Robust Control for a Class of Nonlinear Switched Systems with Mixed Delays

Yongzhao Wang, Wenqiong Hou, Jiangrui Ding

Abstract—**In this paper, we consider the problem of exponential H_∞ control for a class of nonlinear switched systems subject to exogenous disturbance and mixed time-varying delays. Specifically, the mixed time-varying delays with upper bound are considered to describe the delay-dependent multi-Lyapunov-Krasovskii functional, and the nonlinear perturbation satisfies the Lipschitz condition. Then, based on Lyapunov stability theory and average dwell time method, new delay-dependent sufficient results are obtained such that switched systems with mixed time-varying delays are exponential stabilization and satisfies a prescribed H_∞ control performance index by resorting to Jensen's inequality. Moreover, the problem of state feedback controllers gains are also derived by solving some linear matrix inequalities. Finally, a numerical example and a practical example of river pollution control are used to show the effectiveness of the proposed method.**

Index Terms—**H_∞ control, Switched systems, Average dwell time, Mixed delays, Multi-Lyapunov-Krasovskii functional.**

I. Introduction

As special classes of hybrid systems, switched systems are widely used in engineering applications, which are composed of a set of subsystems and a special switching law[1-4]. In accordance with switching among the subsystems, switching technique ensures the stability of systems as well as specified performance. With the deepening of research, scholars have found that many real-world models can be represented by switched systems, for instance, tracking control systems[5], networked control systems[6], power electronic and robot control device[7], hot metal processing system[8]. In addition, some significant achievements of switched systems get more and more attention in theory as well as practical applications due to their advantages of increased system flexibility and high reliability. For example, the problems of exponentially mean-square stable and H_∞ filter of switched stochastic systems are addressed in [9]. The robust stabilization issue for fuzzy switched system in the presence of actuator dead-zone is considered in [10]. [11] investigated the stability of original switched nonlinear systems subject to delays by utilizing trajectory-based comparison method.

The stability analysis and control synthesis are the key issues in the study of switched systems with time-varying delays. In practical implementations, it is noticed that constant delays and mixed delays inevitably appear in some systems. These factors affect the normal operation and desired performance index of the system. Therefore, it is naturally important to formulate some methods and techniques for dealing with the effect of time-varying delay. Subsequently, in order to overcome these disadvantages, some significant achievements have been made regarding the issue of switched systems subject to time-varying delay in the past few decades. With respect to these problems, we just mention here some representative works. Based on a novel multi-Lyapunov-Krasovskii functional, [12] studies the issue of exponential stabilization for switched nonlinear systems in the presence of time-varying delay and uncertain terms by the average dwell time approach. Considering small uncertainties in controllers, [13] addresses dissipative-based stability analysis and robust mixed non-fragile reliable control design for an uncertain nonlinear switched system. [14] addresses the problem of H_∞ control for switched neural networks systems with distributed time-varying delays based on the Jensen's inequality and bounded average dwell time method. In addition, with asynchronous switching and Lyapunov stability theory, mean-square exponential stabilization of stochastic switched systems with interval time-varying delay is considered in [15]. With the observation mentioned above, it is noted that few papers have reported the study of switched systems with mixed time-varying delays. Moreover, the conservatism of systems can not be guaranteed among the existing results. In view of this, we will consider the issue of mixed time-varying delays with upper bound, and use inequality technique to reduce the conservatism.

In addition, the existence of external disturbances and nonlinear perturbations often cause undesirable performance of dynamical systems such as unstable and performance degradation. It is necessary to pay attention to the impact of these factors in most practical systems. During the last two decades, H_∞ control strategies are widely applied to deal with the stability analysis and control synthesis of switched systems subject to exogenous disturbance. The concept of H_∞ finite-time boundedness is first introduced in [16], and based on the average dwell time method, some new delay-dependent criteria are obtained to ensure the H_∞ finite-time boundedness of discrete-time switched delay systems. The issue of finite-time L_∞ filter design for networked Markov switched singular systems is addressed and a satisfied H_∞ performance is achieved by designing a unified method in [17]. The maximum and minimum dwell time scheme and multi-Lyapunov-Krasovskii functional technique are considered in [18]. By applying these combinations, the exponential stabilization and non-weighted H_∞ control performance of switched control systems are addressed and criteria on external stability are also obtained, respectively.

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The H_∞ control for two-dimensional switched systems with time-delayed are investigated in [19] by using the discrete Jensen inequality and the Lyapunov method. To the best of our knowledge, the issue of H_∞ control of switched systems with mixed time-varying delays and exogenous disturbance has not been yet completely solved and results are relatively infrequent.

Inspired by the results mentioned above, the objective of this paper is to study H_∞ control of nonlinear switched systems in the presence of exogenous disturbance and mixed time-varying delays. Our contributions to the literature are four-fold.

• Time delay and nonlinear are often coexisting in practical implementations, which are often give rise to instability and oscillation. In this paper, the mixed time-varying delays with upper bound and the nonlinear perturbation satisfies the Lipschitz condition are considered.

• Based on Lyapunov stability theory and average d-well time scheme, a novel delay-dependent multi-Lyapunov-Krasovskii functional is constructed that utilizes the complete available information about the upper bound of mixed time-varying delays.

• New delay-dependent exponentially stable criteria and a prescribed H_∞ control performance index of switched systems with mixed time-varying delays are obtained by resorting to Jensen's inequality. Moreover, the design of the proposed the state feedback controllers of switched systems subject to exogenous disturbance and mixed time-varying delays is given through deformation of Linear Matrix Inequalities.

• On the basis of the proposed schemes, a practical example of river pollution control is carried out to confirm the validity and potential of the developed results.

The remainder of the paper is organized as follows. Section 2 presents the problem description and preliminaries. Section 3 derives the results on exponential stabilization and predefined H_∞ control performance of switched systems in the presence of exogenous disturbance and mixed time-varying delays. In Section 4, two examples are carried out to illustrate the effectiveness of the proposed approach. Concluding remarks and directions for future research are proposed in Section 5.

II. Problem formulation and preliminaries

In this section, we consider nonlinear switched systems in the presence of exogenous disturbance and mixed time-varying delays as follows:

$$\begin{cases} \dot{x}(t) = A_{1\sigma(t)}x(t) + A_{2\sigma(t)}x(t-d(t)) + B_{\sigma(t)}u(t) \\ \quad + C_{\sigma(t)}\displaystyle\int_{t-\tau(t)}^{t} x(s)ds + g(t, x(t-\tau(t)) + D_{\sigma(t)}\nu(t) \\ z(t) = E_{\sigma(t)}x(t) + F_{\sigma(t)}\nu(t), \\ x(t_0+s) = \phi(s), s \in [-max(d_M, \tau_M), 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$, $\varphi(s) \in R^n$, $\nu(t) \in R^q$ and $u(t) \in R^m$ denote the state vector, initial condition, exogenous disturbance, and the control input, respectively. $z(t) \in R^n$ is the measured output, the switching signal $\sigma(t) : [0, \infty] \to M = \{1, 2, \ldots, n\}$ is a piecewise continuous (from the right) function, where n is the number of subsystems. Specifically, we denote $\Sigma : \{(t_0, \sigma(t)), \cdots, (t_k, \sigma(t)), \cdots, k = 0, 1, 2, \cdots\}$, where t_0 is the initial switching instant and t_k denotes the kth switching instant.

For $t \in [t_k, t_{k+1})$, we assume that the ith subsystem is activated. $A_{1i}, A_{2i}, B_i, C_i, D_i, E_i, F_i$ are constant matrices. $d(t)$ and $\tau(t)$ denote the time-varying delay satisfying

$$\begin{aligned} 0 \le d(t) \le d_M, \quad \dot{d}(t) \le \bar{d} < 1; \\ 0 \le \tau(t) \le \tau_M, \quad \dot{\tau}(t) \le \bar{\tau} < 1. \end{aligned} \tag{2}$$

$g(t, x(t-\tau(t)))$ is an nonlinear perturbation function, satisfying

$$\|g(t, x(t-\tau(t)))\| \le \delta \|x(t-\tau(t)))\|, \tag{3}$$

where δ is real constant.

Remark 1. External disturbances and nonlinear perturbations often cause undesirable performance such as unstable and performance degradation in many control applications. In this paper, we consider the mixed time-varying delays with upper bound and the nonlinear perturbation satisfying the Lipschitz condition. Compared with the existing results, the switched model in this paper is more comprehensive and practical in engineering.

For switched systems (1), the state feedback is described as:

$$u(t) = K_{\sigma(t)}x(t). \tag{4}$$

For convenience of calculation and analysis, we recorded as $\tilde{A}_{1i} = A_{1i} + B_iK_i$ such that the resulting closed-loop (1) by following:

$$\begin{aligned} \dot{x}(t) = \tilde{A}_{1i}x(t) + A_{2i}x(t-d(t)) + C_{\sigma(t)}\int_{t-\tau(t)}^{t} x(s)ds \\ + g(t, x(t-\tau(t))) + D_i\nu(t) \end{aligned} \tag{5}$$

Before proving theorem, the following definitions and lemmas are crucial for the development of our main results.

Definition 1.([20]) *$N_\sigma(t, T)$ is the switching number of $\sigma(t)$ on an interval (t, T). For any $T > t \ge 0$, if*

$$N_\sigma(t, T) \le N_0 + (T-t)/\tau_\alpha, \tag{6}$$

holds for given $N_0 \ge 0$ and $\tau_\alpha \ge 0$, the constant τ_α is called the average dwell time. In this paper, $N_0 = 0$.

Remark 2. Average dwell time method is an available scheme to obtain a satisfied performance by designing the maximum switching numbers over a operating interval. Moreover, with the deepening of research, the concept of maximum and minimum dwell time scheme is also considered in [18]. It is worth noting that the weighted term $e^{-\alpha s}$ can be canceled in [18] when applying maximum and minimum dwell time method and the results obtained also increase conservatism. In order to obtain the lower bound for the dwell time (i.e., $\tau_a > \tau_a^* = \frac{\ln \mu}{\alpha}$), the average dwell time method will be applied in this paper.

Definition 2.([21]) *The equilibrium $x^\star = 0$ of the system (1) with $\nu(t) = 0$ is said to be exponentially stable under switching signal $\sigma(t)$ if the solution $x(t)$ satisfies*

$$\|x(t)\|^2 \le re^{-\alpha(t-t_0)}\|x_{t_0}\|_c, \quad \forall t \ge t_0 \tag{7}$$

for constants $r \geq 1, \alpha > 0$, *where* $\|x_{t_0}\|_c = \sup_{-max(d_M,\tau_M)\leq\theta\leq 0}\{\|x(t_0+\theta)\|, \|\dot{x}(t_0+\theta)\|\}$.

Definition 3.([22]) *For a given performance level* $\gamma > 0$, *switched system (1) is said to be exponential stabilization with an* H_∞ *disturbance attenuation level* γ, *if the following conditions hold:*
(1) Switched system (1) is exponentially stabilizable when the exogenous disturbance $\nu(t) = 0$.
(2) Under the zero initial condition, for any nonzero exogenous disturbance, there is

$$\int_0^\infty e^{-\lambda t} z^T(t)z(t)dt \leq \gamma^2 \int_0^\infty \nu^T(t)\nu(t)dt \tag{8}$$

where $\lambda \geq 0, \gamma > 0$.

Lemma 1.([23]) *For a given matrix* $S = \begin{pmatrix} S_{11} & S_{12} \\ * & S_{22}\end{pmatrix}$ *with* $S_{11} = S_{11}^T, S_{22} = S_{22}^T$, *then the following conditions are equivalent:*

(1) $S < 0$,

(2) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$,

(3) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2.([17]) *For any positive definite constant matrix* M, *scalars* a *and* b: $b < a$, *vector function* $\gamma : [b, a] \to R^l$ *such that the integration concerned are well defined, then:*

$$(a-b)\int_b^a \varpi^T(s)M\varpi(s)ds \geq (\int_b^a \varpi(s)ds)^T M(\int_b^a \varpi(s)ds).$$

III. Main results

The exponential stable and H_∞ control of nonlinear switched systems (1) in the presence of mixed time-varying delays and exogenous disturbance are investigated by resorting to the average dwell time scheme and Jensens Inequality technique in this section. In view of this, we firstly consider that the resulting closed-loop system (5) is exponential stable when $\nu(t) = 0$.

A. Stability analysis

Theorem 1. *For given positive constants* α, d_M, τ_M, *and* $\mu \geq 1$, *if there exist positive constant* ε_i *and symmetric and positive definite matrices* $P_i, S_{1i}, S_{2i}, T_{1i}, T_{2i}$ *such that the following matrix inequalities hold for all* $i, j \in M, i \neq j$,

$$\begin{aligned} &P_i \leq \mu P_j, S_{1i} \leq \mu S_{1j}, S_{2i} \leq \mu S_{2j}, \\ &T_{1i} \leq \mu T_{1j},\ T_{2i} \leq \mu T_{2j}, \end{aligned} \tag{9}$$

$$\Xi_i = \begin{pmatrix} \phi_{11}^i & \phi_{12}^i & 0 & P_i & 0 & \phi_{16}^i \\ * & \phi_{22}^i & 0 & 0 & 0 & 0 \\ * & * & \phi_{33}^i & 0 & 0 & 0 \\ * & * & * & \phi_{44}^i & 0 & 0 \\ * & * & * & * & \phi_{55}^i & 0 \\ * & * & * & * & * & \phi_{66}^i \end{pmatrix} < 0. \tag{10}$$

where

$\phi_{11}^i = P_i\tilde{A}_{1i} + \tilde{A}_{1i}^T P_i + S_{1i} + S_{2i} + d_M^2 T_{1i} + \tau_M^2 T_{2i} + \alpha P_i$,

$\phi_{12}^i = P_i A_{2i}, \quad \phi_{16}^i = P_i C_i, \quad \phi_{22}^i = -(1-\bar{d})e^{-\alpha h_M} S_{1i}$,

$\phi_{33}^i = \delta^2 I - (1-\bar{\tau})e^{-\alpha\tau_M} S_{2i}, \quad \phi_{44}^i = -I$,

$\phi_{55}^i = -e^{-\alpha h_M} T_{1i}, \quad \phi_{66}^i = -e^{-\alpha\tau_M} T_{2i}$.

Then, the resulting closed-loop system (5) with $\omega(t) = 0$ *is exponentially stabilizable for any switching signal with the average dwell time satisfying*

$$\tau_a > \tau_a^* = \frac{\ln \mu}{\alpha}. \tag{11}$$

Proof: When $t \in [t_k, t_{k+1})$ and $\omega(t) = 0$, we assume that the ith subsystem is activated, the delay-dependent Lyapunov-Krasovskii functional is constructed as follows:

$$V(t) = V_i(t) = \sum_{s=1}^{5} V_{si}(t), \tag{12}$$

where

$$\begin{aligned}
V_{1i}(t) &= x^T(t)P_i x(t),\\
V_{2i}(t) &= \int_{t-d(t)}^t e^{\alpha(s-t)} x^T(s)S_{1\sigma(t)}x(s)ds,\\
V_{3i}(t) &= \int_{t-\tau(t)}^t e^{\alpha(s-t)} x^T(s)S_{2\sigma(t)}x(s)ds,\\
V_{4i}(t) &= d_M\int_{-d_M}^0\int_{t+\theta}^t e^{\alpha(s-t)} x^T(s)T_{1\sigma(t)}x(s)dsd\theta,\\
V_{5i}(t) &= \tau_M\int_{-\tau_M}^0\int_{t+\theta}^t e^{\alpha(s-t)} x^T(s)T_{2\sigma(t)}x(s)dsd\theta.
\end{aligned}$$

The time derivative of (12) yields

$$\begin{aligned}
\dot{V_{1i}}(t) &= 2x^T(t)P_i\dot{x}(t),\\
\dot{V_{2i}}(t) &= -(1-\dot{d}(t))e^{-\alpha d(t)}x(t-d(t))S_{1i}x(t-d(t))\\
&\quad + x(t)S_{1i}x(t) - \alpha V_{2i}(t)\\
&\leq -(1-\bar{h})e^{-\alpha d_M}x(t-d(t))S_{1i}x(t-d(t))\\
&\quad + x(t)S_{1i}x(t) - \alpha V_{2i}(t),\\
\dot{V_{3i}}(t) &= -(1-\dot{\tau}(t))e^{-\alpha\tau(t)}x(t-d\tau(t))S_{2i}x(t-\tau(t))\\
&\quad + x(t)S_{2i}x(t) - \alpha V_{3i}(t)\\
&\leq -(1-\bar{\tau})e^{-\alpha\tau_M}x(t-\tau(t))S_{2i}x(t-\tau(t))\\
&\quad + x(t)S_{2i}x(t) - \alpha V_{3i}(t),\\
\dot{V_{4i}}(t) &= -d_M\int_{-d_M}^0 e^{\alpha\theta}x^T(t+\theta)T_{1i}x(t+\theta)d\theta\\
&\quad + d_M^2 x^T(t)T_{1i}x(t) - \alpha V_{4i}(t)\\
&\leq -d_M\int_{t-d_M}^t e^{-\alpha d_M}x^T(s)T_{1i}x(s)ds - \alpha V_{5i}(t)\\
&\quad + d_M^2 x^T(t)T_{1i}x(t),\\
\dot{V_{5i}}(t) &= -\tau_M\int_{-\tau_M}^0 e^{\alpha\theta}x^T(t+\theta)T_{2i}x(t+\theta)d\theta\\
&\quad + \tau_M^2 x^T(t)T_{2i}x(t) - \alpha V_{5i}(t)\\
&\leq -\tau_M\int_{t-\tau_M}^t e^{-\alpha\tau_M}x^T(s)T_{2i}x(s)ds - \alpha V_{5i}(t)\\
&\quad + \tau_M^2 x^T(t)T_{2i}x(t).
\end{aligned} \tag{13}$$

According (3), we have

$$\begin{aligned} &\delta^2 x^T(t-\tau(t))x(t-\tau(t))\\ &\quad - g^T(t, x(t-\tau(t)))g(t, x(t-\tau(t))) \geq 0. \end{aligned} \tag{14}$$

Form Jensens inequality, we get

$$\begin{aligned}&-\tau_M\int_{t-\tau_M}^{t} e^{-\alpha\tau_M}x^T(s)T_{2i}x(s)ds\\&\quad\le -e^{-\alpha\tau_M}\left(\int_{t-\tau_M}^{t}x^T(s)ds\right)T_{2i}\left(\int_{t-\tau_M}^{t}x(s)ds\right)\\&\quad\le -e^{-\alpha\tau_M}\left(\int_{t-\tau(t)}^{t}x^T(s)ds\right)T_{2i}\left(\int_{t-\tau(t)}^{t}x(s)ds\right);\\&-d_M\int_{t-d_M}^{t} e^{-\alpha d_M}x^T(s)T_{1i}x(s)ds\\&\quad\le -e^{-\alpha d_M}\left(\int_{t-d_M}^{t}x^T(s)ds\right)T_{1i}\left(\int_{t-d_M}^{t}x(s)ds\right).\end{aligned}\tag{15}$$

Considering (13), (14), and (15)

$$\begin{aligned}&\dot V_i(t)+\alpha V_i(t)\\&\le x^T(t)[P_i\tilde A_{1i}+\tilde A_{1i}^TP_i+S_{1i}+S_{2i}+d_M^2T_{1i}+\alpha P_i\\&\quad+\tau_M^2T_{2i}]x(t)-g^T(t,x(t-\tau(t)))g(t,x(t-\tau(t)))\\&\quad+x^T(t-\tau(t))[\delta_i^2I-(1-\bar\tau)e^{-\alpha\tau_M}S_{2i}]x(t-\tau(t))\\&\quad-(1-\bar d)e^{-\alpha d_M}x^T(t-d(t))S_{1i}x(t-d(t))\\&\quad+x^T(t-d(t))A_{2i}^TP_ix(t)+x^T(t)P_iC_i\int_{t-\tau(t)}^{t}x(s)ds\\&\quad+x^T(t)P_ig(t,x(t-\tau(t)))+g^T(t,x(t-\tau(t)))P_ix(t)\\&\quad+x^T(t)P_iA_{2i}x(t-d(t))+\int_{t-\tau(t)}^{t}x^T(s)dsC_i^TP_ix(t)\\&\quad-e^{-\alpha\tau_M}\left(\int_{t-\tau(t)}^{t}x^T(s)ds\right)T_{2i}\left(\int_{t-\tau(t)}^{t}x(s)ds\right)\\&\quad-e^{-\alpha d_M}\left(\int_{t-d_M}^{t}x^T(s)ds\right)T_{1i}\left(\int_{t-d_M}^{t}x(s)ds\right)\end{aligned}\tag{16}$$

Then,

$$\dot V_i(t)+\alpha V_i(t)\le\phi^T(t)\tilde\Xi_i\phi(t),\tag{17}$$

where

$$\begin{aligned}\psi(t)=\big(&x^T(t)\quad x^T(t-d(t))\quad x^T(t-\tau(t))\\&g^T(t,x(t-\tau(t)))\quad \textstyle\int_{t-d_M}^{t}x^T(s)ds\quad \int_{t-\tau(t)}^{t}x^T(s)ds\big)^T,\end{aligned}$$

According (10), we have

$$\dot V_i(t)-\alpha V_i(t)\le 0.\tag{18}$$

When $t\in[t_k,t_{k+1})$, simultaneously integrating from t_k to t on both sides of the above formula, we can get

$$V(t)=V_{\sigma(t)}(t)\le e^{-\alpha(t-t_k)}V_{\sigma(t_k)}(t_k),\ t_k\le t<t_{k+1}.\tag{19}$$

Form (9), (19) and $k=N_\sigma(t,t_0)\le(t-t_0)/\tau_a$, we have

$$\begin{aligned}V(t)&\le e^{-\alpha(t-t_k)}\mu V_{\sigma(t_k{}^-)}(t_k{}^-)\\&\le\cdots\le e^{-\alpha(t-t_0)}\mu^kV_{\sigma(t_0{}^-)}(t_0{}^-)\\&\le e^{-(\alpha-\ln\mu/\tau_a)(t-t_0)}V_{\sigma(t_0)}(t_0).\end{aligned}\tag{20}$$

According Lyapunov-Krasovskii functional (12), we can get

$$\begin{aligned}V(t)&\ge a\|x(t)\|^2,\\V(t_0)&\le b\|x_{t_0}\|_c,\end{aligned}$$

where

$$\begin{aligned}a&=\min_{i\in N}\lambda_{\min}(P_i),\\b&=\max_{i\in N}\lambda_{\max}(P_i)+h_M\max_{i\in N}\lambda_{\max}(Q_{1i})\\&\quad+\tau_M\max_{i\in N}\lambda_{\max}(Q_{2i})+\tfrac12h_M^3\max_{i\in N}\lambda_{\max}(R_{1i})\\&\quad+\tfrac12\tau_M^3\max_{i\in N}\lambda_{\max}(R_{2i}),\end{aligned}$$

$\|x_{t_0}\|_c=\sup_{-max(h_M,d_M)\le\theta\le0}\{\|x(t_0+\theta)\|,\|\dot x(t_0+\theta)\|\}$. Hence,

$$\|x(t)\|\le\sqrt{\frac{b}{a}}\|x_{t_0}\|_ce^{-\frac12(\alpha-\ln\mu/\tau_a)(t-t_0)}.\tag{21}$$

By Definition 2, system (1) is exponentially stability. The proof is completed.

Remark 3. It should be pointed out that if $\mu=1$, switching signals of switched systems degenerates to arbitrary switching, which implies that $\tau_a^*=0$. However, in the mathematical model of the actual system, this phenomenon is often undesirable, because in practice it may cause the entire switched systems to be unstable due to fast switching (see [29] for more details).

Remark 4. If the matrix C_i is a 0 matrix, the term of $\int_{t-\tau(t)}^{t}x(s)ds$ will not exist in (1). In this case, we get a special nonlinear switched time-delay system. Thus, the term of $\int_{t-\tau(t)}^{t}x(s)ds$ not only makes the switched system more general than the model created in [11, 15]. In addition, the mixed time-delays with upper bound are considered in (12) to describe the delay-dependent multi-Lyapunov-Krasovskii functional and free-weighting matrices and Jensens inequality are also introduced in the process of calculation. Compared with [30], the results obtained in this paper are less conservative by the proposed approach.

B. H_∞ control

In the following subsection, we considered the issue of H_∞ control for the resulting system (5) with respect to the exogenous disturbance input $\nu(t)\ne0$, and some new delay-dependent exponentially stable criteria with a prescribed weighted attenuation performance index γ are provided.

Theorem 2. *For given positive constants α,γ,d_M, and τ_M, if there exist symmetric and positive definite matrices $P_i,Q_{1i},Q_{2i},R_{1i},R_{2i}$ and K_i, such that the following matrix inequalities hold for all $i,j\in M$,*

$$\begin{aligned}&P_i\le\mu P_j,\ S_{1i}\le\mu S_{1j},\ S_{2i}\le\mu S_{2j},\\&T_{1i}\le\mu T_{1j},\ T_{2i}\le\mu T_{2j},\end{aligned}\tag{22}$$

$$\tilde\Xi_i=\begin{pmatrix}\tilde\phi_{11}^i&\phi_{12}^i&0&P_i&0&\phi_{16}^i&\phi_{17}^i\\ *&\phi_{22}^i&0&0&0&0&0\\ *&*&\phi_{33}^i&0&0&0&0\\ *&*&*&\phi_{44}^i&0&0&0\\ *&*&*&*&\phi_{55}^i&0&0\\ *&*&*&*&*&\phi_{66}^i&0\\ *&*&*&*&*&*&\phi_{77}^i\end{pmatrix}<0\tag{23}$$

where

$$\begin{aligned}&\tilde\phi_{11}^i=P_i\bar A_{1i}+\bar A_{1i}^TP_i+S_{1i}+S_{2i}+d_M^2T_{1i}+\tau_M^2T_{2i}+\alpha P_i,\\&\phi_{77}^i=F_i^TF_i-\gamma^2I,\ \phi_{17}^i=P_iD_i+E_i^TF_i.\end{aligned}$$

Then, the resulting system (5) is exponentially stabilizable and has weighted attenuation performance γ for any switching signal with the average dwell time satisfies (11).

Proof: For $t \in [t_k, t_{k+1})$, we choose Lyapunov-Krasovskii functional as (12), thus, we can get

$$\begin{aligned}
&\dot{V}(t) + \alpha V(t) + z^T(t)z(t) - \gamma^2\nu^T(t)\nu(t) \\
&\le x^T(t)[P_i\tilde{A}_{1i} + \tilde{A}_{1i}^T P_i + S_{1i} + S_{2i} + d_M^2 T_i + \tau_M^2 T_i \\
&+ \alpha P_i + D_i^T D_i]x(t) + \nu^T(t)(D_i^T P_i + F_i^T E_i)x(t) \\
&- (1-\bar{\tau})x^T(t-\tau(t))e^{-\alpha\tau_M} S_{2i}x(t-\tau(t)) \\
&+ x^T(t)P_i g(t, x(t-\tau(t))) + \int_{t-\tau(t)}^t x^T(s)ds C_i^T P_i x(t) \\
&+ x^T(t)P_i A_{2i}x(t-d(t)) + \nu^T(t)(F_i^T F_i - \gamma^2 I)\nu(t) \\
&+ g^T(t, x(t-\tau(t)))P_i x(t) + x^T(t)(P_i D_i + E_i^T F_i)\nu(t) \\
&+ [\delta^2 I - (1-\bar{d})]e^{-\alpha d_M} x^T(t-d(t))S_{1i}x(t-d(t)) \\
&+ x^T(t)P_i C_i \int_{t-\tau(t)}^t x(s)ds + x^T(t-d(t))A_{2i}^T P_i x(t) \\
&- e^{-\alpha d_M}\left(\int_{t-d_M}^t x^T(s)ds\right)T_{1i}\left(\int_{t-d_M}^t x(s)ds\right) \\
&- e^{-\alpha\tau_M}\left(\int_{t-\tau(t)}^t x^T(s)ds\right)^T T_{2i}\left(\int_{t-\tau(t)}^t x(s)ds\right) \\
&- g^T(t, x(t-\tau(t)))g(t, x(t-\tau(t))).
\end{aligned}$$

Defined

$$\begin{aligned}
\tilde{\psi}(t) = [&x^T(t) \quad x^T(t-d(t)) \quad x^T(t-\tau(t)) \\
&g^T(t, x(t-\tau(t))) \quad \left(\int_{t-d_M}^t x(s)ds\right)^T \\
&\left(\int_{t-\tau(t)}^t x(s)ds\right)^T \quad \nu^T(t)]^T.
\end{aligned}$$

So

$$\dot{V}(t) + \alpha V(t) + z^T(t)z(t) - \gamma^2\nu^T(t)\nu(t) \le \tilde{\psi}^T(t)\tilde{\Xi}_i\tilde{\psi}(t).$$

We have

$$\dot{V}(t) + \alpha V(t) + z^T(t)z(t) - \gamma^2\nu^T(t)\nu(t) \le 0. \tag{24}$$

Integrating from t_k to t on both sides of (24), we have

$$V(t) \le e^{-\alpha(t-t_k)}V(t_k) - \int_{t_k}^t e^{-\alpha(t-s)}\Gamma(s)ds. \tag{25}$$

where $\Gamma(t) = z^T(t)z(t) - \gamma^2\nu^T(t)\nu(t)$.

Combining (9) and (25), we obtain

$$\begin{aligned}
V(t) &\le e^{-\alpha(t-t_k)}V(t_k) - \int_{t_k}^t e^{-\alpha(t-s)}\Gamma(s)ds \\
&\le \mu^k V(t_0)e^{-\alpha t} - \mu^k\int_0^{t_1} e^{-\alpha(t-s)}\Gamma(s)ds \\
&- \mu^{k-1}\int_{t_1}^{t_2} e^{-\alpha(t-s)}\Gamma(s)ds \\
&- \cdots - \mu^{k-1}\int_{t_k}^t e^{-\alpha(t-s)}\Gamma(s)ds \\
&\le e^{-\alpha t + N_\sigma(0,t)\ln u}V(0) - \int_0^t e^{-\alpha t + N_\sigma(s,t)\ln u}\Gamma(s)ds.
\end{aligned} \tag{26}$$

Then,

$$0 \le -\int_0^t e^{-\alpha(t-s) + N_\sigma(s,t)\ln u}\Gamma(s)ds. \tag{27}$$

Using $e^{-N_\sigma(0,t)\ln\mu}$ to pre-multiply and post-multiply the left term of (27), we have

$$\begin{aligned}
&\int_0^t e^{-\alpha(t-s) - N_\sigma(0,s)\ln u}z^T(s)z(s)ds \\
&\quad \le \int_0^t e^{-\alpha(t-s) - N_\sigma(0,s)\ln u}\gamma^2\nu^T(s)\nu(s)ds.
\end{aligned} \tag{28}$$

When $N_\sigma(0,s) \le \frac{s}{\tau_a}$ and $\tau_a > \tau_a^* = \frac{\ln u}{\alpha}$, it is easy to obtain $N_\sigma(0,s)\ln u \le \alpha s$. So

$$\int_0^t e^{-\alpha(t-s)-\alpha s}z^T(s)z(s)ds \le \int_0^t e^{-\alpha(t-s)}\gamma^2\omega^T(s)\omega(s)ds. \tag{29}$$

Integrating (29) from 0 to ∞, we have

$$\int_0^\infty e^{-\alpha s}z^T(s)z(s)ds \le \int_0^\infty \gamma^2\omega^T(s)\omega(s)ds.$$

The proof is completed.

C. Controller design

The design of the proposed the state feedback controllers of switched systems subject to exogenous disturbance and mixed time-varying delays is given through deformation of Linear Matrix Inequalities in this section.

Theorem 3. *For given positive constants α, γ, d_M and τ_M, if there exist symmetric and positive definite matrices $X_i, G_{1i}, G_{2i}, O_{1i}, O_{2i}$ and K_i, such that the following matrix inequalities hold for all $i, j \in M$,*

$$\begin{aligned}
&N_i \le \mu N_j,\ G_{1i} \le \mu G_{1j},\ G_{2i} \le \mu G_{2j}, \\
&O_{1i} \le \mu O_{1j},\ O_{2i} \le \mu O_{2j},
\end{aligned} \tag{30}$$

$$\bar{\Xi}_i = \begin{pmatrix} \Delta_{11}^i & \Delta_{12}^i \\ * & \Delta_{22}^i \end{pmatrix} < 0. \tag{31}$$

where

$$\Delta_{11}^i = \begin{pmatrix} \bar{\phi}_{11}^i & \bar{\phi}_{12}^i & 0 & I & 0 & \bar{\phi}_{16}^i \\ * & \bar{\phi}_{22}^i & 0 & 0 & 0 & 0 \\ * & * & \bar{\phi}_{33}^i & 0 & 0 & 0 \\ * & * & * & \bar{\phi}_{44}^i & 0 & 0 \\ * & * & * & * & \bar{\phi}_{55}^i & 0 \\ * & * & * & * & * & \bar{\phi}_{66}^i \end{pmatrix},$$

$$\Delta_{12}^i = \begin{pmatrix} \phi_{17}^i & 0 & d_M N_i & \tau_M N_i & N_i & N_i \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta N_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\Delta_{22}^i = diag\left\{\phi_{77}^i \ -I \ -G_{1i} \ -G_{2i} \ -O_{1i} \ -O_{2i}\right\},$$

where

$\bar{\phi}_{11}^i = A_{1i}N_i + B_i Y_i + (A_{1i}N_i + B_i Y_i)^T + \alpha N_i$,

$\bar{\phi}_{12}^i = A_{2i}N_i,\ \bar{\phi}_{16}^i = C_i,\ \bar{\phi}_{22}^i = (1-\bar{d})e^{-\alpha d_M}(O_{1i} - 2N_i)$,

$\bar{\phi}_{33}^i = (1-\bar{\tau})e^{-\alpha\tau_M}(O_{2i} - 2N_i), \quad \bar{\phi}_{44}^i = -I$,

$\bar{\phi}_{55}^i = e^{-\alpha d_M}(G_{1i} - 2N_i), \quad \bar{\phi}_{66}^i = e^{-\alpha\tau_M}(G_{2i} - 2N_i)$.

Then the system (1) is exponentially stabilizable and has weighted attenuation performance γ for any switching signal with the average dwell time satisfies (11). Furthermore, the controller can be designed by the following formula:

$$K_i = Y_i N_i^{-1} \tag{32}$$

Proof: From $G_{1i} > 0, G_{2i} > 0, O_{1i} > 0$, and $O_{2i} > 0$, we can get

$$(G_{1i} - N_i)^T G_{1i}^{-1}(G_{1i} - N_i) \geq 0,$$
$$(G_{2i} - N_i)^T G_{2i}^{-1}(G_{2i} - N_i) \geq 0,$$
$$(O_{1i} - N_i)^T O_{1i}^{-1}(O_{1i} - N_i) \geq 0,$$
$$(O_{2i} - N_i)^T O_{2i}^{-1}(O_{2i} - N_i) \geq 0.$$

Then the following inequality can be obtained:

$$\begin{aligned} G_{1i} - 2N_i &\geq -N_i G_{1i}^{-1} N_i, \\ G_{2i} - 2N_i &\geq -N_i G_{2i}^{-1} N_i, \\ O_{1i} - 2N_i &\geq -N_i O_{1i}^{-1} N_i, \\ O_{2i} - 2N_i &\geq -N_i O_{2i}^{-1} N_i. \end{aligned} \tag{33}$$

Substituting (33) into (31) and multiplying both sides of (31) by

$$diag\{N_i^{-1}, N_i^{-1}, N_i^{-1}, I, N_i^{-1}, N_i^{-1}, I, I, I, I, I, I\}$$

The following inequality is obtained,

$$\begin{pmatrix} \hat{\Delta}_{11}^i & \hat{\Delta}_{12}^i \\ * & \Delta_{22}^i \end{pmatrix} < 0, \tag{34}$$

where

$$\hat{\Delta}_{11}^i = \begin{pmatrix} \hat{\phi}_{11}^i & \hat{\phi}_{12}^i & 0 & I & 0 & \hat{\phi}_{16}^i \\ * & \hat{\phi}_{22}^i & 0 & 0 & 0 & 0 \\ * & * & \hat{\phi}_{33}^i & 0 & 0 & 0 \\ * & * & * & \hat{\phi}_{44}^i & 0 & 0 \\ * & * & * & * & \hat{\phi}_{55}^i & 0 \\ * & * & * & * & * & \hat{\phi}_{66}^i \end{pmatrix}$$

$$\hat{\Delta}_{12}^i = \begin{pmatrix} \phi_{17}^i & 0 & d_M I & \tau_M I & I & I \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\hat{\phi}_{11}^i = N_i^{-1}A_{1i} + N_i^{-1}B_i Y_i N_i^{-1} + (N_i^{-1}A_{1i} + N_i^{-1}B_i Y_i N_i^{-1})^T + \alpha N_i^{-1},\ \hat{\phi}_{12}^i = N_i^{-1}A_{2i},\ \hat{\phi}_{16}^i = N_i^{-1}C_i,$$
$$\hat{\phi}_{22}^i = -(1-\bar{d})e^{-\alpha h_M} S_{1i}^{-1}, \hat{\phi}_{44}^i = -I, \hat{\phi}_{55}^i = -e^{-\alpha d_M} T_{1i}^{-1},$$
$$\hat{\phi}_{33}^i = -(1-\bar{\tau})e^{-\alpha\tau_M} S_{2i}^{-1},\quad \hat{\phi}_{66}^i = e^{-\alpha\tau_M} T_{2i}^{-1}.$$

Then setting

$$\begin{aligned} Y_i = K_i N_i, \quad N_i^{-1} = P_i, \quad G_{1i}^{-1} = S_{1i}, \\ G_{2i}^{-1} = S_{2i}, \quad O_{1i}^{-1} = T_{1i}, \quad O_{2i}^{-1} = T_{2i}. \end{aligned} \tag{35}$$

and using Lemma 2.2 in (34), it can be concluded that (24) holds. This means that (31) implies (24). Moreover, the controller gains are given by (32). The proof is completed.

Remark 5. In fact, H_∞ control of switched systems is an important research issue in many engineering systems when switched systems are constantly disturbed by the external behavior, e.g., tracking process, networked control systems, circuit systems. Specifically, the issue of H_∞ is considered such that the switched systems is exponential stabilization and satisfies a prescribed H_∞ control performance level in this paper. Moreover, on the basis of Jensen's Inequality, the average dwell time method and free weighting matrix technique, some new delay-dependent sufficient results are obtained. Compared with the existing of H_∞ control, it is noted that the results in [5,22] can be further improved by our method.

Remark 6. A problem of finite-time H_∞ control for uncertain switched neural networks is addressed in [14], However, to the best of our knowledge, the finite-time H_∞ control behavior can be termed as short-time stability or finite-time stability (see [16] for more details). However, in practical engineering systems, we need the system to be asymptotic stable in most cases. Motivated by this, We consider the H_∞ control problem of the switched systems in $[0, \infty)$ instead of the interval of $[0, T]$.

Remark 7. In [31], the issue of dynamic output feedback controller was addressed for a linear switched systems. Moreover, as the switched systems state $x(t)$ is not always available and exogenous disturbance often exist in many practical systems. For this reason, H_∞ control scheme is proposed in Theorem 2. Specifically, when we do not consider the effects of exogenous disturbance and nonlinear perturbation, the model of switched systems considered in [31] is a special case of this paper. It is shown that our results have a larger application range.

IV. Numerical examples

In this section, on the basis of the proposed schemes, we present a numerical example and a practical example of river pollution control to demonstrate the effectiveness of the proposed approach.

Example 1. Consider switched system (1) composed of two subsystems with the following parameters:

$$A_{11} = \begin{bmatrix} -1.8 & 0.2 \\ 0.3 & -1.5 \end{bmatrix},\ A_{21} = \begin{bmatrix} -0.9 & 0.1 \\ 0.1 & -1.4 \end{bmatrix},$$
$$A_{12} = \begin{bmatrix} -2.1 & 0.3 \\ 0.1 & -1.7 \end{bmatrix},\ A_{22} = \begin{bmatrix} -0.6 & 0.2 \\ 0.1 & -1.7 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.7 \\ 0.3 \end{bmatrix},\quad C_1 = \begin{bmatrix} -0.2 & 0.3 \\ 0.1 & -0.3 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.1 \\ 0.8 \end{bmatrix},\quad C_2 = \begin{bmatrix} -0.1 & 0.1 \\ 0.2 & -0.1 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} -0.9 & 0.2 \\ 0.2 & -1.6 \end{bmatrix},\ D_2 = \begin{bmatrix} -1.1 & 0.1 \\ 0.1 & -1.8 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} -0.7 & 0.1 \\ 0.1 & -0.8 \end{bmatrix},\ E_2 = \begin{bmatrix} -1.2 & 0.1 \\ 0.2 & -1.3 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} -1.1 & 0.2 \\ 0.1 & -1.9 \end{bmatrix},\ F_2 = \begin{bmatrix} -1.1 & 0.2 \\ 0.1 & -1.2 \end{bmatrix}.$$

Let $\alpha = 0.8$, $d_M = 0.6$, $\tau_M = 0.9, \bar{d} = 0.3$, $\bar{\tau} = 0.5, \mu = 1.7, \gamma = 0.6, d(t) = 0.3 + 0.3\sin(t)$, $\tau(t) = 0.4 + 0.5\sin(t)$, and $\nu(t) = (0.1e^{-2t}\ \ 0.2e^{-3t})^T$. By calculating, we can get the average dwell time is $\tau_a > 0.6633$. Moreover, we consider $g(t, x(t-\tau(t))) = ((0.3sin(x(t))\ \ 0.2cosx(t-\tau(t)))^T$.

By solving (23) and (24), we can get

$$P_1=\begin{bmatrix}0.9762 & -0.0175\\ -0.0175 & 1.0073\end{bmatrix},$$
$$P_2=\begin{bmatrix}0.8256 & -0.2334\\ -0.2334 & 0.9011\end{bmatrix},$$
$$S_{11}=\begin{bmatrix}1.0122 & -0.1325\\ -0.1325 & 1.2117\end{bmatrix},$$
$$S_{21}=\begin{bmatrix}1.0231 & -0.1332\\ -0.1332 & 1.1239\end{bmatrix},$$
$$S_{12}=\begin{bmatrix}0.9263 & -0.1286\\ -0.1286 & 1.0092\end{bmatrix},$$
$$S_{22}=\begin{bmatrix}1.0128 & -0.2176\\ -0.2176 & 1.1254\end{bmatrix},$$
$$T_{11}=\begin{bmatrix}0.9729 & -0.2825\\ -0.2825 & 1.0726\end{bmatrix},$$
$$T_{21}=\begin{bmatrix}1.0127 & -0.2903\\ -0.2903 & 1.2527\end{bmatrix},$$
$$T_{12}=\begin{bmatrix}1.7822 & -0.3037\\ -0.3037 & 1.6735\end{bmatrix},$$
$$T_{22}=\begin{bmatrix}0.9923 & -0.3826\\ -0.3826 & 1.0771\end{bmatrix},$$
$$Y_1=\begin{bmatrix}1.2733 & 1.8682\end{bmatrix},$$
$$Y_2=\begin{bmatrix}0.9678 & 1.0245\end{bmatrix}.$$

Then, the controller gains are

$$K_1=\begin{bmatrix}1.3380 & 1.8779\end{bmatrix},$$
$$K_2=\begin{bmatrix}1.6117 & -1.5544\end{bmatrix}.$$

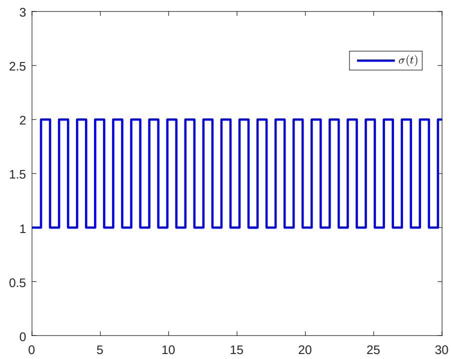

Fig. 1: The switching law.

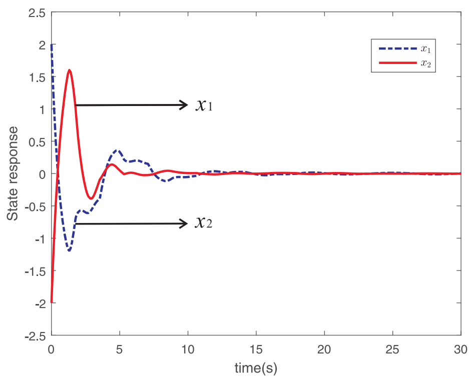

Fig. 2: State response of the closed-loop system.

The trajectories of the closed-loop switched system (5) with $x(0)=(2,-2)^T$ the initial state and switching signal $\sigma(t)$ are shown in Fig.1 and Fig.2, respectively. Fig.2 shows that the trajectory of subsystem 1 and subsystem 2 gradually tend to equilibrium position under the designed switching signal. As a consequence, The implication of this numerical example is to illustrate the effectiveness of the approach proposed.

Example 2. It is worth mentioning that the safe water resources are important for social development. In fact, the current problem of water pollution is also an urgent issue for every country. Next, on the basis of the proposed H_∞ control schemes in this paper, we present a practical example of river pollution control to confirm the effectiveness of the approach proposed.

According to [27,28], the concentrations per unit volume of biochemical oxygen demand (BOD) and dissolved oxygen (DO) are denoted as $m(t)$ and $l(t)$, respectively. m^* and l^* denote the desired steady values of $m(t)$ and $l(t)$, respectively. Let

$$x_1(t)=m(t)-m^*, x_2(t)=l(t)-l^*,$$
$$x(t)=[x_1^T(t)\, x_2^T(t)]^T\,.$$

Subsequently, the dynamic equation for $x(t)$ is given by

$$\dot{x}(t)=Ax(t)+\bar{A}x(t-d(t))+Bu(t)+\nu(t) \tag{36}$$

where

$$A=\begin{bmatrix}-\xi_1-\varepsilon_1-\varepsilon_2 & 0\\ -\xi_3 & -\xi_2-\varepsilon_1-\varepsilon_2\end{bmatrix},$$
$$\bar{A}=\begin{bmatrix}\varepsilon_2 & 0\\ 0 & \varepsilon_2\end{bmatrix},\quad B=\begin{bmatrix}\varepsilon_1\\ 1\end{bmatrix},$$

$x(t)\in R^2, u(t)\in R^2, \nu(t)\in R^2$ are the state vector, control input and exogenous disturbance of water pollution control system, respectively. In addition, $\xi_i(i=1,2,3)$ and $\varepsilon_j(j=1,2)$ are known constants and the physical meaning of parameters can be found in [27,28]. Moreover, in order to match the actual situation, we considered nonlinear term and conservativeness during the modeling process. Simultaneously, we assume that there are two actuators, which correspond to no failures occur and failures occur situations, respectively. Therefore, water pollution control system can be described as the following switched system:

$$\begin{cases}\dot{x}(t)=A_1x(t)+\bar{A}_2x(t-d(t))+Bu(t)+g(t,x(t-\tau(t))\\ \qquad +C\displaystyle\int_{t-\tau(t)}^{t}x(s)ds+D\nu(t), \quad \text{no failures occur;}\\ \dot{x}(t)=A_3x(t)+\bar{A}_4x(t-d(t))+Bu(t)+g(t,x(t-\tau(t))\\ \qquad +C\displaystyle\int_{t-\tau(t)}^{t}x(s)ds+D\nu(t), \quad \text{failures occur.}\end{cases} \tag{37}$$

Next, we choose $\xi_1=1.5, \xi_2=0.6, \xi_3=1.4, \varepsilon_1=0.4, \varepsilon_2=0.6$, and get that

$$A=\begin{bmatrix}-2.5 & 0\\ -1.4 & -0.6\end{bmatrix}, \bar{A}=\begin{bmatrix}0.6 & 0\\ 0 & 0.6\end{bmatrix}, B=\begin{bmatrix}0.4\\ 1\end{bmatrix}.$$

Let $A_1=A$, $\bar{A}_2=\bar{A}$,

$$A_3=A+\begin{bmatrix}3 & 0\\ 0 & 1\end{bmatrix}, \bar{A}_4=\bar{A}+\begin{bmatrix}-0.2 & 0\\ 0 & -0.2\end{bmatrix},$$
$$C=D=\begin{bmatrix}0.11 & 0\\ 0 & 0.11\end{bmatrix}.$$

In addition, we choose $\alpha = 0.45, d_M = 0.8, \tau_M = 0.7, \bar{d} = 0.4, \bar{\tau} = 0.6, \mu = 1.85, \gamma = 0.7, d(t) = 0.4 + 0.4\sin(t)$, $\tau(t) = 0.1 + 0.6\sin(t), \nu(t) = (0.02e^{-3t} \ \frac{0.01}{1+e^{5t}})^T$,

$$g(t, x(t-\tau(t))) = \begin{pmatrix} 0.01\sin(x_1(t-\tau(t))) \\ 0.02\sin(x_2(t-\tau(t))) \end{pmatrix}.$$

By solving (11), (23) and (24), we can get average dwell time $\tau_a > 1.3671$ and

$$\begin{aligned}
P_1 &= \begin{bmatrix} 1.1552 & -1.4957 \\ -1.4957 & 2.3321 \end{bmatrix}, \\
P_2 &= \begin{bmatrix} 7.3874 & -3.7818 \\ -3.7818 & 4.1623 \end{bmatrix}, \\
S_{11} &= \begin{bmatrix} 0.3307 & -0.3013 \\ -0.3013 & 0.5532 \end{bmatrix}, \\
S_{21} &= \begin{bmatrix} 0.3132 & -0.1248 \\ -0.1248 & 0.3584 \end{bmatrix}, \\
S_{12} &= \begin{bmatrix} 0.8228 & -0.1626 \\ -0.1626 & 0.6926 \end{bmatrix}, \\
S_{22} &= \begin{bmatrix} 0.5098 & -0.0753 \\ -0.0753 & 0.4517 \end{bmatrix}, \\
T_{11} &= \begin{bmatrix} 0.5442 & -0.5975 \\ -0.5975 & 0.9310 \end{bmatrix}, \\
T_{21} &= \begin{bmatrix} 0.6156 & -0.6971 \\ -0.6971 & 1.0827 \end{bmatrix}, \\
T_{12} &= \begin{bmatrix} 1.9692 & -0.7040 \\ -0.7040 & 1.4063 \end{bmatrix}, \\
T_{22} &= \begin{bmatrix} 2.2799 & -0.8392 \\ -0.8392 & 1.5794 \end{bmatrix}, \\
Y_1 &= \begin{bmatrix} 0.0648 & 0.9883 \end{bmatrix}, \\
Y_2 &= \begin{bmatrix} 0.5828 & 0.4235 \end{bmatrix}.
\end{aligned}$$

Then, the controller gains are

$$\begin{aligned}
K_1 &= \begin{bmatrix} -1.4034 & 2.2080 \end{bmatrix}, \\
K_2 &= \begin{bmatrix} 2.7037 & -0.4413 \end{bmatrix}.
\end{aligned}$$

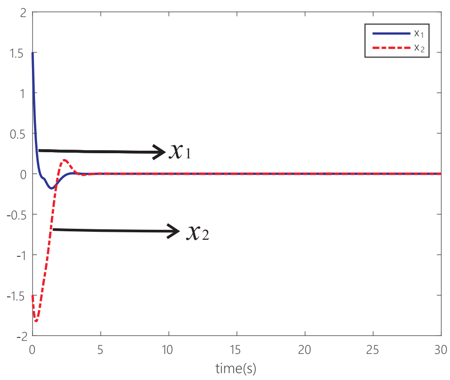

Fig. 3: State response of subsystem1.

Fig.3 and Fig.4 correspond to the states response of subsystem 1 and subsystem 2 with the initial condition $x(0) = (1.5, -1.5)^T$, respectively. Moreover, we can also observe from Fig. 4 that subsystem 2 is unstable when the actuator occur failures. To overcome this obstacle, we design switching signal and use the proposed H_∞ control schemes to achieve the system (37) with the initial condition $x(0) = (1.5, -1.5)^T$ is exponentially stabilizable. Form Fig.5 and Fig.6, we can obviously see that the developed method is correct in the control of water pollution process.

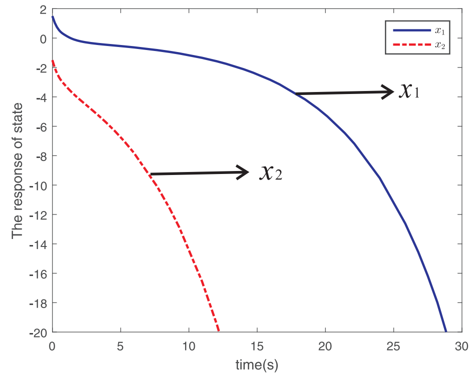

Fig. 4: State response of subsystem2.

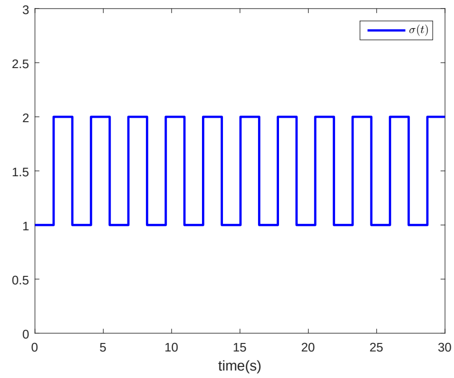

Fig. 5: The switching law of (37).

Remark 8. Compared with some existing the stability results of nonlinear switched systems, the proposed method has a wider application range from the physical point of view by example of water pollution control. Specifically, it should be pointed out our method can also handle the case that the switched systems contains unstable subsystems. Therefore, our results have improved in terms of conservatism.

V. Conclusions

This paper mainly studies the issue of exponential H_∞ control for a class of nonlinear switched systems subject to exogenous disturbance and mixed time-varying delays. Considering the mixed time-varying delays with upper bound and the nonlinear perturbation satisfies the Lipschitz condition, a novel delay-dependent multi-Lyapunov-Krasovskii functional is constructed that utilizes the complete available information about the upper bound of mixed time-varying delays. Subsequently, based on Lyapunov stability theory and average dwell time method, some delay-dependent exponentially stable criteria and a prescribed H_∞ control performance of switched systems with mixed time-varying delays are obtained by resorting to Jensen's inequality. Specifically, compared with the existing literature, our conservatism is guaranteed by simulation results, and the proposed control scheme have wide practical range by example of water pollution control. Finally, the state feedback controllers gains are also derived by solving some linear matrix inequalities.

In addition, control system applications are becoming widespread [24-26], as we all know that random delay is caused by network constraints in some microprocessors control systems. It is important to deal with the effects of

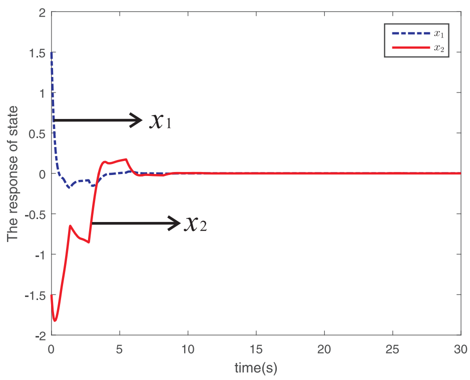

Fig. 6: State response of system (37).

random delay while maintaining the control performance. Therefore, the time-varying random delay is my next research plan.

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