scheme with $D_f = 10 \times 10^{-5}\,(mm)^2\,s^{-1}$

time.

In addition, we found that the distribution of moisture content from the centre to boundary surface of the rough rice would spread faster as the diffusivity, D_f is increases. At fixed time, for example at $t = 10$ hr., we can see that the moisture at the surface of rough rice would higher as the diffusivity, D_f increases.

The results from this work show that the moisture content of rough rice at 14-15% is suitable for harvesting and storing rough rice according to the experimental data in Thailand.

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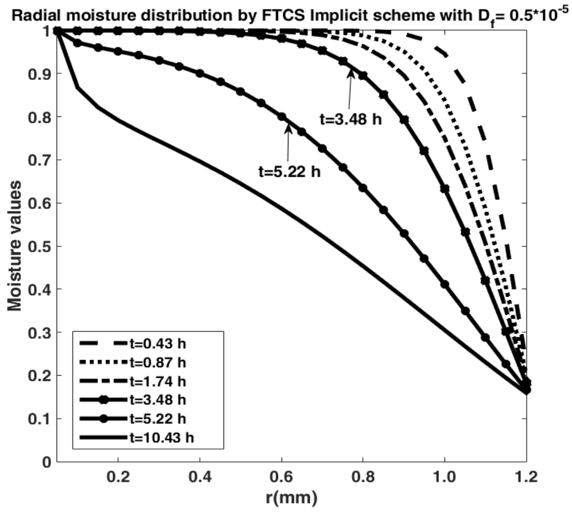

Fig. 13. Radial moisture distribution by implicit scheme with $D_f = 0.5 \times 10^{-5}\,(mm)^2\,s^{-1}$

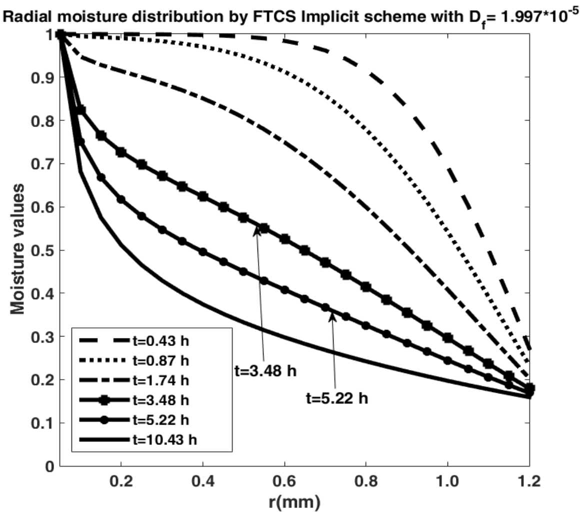

Fig. 14. Radial moisture distribution by implicit scheme with $D_f = 1.997 \times 10^{-5}\,(mm)^2\,s^{-1}$

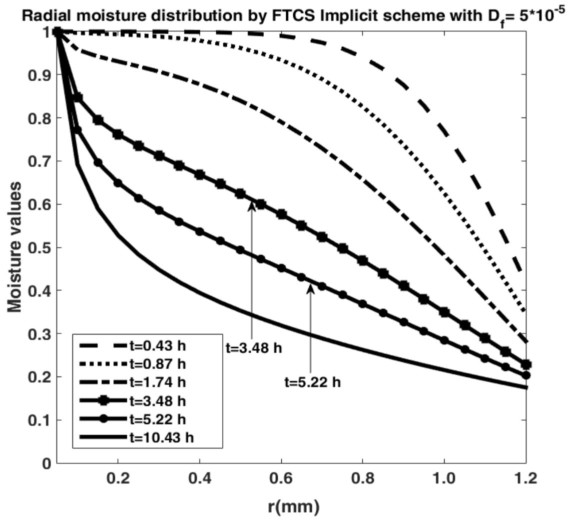

Fig. 15. Radial moisture distribution by implicit scheme with $D_f = 5 \times 10^{-5}\,(mm)^2\,s^{-1}$

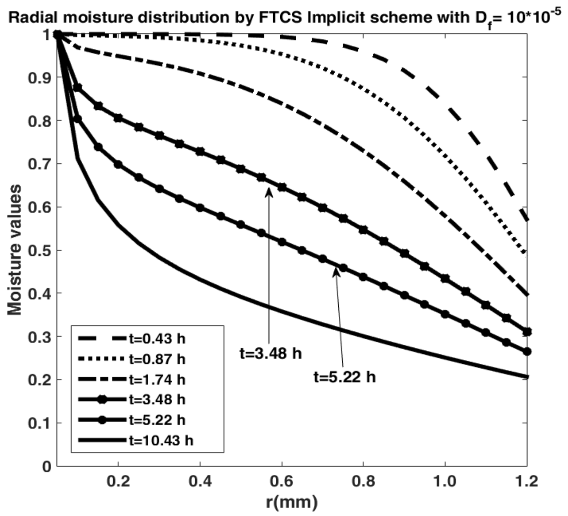

Fig. 16. Radial moisture distribution by implicit scheme with $D_f = 10 \times 10^{-5}\,(mm)^2\,s^{-1}$

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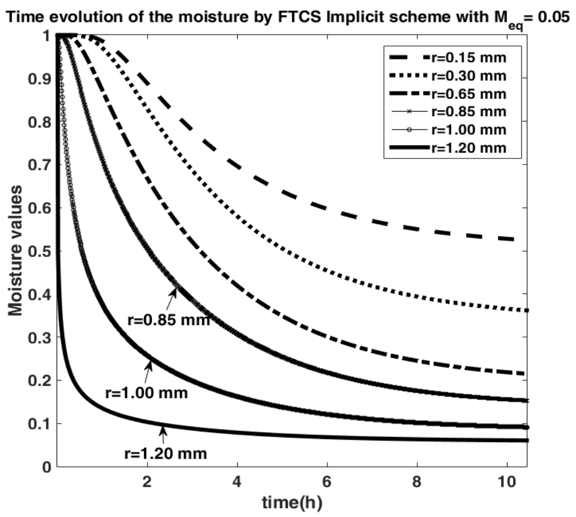

Fig. 17. Time evolution of the moisture content by implicit scheme with $M_{eq} = 0.05$

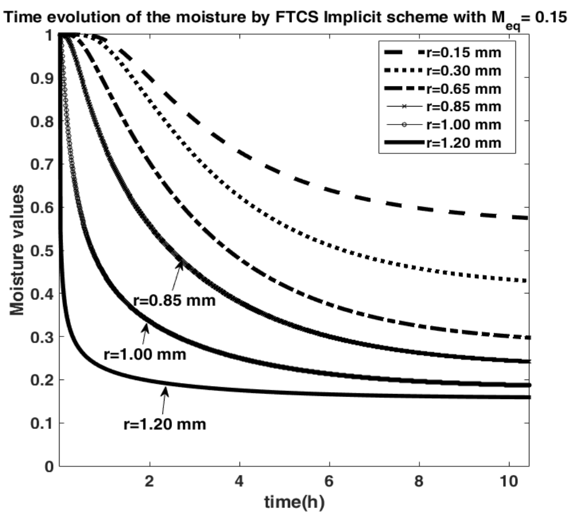

Fig. 18. Time evolution of the moisture content by implicit scheme with $M_{eq} = 0.15$

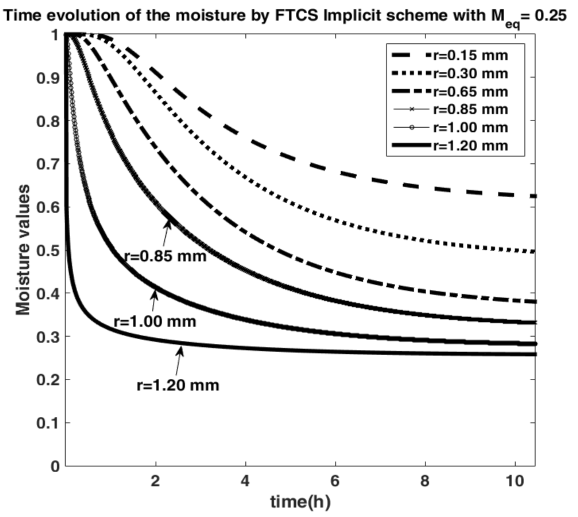

Fig. 19. Time evolution of the moisture content by implicit scheme with $M_{eq} = 0.25$

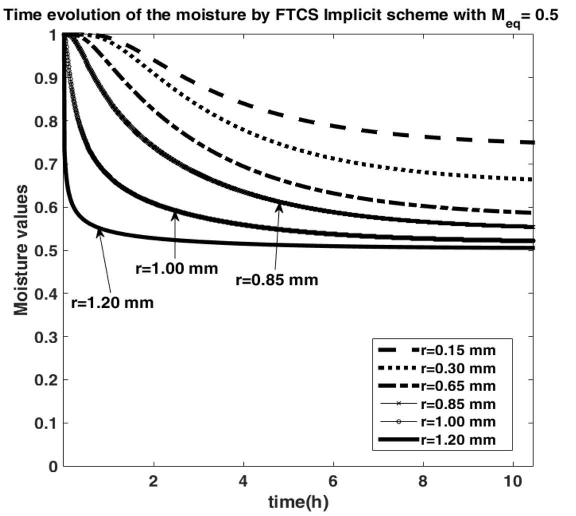

Fig. 20. Time evolution of the moisture content by implicit scheme with $M_{eq} = 0.5$

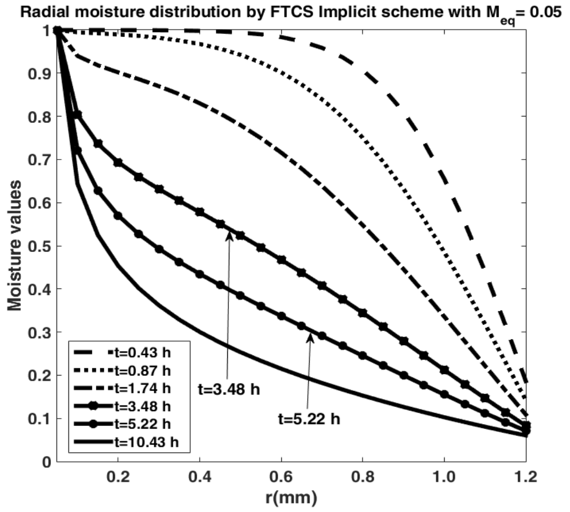

Fig. 21. Radial moisture distribution by implicit scheme with $M_{eq} = 0.05$

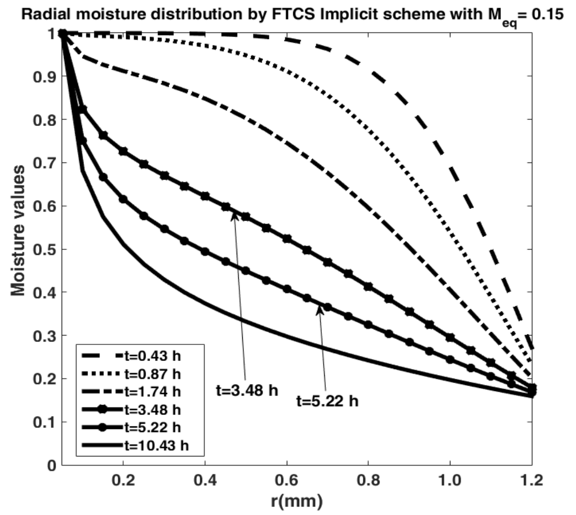

Fig. 22. Radial moisture distribution by implicit scheme with $M_{eq} = 0.15$

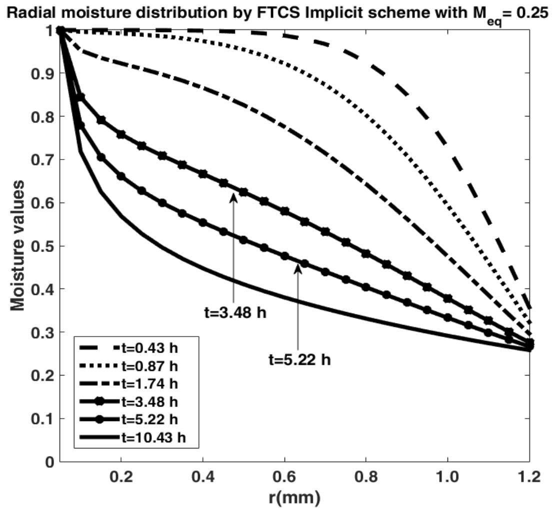

Fig. 23. Radial moisture distribution by implicit scheme with $M_{eq} = 0.25$

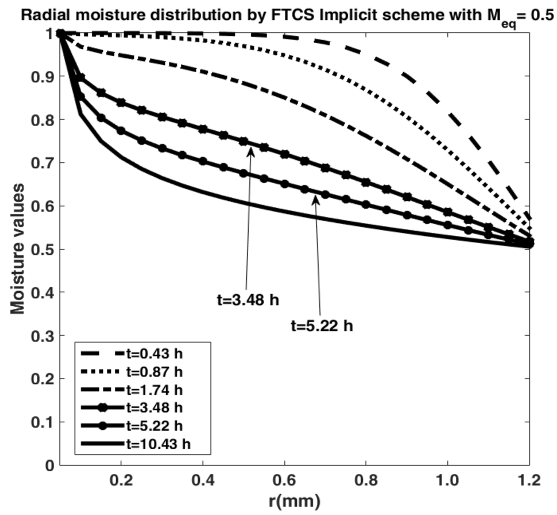

Fig. 24. Radial moisture distribution by implicit scheme with $M_{eq} = 0.5$