Investigation of Particle Multi-Swarm Optimization with Diversive Curiosity

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***Abstract*—This paper presents a new series of search methods of particle multi-swarm optimization (PMSO), which have intelligent judgment function in search process. The key idea, here, is first time systematically to create a psychological concept of diversive curiosity into the existing particle multi-swarm optimizers as an internal indicator. According to the idea, four search methods of PMSO with diversive curiosity, i.e. multiple particle swarm optimizers with information sharing and diversive curiosity (*MPSOISDC*), multiple particle swarm optimizers with inertia weight with information sharing and diversive curiosity (*MPSOIWISDC*), multiple canonical particle swarm optimizers with information sharing and diversive curiosity (*MCPSOISDC*), and hybrid particle swarm optimizers with information sharing and diversive curiosity (*HPSOISDC*) are proposed. This is a new technical expansion of PMSO in search framework for overcoming initial stagnation and avoiding boredom behavior to enhance search efficiency. In computer experiments, with adjusting the values of two parameters, i.e. duration of judgment and sensitivity, of the internal indicator, we inspect the performance index of the proposed methods by dealing with a suite of benchmark problems in search process. Based on detail analysis of the obtained experimental results, we reveal the outstanding search capabilities and characteristics of *MPSOISDC*, *MPSOIWISDC*, *MCPSOISDC*, and *HPSOISDC*, respectively.**

***Index Terms*—swarm intelligence, particle multi-swarm optimization, information sharing, diversive curiosity, initial stagnation, parallel computation**

I. Introduction

NEEDLESS to say, a lot of traditional search methods, e.g., steepest descent method, conjugate gradient method, and quasi-newton method etc., these ones may be in better search accuracy and exact computation. However, they have brittle operations and necessary information to search subject, computing condition, and search environment, in contrast to many population-based stochastic search methods of genetic and evolutionary computation (GEC). But those non-traditional search methods can provide more robust, efficient, and expandable approach with different genetic operators to handle high-grade nonlinear, multimodal, and complex real-world problems [10], [11], [25].

As a new member of population-based stochastic search methods of GEC, the technique of particle swarm optimization (PSO) has been widely applied in different areas of science, technology, engineering, communication, traffic control, and applications etc. to demonstrate its search performance and adaptability. These good accomplishments are because of which the mechanism of PSO itself has the distinguishing features: information exchange, intrinsic memory, and directional search compared to the other members such as genetic algorithms (GAs), evolutionary programming (EP), differential evolution (DE), and so on [8], [16], [24].

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Nevertheless, during the rudimentary stage of PSO development for improving search convergence, solution accuracy, and search efficiency of the original particle swarm optimizer (called as the *PSO*) [14] in mechanism, many variants and remodels of the *PSO* such as particle swarm optimizer with inertia weight (*PSOIW*) [19], canonical particle swarm optimizer (*CPSO*) [3], [4] etc. were created during several years after the *PSO* published.

In recent years, especially, many studies, papers, and reports etc. about particle multi-swarm optimization (PMSO) in relation to symbiosis, swarm behavior, and synergy are in the researcher's spotlight. For instance, hybrid PSO, multi-layer PSO, and multiple PSO with decision-marking strategy etc. are published [1], [7], [17], [30] one after another for deepening on population-based search and capability to attain superior search performance. Owing to the exceptional advantage of PMSO, it has been receiving increasing attention and research.

PMSO belongs to a main branch of technical development of PSO. The combinatorial technique of PMSO is very important and useful, because it can allow various expressions and combination to properly deal with the given different and complex optimization problems. As previous studies on the above composition, we proposed four basic search methods of PMSO, i.e. multiple particle swarm optimizers with information sharing (*MPSOIS*), multiple particle swarm optimizers with inertia weight with information sharing (*MPSOIWIS*), multiple canonical particle swarm optimizers with information sharing (*MCPSOIS*), and hybrid particle swarm optimizers with information sharing (*HPSOIS*) [21], [22], and exhibited their strengths in mechanisms and characteristics in search process.

Therefore, utilizing the non-tradition search methods, i.e. the techniques of population-based search, stochastic search, parallel computation, and intelligent judgment function have become one of extremely important approaches to deal with the complicated optimization problems. However, although the above-mentioned search methods have high-performance than those methods without information sharing and single swarm one, in general, the basic characteristics of them have not been changed totally in search process.

For lightening the above issues, i.e. overcoming initial stagnation and avoiding boredom behavior, and enhancing search efficiency of PMSO approach itself, in this paper, we are first time systematically to create a psychological concept [2], [5] of diversive curiosity (DC) into the above-mentioned four basic search methods as an internal indicator to build a new series of search methods of PMSO. These proposed methods are multiple particle swarm optimizers with information sharing and diversive curiosity (*MPSOISDC*), multiple particle swarm optimizers with inertia weight with information sharing and diversive curiosity (*MPSOIWISDC*), multiple canonical particle swarm optimizers with information sharing and diversive curiosity (*MCPSOISDC*), and hybrid particle swarm optimizers with information sharing and diversive curiosity (*HPSOISDC*), respectively.

This is a new technical expansion of PMSO in search framework by the combination of two ways: One is implementation of parallel computation, and the other one is introduction of concept on diversive curiosity. In order to confirm the characteristics of these proposed methods, we inspect their search capabilities and performance through executing many computer experiments. Based on the obtained search results and knowledge acquired about the existing four basic search methods of PMSO, meanwhile, in this paper we carry out a number of computer experiments for measuring these proposed methods how to deal with a suite of benchmark problems, which are unlike difficulty of the given optimization problems.

For revealing the outstanding search capabilities and performance of these new search methods, i.e. *MPSOISDC*, *MPSOIWISDC*, *MCPSOISDC*, and *HPSOISDC*, furthermore, we take more detailed results comparison to clarify the characteristics of each proposed method for accelerating technical development of PMSO to acquire swarm intelligence with high-level.

The rest of this paper is organized as follows: Section II briefly introduces the basic search mechanisms and the built-in characteristics of both PSO and PMSO, respectively. Section III describes the basic judgment function and the concept of diversive curiosity, and proposes newly four search methods of PMSO with DC. Section IV provides the experimental results obtained by implementing the proposed methods in our computer experiments, shows and analyzes these results for confirming the search capabilities and characteristics of the proposed methods, respectively. Finally, the concluding remarks are given in Section V.

II. Introduction of Particle Swarm Optimization and Particle Multi-Swarm Optimization

Until now, there are many kinds of the search methods on PSO and PMSO [7], [12]. For the sake of convenience to the following description of both PSO and PMSO as to their mechanisms, here, let the search space be N-dimensional, $\Omega \in \Re^N$, the number of particle swarms to explore be S, and the number of particles in each swarm be Z.

A. Particle Swarm Optimization

In this section, as a basis for technical development of PSO, the existing three basic search methods, i.e. the *PSO* [14], *PSOIW* [6], [19], and *CPSO* [3], [4], are briefly described. These search methods are the most commonly used in PSO community.

1) About the PSO: The original search method of PSO was proposed by Kennedy and Eberhart (1995). Its mechanism for searching is very simple to effectively deal with a given optimization problems. Specifically, in beginning of a particle swarm search, position (i.e. solution) and velocity (i.e. amount of change for finding the best solution) of the i-th particle in the particle swarm are generated at random, then they are updated continuously as follows:

$$\vec{x}^{\,i}_{k+1} = \vec{x}^{\,i}_k + \vec{v}^{\,i}_{k+1} \tag{1}$$

$$\vec{v}^{\,i}_{k+1} = w_0 \vec{v}^{\,i}_k + w_1 \vec{r}_1 \otimes (\vec{p}^{\,i}_k - \vec{x}^{\,i}_k) + w_2 \vec{r}_2 \otimes (\vec{q}_k - \vec{x}^{\,i}_k) \tag{2}$$

where $\vec{x}^{\,i}_k$ refers to the solution of the i-th particle in the given search space, and $\vec{v}^{\,i}_k$ refers to its velocity at iteration k, respectively. w_0 is an inertia weight, w_1 is a coefficient for individual confidence, w_2 is a coefficient for swarm confidence. $\vec{r}_1,\ \vec{r}_2 \in \Re^N$ are two random vectors in which each element is uniformly distributed over the range $[0, 1]$, and the symbol $\otimes$ is an element-wise operator for vector multiplication. $\vec{p}^{\,i}_k (= arg \max\limits_{j=1,\cdots,k} \{g(\vec{x}^{\,i}_j)\}$, where $g(\cdot)$ is the criterion value of the i-th particle at iteration k.) is the local best solution of the i-th particle up to now, and $\vec{q}_k (= arg \max\limits_{i=1,2,\cdots} \{g(\vec{p}^{\,i}_k)\})$ is the global best solution among whole particle swarm.

2) About PSOIW: For improving the convergence, search capability, and performance of the *PSO*, Shi and Eberhart (1998) proposed the modified method, *PSOIW*. The updating rule of the i-th particle's velocity shown in Eq.(2) by constant reduction of the inertia weight over iteration is given as follows:

$$\vec{v}^{\,i}_{k+1} = w(k)\, \vec{v}^{\,i}_k + w_1 \vec{r}_1 \otimes (\vec{p}^{\,i}_k - \vec{x}^{\,i}_k) + w_2 \vec{r}_2 \otimes (\vec{q}_k - \vec{x}^{\,i}_k) \tag{3}$$

where $w(k)$ is a variable inertia weight, which is linearly reduced from starting value, w_s, to terminal value, w_e, with the increment of iteration k, is given by

$$w(k) = w_s + \frac{w_e - w_s}{K}\, k \tag{4}$$

where K is the maximum number of iteration k, and also is a stop condition for implementing *PSOIW*.

3) About CPSO: For same purpose as the above-mentioned *PSOIW*, Clerc and Kennedy (2002) proposed the modified method, *CPSO*. The updating rule of the i-th particle's velocity shown in Eq.(2) by a constant inertia weight over iteration is given as follows:

$$\vec{v}^{\,i}_{k+1} = \Phi \left(\vec{v}^{\,i}_k + w_1 \vec{r}_1 \otimes (\vec{p}^{\,i}_k - \vec{x}^{\,i}_k) + w_2 \vec{r}_2 \otimes (\vec{q}_k - \vec{x}^{\,i}_k) \right) \tag{5}$$

where Φ is an inertia weight according to the inertia weight w_0 in Eq.(2).

As to implement the above-mentioned three basic search methods of PSO, the original parameters of them are set as follows.

- In performing the *PSO* case, w_0 = 1.0 and w_1 = w_2 = 2.0 are used to search. Because the value of the inertia weight w_0 is set to be 1.0, the convergence of the *PSO* is not well in search process. So it can be considered that the *PSO* has the characteristics of global search.
- In performing *PSOIW* case, both of the boundary values are w_s = 0.9 and w_e = 0.4, respectively, and w_1 = w_2 = 2.0 are still used. Since the values of w_s and w_e of the variable inertia weight $w(k)$ are smaller than 1.0, respectively. So *PSOIW* has the characteristics of asymptotic/local search.
- In performing *CPSO* case, Φ = 0.729, and w_1 = w_2 = 2.05 are used to search. Since the value of Φ is smaller than 1.0, *CPSO* has the characteristics of local search.

B. Particle Multi-Swarm Optimization

As a basis for technical development of PMSO, we have proposed four basic search methods, i.e. *MPSOIS*, *MPSOIWIS*, *MCPSOIS*, and *HPSOIS** [21], [22]. In order to understand the formation of these existing search methods, we assume primarily that the particle multi-swarm consists of multiple particle swarms. Note that in these search methods, every particle swarm in search process is dependent with each other for introducing the most best solution of whole particle multi-swarm. And the mechanisms of them in search process are changed contrast to those search methods without information sharing.

As the characteristics of them, a special confidence term is added into the updating rule of the particle's velocity by the most best solution found by the particle multi-swarm, respectively. Based on the technical innovation strategy of information sharing (IS) in whole particle multi-swarm, the mechanisms of the existing four basic search methods of PMSO are shortly introduced below.

1) About MPSOIS: As the mechanism of *MPSOIS*, the updating rule of the i-th particle's velocity in every swarm is given as follows:

$$\begin{aligned}\vec{v}^{\,i}_{k+1} = w_0\,\vec{v}^{\,i}_k + w_1\vec{r}_1\otimes(\vec{p}^{\,i}_k-\vec{x}^{\,i}_k)+w_2\vec{r}_2\otimes(\vec{q}_k-\vec{x}^{\,i}_k)\\ +w_3\vec{r}_3\otimes(\vec{s}_k-\vec{x}^{\,i}_k)\end{aligned} \quad (6)$$

where $\vec{s}_k$ $(=arg \max_{j=1,\cdots,S} \{g(\vec{q}_k)_j\})$ is the most best solution chosen from the best solution set of whole particle multi-swarm, w_3 is a confidence coefficient for the particle multi-swarm, and $\vec{r}_3$ is a random vector which likes $\vec{r}_1$ and $\vec{r}_2$ described in Section II-A.

2) About MPSOIWIS: In same way as to the mechanism of *MPSOIS*, the updating rule of the i-th particle's velocity in every swarm is given as follows:

$$\begin{aligned}\vec{v}^{\,i}_{k+1} = w(k)\,\vec{v}^{\,i}_k + w_1\vec{r}_1\otimes(\vec{p}^{\,i}_k-\vec{x}^{\,i}_k)+w_2\vec{r}_2\otimes(\vec{q}_k-\vec{x}^{\,i}_k)\\ +w_3\vec{r}_3\otimes(\vec{s}_k-\vec{x}^{\,i}_k)\end{aligned} \quad (7)$$

Since Eq.(2), (3), (4), and Eq.(6) are alike in formulation, the description of the symbols in Eq.(7) is omitted.

3) About MCPSOIS: Similar to the mechanism of *MPSOIS*, the updating rule of the i-th particle's velocity in every swarm is given as follows:

$$\begin{aligned}\vec{v}^{\,i}_{k+1} = \Phi\Big(\vec{v}^{\,i}_k + w_1\vec{r}_1\otimes(\vec{p}^{\,i}_k-\vec{x}^{\,i}_k)+w_2\vec{r}_2\otimes(\vec{q}_k-\vec{x}^{\,i}_k)\\ +w_3\vec{r}_3\otimes(\vec{s}_k-\vec{x}^{\,i}_k)\Big)\end{aligned} \quad (8)$$

Likewise, the description of the symbols in Eq.(8) is omitted.

4) About HPSOIS: Based on the composition of three basic particle swarm optimizers described in Section II-A, there are three updating rules of the i-th particle's velocity, which are added, respectively. Thus, the mechanism of *HPSOIS* is determined by Eqs.(6), (7), and (8), respectively. So HPSOIS has common characteristics of the above-mentioned basic search methods, i.e. the *PSO*, *PSOIW*, and *CPSO* in Section II-A.

Note that since the strategy of information sharing is adopted, the search behavior of every particle swarm is not absolutely independent with each other. In performing the above-mentioned four basic search methods of PMSO, the confidence coefficient $w_3 = 0.3$ (an empirical value) is set in this paper for whole particle multi-swarm to explore. Other parameters of performing every particle multi-swarm are the same as those original parameters described in Section II-A. By this cause, the search capabilities of the *PSO* and *MPSOIS*, *PSOIW* and *MPSOIWIS*, *CPSO* and *MCPSOIS* are quite different, but the characteristics between them are basically similar in search process, respectively.

*HPSOIS is a mixed search method that is basically constructed by the *PSO*, *PSOIW*, and *CPSO*. Thus, it has the characteristics of these search methods.

III. A Proposal of Particle Multi-Swarm Optimization with Diversive Curiosity

Although the existing four basic search methods of PMSO described in Section II-B have high-performance to explore, but they still have drawbacks in search process, which be called as boredom behavior being a kind of stagnation phenomenon. For enhancing the search performance of PMSO to overcome initial stagnation and to avoid boredom behavior, furthermore, it is necessary to superpose a special strategy, called as diversive curiosity (DC) [30], to improve the efficiency and search performance of the above-mentioned search methods of PMSO.

A. Presentation of Diversive Curiosity

Curiosity is an emotion related to natural inquisitive behavior for humans and animals, and its importance and effect in motivating search cannot be ignored [5], [18]. Berlyne categorized it into two types: one is diversive curiosity, an other is specific curiosity [2]. About the former, Loewenstein (1994) insisted that "diversive curiosity occupies a critical position at the crossroad of cognition and motivation" in [15].

According to the assumption of the "cognition" is the act of exploitation, and the "motivation" is the intention to exploration, Zhang and Ishikawa (2008) created the following internal indicator to distinguish the above-mentioned behavioral activities in search process [28], [29].

$$y_k(L,\varepsilon) = max\Big(\varepsilon - \sum_{l=1}^{L}\frac{\big|g(\vec{s}^{\,b}_k)-g(\vec{s}^{\,b}_{k-l})\big|}{L},\ 0\Big) \quad (9)$$

where $\vec{s}^{\,b}_k$ is the most best solution found by whole particle multi-swarm at iteration k. As two adjustable parameters of the internal indicator, L refers to duration of judgment, and ε refers to a positive tolerance coefficient (i.e. sensitivity).

As to the computing mode of the internal indicator, it is obvious that the smaller the value of the sensitivity ε is, the higher the probability of attaining the most best solution is. On the other hand, the longer of the duration of judgment L is, the higher the probability of attaining the most best solution is. Therefore, the change of output of the internal indicator, $y_k(L,\varepsilon)$, reflects the result of population-based decision-making generated by whole particle multi-swarm until the situation of the present search.

Since boredom behavior and useless attempt of the particle multi-swarm in search process is overcome by the reliable way of alleviating initial stagnation and boredom behavior, so the efficiency and search performance of PMSO with DC in exploring could be greatly enhanced in a fixed search period, K.

TABLE I
FUNCTIONS AND CRITERIA OF THE GIVEN SUITE OF BENCHMARK PROBLEMS. THE SEARCH SPACE FOR EACH BENCHMARK PROBLEM IS LIMITED TO $\boldsymbol{\Omega} \in (-5.12, 5.12)^N$.

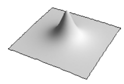
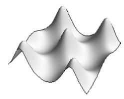
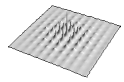
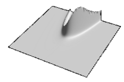
Problem	Function	Criterion	Distribution in 2D
Sphere (*Sp*)	$f_{Sp}(\vec{x}) = \sum_{d=1}^{N} x_d^2$	$g_{Sp}(\vec{x}) = \frac{1}{f_{Sp}(\vec{x})+1}$	
Griewank (*Gr*)	$f_{Gr}(\vec{x}) = \frac{1}{4000}\sum_{d=1}^{N} x_d^2 - \prod_{d=1}^{N} cos\left(\frac{x_d}{\sqrt{d}}\right) + 1$	$g_{Gr}(\vec{x}) = \frac{1}{f_{Gr}(\vec{x})+1}$	
Rastrigin (*Ra*)	$f_{Ra}(\vec{x}) = \sum_{d=1}^{N} \Big(x_d^2 - 10\, cos\,(2\pi x_d) + 10\Big)$	$g_{Ra}(\vec{x}) = \frac{1}{f_{Ra}(\vec{x})+1}$	
Rosenbrock (*Ro*)	$f_{Ro}(\vec{x}) = \sum_{d=1}^{N-1} \Big[100\big(x_{d+1} - x_d^2\big)^2 + \big(1 - x_d\big)^2\Big]$	$g_{Ro}(\vec{x}) = \frac{1}{f_{Ro}(\vec{x})+1}$	

TABLE II
THE MAJOR PARAMETERS USED IN IMPLEMENTING THE PROPOSED METHODS TO EXPLORE THE BEST SOLUTION

Parameter	Value	Parameter	Value
number of individuals, M	30	number of iterations, K	400
number of particle swarms, S	3	duration of judgment, L	10, 20, $\cdots$, 90
number of particles in each swarm, Z	10	tolerance coefficient, ε	$10^{-6} \sim 10^{-2}$

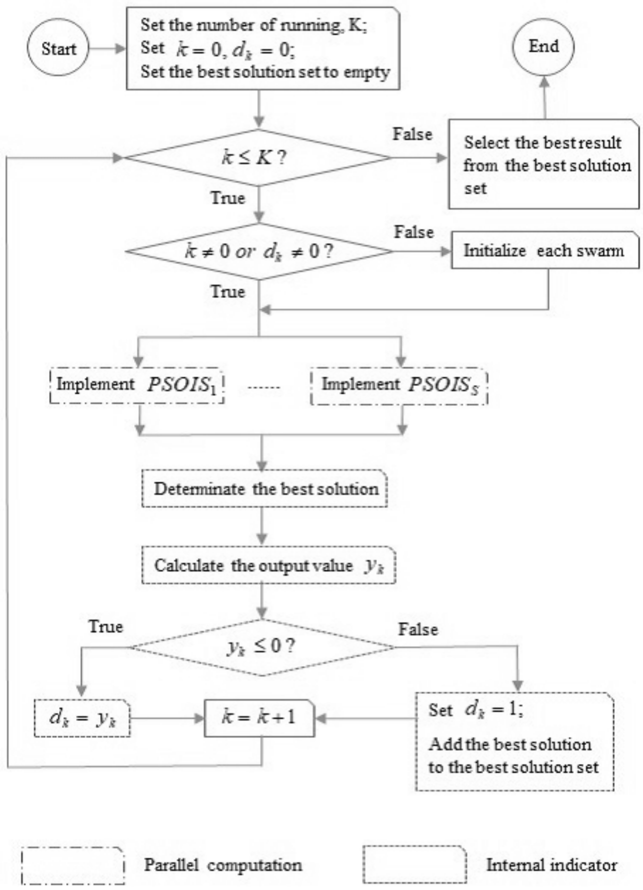


Fig. 1. A common flowchart of PMSO with DC. The parts of dotted lines show the parallel computation of particle multi-swarm. The parts of dashed lines show that internal indicator monitors the changed situation in search process for overcoming initial stagnation.

B. Realization of the Search Methods of PMSO with DC

We know that the internal indicator is to monitor whether the variable status of the most best solutions, $\vec{s}_k^{\,b}$, continues to change or not in duration of judgment for exhibiting the mechanism of diversive curiosity of whole particle multi-swarm to explore. According to the formation of the internal indicator, a construction of PMSO with DC can be realized easily.

Fig. 1 specifically gives a common construction (i.e. a flowchart of PMSO) for implementing these proposed methods, i.e. *MPSOISDC*, *MPSOIWISDC*, *MCPSOISDC*, and *HPSOISDC* to accomplish parallel computation and to realize the strategy of diversive curiosity for acquiring better search performance of PMSO with high-level.

Consequently, it is clear that each proposed method can be realized by replacing each search mechanism which is described in Section II-B at the place of $PSOIS_i$ ($i = 1, 2, \cdots, S$) in the yellow parts of Fig. 1. Specially, when $S = 1$ is set, the construction of PMSO with DC will become the one of PSO with DC [28], [29].

Based on the above-mentioned introduction of the DC strategy and the common flowchart of whole implementing process, we newly propose four search methods of PMSO with DC, which are multiple particle swarm optimizers with information sharing and diversive curiosity (*MPSOISDC*), multiple particle swarm optimizers with inertia weight with information sharing and diversive curiosity (*MPSOIWISDC*), multiple canonical particle swarm optimizers with information sharing and diversive curiosity (*MCPSOISDC*), and hybrid particle swarm optimizers with information sharing and diversive curiosity (*HPSOISDC*), respectively.

Overall, this is a new technical expansion of PMSO in search framework by the combination of two ways, i.e. implementation of parallel computation and introduction of concept on diversive curiosity. Even if both of them are not new ways (the existing technical methods [21], [29]), an innovative search method can be created with their combination. According to this feature, the efficiency and search performance of the used search method could be greatly enhanced in a fixed search period, K.

IV. Computer Experiments

In the following computer experiments, without loss of generality, we adopt a suite of benchmark problems [23] (including two unimodal and two multimodal problems) with 5 dimensions (5D) to facilitate data comparison and analysis of performance index of the proposed methods, and these criteria to test are shown in TABLE I. And the search range of all simulation cases is $\mathbf{\Omega} \in (-5.12, 5.12)^5$.

The criterion of the given benchmark problem (i.e. finding the global minimum of the given function) is expressed as follows:

$$g_\star(\vec{s}_k) = \frac{1}{1+f_\star(\vec{s}_k)} \tag{10}$$

where the symbol $\star$ denotes each given benchmark problem, i.e. *Sphere* (Sp), *Griewank* (Gr), *Rastrigin* (Ra), and *Rosenbrock* (Ro), respectively.

According to the definition of this criterion, when the most best solution, $\vec{s}_k^b$, is found by whole particle multi-swarm search, the result of the given function $f_\star(\vec{s}_k^b)$ approaches 0. Thus, the corresponding output value of the criterion, $g_\star(\vec{s}_k^b)$, approaches 1.

TABEL II shows the major parameters used to explore for dealing with the given optimization problems by implementing the proposed methods, respectively.

A. Characteristics of the Proposed Methods

Due to performance comparison and characteristics observation between the existing search method *HPSOIS* and the proposed method *HPSOISDC*, as an instance, Fig. 2 shows the different fitness values of every particle swarm searching for successfully dealing with *Rastrigin* problem (multimodal) by performing *HPSOIS*†. The change of each best fitness value (i.e. Best1, Best2, and Best3) of the particle multi-swarm in whole search process can be confirmed to attain the position of the best solution. Therefore, the obtained experimental result (mBest) of *HPSOIS* is converged to 1.0.

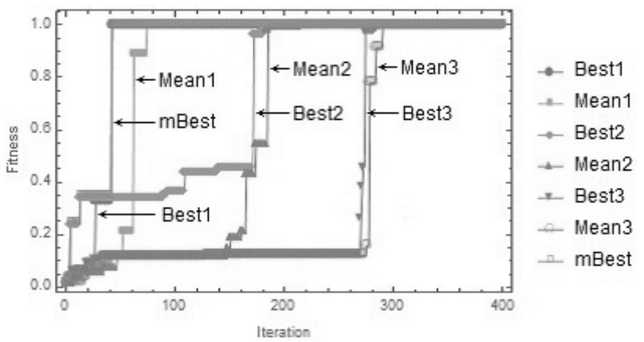


Fig. 2. Various change of fitness values in a search process for dealing with *Rastrigin* problem by performing *HPSOIS*. Best1 and Mean1 are the best fitness value and average fitness value of the 1st particle swarm, respectively. Best2 and Mean2 are the best fitness value and average fitness value of the 2nd particle swarm, respectively. Best3 and Mean3 are the best fitness value and average fitness value of the 3rd particle swarm, respectively. mBest is the most best fitness value of whole particle multi-swarm.

Although one of the existing search methods, *HPSOIS*, has powerful search capability, but the better search results obtained are not always guaranteed to be achieved. As an other instance, Fig. 3 shows the search results, i.e. the fitness values, of continuously dealing with *Rastrigin* problem by performing *HPSOIS* in 10 trials under the same computing condition. The obtained experimental result shows that the search results of whole particle multi-swarm falls into local minimum (i.e. initial stagnation) 3 times. Therefore, the success rate of *HPSOIS* in exploring is only 70% in 10 trials. In this case, as to statistical data, the average of the obtained fitness values is about 0.72, and the value of standard deviation is about 0.45, respectively. The mean value for the obtained experimental results is appeared as the final search results.

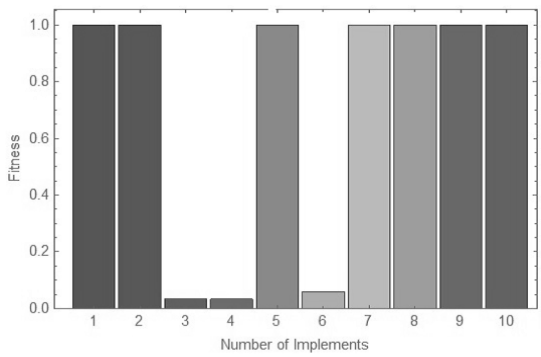


Fig. 3. A search result of continuously dealing with *Rastrigin* problem by performing *HPSOIS* in 10 trials

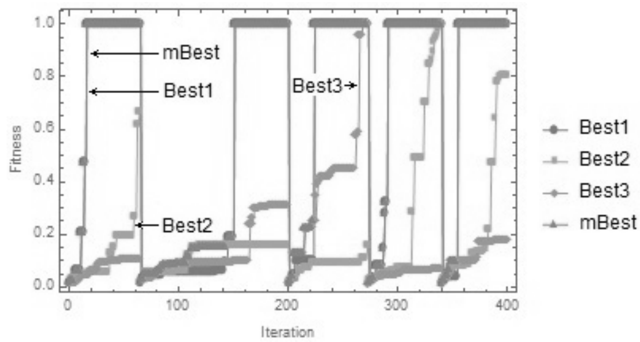


Fig. 4. Various change of fitness values in search process for dealing with *Rastrigin* problem by performing *HPSOISDC*.

Concretely, under the same experimental conditions (i.e. duration of judgment L=50 and sensitivity ε=0.001), the proposed method, *HPSOISDC*, is executed for dealing with the same problem. As an obtained search result, Fig. 4 shows the change of different fitness values of the particle multi-swarm in whole search process, which include the most best solution and the best solutions obtained by each particle swarm search. We can see clearly that the search process is repeated several times to initialization under the same search number, K, for finding the most best solution. This is an important point to overcome initial stagnation, and to avoid boredom behavior, which increases the number of initializing search to gain availability.

Because the proposed method, *HPSOISDC*, is performed several times within a constant iteration K = 400, it is obvious that the success rate of *HPSOISDC* in search capability is massively improved. Due to this cause, it is obvious that the created strategy of diversive curiosity takes an active role in whole search process.

With reference to the obtained search results, the exper-

†Computing environment – hardware: DELL OPTIPLEX 3020, Intel(R) core(TM) i5-4590, CPU 3.3GHz, RAM 8.0GB; Software: Mathematica 11.3; Search time: about 9.45 sec.

imental results of performing the proposed methods, i.e. MPSOISDC, MPSOIWISDC, and MCPSOISDC, for dealing with the given *Rastrigin* problem are shown in Fig. 5, Fig. 6, and Fig. 7, respectively.

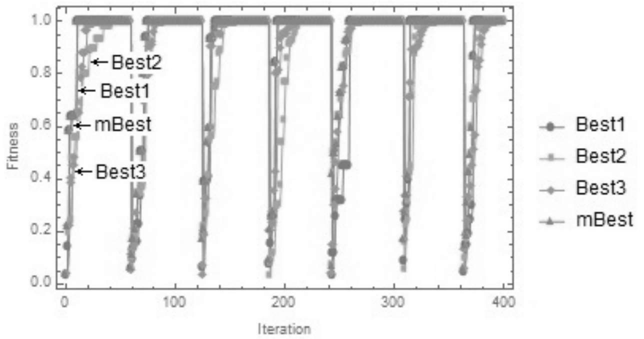


Fig. 5. Various change of fitness values in search process for dealing with *Rastrigin* problem by performing *MPSOISDC*.

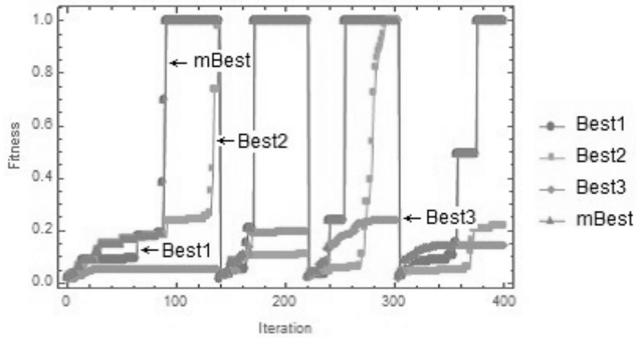


Fig. 6. Various change of fitness values in search process for dealing with *Rastrigin* problem by performing *MPSOIWISDC*.

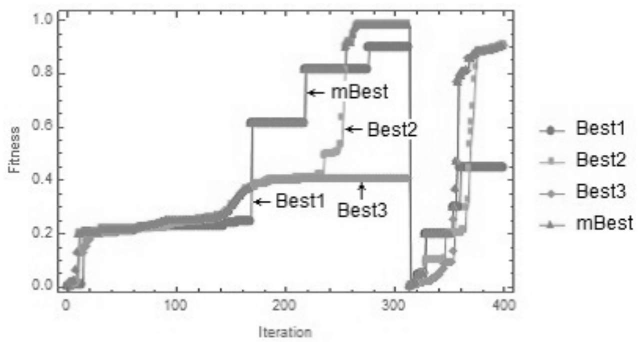


Fig. 7. Various change of fitness values in search process for dealing with *Rastrigin* problem by performing *MCPSOISDC*.

In these search processes, the search capability of each proposed method differs depending on the change of fitness values of the most best solution. Then, considering the probability of exploring, the following relationship is established for the search capability and performance of each proposed method to deal with *Rastrigin* problem.

$$MPSOISDC \succ HPSOISDC \succ MPSOIWISDC \succ MCPSOISDC$$

From the objective viewpoint, the search capability of each proposed method is predicted to deal with other benchmark problems.

B. The Search Performance of the Proposed Methods

Based on the above-mentioned experimental results in Section IV-A, the search capability and characteristics of *HPSOISDC* for exploring are confirmed. We furthermore investigate the search capabilities and performance of the proposed methods with adjusting two parameters, i.e. duration of judgment L and sensitivity ε, of the internal indicator in detail.

As the obtained main experimental results, Fig. 8 shows four figures in where the obtained four surfaces of the average fitness values of performing the proposed methods, i.e. *MPSOISDC* (M1), *MPSOIWISDC* (M2), *MCPSOISDC* (M3), and *HPSOISDC* (M4), respectively, for dealing with each given benchmark (i.e. Sp, Gr, Ra, and Ro) problem in 10 trials by adjusting two parameters, L and ε, of the internal indicator.

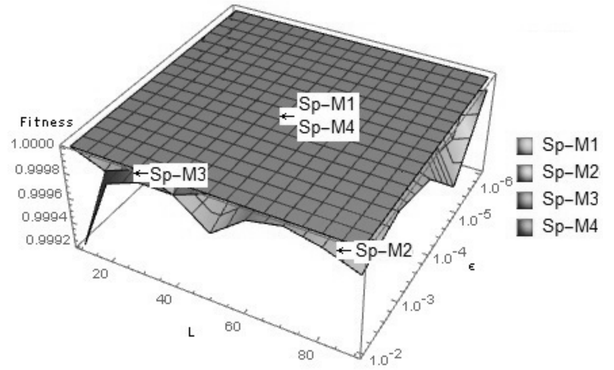
And for the sake of cross comparison and observation with ease, Fig. 9 gives four figures in where the obtained four surfaces of the average fitness values of performing each proposed method, i.e. M1, M2, M3, and M4, to deal with the given different benchmark problems, respectively.

First of all, by comparing the obtained experimental results shown in Fig. 8(a) and the corresponding Fig. 9(a), (b), (c), and (d), respectively, we can see that all of the proposed methods have very high-performance for dealing with *Sphere* (Sp) problem. In spite of that the search results of *MPSOIWISDC* (M2) and *MCPSOISDC* (M3) are few affected by the variation of two parameters, L and ε. Since *Sphere* (Sp) problem is a simple unimodal, each proposed method has very high-performance to deal with the given optimization problem.

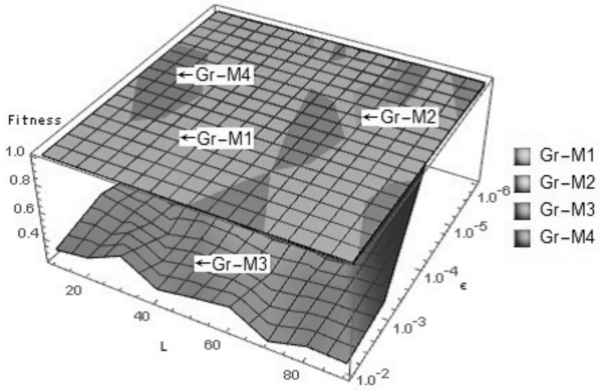
Second, by comparing the obtained experimental results shown in Fig. 8(b) and the corresponding Fig. 9(a), (b), (c), and (d), respectively, we can see that three proposed methods, i.e. *MPSOISDC* (M1), *MPSOIWISDC* (M2), and *HPSOISDC* (M4), have very high-performance to deal with *Griewank* (Gr) problem beside the search result of *MCPSOISDC* (M3). However, in the case of performing *MCPSOISDC* (M3) with increment of L and decrement of ε, the search result of it still found the best solution with high probability.

Third, by comparing the obtained experimental results shown in Fig. 8(c) and the corresponding Fig. 9(a), (b), (c), and (d), respectively, we can see that two proposed methods, i.e. *MPSOISDC* (M1) and *HPSOISDC* (M4), have very high-performance to deal with *Rastrigin* (Ra) problem. In the case of performing *MPSOIWISDC* (M2) with increment of L, the search result gently varies from better situation to bad one. On one hand, in the case of performing *MCPSOISDC* (M3) without increment of L and decrement of ε, the search result of it shows that the best solution cannot be found, because the obtained surfaces of the average fitness values are very lower.

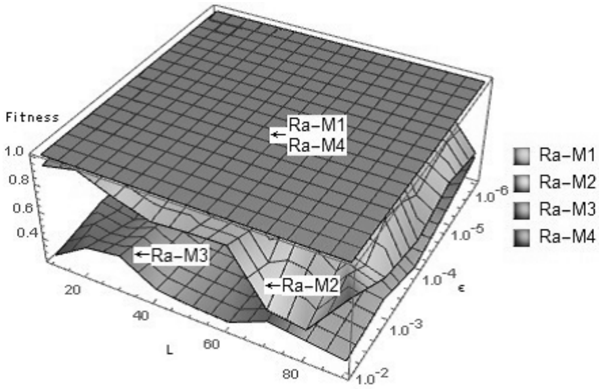
Fourth, by comparing the obtained experimental results shown in Fig. 8(d) and the corresponding Fig. 9(a), (b), (c), and (d), respectively, we can see that the proposed method *MCPSOISDC* (M3) has very high-performance for dealing with *Rosenbrock* (Ro) problem only. The search results of the other search methods, i.e. *MPSOISDC* (M1), *MPSOIWISDC* (M2), and *HPSOISDC* (M4) are not so well in search performance.



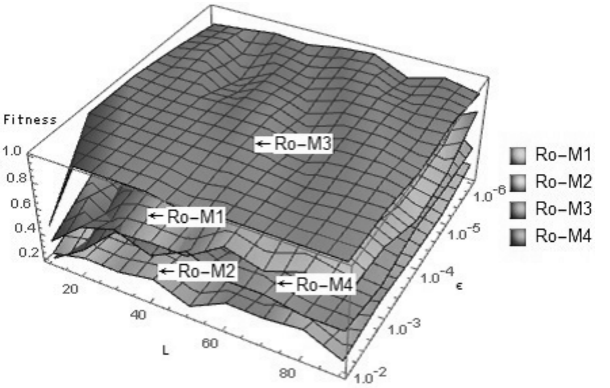
(a)



(b)

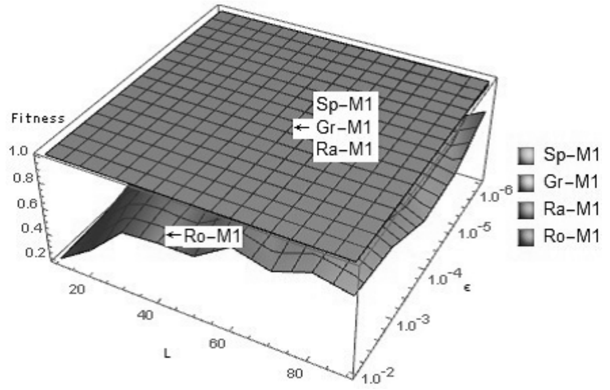


(c)

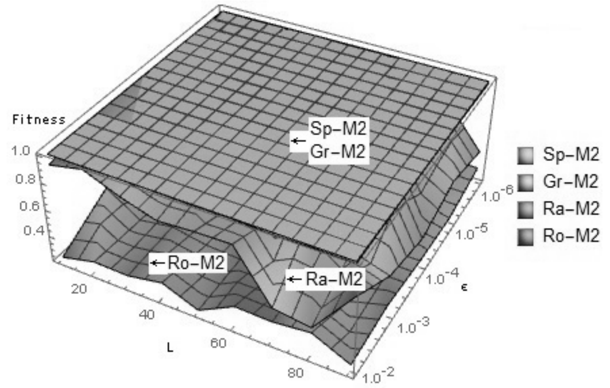


(d)

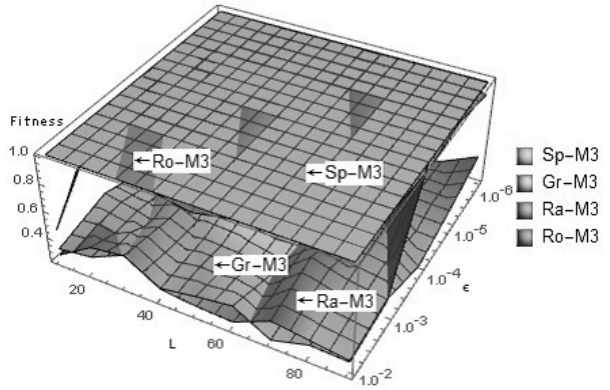
Fig. 8. Search results (i.e. the surfaces of the average fitness values) of performing the different proposed methods with adjusting two parameters, L and ε, of the internal indicator. (a) *Sphere* (Sp) problem, (b) *Griewank* (Gr) problem, (c) *Rastrigin* (Ra) problem, (d) *Rosenbrock* (Ro) problem.



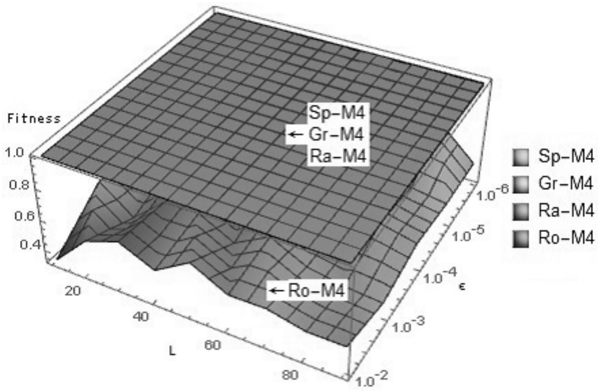
(a)



(b)



(c)



(d)

Fig. 9. Search results (i.e. the surfaces of the average fitness values) of performing the same proposed method with adjusting two parameters, L and ε, of the internal indicator. (a) *MPSOISDC* (M1) case, (b) *MPSOIWISDC* (M2) case, (c) *MCPSOISDC* (M3) case, (d) *HPSOISDC* (M4) case.

Considering the obtained experimental results of dealing with *Sphere* (Sp), *Griewank* (Gr), and *Rastrigin* (Ra) problems, comprehensively, it can be found that the search capability of *MPSOISDC* (M1) is significantly increased. In addition, regarding the obtained experimental result of dealing with *Rosenbrock* (Ro) problem, the search capability of *MPSOISDC* (M1) is influenced by the change of the adjustment parameters, L and ε, of the internal indicator.

C. Observation of the Hidden Experimental Results

For clearly observing the effect of the internal indicator, here, the experimental results of Fig. 8 and Fig. 9, which are the important search results hidden in both of the figures are examined.

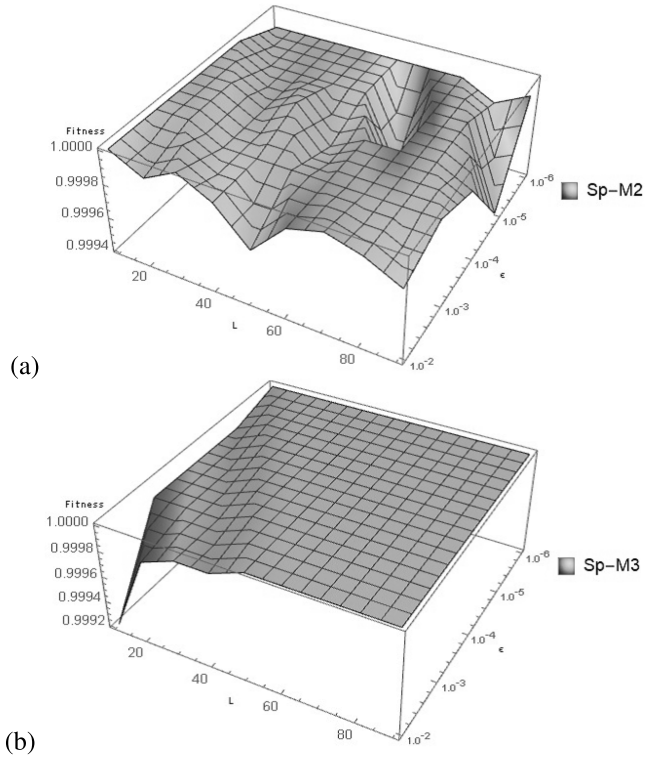


Fig. 10. Search results (i.e. the average fitness values) with adjusting two parameters, L and ε, of the internal indicator for dealing with *Sphere* problem. (a) *MPSOIWISDC* (M2) case, (b) *MCPSOISDC* (M3) case.

Fig. 10 shows the obtained experimental results of the proposed methods, i.e. *MPSOIWISDC* (M2) and *MCPSOISDC* (M3), for dealing with *Sphere* (Sp) problem. We can see that both of the surfaces of the average fitness values by adjusting two parameters, L and ε. They are found to have very high search capabilities by adjusting two parameters, L and ε, of the internal indicator.

Fig. 11 shows the obtained experimental results of the proposed methods, i.e. *MPSOIWISDC* (M2) and *MCPSOISDC* (M3), for dealing with *Griewank* (Gr) problem. We can see that both of the surfaces of the average fitness values by adjusting two parameters, L and ε. They are found to have search capabilities with adjusting two parameters, L and ε. Both are found to have search capabilities, although the search effect of *MPSOIWISDC* (M2) is lightened with the increment of sensitivity ε.

Fig. 12 shows the obtained experimental results of the proposed methods, i.e. *MPSOIWISDC* (M2) and *MCPSOISDC* (M3), for dealing with *Rastrigin* (Ra) problem. We can see that both of the surfaces of the average fitness values by adjusting two parameters, L and ε. They are found to have search capabilities with adjusting two parameters, L and ε. Both are found to have search capabilities, although the search effect of *MPSOIWISDC* (M2) is lightened with the increment of sensitivity ε.

Fig. 13 shows the obtained experimental results of the proposed methods, i.e. *MPSOISDC* (M1), *MPSOIWISDC* (M2), and *HPSOISDC* (M4) for dealing with *Rosenbrock* (Ro) problem. We can see that both of the surfaces of the average fitness values by adjusting two parameters, L and ε. They are found to have better search capabilities. The *MPSOISDC*

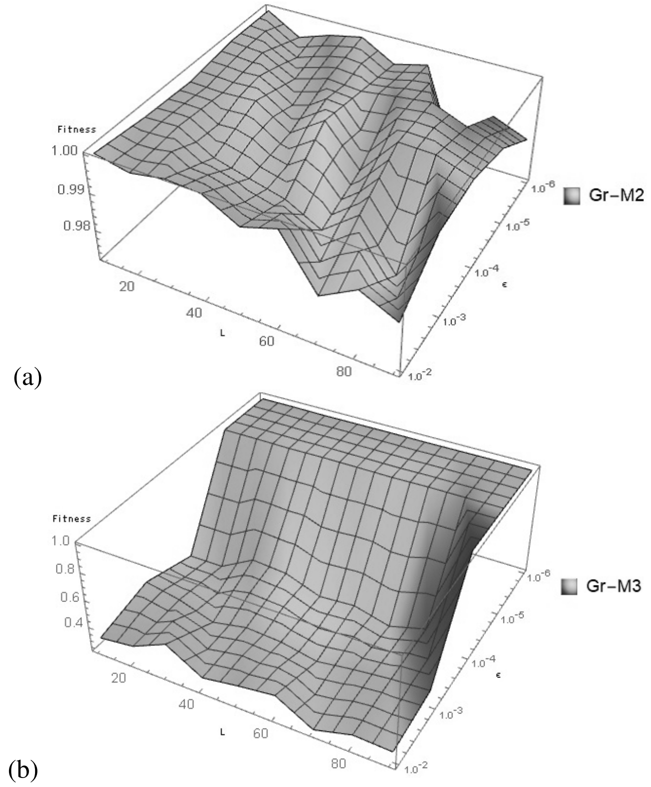


Fig. 11. Search results (i.e. the average fitness values) with adjusting two parameters, L and ε, of the internal indicator for dealing with *Griewank* problem. (a) *MPSOIWISDC* (M2) case, (b) *MCPSOISDC* (M3) case.

(M1) is found to have search capabilities, although the search effect of *MPSOIWISDC* (M2) is lightened with the increment of sensitivity ε.

By observing these obtained experimental results of Fig. 13, although the obtained search results, i.e. these surfaces of the average fitness values, are unstable, in special, for implementing *MPSOIWISDC* (M2) and *HPSOISDC* (M4), but the obtained experimental results can be confirmed that with the value of sensitivity ε becomes small, and the duration of judgment L becomes big, the probability of attaining better solutions becomes high. This affection is broadly consistent with theoretical analysis described in Section III.

D. Discussion for Methodology

In this paper, the initialization scheme, i.e. the strategy of diversive curiosity, is designed to help the used particle multi-swarm jump out a local optimum in the infeasible area for dealing with complex optimization problems.

It is well known that some search methods of GEC can do it. For example, Real-Coded GA [10], [27], the improved DE [26] and so on. These search methods are designed to be stochastic direct search ones which have the advantage of being easily applied to experimental maximization (or minimization). Generally, by adding some strategies, they have high-efficiency and high-performance for dealing with the given benchmark problems yet.

However, the disadvantage of the above-mentioned search methods is that a large number of individuals and time attaining to the best solution are required for exploring. Thus, for obtaining the most best solution, they will take much time to generate the next population.

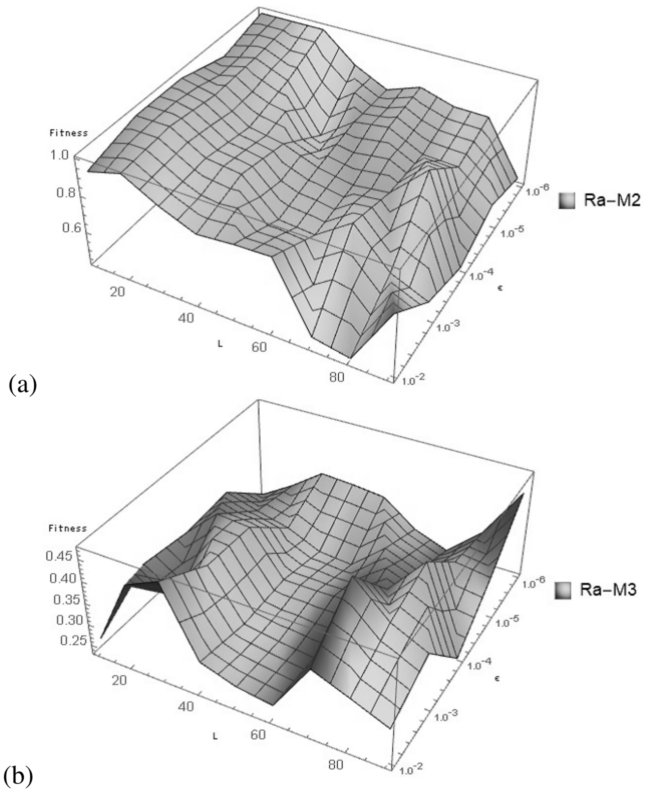


Fig. 12. Search results (i.e. the average fitness values) with adjusting two parameters, L and ε, of the internal indicator for dealing with *Rastrigin* problem. (a) *MPSOIWISDC* (M2) case, (b) *MCPSOISDC* (M3) case.

Hereby, this is one of the main reasons for choosing and developing PMSO methods to deal with complex optimization problems.

V. Conclusion

In this paper, we proposed a new series of search methods of particle multi-swarm optimization (PMSO) with diversive curiosity (DC), i.e. *MPSOISDC*, *MPSOIWISDC*, *MCPSOISDC*, and *HPSOISDC*, which have intelligent judgment function in their search process. The key idea, here, is first time systematically to create a psychological concept of diversive curiosity into the existing four basic search methods of PMSO as an internal indicator. This is a new technical expansion of PMSO in search framework for overcoming initial stagnation and avoiding boredom behavior to enhance search efficiency, and to improve the search capabilities and performance of the search methods of PMSO.

In our computer experiments, with adjusting the values of two parameters, i.e. duration of judgment and sensitivity, in the internal indicator, we inspect the search capabilities and characteristics of these new search methods for dealing with a suite of benchmark problems with 5D. Concretely, the observation and application of PMSO with DC to handle the given different optimization problems well demonstrated their effectiveness. Based on detail analysis to the obtained experimental results, we reveal the outstanding search capabilities and characteristics of *MPSOISDC*, *MPSOIWISDC*, *MCPSOISDC*, and *HPSOISDC*, respectively. Despite the fact that these mechanisms of PMSO with DC are very simple, the efficiency of each proposed method itself is still massively improved.

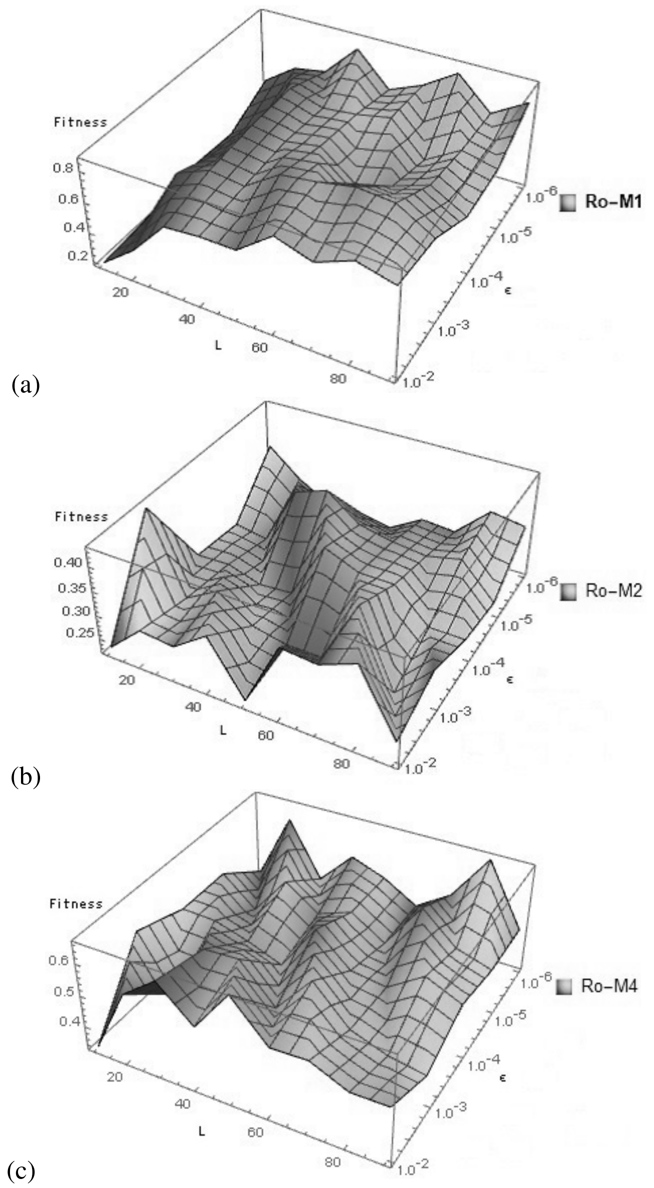


Fig. 13. Search results (i.e. the average fitness values) with adjusting two parameters, L and ε, of the internal indicator for dealing with *Rosenbrock* problem. (a) *MPSOISDC* (M1) case, (b) *MPSOIWISDC* (M2) case, (c) *HPSOISDC* (M4) case.

According to the comparison of experimental results and observation of these proposed methods, four search methods of PMSO with DC are confirmed in search capabilities and performance. As major goal in this study, it can be considered to have taken a step toward realizing PMSO to acquire swarm intelligence with high-level.

It is left for further study to apply the proposed methods, i.e. the technical expansion of PMSO in search framework, to data mining, system identification, multi-objective optimization, evolutionary neural network, and practical optimization problems in the real-world.

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