Adaptive Fuzzy Control for a Class of MIMO Nonlinear Systems with Bounded Control Inputs

Zhiqiang Song, Wu Fang, Xiaozhao Liu, and Aihong Lu

Abstract—**An adaptive fuzzy control scheme is proposed for a class of unknown multi-input multi-output (MIMO) nonlinear dynamic systems with bounded control inputs. A regularized inverse matrix was adopted to obtain a stable controller. Using a hyperbolic tangent smoothing function, the control signals can be limited to the amplitude range of actuators. This solves the problem of an arbitrary amplitude of control signals, which is very disadvantageous for the actuator. The feasibility of the proposed approach is demonstrated by simulations.**

Index Terms—**Adaptive fuzzy control, MIMO nonlinear systems, bounded control inputs, simulation**

I. INTRODUCTION

In the past two decades, controller design for nonlinear systems has been given sustained attention. Traditional feedback control theory is inadequate for designing a controller in nonlinear systems that can meet performance requirements, while the newer adaptive control technology can ensure the stability of the system globally. In 1998, Ye and Jiang [1] reported an adaptive scheme for nonlinear systems that did not require prior knowledge of control directions. Fuzzy control is also an effective method for nonlinear system design, and has been widely used in many industrial systems [2-3]. Al-Hadithi et al. [4] in 2015 developed a fuzzy optimal control scheme using a generalized Takagi-Sugeno model for nonlinear multivariable systems, which was improved through a weighting parameters approach.

With advances in intelligent theory, many adaptive control systems have been generated, including those based on fuzzy logic or neural networks. Indeed, for complex nonlinear systems, adaptive fuzzy control (AFC) is superior to traditional methods for dealing with parameter changes, unmodeled dynamics, and external disturbances [5]. Ghavidel and Kalat [6] proposed an observer-based adaptive fuzzy controller for nonlinear systems, with a feedback error function to approximate and compensate for unknown uncertainties and external disturbances. Liu et al. [7] developed an adaptive fuzzy controller for nonlinear discrete-time systems with dead zone and input constraints. The backstepping method has also been applied to the design of nonlinear systems. Lin et al. [8] designed an AFC scheme using backstepping for nonlinear pure-feedback systems with unknown dead zone output and external disturbance. Tong et al. [9] developed a fuzzy adaptive backstepping output feedback control scheme for MIMO (multiple input, multiple output) nonlinear systems with immeasurable states. Tong et al. [10] described an adaptive fuzzy backstepping dynamic surface control algorithm, with a fuzzy state observer adopted to estimate immeasurable states.

An adaptive fuzzy output tracking control approach was also proposed for SISO (single input, single output) unknown nonlinear systems [11], and furthermore, for SISO nonlinear systems with unstructured uncertainties and unknown dead zone [12]. In the later, fuzzy logic systems were used to approximate unstructured uncertainties.

In recent years, many scholars have studied the AFC of MIMO nonlinear systems [13-24]. Labiod et al. [13] reported an AFC method for uncertain MIMO nonlinear systems, in which the non-singularity of the fuzzy controller is guaranteed based on a generalized inverse of the matrix. To avoid the singularity problem of fuzzy controllers, Tong et al. [14] adopted singular value decomposition of matrices. Labiod et al. [15] offered a direct AFC law to approximate an unknown ideal controller; the parameters of the fuzzy systems were adjusted using a gradient descent algorithm. Three AFC schemes were proposed by Boulkroune et al. [16] for MIMO nonlinear systems with known or unknown control directions. A Nussbaum-type function was incorporated to accommodate control gain matrices of unknown sign. For MIMO nonlinear time-delay systems, this group also developed an AFC of variable-structure using matrix decomposition [17]. For MIMO non-affine systems, a fuzzy indirect adaptive control, based on approximation, was investigated by Boulkroune et al. [18], also with the Nussbaum gain function. Nekoukar and Erfanian [19] adopted an adaptive fuzzy sliding mode control for MIMO uncertain nonlinear systems, to identify the dynamics of the

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plant. For unknown input nonlinearities such as backlash-like hysteresis or dead-zone, an adaptive fuzzy output feedback controller was presented by Reza [20]. Shi [21] offered an indirect AFC approach for MIMO nonlinear systems with asymmetric control gain matrix and unknown control direction. For systems with asymmetric control gain matrix and unknown dead-zone inputs, the same author developed an AFC scheme using matrix decomposition [22]. Furthermore, Shi et al. [23] designed an indirect adaptive fuzzy-prescribed performance control scheme for MIMO feedback linearizable systems with unknown control direction, and Shi and Li [24] proposed an AFC scheme using prescribed performance bounds and Nussbaum-type gain function for MIMO nonlinear systems with unknown control direction and external disturbances.

Although the contributions of the above authors have been valuable, yet systems in which the control input is limited have not been considered. In actual systems, the output amplitude of the actuator is bounded within limits and control law cannot be applied. The present study formulated a control goal for MIMO nonlinear systems, and presents an AFC scheme with bounded control inputs. The control scheme is then demonstrated via simulation with a two-link rigid robot manipulator.

This paper is organized as follows. Section II formulates a class of MIMO nonlinear systems and the control goal. In Section III, an adaptive fuzzy control scheme with bounded control inputs is presented. In Section IV, the control scheme is applied to a two-link rigid robot manipulator. Section V concludes this paper.

II. Problem Formulation

Consider the following MIMO nonlinear system [13]:

$$\begin{gathered} y_1^{(r_1)} = f_1(\boldsymbol{x}) + \sum_{j=1}^{p} g_{1j}(\boldsymbol{x})u_j \\ \vdots \\ y_p^{(r_p)} = f_p(\boldsymbol{x}) + \sum_{j=1}^{p} g_{pj}(\boldsymbol{x})u_j \end{gathered} \quad , \tag{1}$$

where $\boldsymbol{x} = \left[y_1, \dot{y}_1, \cdots, y_1^{(r_1-1)}, \cdots, y_p^{(r_p-1)} \right]^{\mathrm{T}}$ is the system state vector which is assumed measurable, $\boldsymbol{u}=[u_1,\ldots, u_p]^{\mathrm{T}}$ is the control input vector, $\boldsymbol{y}=[y_1,\ldots,y_p]^{\mathrm{T}}$ is the output vector, and $f_i(\boldsymbol{x})$, $g_{ij}(\boldsymbol{x})$, $i,j=1,2,\cdots,p$ are unknown smooth nonlinear functions.

Let

$$\boldsymbol{y}^{(r)} = \left[y_1^{(r_1)}, \cdots, y_p^{(r_p)} \right]^{\mathrm{T}},$$

$$\boldsymbol{F}(\boldsymbol{x}) = \left[f_1(\boldsymbol{x}), \cdots, f_p(\boldsymbol{x}) \right]^{\mathrm{T}},$$

$$\boldsymbol{G}(\boldsymbol{x}) = \begin{bmatrix} g_{11}(\boldsymbol{x}) & \cdots & g_{1p}(\boldsymbol{x}) \\ \vdots & \ddots & \vdots \\ g_{p1}(\boldsymbol{x}) & \cdots & g_{pp}(\boldsymbol{x}) \end{bmatrix},$$

Then (1) can be rewritten as:

$$\boldsymbol{y}^{(r)} = \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}. \tag{2}$$

By designing the control law $\boldsymbol{u}(t)$, it is guaranteed that all variables of the closed-loop system are bounded, and the output of the system can track the desired trajectory: $\boldsymbol{y}_{\mathrm{d}}(t)=[y_{\mathrm{d}1}(t),\ldots,y_{\mathrm{d}p}(t)]^{\mathrm{T}}$.

We make two assumptions throughout this report, as follows.

Assumption 1. $G(x)$ is a positive definite matrix, and there exists a real number $\sigma_0>0$ such that $G(x)\geq\sigma_0\boldsymbol{I}_p$.

Assumption 2. The desired trajectory $y_{\mathrm{d}i}(t)$, $i=1,\ldots,p$ is bounded, and r_i-order derivatives are bounded.

Trajectory tracking errors are defined as:

$$\begin{gathered} e_1(t) = y_{\mathrm{d}1}(t) - y_1(t) \\ \vdots \\ e_p(t) = y_{\mathrm{d}p}(t) - y_p(t) \end{gathered} \quad . \tag{3}$$

Filter tracking errors are defined as:

$$\begin{gathered} s_1(t) = (\frac{\mathrm{d}}{\mathrm{d}t} + \lambda_1)^{r_1-1} e_1(t), \ \ \lambda_1 > 0 \\ \vdots \\ s_p(t) = (\frac{\mathrm{d}}{\mathrm{d}t} + \lambda_p)^{r_p-1} e_p(t), \ \ \lambda_p > 0 \end{gathered} \quad . \tag{4}$$

According to Newton's binomial theorem:

$$(a+b)^n = \sum_{i=0}^{n} C_n^i a^i b^{n-i} , \tag{5}$$

where $C_n^i = \dfrac{n!}{(n-i)!i!}$ are binomial expansion coefficients.

Applying (5) to (4), one obtains:

$$\begin{aligned} s_i(t) &= (\frac{\mathrm{d}}{\mathrm{d}t} + \lambda_i)^{r_i-1} e_i(t) = \sum_{j=0}^{r_i-1} \frac{(r_i-1)!}{(r_i-1-j)!\,j!} (\frac{\mathrm{d}}{\mathrm{d}t})^j e_i(t) \lambda_i^{r_i-1-j} \\ &= \sum_{j=1}^{r_i} \frac{(r_i-1)!}{(r_i-j)!(j-1)!} (\frac{\mathrm{d}}{\mathrm{d}t})^{j-1} e_i(t) \lambda_i^{r_i-j} \end{aligned} , \tag{6}$$

Then

$$\begin{aligned} \dot{s}_i(t) &= \sum_{j=1}^{r_i} \frac{(r_i-1)!}{(r_i-j)!(j-1)!} e_i^{(j)}(t)\lambda_i^{r_i-j} = e_i^{(r_i)} + \sum_{j=1}^{r_i-1} \frac{(r_i-1)!}{(r_i-j)!(j-1)!} e_i^{(j)}(t)\lambda_i^{r_i-j} \\ &= y_{\mathrm{d}i}^{(r_i)} - y_i^{(r_i)} + \sum_{j=1}^{r_i-1} \frac{(r_i-1)!}{(r_i-j)!(j-1)!} e_i^{(j)}(t)\lambda_i^{r_i-j} \\ &= y_{\mathrm{d}i}^{(r_i)} - f_i(x) - \sum_{j=1}^{i} g_{ij}(x)u_j + \sum_{j=1}^{r_i-1} \frac{(r_i-1)!}{(r_i-j)!(j-1)!} e_i^{(j)}(t)\lambda_i^{r_i-j} \end{aligned} \tag{7}$$

The time derivatives of the filtered errors can be rewritten as:

$$\begin{gathered} \dot{s}_1 = v_1 - f_1(x) - \sum_{j=1}^{p} g_{1j}(x)u_j \\ \vdots \\ \dot{s}_p = v_p - f_p(x) - \sum_{j=1}^{p} g_{pj}(x)u_j \end{gathered} \quad , \tag{8}$$

where $v_1,\ldots,v_p$ are as follows:

$$\begin{aligned} v_1 &= y_{d1}^{(r_1)} + \beta_{1,r_1-1} e_1^{(r_1-1)} + \cdots + \beta_{1,1}\dot{e}_1 \\ &\vdots \\ v_p &= y_{dp}^{(r_p)} + \beta_{p,r_p-1} e_p^{(r_p-1)} + \cdots + \beta_{p,p}\dot{e}_p \end{aligned} \quad (9)$$

$$\beta_{i,j} = \frac{(r_i-1)!}{(r_i-j)!(j-1)!}\lambda_i^{r_i-j},\ i=1,\cdots,p,\ j=1,\cdots,r_i-1$$

Denote $\boldsymbol{s}(t)=[s_1(t),\ldots,s_p(t)]^T$, $\boldsymbol{v}(t)=[v_1(t),\ldots,v_p(t)]^T$, then (8) can be expressed as:

$$\dot{\boldsymbol{s}} = \boldsymbol{v} - \boldsymbol{F}(\boldsymbol{x}) - \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}. \quad (10)$$

If the nonlinear functions $\boldsymbol{F}(\boldsymbol{x})$ and $\boldsymbol{G}(\boldsymbol{x})$ are given, the following control law can be used:

$$\boldsymbol{u} = \boldsymbol{G}^{-1}(\boldsymbol{x})(-\boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{v} + \boldsymbol{K}_0\boldsymbol{s}), \quad (11)$$

where $\boldsymbol{K}_0=\mathrm{diag}[k_{01},\ldots,k_{0p}]$, $k_{0i}>0$, $i=1,\ldots,p$.

Substituting (11) into (10) yields:

$$\dot{\boldsymbol{s}}(t) = -\boldsymbol{K}_0\boldsymbol{s}(t), \quad (12)$$

or, equivalently

$$\dot{s}_i(t) = -K_{0i}s_i(t),\ i=1,\ldots,p. \quad (13)$$

Solving differential equations yields:

$$s_i(t) = s_i(0)e^{-K_{0i}t},\ i=1,\ldots,p. \quad (14)$$

which implies that $s_i(t)\to 0$ as $t\to\infty$.

Therefore $e_i(t)$ and its r_i-1 order derivative converge to zero uniformly. When $f_i(\boldsymbol{x})$ and $g_{ij}(\boldsymbol{x})$ are known, the control law (11) is easily obtained, but in the actual system, the nonlinear functions $f_i(\boldsymbol{x})$ and $g_{ij}(\boldsymbol{x})$ are unknown, and the control law (11) cannot be designed. Fuzzy systems can be used to approximate the nonlinear functions $f_i(\boldsymbol{x})$ and $g_{ij}(\boldsymbol{x})$.

III. Adaptive Fuzzy Control Scheme

A. Fuzzy Logic Systems

The fuzzy system applied in this paper consists of a fuzzifier, some fuzzy IF-THEN rules, a fuzzy inference engine, and a defuzzifier. The fuzzy inference engine utilizes the fuzzy IF-THEN rules to perform a mapping from an input vector $\boldsymbol{x}=[x_1,\ldots,x_n]^T \in \mathbf{R}^n$, to an output variable $y\in\mathbf{R}$. The lth fuzzy rule can be written as:

$R^{(l)}$:IF x_1 is F_1^l and… and x_n is F_n^l,Then y is G^l,

where F_i^l and G^l are fuzzy sets associating with fuzzy member functions $\mu_{F_i^l}(x_i)$ and $\mu_{G^l}(y)$,respectively, and l=1,…,M; M is the number of rules.

Applying singleton fuzzification, product inference, and center-average defuzzification to design the fuzzy system, the output of the fuzzy system is expressed as follows:

$$y(\boldsymbol{x}) = \frac{\sum_{l=1}^{M} y^l \left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right)}, \quad (15)$$

where $y^l = \max_{y\in\boldsymbol{R}}\mu_{G^l}(y)$.

Define the fuzzy basis function as:

$$\xi_l(\boldsymbol{x}) = \frac{\prod_{i=1}^{n}\mu_{F_i^l}(x_i)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right)}, \quad (16)$$

then (15) can be rewritten as:

$$y(\boldsymbol{x}) = \boldsymbol{\xi}^T(\boldsymbol{x})\boldsymbol{\theta}, \quad (17)$$

where $\boldsymbol{\xi}(\boldsymbol{x}) = [\xi_1(\boldsymbol{x}),\cdots,\xi_M(\boldsymbol{x})]^T$; $\theta=[y^1,\ldots,y^M]^T$.

B. Adaptive Fuzzy Control Scheme

If fuzzy systems are used to approximate the unknown nonlinear functions $f_i(x)$ and $g_{ij}(x)$, then these approximations are employed to design the adaptive control laws, to meet the control goal. Let the nonlinear functions $f_i(x)$ and $g_{ij}(x)$ be approximated by fuzzy systems as follows:

$$\hat{f}_i(\boldsymbol{x},\boldsymbol{\theta}_{f_i}) = \boldsymbol{\xi}_{f_i}^T(\boldsymbol{x})\boldsymbol{\theta}_{f_i},\ i=1,\cdots,p, \quad (18)$$

$$\hat{g}_{ij}(\boldsymbol{x},\boldsymbol{\theta}_{g_{ij}}) = \boldsymbol{\xi}_{g_{ij}}^T(\boldsymbol{x})\boldsymbol{\theta}_{g_{ij}},\ i=1,\cdots,p, \quad (19)$$

where $\boldsymbol{\xi}_{f_i}(\boldsymbol{x})$ and $\boldsymbol{\xi}_{g_{ij}}(\boldsymbol{x})$ are fuzzy basis vectors, then $\boldsymbol{\theta}_{f_i}$ and $\boldsymbol{\theta}_{g_{ij}}$ are the adjustable parameter vectors of each fuzzy system, respectively.

Define

$$\boldsymbol{\theta}_{f_i}^* = \arg\min_{\boldsymbol{\theta}_{f_i}}\{\sup_{x\in D_x}|f_i(x) - \hat{f}_i(x,\boldsymbol{\theta}_{f_i})|\}, \quad (20)$$

$$\boldsymbol{\theta}_{g_{ij}}^* = \arg\min_{\boldsymbol{\theta}_{g_{ij}}}\{\sup_{x\in D_x}|g_{ij}(x) - \hat{g}_{ij}(x,\boldsymbol{\theta}_{g_{ij}})|\}, \quad (21)$$

as the optimal approximation parameters of $\boldsymbol{\theta}_{f_i}$ and $\boldsymbol{\theta}_{g_{ij}}$, respectively; $\boldsymbol{\theta}_{f_i}^*$ and $\boldsymbol{\theta}_{g_{ij}}^*$ are only for analytical purposes.

Define

$$\tilde{\boldsymbol{\theta}}_{f_i} = \boldsymbol{\theta}_{f_i}^* - \boldsymbol{\theta}_{f_i},\ \tilde{\boldsymbol{\theta}}_{g_{ij}} = \boldsymbol{\theta}_{g_{ij}}^* - \boldsymbol{\theta}_{g_{ij}}, \quad (22)$$

$$\varepsilon_{f_i}(\boldsymbol{x}) = f_i(\boldsymbol{x}) - \hat{f}_i(\boldsymbol{x},\boldsymbol{\theta}_{f_i}^*), \quad (23)$$

$$\varepsilon_{g_{ij}}(\boldsymbol{x}) = g_{ij}(\boldsymbol{x}) - \hat{g}_{ij}(\boldsymbol{x},\boldsymbol{\theta}_{g_{ij}}^*), \quad (24)$$

where $\varepsilon_{f_i}(\boldsymbol{x})$ and $\varepsilon_{g_{ij}}(\boldsymbol{x})$ are the minimum fuzzy approximation errors.

We assume that the compact set D_x is large enough so that it can guarantee that $x\in D_x$. It is fair to assume that the minimum approximation errors are bounded for $x\in D_x$, that is,

$$\left|\varepsilon_{f_i}(\boldsymbol{x})\right| \le \bar{\varepsilon}_{f_i},\ \left|\varepsilon_{g_{ij}}(\boldsymbol{x})\right| \le \bar{\varepsilon}_{g_{ij}},\ \forall x\in D_x,$$

where $\bar{\varepsilon}_{f_i}$ and $\bar{\varepsilon}_{g_{ij}}$ are known constants.

Denote

$$\hat{\boldsymbol{F}}(x,\boldsymbol{\theta}_f) = \left[\hat{f}_1(\boldsymbol{x}),\cdots,\hat{f}_p(\boldsymbol{x})\right]^T,$$

$$\hat{\boldsymbol{G}}(x,\boldsymbol{\theta}_g) = \begin{bmatrix} \hat{g}_{11}(\boldsymbol{x}) & \cdots & \hat{g}_{1p}(\boldsymbol{x}) \\ \vdots & \ddots & \vdots \\ \hat{g}_{p1}(\boldsymbol{x}) & \cdots & \hat{g}_{pp}(\boldsymbol{x}) \end{bmatrix},$$

$$\boldsymbol{\varepsilon}_{\mathrm{f}}(\boldsymbol{x}) = \left[\varepsilon_{\mathrm{f}_1}(\boldsymbol{x}), \quad \cdots, \quad \varepsilon_{\mathrm{f}_p}(\boldsymbol{x})\right]^{\mathrm{T}},$$

$$\boldsymbol{\varepsilon}_{\mathrm{g}}(\boldsymbol{x}) = \begin{bmatrix} \varepsilon_{\mathrm{g}_{11}}(\boldsymbol{x}) & \cdots & \varepsilon_{\mathrm{g}_{1p}}(\boldsymbol{x}) \\ \vdots & \ddots & \vdots \\ \varepsilon_{\mathrm{g}_{p1}}(\boldsymbol{x}) & \cdots & \varepsilon_{\mathrm{g}_{pp}}(\boldsymbol{x}) \end{bmatrix},$$

$$\bar{\boldsymbol{\varepsilon}}_{\mathrm{f}}(\boldsymbol{x}) = [\bar{\varepsilon}_{\mathrm{f}_1}(\boldsymbol{x}), \cdots, \bar{\varepsilon}_{\mathrm{f}_p}(\boldsymbol{x})]^{\mathrm{T}},$$

$$\bar{\boldsymbol{\varepsilon}}_{\mathrm{g}}(\boldsymbol{x}) = \begin{bmatrix} \bar{\varepsilon}_{\mathrm{g}_{11}}(\boldsymbol{x}) & \cdots & \bar{\varepsilon}_{\mathrm{g}_{1p}}(\boldsymbol{x}) \\ \cdots & \ddots & \vdots \\ \bar{\varepsilon}_{\mathrm{g}_{p1}}(\boldsymbol{x}) & \cdots & \bar{\varepsilon}_{\mathrm{g}_{pp}}(\boldsymbol{x}) \end{bmatrix}.$$

By using $\hat{\boldsymbol{F}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{f}})$ and $\hat{\boldsymbol{G}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})$ respectively, instead of $\boldsymbol{F}(\boldsymbol{x})$ and $\boldsymbol{G}(\boldsymbol{x})$ in (11), then from the above analysis, we have:

$$\boldsymbol{F}(\boldsymbol{x}) - \hat{\boldsymbol{F}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{f}}) = \hat{\boldsymbol{F}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{f}}^*) - \hat{\boldsymbol{F}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{f}}) + \boldsymbol{\varepsilon}_{\mathrm{f}}(\boldsymbol{x}), \quad (25)$$

$$\boldsymbol{G}(\boldsymbol{x}) - \hat{\boldsymbol{G}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}}) = \hat{\boldsymbol{G}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}}^*) - \hat{\boldsymbol{G}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}}) + \boldsymbol{\varepsilon}_{\mathrm{g}}(\boldsymbol{x}). \quad (26)$$

Consider the control law $\boldsymbol{u}=\boldsymbol{u}_{\mathrm{c}}$, where $\boldsymbol{u}_{\mathrm{c}}$ is the control term defined as

$$\boldsymbol{u}_{\mathrm{c}} = \hat{\boldsymbol{G}}^{-1}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})(-\hat{\boldsymbol{F}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{f}}) + \boldsymbol{v} + \boldsymbol{K}_0\boldsymbol{s}). \quad (27)$$

Since the matrix $\hat{\boldsymbol{G}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})$ is generated on-line by estimating the parameters $\boldsymbol{\theta}_{\mathrm{g}}$, it is difficult to guarantee the non-singularity $\hat{\boldsymbol{G}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})$.

For this reason, the regularized inverse: $\hat{\boldsymbol{G}}^{\mathrm{T}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})[\varepsilon_0\boldsymbol{I}_p + \hat{\boldsymbol{G}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})\hat{\boldsymbol{G}}^{\mathrm{T}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})]^{-1}$ is used instead of $\hat{\boldsymbol{G}}^{-1}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})$, and (27) can be expressed as:

$$\boldsymbol{u}_{\mathrm{c}} = \hat{\boldsymbol{G}}^{\mathrm{T}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})[\varepsilon_0\boldsymbol{I}_p + \hat{\boldsymbol{G}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})\hat{\boldsymbol{G}}^{\mathrm{T}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})]^{-1}(-\hat{\boldsymbol{F}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{f}}) + \boldsymbol{v} + \boldsymbol{K}_0\boldsymbol{s}), \quad (28)$$

where ε_0 is a small positive constant and $\boldsymbol{I}_p$ is an identity matrix.

To reduce the reconstruction errors, the robust control term $\boldsymbol{u}_{\mathrm{r}}$ is introduced to the control law:

$$\boldsymbol{u}=\boldsymbol{u}_{\mathrm{c}}+\boldsymbol{u}_{\mathrm{r}}, \quad (29)$$

$$\boldsymbol{u}_{\mathrm{r}} = \frac{\boldsymbol{s}\,|\,\boldsymbol{s}^{\mathrm{T}}\,|\,(\bar{\boldsymbol{\varepsilon}}_{\mathrm{f}} + \bar{\boldsymbol{\varepsilon}}_{\mathrm{g}}\,|\,\boldsymbol{u}_{\mathrm{c}}\,| + |\,\boldsymbol{u}_0\,|)}{\sigma_0\|\boldsymbol{s}\|^2 + \delta}, \quad (30)$$

$$\boldsymbol{u}_0 = \varepsilon_0[\varepsilon_0\boldsymbol{I}_p + \hat{\boldsymbol{G}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})\hat{\boldsymbol{G}}^{\mathrm{T}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{g}})]^{-1}(-\hat{\boldsymbol{F}}(\boldsymbol{x},\boldsymbol{\theta}_{\mathrm{f}}) + \boldsymbol{v} + \boldsymbol{K}_0\boldsymbol{s}), \quad (31)$$

where δ is a time-varying parameter, defined below.

To meet the control goal, the design parameter δ and the adaptive parameters $\boldsymbol{\theta}_{\mathrm{f}_i}$ and $\boldsymbol{\theta}_{\mathrm{g}_{ij}}$ are updated by the following adaptive laws:

$$\dot{\delta} = -\eta_0 \frac{|\,\boldsymbol{s}^{\mathrm{T}}\,|\,(\bar{\boldsymbol{\varepsilon}}_{\mathrm{f}} + \bar{\boldsymbol{\varepsilon}}_{\mathrm{g}}\,|\,\boldsymbol{u}_{\mathrm{c}}\,| + |\,\boldsymbol{u}_0\,|)}{\sigma_0\|\boldsymbol{s}\|^2 + \delta}, \quad (32)$$

$$\dot{\boldsymbol{\theta}}_{\mathrm{f}_i} = -\eta_{\mathrm{f}_i}\boldsymbol{\xi}_{\mathrm{f}_i}(\boldsymbol{x})s_i, \quad (33)$$

$$\dot{\boldsymbol{\theta}}_{\mathrm{g}_{ij}} = -\eta_{\mathrm{g}_{ij}}\boldsymbol{\xi}_{\mathrm{g}_{ij}}(\boldsymbol{x})s_i u_{\mathrm{c}_j}, \quad (34)$$

where $\eta_{\mathrm{f}_i} > 0$, $\eta_{\mathrm{g}_{ij}} > 0$, $\eta_0 > 0$, and $\delta(0) > 0$.

If the system (1) satisfies the Assumptions 1 and 2, and adopts the control laws defined by (28)-(30) with the adaptive laws designed by (32)-(34), then all signals in the closed loop system are bounded, and the tracking errors and its derivatives asymptotically converge to zero [13].

However, the above control laws do not consider the problem of limited control inputs. In practical control applications, the output amplitude of the actuator, due to its own physical characteristics, is limited. Thus, the problem of input limitation must be considered.

To satisfy $|u(t)| \le u_M$, consider the following hyperbolic tangent smoothing function:

$$f(u) = u_{\mathrm{M}}\tanh(\frac{u}{u_{\mathrm{M}}}) = u_{\mathrm{M}}\frac{e^{u/u_{\mathrm{M}}} - e^{-u/u_{\mathrm{M}}}}{e^{u/u_{\mathrm{M}}} + e^{-u/u_{\mathrm{M}}}}, \quad (35)$$

which has the following characteristics:

$$|\,f(u)\,| = u_{\mathrm{M}}\left|\tanh(\frac{u}{u_{\mathrm{M}}})\right| \le u_{\mathrm{M}},$$

$$0 < \frac{\partial f(u)}{\partial u} = \frac{4}{(e^{u/u_{\mathrm{M}}} + e^{-u/u_{\mathrm{M}}})} \le 1.$$

Control law (29) is then modified as follows:

$$\boldsymbol{u}_{\mathrm{a}} = u_M\tanh(\frac{\boldsymbol{u}}{u_M}), \quad (36)$$

where $\boldsymbol{u}_{\mathrm{a}}$ is the real control input.

IV. Simulation

A. Control Inputs Without Restriction

For a two-link rigid robot manipulator moving in a horizontal plane, the dynamic equations can be written as [13]:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1+\dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad (37)$$

Equation (37) can be written as

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1}\left\{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} - \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1+\dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}\right\}, \quad (38)$$

where

$$M_{11} = a_1 + 2a_3\cos q_2 + 2a_4\sin q_2,$$
$$M_{21} = M_{12} = a_2 + a_3\cos q_2 + a_4\sin q_2,$$
$$M_{22} = a_2,$$
$$h = a_3\sin q_2 - a_4\cos q_2$$

with

$$a_1 = I_1 + m_1 l_{c1}^2 + I_e + m_e l_{ce}^2 + m_e l_1^2,$$
$$a_2 = I_e + m_e l_{ce}^2,$$
$$a_3 = m_e l_1 l_{ce}\cos\delta_e,$$

$$a_4 = m_e l_1 l_{ce} \sin\delta_e$$

Let m_1=1, m_e=2, l_1=1, l_{c1}=0.5, l_{ce}=0.5, I_1=0.12, I_e=0.25, δ_e =π/6, and let $\boldsymbol{y}=[q_1, q_2]^T$, $\boldsymbol{u}=[u_1, u_2]^T$, $\boldsymbol{x}=[q_1, \dot{q}_1, q_2, \dot{q}_2]^T$. Then:

$$\boldsymbol{F}(\boldsymbol{x}) = \begin{bmatrix} f_1(\boldsymbol{x}) \\ f_2(\boldsymbol{x}) \end{bmatrix} = -M^{-1} \begin{bmatrix} -h\dot{q}_2 & -h(\dot{q}_1+\dot{q}_2) \\ h\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\boldsymbol{G}(\boldsymbol{x}) = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = M^{-1} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}^{-1}$$

and the robot system described by (38) can be express as

$$\ddot{\boldsymbol{y}} = \boldsymbol{F}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}. \quad (39)$$

The control goal is to make the system outputs q_1 and q_2 track the desired trajectories y_{d1}=sin$t$ and y_{d2}=sin$t$, respectively. With the simulation, $\boldsymbol{F}(\boldsymbol{x})$ and $\boldsymbol{G}(\boldsymbol{x})$ are assumed to be unknown, i.e., the adaptive fuzzy controller does not need the knowledge of the system's model, and the dynamic model is only required for simulation. For $\boldsymbol{x}=[q_1, \dot{q}_1, q_2, \dot{q}_2]^T$, we define three Gaussian membership functions as:

$$\mu_{F_i^1}(x_i) = \exp(-\frac{1}{2}(\frac{x_i+1.25}{0.6})^2),$$

$$\mu_{F_i^2}(x_i) = \exp(-\frac{1}{2}(\frac{x_i}{0.6})^2),$$

$$\mu_{F_i^3}(x_i) = \exp(-\frac{1}{2}(\frac{x_i-1.25}{0.6})^2),\ i=1,2,3,4.$$

First, the control law (29) is employed. The initial conditions of the robot: $x(0)=[0.5\ 0\ 0.25\ 0]^T$, and the design parameters of the simulation are selected as follows: $\varepsilon_0 = 0.1$, $\eta_{f_i} = 0.5$, $\eta_{g_{ij}} = 0.5$, $\eta_0 = 0.001$, $\sigma_0 = 0.2$, $\delta(0) = 0$, $\bar{\boldsymbol{\varepsilon}}_g = \begin{bmatrix} 0.2 & 0.2 \\ 0.2 & 0.2 \end{bmatrix}$, $\bar{\boldsymbol{\varepsilon}}_f = [0.2,\ \ 0.2]^T$.

When λ_1=25, λ_2=25, and $\boldsymbol{K}_0$=15I_2, the simulation results for the first and second links are shown in Figs. 1 and 2, respectively, and for the control inputs are shown in Fig. 3. From the simulation experiments, we conclude that the values of λ_1, λ_2, and $\boldsymbol{K}_0$ have great influence on the control input $\boldsymbol{u}$, and the control input $\boldsymbol{u}$ increases with increases in λ_1, λ_2, and $\boldsymbol{K}_0$.

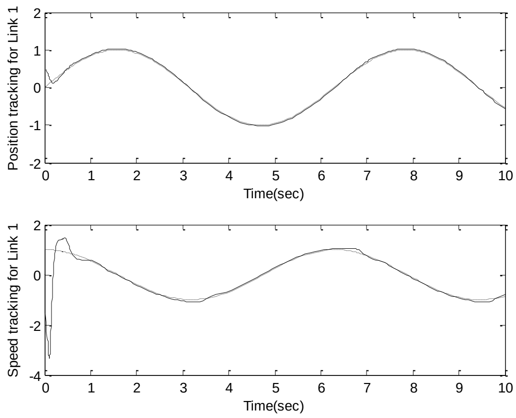

Fig. 1. Tracking curves of Link 1: actual(-) and desired(--)

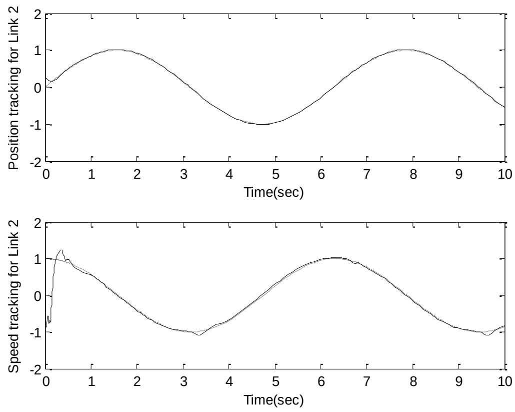

Fig. 2. Tracking curves of Link 2: actual(-) and desired(--)

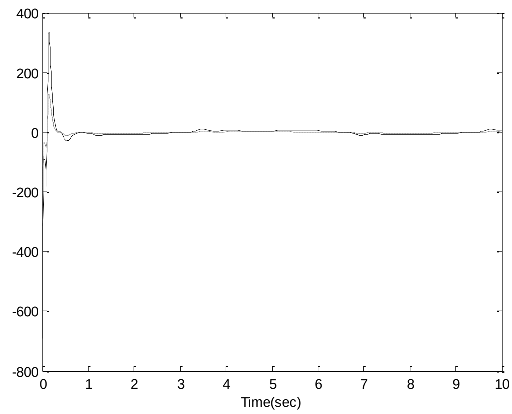

Fig. 3. Control inputs: u_1(-) and u_2(--)

From the simulation results, we can conclude that the control laws are effective for the unknown nonlinear systems. However, the maximum of $\boldsymbol{u}$ is 333.7, and the minimum of $\boldsymbol{u}$ is -291.4. It is unrealistic to consider that an actual control signal will be free from restrictions or constraints.

B. Control Inputs With Restriction

For restricted control inputs, the control law (36) is exploited. When u_M =150 and all other parameters are the same as those in Section IV, the simulation results are shown in Figs. 4-5.

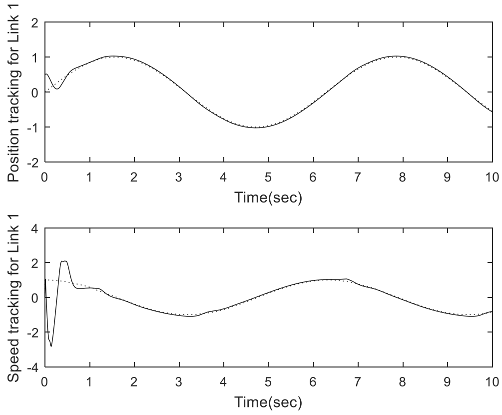

Fig. 4. Tracking curves of Link 1 with bounded control inputs (u_M =150): actual(-) and desired(--)

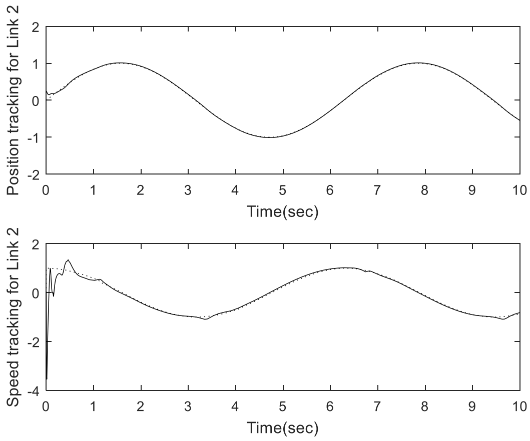

Fig. 5. Tracking curves of Link 2 with bounded control inputs ($u_{M=}$150): actual(-) and desired(--)

When u_M =100 and all other parameters are the same as those in Section IV, the simulation results are shown in Figs. 6-7.

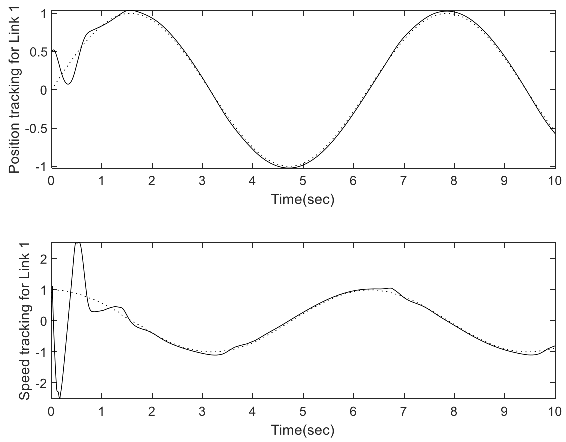

Fig. 6. Tracking curves of Link 1 with bounded control inputs (u_M =100): actual(-) and desired(--)

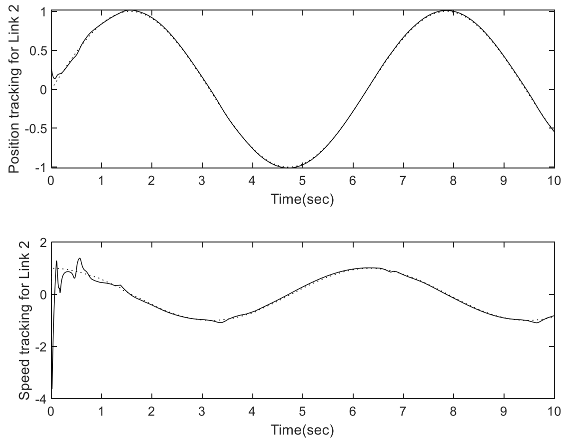

Fig. 7. Tracking curves of Link 2 with bounded control inputs ($u_{M=}$100): actual(-) and desired(--)

When u_M =50 and all other parameters are the same as those in Section IV, the simulation results are shown in Figs. 8-10.

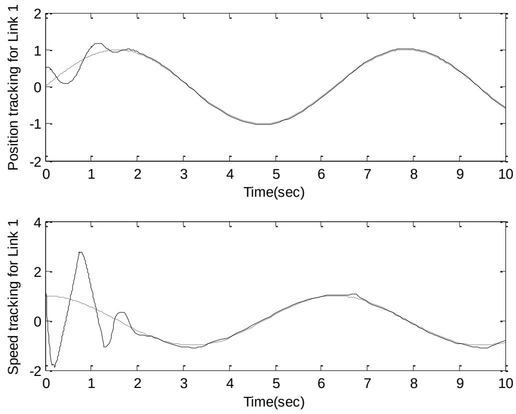

Fig. 8. Tracking curves of Link 1 with bounded control inputs (u_M =50): actual(-) and desired(--)

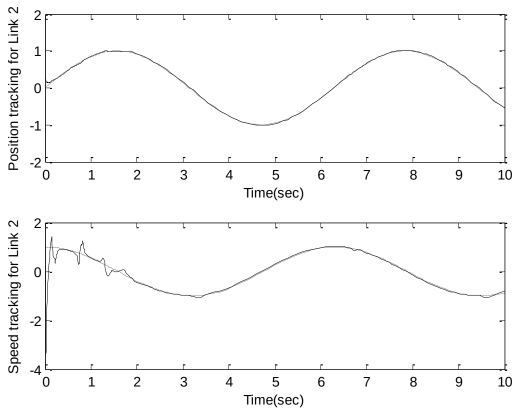

Fig. 9. Tracking curves of Link 2 with bounded control inputs (u_M =50): actual(-) and desired(--)

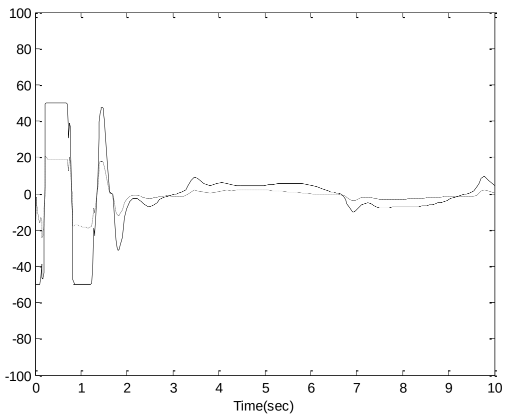

Fig. 10. Control inputs with restriction (u_M =50): u_1(-) and u_2(--)

The simulation results show that tracking capability of the proposed controller is acceptable, when the control inputs have restricted amplitudes.

V. Conclusion

In this paper, an adaptive fuzzy control scheme with bounded control inputs is proposed for a class of MINO nonlinear systems. The proposed approach does not know the mathematical model of the plant with the help of fuzzy systems. To avoid a situation in which the control signal exceeds its amplitude, a hyperbolic tangent smoothing function is incorporated in the control terms to deal with the unknown amplitude of control signals. Simulation results demonstrate the feasibility and capability of the proposed approach. The proposed approach is of practical significance for the control of real nonlinear systems.

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