Multi-objective Optimal Control of Resources Applied to an Electric Power Distribution System

Jairo A. Villegas-Florez, Byron S. Hernandez-Osorio, Eduardo Giraldo

***Abstract*—In this article is presented a novel strategy for multi-objective optimal control of resources in a power distribution system for service restoration. The distribution system is modeled as a discrete Markov chain in state space by considering the probability of failure and operation of a node. A multi-objective cost function is proposed by considering the dynamical model of the distribution system as well as the failure and repair rates of each specialized crew, where the optimal multi-objective optimal control solution is obtained by using an exhaustive search algorithm. As a result, the obtained optimal control signal is the adequate working group for each failure that minimizes the repair rate and the travel time and therefore guarantees the minimum time of permanence in the failure state for the distribution system. A simulation by considering real conditions and real data from a distribution system is performed where a comparison among several scenarios is analyzed.**

***Index Terms*—Multi-objective optimization, optimal control, resource allocation.**

I. Introduction

In order to guarantee service continuity, utilities have to allocate significant resources to failure response and maintenance, namely: technical and professional staff, transportation and specialized equipment, tools and supplies [1]. Any disruption in the electricity service has operational and economic consequences for the community but also implies a cost relative to the technical and professional staff (crews) required to move from the control center to the failure node and their corresponding requirements in order to achieve a restoration of the service [2].

In [3] is proposed a scheme of segmentation for maintenance areas with their corresponding maintenance crews for a power distribution system. However, an optimal maintenance protocol should include a reasonable total time of the process and quick restoration of the service. In [4] a static solution based on a Vehicle Routing Problem is proposed but using a static model and by using genetic algorithms and simulated annealing methods as described in [5]. Also, in [6] the vehicle routing problem with hard time windows and stochastic service times. In addition, among the factors that affect the aforementioned quality of maintenance, we could report the criteria of resource allocation and the displacement time of the maintenance crew. However, the aforementioned methods only consider static models that ignore the implicit dynamics of the process [4].

The selection of optimal crews for restoration pf the service in a distribution system can be solved by applying optimal multivariable control techniques, where a dynamical model of the system is required [7]. Since the problem has multiple constraints, a multi-objective cost function that involves the required constraints is also needed [8].

In this paper is shown a novel strategy for multi-objective optimal control of resources in a power distribution system for service restoration. Two contributions are presented: the first contribution is a discrete state-space model as a discrete Markov chain that describes the implicit dynamics, the repair time, and the distance to the failure. It is worth noting that the model considers the failure rate and repair rate of the crews based on real data. The second contribution is an optimal multi-objective control solution based on an exhaustive search algorithm where the obtained control signal is the crew assignation to attend the failure. The proposed approach is evaluated for several scenarios considering real data. The paper is organized as follows: in section II, a theoretical framework of the proposed approach, which includes the dynamical state-space model, is presented. In section III are presented the results and discussions, and finally, in section IV, the conclusions, and future works are presented.

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II. Theoretical framework

A. Problem Statement

Consider the probability of failure and operation of a node in a power distribution system as follows:

$$\begin{bmatrix} \dot{P}_o \\ \dot{P}_f \end{bmatrix} = \begin{bmatrix} -\lambda & \mu \\ \lambda & -\mu \end{bmatrix} \begin{bmatrix} P_o(t) \\ P_f(t) \end{bmatrix} \tag{1}$$

being $P_o(t)$ the probability of operation and $P_f(t)$ the probability of failure of each node, and λ the failure rate (failures per unit of time) and μ the repair rate (repairs per unit of time). A discrete model can be obtained from (1) as follows [9]:

$$\frac{P_o[k+1] - P_o[k]}{\Delta t} = -\lambda P_o[k] + \mu P_f[k] \tag{2}$$

being Δt the sample time, and $P_o[k]$ the probability of operation at sample k.

It can be seen that by considering that $P_f[k] = 1 - P_o[k]$ the following difference equation can be obtained to describe the probability of failure for each node in terms of λ and μ

$$P_o[k+1] = (1 - (\lambda + \mu)\,\Delta t)\, P_o[k] + \mu \Delta t \tag{3}$$

Equation (3) describes the implicit dynamics of the operation of each node of an electric distribution system.

If the node is not operational, a crew must be sent in order to change the current state. There is a distance effect related to the displacement of the crew from the control center to the failure point. Two matrices are defined, a location matrix of the crews U in (4) and a distance matrix D in (5).

$$U = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & \cdots \end{bmatrix} \quad (4)$$

from (4) it can be interpreted that crews 1, 2 and 3 are of type 1 and are located in control center 1, meanwhile crews 4, 5 and 6 are located in control center 2.

$$D = \begin{bmatrix} d_{1,1} & d_{1,2} & \cdots & d_{1,L} \\ d_{2,1} & d_{2,2} & \cdots & d_{2,L} \\ \vdots & \vdots & \ddots & \vdots \\ d_{I,1} & d_{I,2} & \cdots & d_{I,L} \end{bmatrix} \quad (5)$$

where $d_{i;l}$ is the distance from the point of failure i to the control center l this distance is a function of the Universal Transverse Mercator (UTM) coordinates of each point. Therefore the distance to the failure i of a selected crew $u[k]$ at sample k (time instant $t_k = k\Delta t$) can be computed as

$$d_i[k] = E_f D U u[k] \quad (6)$$

being E_f defined as a row vector of 67 elements where $E_{f_i} \in \{0, 1\}$ indicates the point of occurrence of the failure.

On the other hand, the repair time that each crew requires for an specific failure depends of the skills of the crew. To this end, a matrix of skills is defined of the following form:

$$H_a = \begin{bmatrix} h_{11} & \cdots & h_{1N} \\ \vdots & \ldots & \vdots \\ h_{J1} & \cdots & h_{JN} \end{bmatrix} \quad (7)$$

where h_{jn} represents the estimated time it takes for a crew type j to repair a type n failure.

Therefore, the repair time that requires the selected crew $u[k]$ at sample k can be computed as

$$t_r = P_{tf} H_a u[k] \quad (8)$$

being P_{tf} a vector of probabilities for the type of failure that follows a negative binomial distribution, where $P_{tf} = [p_{i1}, \ldots, p_{in}, \ldots, p_{i5}]$ is the probability of occurrence for a failure at point i. This model describes the probability that a failure at point i be type n.

B. Proposed dynamical model

The following model $x_i[k+1] = f(x_i[k], u[k])$ has been proposed to consider the implicit dynamic, the distance effects and the repair times for each i node:

$$\begin{aligned} P_i[k+1] =& \alpha((1 - \Delta t(\lambda + \mu))P_i[k] + \mu \Delta t) \\ &+ \beta(P_i[k] + \Delta t \cdot f_1(E_f D U u[k])) \\ &+ \theta(P_i[k] + \Delta t \cdot f_2(P_{tf} H_a u[k])) \end{aligned} \quad (9)$$

where $u[k]$ is the control signal defined as a column vector where $u_l \in \{0, 1\}$ indicates the crew that is assigned to repair a new failure in the system at time instant k. The parameters α, β, θ are the weights of the model related to the implicit dynamic, displacement time and repair time respectively.

The functions f_1 and f_2 are defined as follows:

$$f_1 = \frac{1}{\Delta t \sqrt{2\pi\sigma_1^2}} \cdot e^{\frac{(ln\Delta t - \mu_1)^2}{2\sigma_1^2}} \quad (10)$$

$$f_2 = \frac{1}{\Delta t \sqrt{2\pi\sigma_2^2}} \cdot e^{\frac{(ln\Delta t - \mu_2)^2}{2\sigma_2^2}} \quad (11)$$

being μ_1, σ_1 are computed as follows:

$$E_{x_1} = \frac{d_i}{50Km/h} \quad (12)$$

$$V_{x_1} = \left[\frac{40d_i}{2100Km^2/h^2}\right]^2 \quad (13)$$

$$\sigma_1^2 = \ln\left(\frac{V_{x_1}}{E_{x_1}^2} + 1\right) \quad (14)$$

$$\mu_1 = \ln E_{x_1} - \frac{\sigma_1^2}{2} \quad (15)$$

and μ_2 and σ_2 are computed as:

$$E_{x_2} = t_r \quad (16)$$

$$V_{x_2} = (t_r)^2 \quad (17)$$

$$\sigma_2^2 = \ln\left(\frac{V_{x_2}}{E_{x_2}^2} + 1\right) \quad (18)$$

$$\mu_2 = \ln E_{x_2} - \frac{\sigma_2^2}{2} \quad (19)$$

with d_i and t_r computed by using (6) and (8).

C. Multi-objective Optimal control

The main aim is to minimize the travel time and the repair time for each new failure that appears holding that the system must be in operational state (operational probability must be equal to one). To this end, an exhaustive search algorithm is proposed. The multi-objective cost function used to this end is a combination of the multiple objectives of the problem and also the state space equation of the model, as follows:

$$\begin{aligned} J =& \alpha((1 - \Delta t(\lambda + \mu))P_i[k] + \mu \Delta t) \\ &+ \beta(P_i[k] + \Delta t \cdot f_1(E_f D U u[k])) \\ &+ \theta(P_i[k] + \Delta t \cdot f_2(P_{tf} H_a u[k])) \end{aligned} \quad (20)$$

where:

- i is the node that is in failure state
- α is the weight related to the state space model
- β is the weight related to the displacement time
- $\theta$is the weight related to the repair time

The solution is obtained by maximizing the probability of operation (maximizing J) in terms of the displacement time and repair time, that are computed from $u[k]$

$$u[k] = \underset{u[k]}{\operatorname{argmax}}\, J \quad (21)$$

It is worth noting that the obtained multi-objective optimal control signal $u[k]$ is the crew that is assigned to repair a failure in the distribution system at each time sample k.

III. Results and Discussion

In order to evaluate the performance of the method, several scenarios are considered. The real data used for this study is the database presented in [3]. The database provides the coordinates (UTMx, UTMy) and includes 67 nodes of failure. From these points, 30 nodes are randomly simulated. For each of these nodes is analyzed t_f and t_r to synthesize the sets λ_i and μ_i for $i = 1, 2, 3, ...67$. Under similar criteria, 5 types of failures and 5 types of crews are defined for the simulation. Using the definition of probability by relative frequency, the following vector of point probabilities is calculated for each type of failure [10]:

$$P_{tfo} = \begin{bmatrix} 0.7446 & 0.0820 & 0.0718 & 0.0582 & 0.0434 \end{bmatrix} \quad (22)$$

The method of solution for this problem is an exhaustive search, as described in [11] and [12]. In this case, the main objective is to find the nearest group to the fail that minimize the displacement time and repair time by considering the dynamic state space model of the distribution system of (20).

For this problem, the simulation has 14 working groups or crews which are divided as follows: 8 are type 1, 2 are type 2, 2 are type 3, 1 is type 4 and 1 is type 5. The travel time between a point of failure and the control center is calculated by the following formula: D_{il}/v where $v \sim \mathcal{N}(50[km/h], 20[km/h])$.

At the top of the graph, in the Fig. 1 a simulation by using 30 nodes for an interval of 48 hours is showed. It is worth noting that in the Fig. 1 the black lines represent the time that a failure is active in an specific node for a certain period of time.

The bottom of the graph in Fig. 1 shows the optimal crew that is selected to attend the failure at each time sample k (control signal $u[k]$).

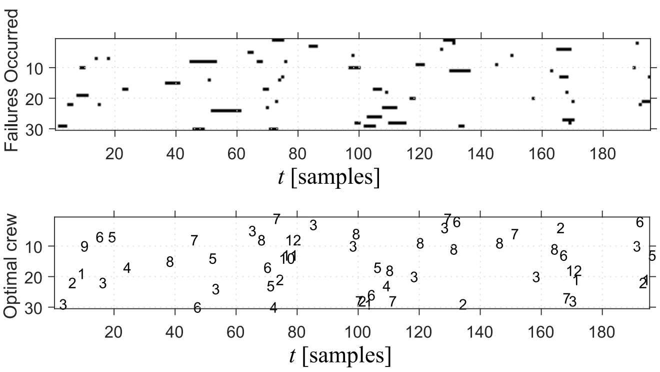

Fig. 1. Simulation of 30 nodes during 48 hours using a $\Delta t = 0.25$ hours, with their corresponding crew optimally selected to attend the failure

The total number of failures generated for this simulation is 56. In the bottom of the Fig. 1 is presented the optimal control signal $u[k]$ obtained for each failure. It is worth noting that the control signal is the optimally selected crew that is available to repair each failure.

In Fig. 2 a segment of the first 30 samples of Fig. 1 is presented where the optimal crew (control signal $u[k]$) and the failure occurred at sample k are superposed. It can be seen clearly which crew is selected optimally to attend each failure, as well as the number of samples required to restore the system to normal operation.

In Fig. 3 is showed the histogram describing the number of failures attended for each crew for the simulation of 30 nodes and 48 hours with a sample time $\Delta t = 0.25$ hours.

Finally, in the Fig. 4, it is showed the probability of operation for the all power electric distribution system after repair the fail

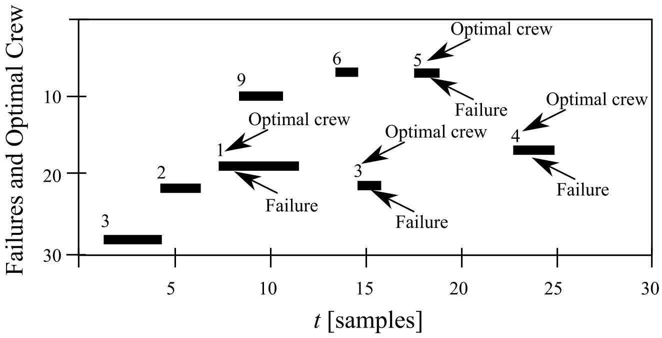

Fig. 2. Segment of 30 samples of Fig. 1 where it is shown the optimal crew (control signal $u[k]$) for each failure occurred at sample k.

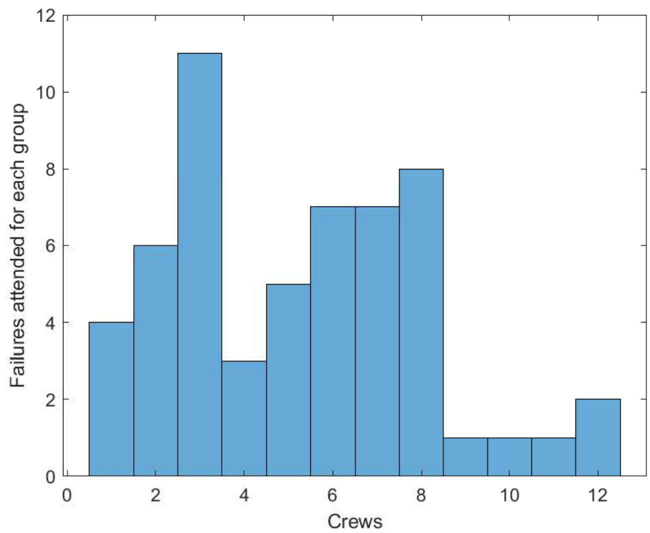

Fig. 3. Histogram describing the number of failures attended for each crew for the simulation of 48 hours

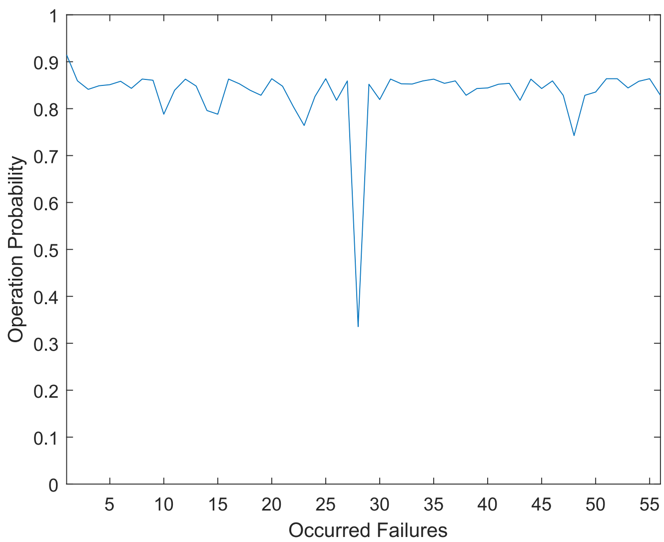

Fig. 4. The probability of operation of each fail attended for 30 nodes and 48 hours

The second simulation is performed for 30 nodes for an interval of 72 hours. The simulation results are presented in Fig. 5. The total number of failures generated in the simulation with the same number of nodes, but with 72 hours is showed at the top of the graphic in the Fig. 5, the number of the failures was 84. In the bottom of Fig. 5 is showed the number of failures attended for each crew for the simulation of 30 nodes and 72 hours with a sample time $\Delta t = 0.25$ hours, in the graph is presented too the optimal control signal $u[k]$ obtained for each failure, where the control signal is the optimally selected crew that is available to repair each failure at each time instant.

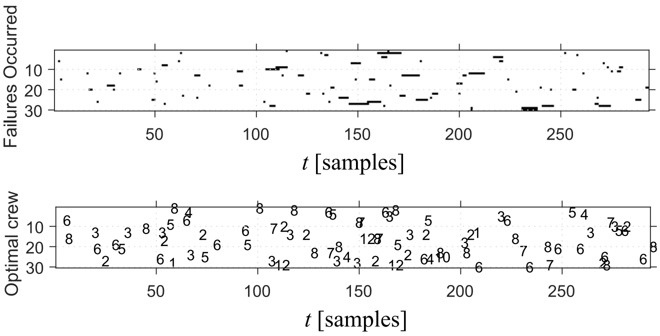

Fig. 5. Simulation of 30 nodes during 72 hours using a $\Delta t = 0.25$ hours, with their corresponding crew optimally selected to attend the failure

In Fig. 6 a segment of the 50 samples starting at sample 125 to sample 175 of Fig. 5 is presented where the optimal crew (control signal $u[k]$) and the failure occurred at sample k are superposed. It can be seen clearly which crew is selected optimally to attend each failure, as well as the number of samples required to restore the system to normal operation.

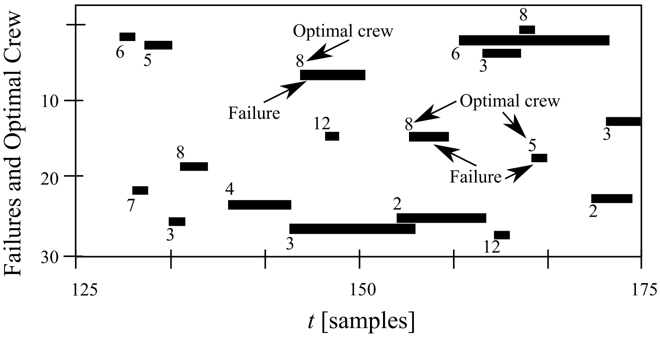

Fig. 6. Segment of 50 samples of Fig. 5 where it is shown the optimal crew (control signal $u[k]$) for each failure occurred at sample k.

In Fig. 7 is showed the histogram describing the number of failures attended for each crew for the simulation of 30 nodes and 72 hours with a sample time $\Delta t = 0.25$ hours.

In the Fig. 8 is showed the probability of operation for the whole electric distribution system after repair the fail for a simulation of 72 hours.

In Fig. 9 and Fig. 10 is showed the repair time mean for the failures occurred in a period of time from 48 hours and 72 hours, respectively

In addition, in Fig. 11 is presented how the objective function improves the operation probability for the electric power distribution system.

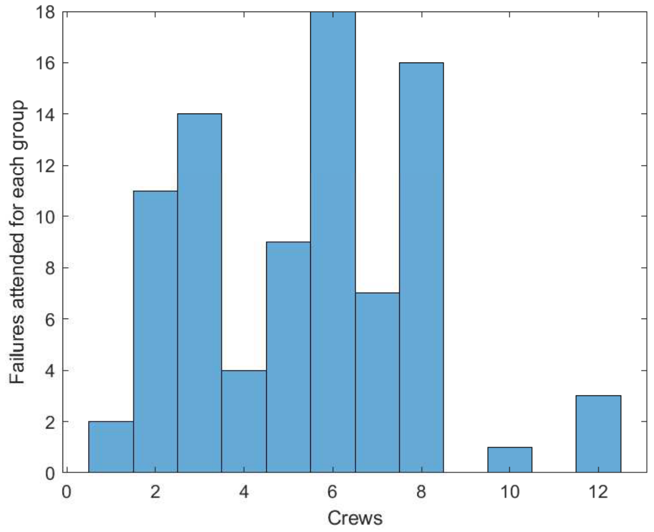

Fig. 7. Histogram describing the number of failures attended for each crew for the simulation of 72 hours

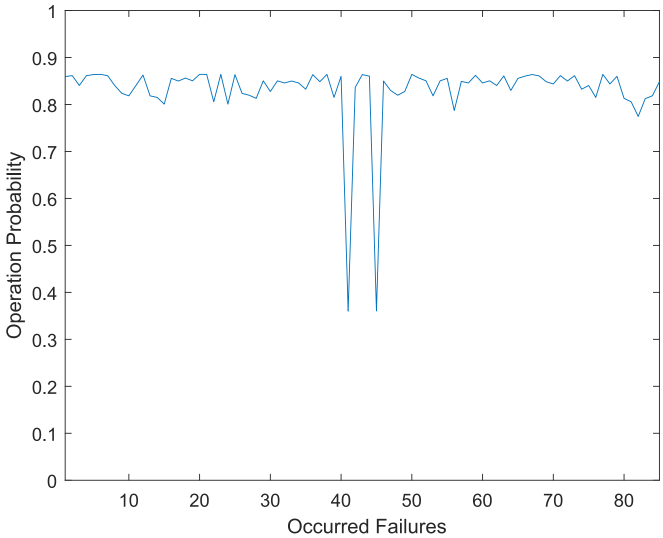

Fig. 8. The probability of operation of each fail attended for 30 nodes and 72 hours

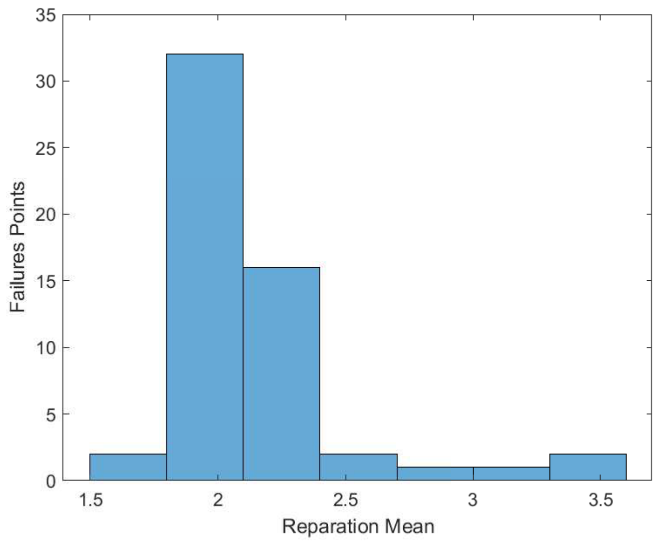

Fig. 9. Repair time mean for a simulation with 30 nodes and 48 hours

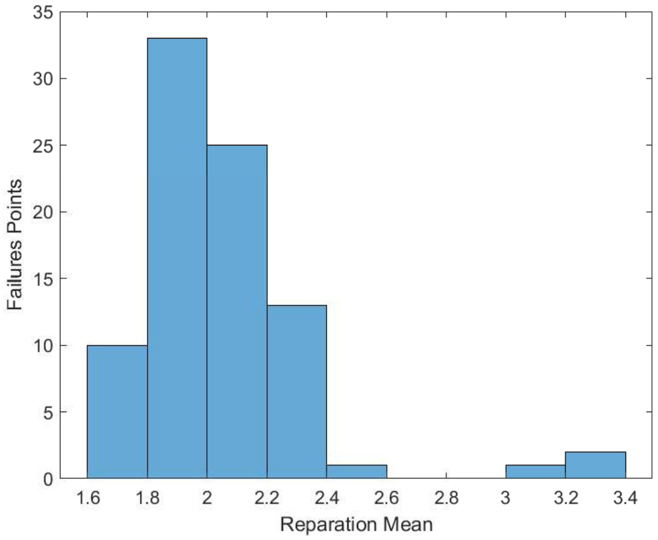

Fig. 10. Repair time mean for a simulation with 30 nodes and 72 hours

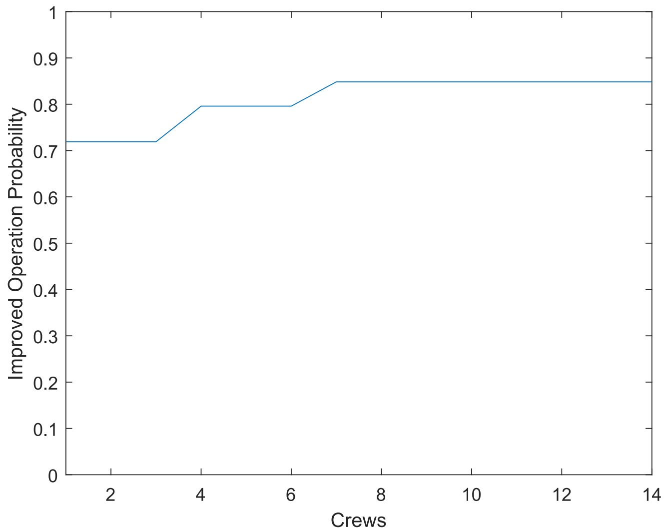

Fig. 11. Probability of improvement for one fail

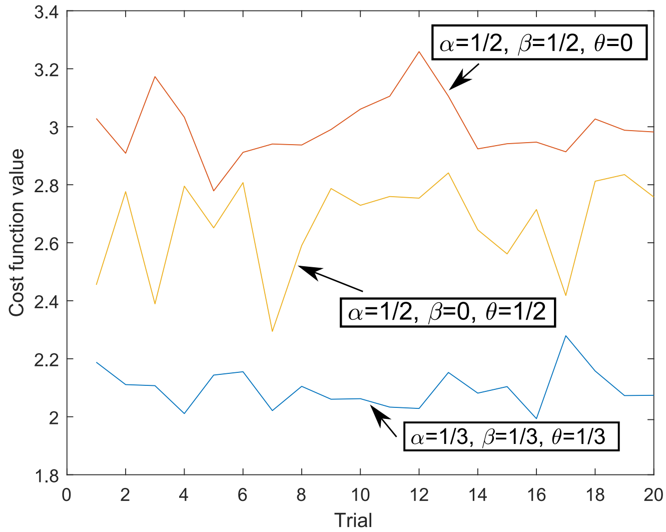

Fig. 12. Simulation of 30 nodes during 48 hours using a $\Delta t = 0.25$ hours, by considering the complete cost function or a reduced cost function without the repair time constraint ($\theta = 0$) or without the displacement time constraint ($\beta = 0$)

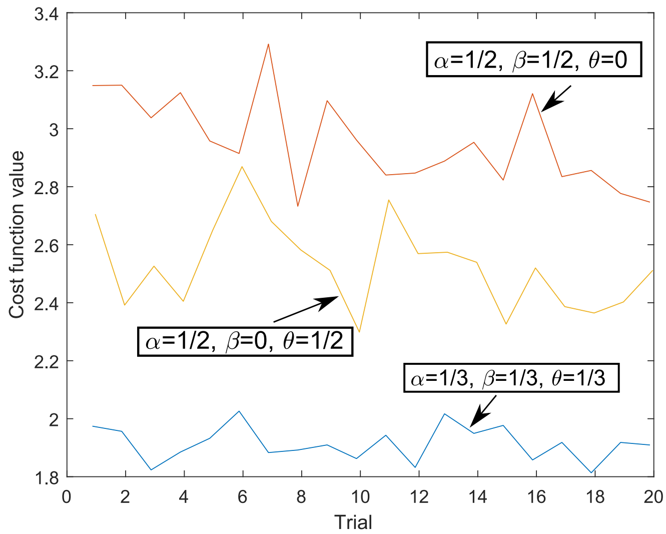

Fig. 13. Simulation of 30 nodes during 72 hours using a $\Delta t = 0.25$ hours, by considering the complete cost function or a reduced cost function without the repair time constraint ($\theta = 0$) or without the displacement time constraint ($\beta = 0$)

Finally, several scenarios are considered in the case of multi-objective optimal control where the cost function considering all the constraints or the cost function considering only one constraint (repair time or displacement time) are considered. To this end, 20 experiments (trials) are performed, where $\theta = 0$ means that the repair time constraint is not considered, and $\beta = 0$ means that the displacement time constraint is not considered. It can be seen that the weights α, β and θ always are selected in order to assign the same weight to each part of the cost function. The corresponding results can be seen in Fig. 12 and Fig. 13.

From Fig. 12 and Fig. 13 it can be seen that the in both cases (48 hours and 72 hours) the lowest cost function value is obtained for the proposed method where all the constraints (repair time, displacement time, and the state space dynamical model) are considered.

From Fig. 2 and Fig 6 and also from Fig. 1 and Fig. 5 it can be seen that the optimal selected crews successfully restore the system to the normal state. However, the repair time and the displacement time is directly related to the crew that is selected for each failure, and therefore, in Fig. 2 and Fig 6 and also from Fig. 1 and Fig. 5 it is noticeable that some failures are active during more time.

IV. Conclusions

A novel strategy for multi-objective optimal control of resources in an electric power distribution system for service restoration is presented. Two contributions are clarified: the first contribution is the selection of a discrete state space model as a discrete Markov chain in order to describe the implicit dynamics, the repair time and the distance to the failure. In this case, is worth noting that the model considers the failure rate and repair rate of the crews based on real data.

The second contribution is an optimal multi-objective control solution based on an exhaustive search algorithm where the obtained control signal is the crew assignation to attend the failure. It can be seen that the proposed approach is adjusted to real data and allows the optimal selection of the crew considering the implicit dynamics of the distribution system, the repair time and the travel time.

As future work, several methods for solution of the multi-objective optimal control can be explored in order to reduce computational cost and to obtain an adequate method for real time implementation.

REFERENCES

[1] S. Wang, B. R. Sarker, L. Mann, and E. Triantaphyllou, "Resource planning and a depot location model for electric power restoration," *European Journal of Operational Research*, vol. 155, no. 1, pp. 22–43, 2004.

[2] C. Parra, W. Salomón *et al.*, "Modelo de dimensionamiento de cuadrillas para la atención de fallas en el sistema de distribución de codensa sa esp," Ph.D. dissertation, Universidad Nacional de Colombia-Sede Bogotá, 2016.

[3] C. Zapata, S. Silva, H. Gonzalez, O. Burbano, and J. Hernández, "Modeling the repair process of a power distribution system," in *Transmission and Distribution Conference and Exposition: Latin America, 2008 IEEE/PES*. IEEE, 2008, pp. 1–7.

[4] A. A. Agudelo-Zapata, E. Giraldo, and J. A. Villegas-Florez, "Application of vrp techniques to the allocation of resources in an electric power distribution system," *Journal of Computational Science*, vol. 35, pp. 102 – 109, 2019. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S187775031830259X

[5] A. Sahu and R. Tapadar, "Solving the assignment problem using genetic algorithm and simulated annealing," *IAENG International Journal of Applied Mathematics*, vol. 36, no. 1, pp. 37–40, 2006.

[6] F. Errico, G. Desaulniers, M. Gendreau, W. Rei, and L.-M. Rousseau, "The vehicle routing problem with hard time windows and stochastic service times," *EURO Journal on Transportation and Logistics*, pp. 1–29, 2013.

[7] F. Osorio-Arteaga, D. Giraldo-Buitrago, and E. Giraldo, "Sliding mode control applied to mimo systems," *Engineering Letters*, vol. 27, no. 4, pp. 802–806, 2019.

[8] E. Giraldo, *Multivariable Control*. Scholar's Press, 2016.

[9] D. Giraldo and E. Giraldo, *Teoría de control digital*. Universidad Tecnológica de Pereira. Facultad de Ingeniería Eléctrica, 2012.

[10] C. H. de Caldas S.A. E.S.P (Chec), "Registro de fallas en el sistema de distribución chec," 2014.

[11] D. Morillo, L. Moreno, and J. Díaz, "Analytic and heuristic methodologies for solving the resource constrained project scheduling problem (rcpsp): a review part 1," *Ingeniería y Ciencia*, vol. 10, no. 19, pp. 247–271, 2014.

[12] ——, "Analytic and heuristic methodologies for solving the resource constrained project scheduling problem (rcpsp): a review. part 2," *Ingeniería y Ciencia*, vol. 10, no. 20, pp. 203–227, 2014.