A Non-dimensional Mathematical Model of Salinity Measurement in the Chaophraya River Using a New Fourth Order Finite Difference Method with the Saulyev Technique

Nataya Camcoon, Nopparat Pochai

Abstract—**Salinity in a river is a measure of the content of salts in water. Salinity intrusion problem pose hazards for a river as well as affecting human health and agriculture. There are two methods to measure the salinity in a river. First, the sampling water method by monitoring stations has been using to collect the actual data. Second, a mathematical model is introduced to predict the salinity in a river. In this research, a mathematical model of salinity measurement in a river with releasing fresh water from a diversion dam effect is proposed. There are two finite difference techniques are introduced to approximate the model solution. The traditional forward time centered space techniques are also introduced. A new fourth order finite difference method is employed to accurately approximate the salinity in a river. A part of the Chaopraya river which is closed to the estuary is experimented. The actual problem is focused in this research. The experiment suggested can be used in many practical measurements of the salinity. The proposed method will predict the salinity level in a period on the future. The computational salinity measurement gives precisely results when the actual salinity and numerical salinity are compared. The proposed numerical simulation can be applied to a salinity forecasting.**

Index Terms—**Non-dimensional model, Salinity, Chaophraya river, Forward Time Central Space finite difference scheme, New fourth-order scheme with the Saulyev method, Finite difference method**.

I. INTRODUCTION

In [1-6] the numerical model were used to solve the water pollution measurement problems. In [7-8], the finite difference method was used to solve the hydrodynamic model with the constant coefficients in the closed uniform reservoir. In [9], an analytical solution to the hydrodynamic model in a closed uniform reservoir was proposed. In [10], the Lax-Wendroff finite difference method was also proposed to approximate the water elevation and water flow velocity. In [11], the fourth-order method for one-dimensional water quality model in a nonuniform flow stream was proposed. In [12], a non-dimensional form of a two-dimensional hydrodynamic model with generalized boundary condition and initial conditions for describing the elevation of water wave in an open uniform reservoir was proposed.

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In Dungun River, the method for water level forecasting where the data collected contain missing values is proposed in [17]. Probabilistic echo state networks (PESN) assessment of the water quality is proposed in [18]. In [19], this study aims to improve the forecasting of water levels at Bedup River with approximates of the absence of precipitation data, both using Artificial Neural Network (ANN). In [13], a one-dimensional mathematical model of salinity measurement in a river is proposed. A modified model of salinity control in a river with a barrage dam is also introduced.

A non-dimensional mathematical model for the calculation of salinity is proposed in this study. The south Chaophraya river, Thailand, is considered. The techniques for setting the physical parameters are also proposed. In order to accurately estimate the salinity in a flow, a new fourth order finite difference approach is employed to obtain accurately approximated salinity in the Chaophraya.

II. MATHEMATICAL MODEL

A. A salinity measurement model

In a stream water quality model, the governing equation is the dynamic one-dimensional advection-dispersion equation. A simplified representation, averaging the equation over the depths, is shown in [13],

$$\frac{\partial C}{\partial T}+u\frac{\partial C}{\partial X}=D\frac{\partial^2 C}{\partial X^2}-Q \tag{1}$$

for all $(X,T)\in\Omega=[0,L]\times[0,\tau]$, u is the flow velocity and D is a given diffusion coefficient and Q is the sink rate function. Assume that the salinity is diluted by the freshwater, then the salinity advection level is reduced by the freshwater velocity. The percentage ability of freshwater to dilute salinity is assumed to be $0 \le k \le 1$.

The one-dimensional salinity water pollution measurement model in a river can be given as follows [13],

$$\frac{\partial C}{\partial T}+(u_s-ku_w)\frac{\partial C}{\partial X}=D\frac{\partial^2 C}{\partial X^2}-Q \tag{2}$$

where $C(X,T)$ is the salinity concentration (kg/m^3), u_s is advective velocity of salinity water (m/s), k is water salinity removal efficiency rate and u_w is the fresh water flow velocity.

B. Initial and boundary condition

a) The initial condition

The initial condition is defined by an interpolation function of measured raw salinity data. It is aligned on the length of the river from the estuary to the end of the considered area. The initial condition is assumed to be

$$C(X,0) = F(X) \tag{3}$$

for all $X \in [0, L]$, where $F(X)$ is an interpolation function of measured salinity data.

b) The left boundary condition

The left boundary condition is an interpolation function of measured raw data. It is based on the salinity of a river at the first station close to the estuary. The boundary condition is assumed to be

$$C(0,T) = G(T) \tag{4}$$

for all $T \in [0, \tau]$, where $G(T)$ is a given interpolation function by measured salinity data at the first monitoring station.

c) The right boundary condition

The right boundary condition is an interpolation function of measured raw data. It is based on the salinity of a river at the end station. The boundary condition is assumed to be

$$C(L,T) = H(T) \tag{5}$$

for all $T \in [0, \tau]$, where $H(T)$ is a given interpolation function by measured salinity data at the end monitoring station.

C. A non-dimensional salinity measurement model

Taking non-dimensional technique [14] into Eq. (2), we get the following discretization:

$$\frac{\partial c}{\partial t} = \frac{\partial^2 c}{\partial x^2} - \left((u_s - ku_w) \cdot \frac{L}{D} \right) \frac{\partial c}{\partial x} - \frac{QL^2}{DC_0} \tag{6}$$

where L represent the length of river area, C_0 some salinity at zero time, $c = \frac{C}{C_0}$, $t = \frac{DT}{L^2}$ and $x = \frac{X}{L}$.

D. Initial and boundary condition of the non-dimensional model

a) The initial condition

The initial condition is defined by an interpolation function of measured raw salinity data. It is aligned on the length of the river from the estuary to the end of the considered area. The initial condition is assumed to be

$$c(x,0) = f(x) \tag{7}$$

for all $x \in [0,1]$, where $f(x)$ is an interpolation function of measured salinity data.

b) The left boundary condition

The left boundary condition is an interpolation function of measured raw data. It is based on the salinity of a river at the first station close to the estuary. The boundary condition is assumed to be

$$c(0,t) = g(t) \tag{8}$$

for all $t \in [0, \Gamma]$, where $g(t)$ is a given interpolation function by measured salinity data at the first monitoring station.

c) The right boundary condition

The right boundary condition is an interpolation function of measured raw data. It is based on the salinity of a river at the end station. The boundary condition is assumed to be

$$c(L,t) = h(t) \tag{9}$$

for all $t \in [0, \Gamma]$, where $h(t)$ is a given interpolation function by measured salinity data at the end monitoring station.

III. Numerical Techniques

We now discretize the domain by dividing the interval [0,1] into M subintervals such that $M\Delta x=1$ and the time interval $[0,\Gamma]$ into N subintervals such that $N\Delta t = \Gamma$. The grid points (x_i, t_n) are defined by $x_i = i\Delta x$ for all $i = 1,2,3,...,M$ and $t_n = n\Delta t$ for all $n = 1,2,3,...,N$, in which M and N are positive integers. We can then approximate $c(x_i, t_n)$ by c_i^n, value of the difference approximation of $c(x,t)$ at point $x = i\Delta x$ and $t = n\Delta t$, where $0 \le i \le M$ and $0 \le n \le N$. We will employ the Forward Time Central Space finite difference scheme (FTCS) and the new fourth order scheme into Eq. (2).

A. Forward Time Central Space explicit finite difference scheme

Taking the Forward Time Central Space technique [13] into Eq. (2), we get the following discretization:

$$c(x_i, t_n) \cong c_i^n, \tag{10}$$

$$\left.\frac{\partial c}{\partial t}\right|(x_i, t_n) \cong \frac{c_i^{n+1} - c_i^n}{\Delta t}, \tag{11}$$

$$\left.\frac{\partial c}{\partial x}\right|(x_i, t_n) \cong \frac{c_{i+1}^n - c_{i-1}^n}{2\Delta x}, \tag{12}$$

$$\left.\frac{\partial^2 c}{\partial x^2}\right|(x_i, t_n) \cong \frac{c_{i+1}^n - 2c_i^n + c_{i-1}^n}{(\Delta x)^2}, \tag{13}$$

$$u_s(x_i, t_n) = u_{si}^n, \tag{14}$$

$$u_w(x_i, t_n) = u_{wi}^n. \tag{15}$$

Substituting Eqs. (10-15) into Eq. (2), we get the finite difference equation:

$$\frac{c_i^{n+1}-c_i^n}{\Delta t}=\frac{c_{i+1}^n-2c_i^n+c_{i-1}^n}{(\Delta x)^2}-\left((u_{si}^n-ku_{wi}^n)\cdot\frac{L}{D}\right)\cdot\frac{c_{i+1}^n-c_{i-1}^n}{2\Delta x}-\frac{QL^2}{DC_0}. \quad (16)$$

Then the explicit finite difference equation becomes

$$c_i^{n+1}=\left(\rho+\frac{1}{2}\gamma_i^n\right)c_{i-1}^n+(1-2\rho)c_i^n+\left(\rho-\frac{1}{2}\gamma_i^n\right)c_{i+1}^n-\frac{QL^2(\Delta t)}{DC_0} \quad (17)$$

for all $i=1,2,3,...,M$, where $\rho=\frac{\Delta t}{(\Delta x)^2}$ and $\gamma_i^n=\frac{(u_{si}^n-ku_{wi}^n)L\Delta t}{D\Delta x}$. The Forward Time Central Space scheme is conditionally stable subject to constraints in Eq. (16). The stability requirements for the scheme are [13], $0<\lambda<\frac{1}{2}$, and $0<\gamma_i^n<1$.

B. A new fourth-order scheme with the Saulyev method for the salinity water measurement model

Taking a new fourth-order technique [11] into Eq. (2), the following discretization can be obtained:

$$c(x_i,t_n)\cong c_i^n, \quad (18)$$

$$\left.\frac{\partial c}{\partial t}\right|(x_i,t_n)\cong\frac{c_i^{n+1}-c_i^n}{\Delta t}, \quad (19)$$

$$\left.\frac{\partial c}{\partial x}\right|(x_i,t_n)\cong F_i^n\frac{c_{i+2}^n-c_i^n}{2\Delta x}+G_i^n\frac{c_i^n-c_{i-2}^n}{2\Delta x}-H_i^n\frac{c_{i+1}^n-c_{i-1}^n}{2\Delta x}, \quad (20)$$

$$\left.\frac{\partial^2 c}{\partial x^2}\right|(x_i,t_n)\cong P_i^n\frac{c_{i+1}^n-2c_i^n+c_{i-1}^n}{(\Delta x)^2}+R_i^n\frac{c_{i+2}^n-2c_i^n+c_{i-2}^n}{(\Delta x)^2}, \quad (21)$$

$$u_s(x_i,t_n)=u_{si}^n, \quad (22)$$

$$u_w(x_i,t_n)=u_{wi}^n \quad (23)$$

where

$$\rho=\frac{D\Delta t}{(\Delta x)^2},$$

$$\gamma_i^n=\frac{(u_{si}^n-ku_{wi}^n)L\Delta t}{D\Delta x},$$

$$F_i^n=\frac{(12\rho+2(\gamma_i^n)^2-3\gamma_i^n-2)}{12},$$

$$G_i^n=\frac{(12\rho+2(\gamma_i^n)^2+3\gamma_i^n-2)}{12},$$

$$H_i^n=\frac{((\gamma_i^n)^2+6\rho-4)}{3},$$

$$P_i^n=\frac{(-(\gamma_i^n)^4+4(\gamma_i^n)^2-12\rho^2-12\rho(\gamma_i^n)^2+8\rho)}{6\rho},$$

$$R_i^n=\frac{((\gamma_i^n)^4-4(\gamma_i^n)^2+12\rho^2+12\rho(\gamma_i^n)^2-2\rho)}{6\rho}.$$

Substituting Eqs. (18-23) into Eq. (2), we obtain:

$$\frac{c_i^{n+1}-c_i^n}{\Delta t}=\left(P_i^n\frac{c_{i+1}^n-2c_i^n+c_{i-1}^n}{(\Delta x)^2}+R_i^n\frac{c_{i+2}^n-2c_i^n+c_{i-2}^n}{(\Delta x)^2}\right)-\left((u_{si}^n-ku_{wi}^n)\frac{L}{D}\right)\left(F_i^n\frac{c_{i+2}^n-c_i^n}{2\Delta x}+G_i^n\frac{c_i^n-c_{i-2}^n}{2\Delta x}-H_i^n\frac{c_{i+1}^n-c_{i-1}^n}{2\Delta x}\right)-\frac{QL^2}{DC_0}. \quad (24)$$

Then the explicit finite difference equation becomes

$$c_i^{n+1}=\left(\frac{1}{2}\gamma_i^nG_i^n+\rho R_i^n\right)c_{i-2}^n+\left(-\frac{1}{2}\gamma_i^nH_i^n+\rho P_i^n\right)c_{i-1}^n+\left(1+\frac{1}{2}\gamma_i^nF_i^n-\frac{1}{2}\gamma_i^nG_i^n-2\rho P_i^n-2\rho R_i^n\right)c_i^n+\left(\frac{1}{2}\gamma_i^nH_i^n+\rho P_i^n\right)c_{i+1}^n+\left(-\frac{1}{2}\gamma_i^nF_i^n+\rho R_i^n\right)c_{i+2}^n-\frac{QL^2\Delta t}{DC_0} \quad (25)$$

for $2\le i\le M-2$ and $0\le n\le N-1$. For $i=1,M-1$ and M, the new fourth-order finite difference Eq. (25) cannot be employed to calculate the value c_i^n on the grid point next to left and right boundaries of the domain of the solution. An alternate approximate appropriate finite difference method, such as the Saulyev method, is employed to approximate their values as discussed in the following section.

a) The employment of a Saulyev method to the left and the right boundary conditions

The Saulyev scheme is unconditionally stable [13,15]. Applying the Saulyev technique [15] to Eq. (2), we obtain the following discretization:

$$c(x_i,t_n)\cong c_i^n, \quad (26)$$

$$\left.\frac{\partial c}{\partial t}\right|(x_i,t_n)\cong\frac{c_i^{n+1}-c_i^n}{\Delta t}, \quad (27)$$

$$\left.\frac{\partial c}{\partial x}\right|(x_i,t_n)\cong\frac{c_{i+1}^n-c_{i-1}^{n+1}}{2\Delta x}, \quad (28)$$

$$\left.\frac{\partial^2 c}{\partial^2 x}\right|(x_i,t_n)\cong\frac{c_{i+1}^n-c_i^n-c_i^{n+1}+c_{i-1}^{n+1}}{(\Delta x)^2}, \quad (29)$$

$$u_s(x_i,t_n)=u_{si}^n, \quad (30)$$

$$u_w(x_i,t_n)=u_{wi}^n. \quad (31)$$

Substituting Eqs. (26-31) into Eq. (2), we get the finite difference equation:

$$\frac{c_i^{n+1}-c_i^n}{\Delta t}=\frac{c_{i+1}^n-c_i^n-c_i^{n+1}+c_{i-1}^{n+1}}{(\Delta x)^2}-\left(\left(u_{si}^n-ku_{wi}^n\right)\cdot\frac{L}{D}\right)\cdot\frac{c_{i+1}^n-c_{i-1}^{n+1}}{2\Delta x}-\frac{QL^2}{DC_0}. \quad (32)$$

Then the explicit finite difference equation becomes

$$c_i^{n+1}=\left(\frac{1}{1+\rho}\right)\left[\left(\rho+\frac{1}{2}\gamma_i^n\right)c_{i-1}^{n+1}+(1-\rho)c_i^n+\left(\rho-\frac{1}{2}\gamma_i^n\right)c_{i+1}^n-\frac{QL^2\Delta t}{DC_0}\right] \quad (33)$$

for all $i=1,2,3,...,M$, where $\rho=\dfrac{\Delta t}{(\Delta x)^2}$ and $\gamma_i^n=\dfrac{(u_{si}^n-ku_{wi}^n)L\Delta t}{D\Delta x}$.

For $i=1$, we put the known value of the left boundary $c_0^{n+1}=r_0^{n+1}$ into Eq. (33) on the right hand side, and we obtain

$$c_1^{n+1}=\left(\frac{1}{1+\rho}\right)\left[\left(\rho+\frac{1}{2}\gamma_i^n\right)r_0^{n+1}+(1-\rho)c_1^n+\left(\rho-\frac{1}{2}\gamma_i^n\right)c_2^n-\frac{QL^2\Delta t}{DC_0}\right]. \quad (34)$$

For $i=M-1$, we obtain an explicit form of Eq. (33). We have

$$c_{M-1}^{n+1}=\left(\frac{1}{1+\rho}\right)\left[\left(\rho+\frac{1}{2}\gamma_i^n\right)c_{M-2}^{n+1}+(1-\rho)c_{M-1}^n+\left(\rho-\frac{1}{2}\gamma_i^n\right)c_M^n-\frac{QL^2\Delta t}{DC_0}\right]. \quad (35)$$

From Eq. (34) and Eq. (35), we see that the technique does not generate fictitious points along either side of the solution domain. It follows that the new fourth-order finite difference Eq. (25), with the employed Saulyev finite difference Eq. (34) and (35), can be used to calculate the values c_i^n on grid points of the solution domain.

IV. Salinity Measurement in Chaopraya River

We consider the salinity measurement in the urban segment of the Chaopraya, Thailand river $0-90$ km from the first monitoring station S_1 to the eighth monitoring station S_8 . There are 8 monitoring stations lie along the considered river segment as show in Table 1 and Fig. 1, respectively.

Salinity actual measurement data on February 28, 2017 [17] are show in Table 2. Setting that the physical parameters are the surface water flow velocity $u_w=0.30-0.40$ m/sec, the salinity discharging rate due to the water bed $Q=0.55\times10^{-6}-0.65\times10^{-6}$ m^3/sec, the diffusion coefficient of salinity water $D=0.25$ m^2/sec, the bottom salinity water velocity $u_s=0.00015|\sin t_n|$ m/sec for all $t_n=n\Delta t$ and the dilution rate of the fresh water to the salinity $k=0.10\times10^{-2}-0.25\times10^{-2}$ m/sec.

Fig. 1. Salinity monitoring Stations for the salinity measurement in the south Chaopraya, Thailand river.

We will employ the FTCS Eq. (17) and the new fourth order method with the Saulyev technique finite difference techniques to approximate the solutions of the model equation Eq. (6). We get the approximated salinity concertations in Table 3-4 in 6 different cases of physical parameters settings, respectively. The approximated salinity solutions are compared with the actual measurement data as show by the absolute error in Table 5-6 at all 6 simulations.

V. Discussion

In the salinity measurement on a segment of south chaopraya river simulation, the increasing of the fresh water flow velocity does affect the reducing of salinity level and the dilution rate at all monitoring stations as show in Table 3-4. The salinity releasing along the riverbank which gives the less effects to salinity levels as also show in Table 3-4. In Table 5-6, we can see that both of numerical techniques and the proposed mathematical model give approximated salinity measurement with their monitored data at all stations. We can see that approximated solutions are closed to the actual data. In the Fig. 2-3, we get the accurately averaged salinity measurement using both of numerical models.

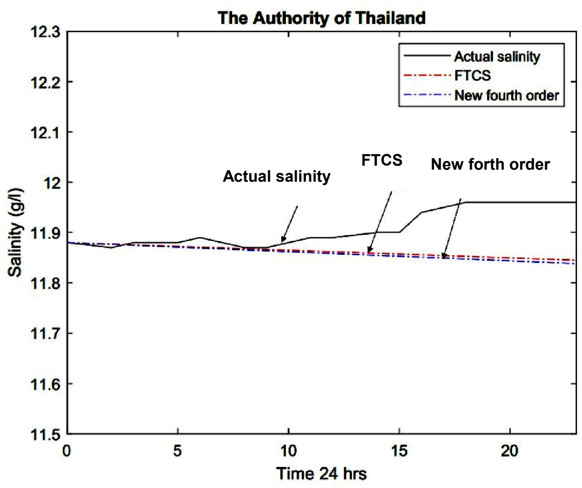

Fig. 2 Comparison of approximated salinity concentrations using the FTCS, the new fourth order method with the Saulyev technique and the actual measurement data at the station S_2 (Port Authority of Thailand Station) along 24 hours when $u_w = 0.30$, $Q = 0.65 \times 10^{-6}$ and $k = 0.02 \times 10^{-2}$.

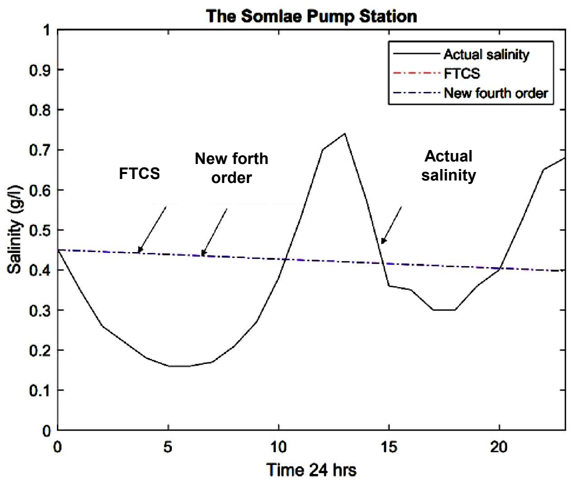

Fig. 3 Comparison of approximated salinity concentrations using the FTCS, the new fourth order method with the Saulyev technique and the actual measurement data at the station S_7 (Somlae Pump Station) along 24 hours when $u_w = 0.30$, $Q = 0.65 \times 10^{-6}$ and $k = 0.02 \times 10^{-2}$.

VI. CONCLUSION

A non-dimensional form of a model for measuring salinity in the river Chaophraya is proposed. It also presents the initial condition and the setting of the boundary conditions. To estimate the salinity rate, the explicit finite difference techniques such as the forward time-centered space method and the new fourth order method using the Saulyev technique are employed. The challenge of calculating salinity is based in a portion of the river Chaophraya. The measured salinity are compared at 8 monitoring stations with actual salinity data. We can achieve that the measured salinity in the river Chaophraya becomes closed for 24 hours to their controlled data. This means that the techniques suggested are basically applicable to calculate salinity in the Chaopraya River. The proposed research can be based on many practical measurements of the salinity. The solution suggested would model the amount of salinity in the future. Salinity estimation produces correct results when opposed to real salinity and salinity in computation.

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REFERENCES

[1] Klaychang, W. and Pochai, N., "Numerical Treatment to a Water-Quality Measurement Model in an Opened-Closed Reservoir," Thai Journal of Mathematics, vol. 13, no. 3, pp. 775-788, 2015.
[2] Pawarisa Samalerk and Pochai, N., "Numerical Simulation of a One-Dimensional Water-Quality Model in a Stream Using a Saulyev Technique with Quadratic Interpolated Initial-Boundary Conditions ," Abstract and Applied Analysis, pp. 7, 2018.
[3] Pochai, N., and Depana, R., "A Numerical Computation of Water Quality Measurement in a Uniform Channel Using a Finite Difference Method", Procedia Engineering, Vol. 8, pp. 85-88, 2011.
[4] Pochai, N., Deepana, R., "An optimal control of water pollution in a stream using a finite difference method," World Academy of Science, Engineering and Technology, vol. 80, pp. 1186-1188, 2011.
[5] Pochai, N., Tangmanee, S., Crane, L.J., Miller, J.J.H., "A mathematical model of water pollution control using the finite element method," in Proceedings in Applied Mathematics and Mechanics, 27–31 March 2006, Berlin, 2006.
[6] Tabuenca, P., Vila, J., Cardona, J., Samartin, A., "Finite element simulation of dispersion in the Bay of Santander," Adv. Eng. Softw., vol. 28, pp. 313–332, 1997.
[7] Kraychang, W., Pochai, N., "Numerical treatment to a water-quality measurement model in an opened-closed reservoir," Journal of Interdisciplinary Mathematics, vol. 13, no. 3, pp. 775-788, 2015.
[8] Pochai, N., Tangmanee, S., Crane, L.J., Miller, J.J.H., "A water quality computation in the uniform channel," J. Interdiscip. Math., vol. 11, pp. 803–814, 2008.
[9] Pochai, N., "A numerical computation of non-dimensional form of stream water quality model with hydrodynamic advection-dispersion-reaction equations," Nonlinear Anal. Hybrid Syst, vol. 13, pp. 666–673, 2009.
[10] Pochai, N., "Unconditional stable numerical techniques for a water-quality model in a non-uniform flow stream," Adv. Differ. Equ. 2017, vol. 286, 2017.
[11] Thongtha, K., Kasemsuwan, J., "Analytical solution to a hydrodynamic model in an open uniform reservoir," Adv. Differ. Equ. 2017, vol. 149, 2017.
[12] Othata, P., Pochai, N., "A one-dimensional mathematical simulation to salinity control in a river with a barrage dam using an unconditionally stable explicit finite difference method," Advances in Difference Equations, vol. 203, 2019.
[13] Smith, G.D., "Numerical Solution of Partial Equations: Finite Difference Method, Oxford," New York, 1985.
[14] Dehghan, M., "Numerircal schemes for one-dimensional parabolic equations with nonstandard initial condition," Appl. Math. Comput, vol. 147, pp. 321-331, 2004.
[15] Bradie, B., "A Friendly Introduction to Numerical Analysis," Prentice Hall, New York, 2005.
[16] MWA (Metropolitan Waterworks Authority). (2019, June 19). Real time water Quality [Online]. Available: http://www.mwa.co.th.
[17] A. Wibowo and S.H. Arbain and N. Z. Abidin, "Combined multiple neural networks and genetic algorithm with missing data treatment: Case study of water level forecasting in Dungun river - Malaysia," IAENG International Journal of Computer Science, vol. 45, no. 2, pp.246–254, 2018.
[18] L. Dongping and Y. Yingchun and W. Jin, "Water quality assessment based on probabilistic echo state networks," IAENG International Journal of Computer Science, vol. 44, no. 3, pp. 270–276, 2017.
[19] R. Bustami and . Bessaih and C. Bong and S. Suhaili, "Artificial neural network for precipitation and water level predictions of bedup river," IAENG International Journal of Computer Science, vol. 34, no. 2, pp.228–233, 2007.

APPENDIX

TABLE I
MONITORING STATION DISTANCE POSITION ALONG THE SEGMENT OF THE URBAN SOUTH CHAOPRAYA, THAILAND RIVER

Salinity Monitoring Station	**Distance** (km)
S_1 : South Bangkok Power Plant Station	0
S_2 : Port Authority of Thailand Station	15
S_3 : Khlong Lat Pho Station	23
S_4 : Phra Phuttha Yodfa Bridge Station	38
S_5 : Phra Nangklao Bridge Station	52
S_6 : Wat Makham Station	79
S_7 : Somlae Pump Station	84
S_8 : Wat Phai Lom Station	90

TABLE II
SALINITY ACTUAL MEASUREMENT AT 8 MONITORING STATIONS, ON FEBRUARY 28, 2017

	Time	Salinity (g/ l)							
		S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
1	00:00	21.52	11.88	14.46	4.41	0.05	1.12	0.45	0.16
2	01:00	20.77	N/A	13.83	4.43	0.05	1.01	0.35	0.16
3	02:00	19.83	11.87	12.86	4.43	0.05	0.78	0.26	0.16
4	03:00	N/A	11.88	11.83	4.43	0.05	0.53	0.22	0.15
5	04:00	18.64	11.88	11.61	4.43	0.05	0.40	0.18	0.15
6	05:00	19.47	11.88	12.40	4.43	N/A	0.31	0.16	0.15
7	06:00	20.64	11.89	13.20	4.44	0.05	0.26	0.16	0.15
8	07:00	21.69	11.88	14.45	4.45	0.05	0.27	0.17	0.14
9	08:00	22.27	11.87	15.13	4.51	0.05	0.38	0.21	0.15
10	09:00	23.22	11.87	16.11	4.52	0.05	0.56	0.27	0.15
11	10:00	23.51	11.88	16.99	4.52	0.05	0.86	0.38	0.16
12	11:00	23.77	11.89	16.92	4.51	0.05	1.04	0.53	0.16
13	12:00	23.80	11.89	16.46	4.51	0.05	1.17	0.70	0.19
14	13:00	23.35	N/A	15.72	4.51	0.05	1.22	0.74	0.16
15	14:00	21.50	11.90	15.46	4.52	0.05	1.17	0.57	0.18
16	15:00	20.24	11.90	14.91	4.52	0.05	1.07	0.36	0.16
17	16:00	20.24	11.94	14.41	4.53	N/A	0.91	0.35	0.17
18	17:00	20.31	N/A	14.73	4.54	3.53	0.77	0.30	0.16
19	18:00	21.21	11.96	15.36	4.54	3.51	0.71	0.30	0.16
20	19:00	21.47	11.96	15.88	4.54	4.01	0.78	0.36	0.16
21	20:00	21.50	11.96	15.71	4.54	4.48	0.87	0.40	0.16
22	21:00	21.51	N/A	16.00	4.54	N/A	1.02	0.52	0.18
23	22:00	22.78	11.96	15.84	4.54	N/A	1.14	0.65	0.19
24	23:00	N/A	11.96	14.93	4.54	N/A	1.19	0.68	0.18
Average		21.51	11.91	14.80	4.50	0.86	0.81	0.39	0.16

TABLE III
ONE DAY-AVERAGED SALINITY MEASUREMENT AT 8 MONITORING STATIONS USING THE FTCS METHOD

u_w	Q	k	Approximation salinity (g/ l)							
(m/ s)	(m^3/ s)	(m/ s)	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
0.30	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8651	14.4249	4.3848	0.0291	1.0949	0.4270	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8642	14.4237	4.3818	0.0283	1.0946	0.4264	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8633	14.4226	4.3788	0.0275	1.0943	0.4259	0.1621
	0.55×10^{-6}	0.25×10^{-2}	21.4427	11.8624	14.4214	4.3758	0.0267	1.0939	0.4253	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8631	14.4228	4.3827	0.0271	1.0928	0.4249	0.1621
	0.60×10^{-6}	0.15×10^{-2}	21.4427	11.8621	14.4217	4.3797	0.0263	1.0925	0.4243	0.1621
	0.65×10^{-6}	0.02×10^{-2}	21.4427	11.8626	14.4225	4.3855	0.0263	1.0912	0.4237	0.1621
0.32	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8650	14.4247	4.3844	0.0290	1.0948	0.4269	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8640	14.4235	4.3812	0.0282	1.0945	0.4263	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8631	14.4223	4.3780	0.0273	1.0942	0.4257	0.1621
	0.55×10^{-6}	0.25×10^{-2}	21.4427	11.8621	14.4210	4.3748	0.0264	1.0938	0.4251	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8630	14.4227	4.3823	0.0270	1.0928	0.4248	0.1621
	0.60×10^{-6}	0.13×10^{-2}	21.4427	11.8624	14.4219	4.3804	0.0264	1.0926	0.4245	0.1621
	0.65×10^{-6}	0.02×10^{-2}	21.4427	11.8625	14.4225	4.3855	0.0263	1.0912	0.4237	0.1621
0.34	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8649	14.4246	4.3840	0.0289	1.0948	0.4268	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8638	14.4233	4.3806	0.0280	1.0944	0.4262	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8628	14.4220	4.3772	0.0271	1.0941	0.4256	0.1621
	0.55×10^{-6}	0.25×10^{-2}	21.4427	11.8618	14.4206	4.3738	0.0262	1.0937	0.4249	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8628	14.4225	4.3819	0.0269	1.0927	0.4248	0.1621
	0.60×10^{-6}	0.13×10^{-2}	21.4427	11.8622	14.4217	4.3799	0.0263	1.0925	0.4244	0.1621
	0.65×10^{-6}	0.02×10^{-2}	21.4427	11.8625	14.4225	4.3854	0.0263	1.0912	0.4237	0.1621
0.36	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8648	14.4244	4.3836	0.0288	1.0947	0.4268	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8637	14.4231	4.3800	0.0278	1.0944	0.4261	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8626	14.4216	4.3764	0.0269	1.0940	0.4254	0.1621
	0.55×10^{-6}	0.23×10^{-2}	21.4427	11.8620	14.4208	4.3742	0.0263	1.0938	0.4250	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8627	14.4224	4.3815	0.0267	1.0927	0.4247	0.1621
	0.60×10^{-6}	0.12×10^{-2}	21.4427	11.8623	14.4218	4.3801	0.0264	1.0925	0.4244	0.1621
	0.65×10^{-6}	0.01×10^{-2}	21.4427	11.8627	14.4227	4.3860	0.0265	1.0912	0.4238	0.1621
0.38	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8646	14.4243	4.3832	0.0287	1.0947	0.4267	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8635	14.4228	4.3794	0.0277	1.0943	0.4260	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8624	14.4213	4.3756	0.0266	1.0939	0.4253	0.1621
	0.55×10^{-6}	0.22×10^{-2}	21.4427	11.8619	14.4207	4.3740	0.0262	1.0938	0.4250	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8626	14.4222	4.3811	0.0266	1.0926	0.4246	0.1621
	0.60×10^{-6}	0.11×10^{-2}	21.4427	11.8623	14.4219	4.3804	0.0264	1.0926	0.4245	0.1621
	0.65×10^{-6}	0.01×10^{-2}	21.4427	11.8627	14.4227	4.3860	0.0265	1.0912	0.4238	0.1621
0.40	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8645	14.4241	4.3828	0.0286	1.0947	0.4266	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8633	14.4226	4.3788	0.0275	1.0943	0.4259	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8621	14.4210	4.3748	0.0264	1.0938	0.4251	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8625	14.4221	4.3807	0.0265	1.0926	0.4245	0.1621
	0.65×10^{-6}	0.01×10^{-2}	21.4427	11.8627	14.4227	4.3859	0.0265	1.0912	0.4238	0.1621

TABLE IV
ONE DAY-AVERAGED SALINITY MEASUREMENT AT 8 MONITORING STATIONS USING THE NEW FOURTH-ORDER METHOD

u_w	Q	k	Approximation salinity (g/ l)							
(m/ s)	(m^3/ s)	(m/ s)	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
0.30	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8618	14.4283	4.3836	0.0279	1.0955	0.4267	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8609	14.4271	4.3806	0.0271	1.0952	0.4261	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8600	14.4260	4.3776	0.0263	1.0948	0.4256	0.1621
	0.55×10^{-6}	0.25×10^{-2}	21.4427	11.8592	14.4247	4.3746	0.0255	1.0945	0.4250	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8598	14.4262	4.3815	0.0259	1.0934	0.4246	0.1621
	0.60×10^{-6}	0.15×10^{-2}	21.4427	11.8588	14.4251	4.3785	0.0251	1.0931	0.4241	0.1621
	0.65×10^{-6}	0.02×10^{-2}	21.4427	11.8592	14.4259	4.3843	0.0251	1.0918	0.4234	0.1621
0.32	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8617	14.4281	4.3832	0.0278	1.0954	0.4266	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8607	14.4269	4.3800	0.0270	1.0951	0.4260	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8598	14.4256	4.3768	0.0261	1.0948	0.4254	0.1621
	0.55×10^{-6}	0.25×10^{-2}	21.4427	11.8589	14.4243	4.3736	0.0253	1.0944	0.4248	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8596	14.4261	4.3811	0.0258	1.0933	0.4245	0.1621
	0.60×10^{-6}	0.13×10^{-2}	21.4427	11.8591	14.4253	4.3792	0.0252	1.0932	0.4242	0.1621
	0.65×10^{-6}	0.02×10^{-2}	21.4427	11.8592	14.4259	4.3842	0.0251	1.0918	0.4234	0.1621
0.34	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8616	14.4280	4.3828	0.0277	1.0954	0.4265	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8606	14.4267	4.3794	0.0268	1.0950	0.4259	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8596	14.4253	4.3760	0.0259	1.0947	0.4253	0.1621
	0.55×10^{-6}	0.25×10^{-2}	21.4427	11.8586	14.4239	4.3726	0.0250	1.0943	0.4246	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8595	14.4259	4.3807	0.0257	1.0933	0.4245	0.1621
	0.60×10^{-6}	0.13×10^{-2}	21.4427	11.8589	14.4251	4.3787	0.0251	1.0931	0.4241	0.1621
	0.65×10^{-6}	0.02×10^{-2}	21.4427	11.8592	14.4259	4.3841	0.0251	1.0918	0.4234	0.1621
0.36	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8615	14.4278	4.3824	0.0276	1.0953	0.4265	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8604	14.4264	4.3788	0.0266	1.0950	0.4258	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8593	14.4250	4.3752	0.0257	1.0946	0.4251	0.1621
	0.55×10^{-6}	0.23×10^{-2}	21.4427	11.8587	14.4241	4.3730	0.0251	1.0944	0.4247	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8594	14.4258	4.3803	0.0255	1.0933	0.4244	0.1621
	0.60×10^{-6}	0.12×10^{-2}	21.4427	11.8590	14.4252	4.3789	0.0252	1.0931	0.4241	0.1621
	0.65×10^{-6}	0.01×10^{-2}	21.4427	11.8594	14.4261	4.3848	0.0253	1.0918	0.4235	0.1621
0.38	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8613	14.4277	4.3820	0.0275	1.0953	0.4264	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8602	14.4262	4.3782	0.0265	1.0949	0.4257	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8591	14.4247	4.3744	0.0255	1.0945	0.4250	0.1621
	0.55×10^{-6}	0.22×10^{-2}	21.4427	11.8587	14.4240	4.3729	0.0251	1.0943	0.4247	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8593	14.4256	4.3799	0.0254	1.0932	0.4243	0.1621
	0.60×10^{-6}	0.11×10^{-2}	21.4427	11.8590	14.4253	4.3792	0.0252	1.0931	0.4242	0.1621
	0.65×10^{-6}	0.01×10^{-2}	21.4427	11.8594	14.4261	4.3847	0.0253	1.0918	0.4235	0.1621
0.40	0.55×10^{-6}	0.10×10^{-2}	21.4427	11.8612	14.4275	4.3816	0.0274	1.0953	0.4263	0.1621
	0.55×10^{-6}	0.15×10^{-2}	21.4427	11.8600	14.4260	4.3776	0.0263	1.0948	0.4256	0.1621
	0.55×10^{-6}	0.20×10^{-2}	21.4427	11.8589	14.4243	4.3736	0.0253	1.0944	0.4248	0.1621
	0.60×10^{-6}	0.10×10^{-2}	21.4427	11.8592	14.4255	4.3795	0.0253	1.0932	0.4242	0.1621
	0.65×10^{-6}	0.01×10^{-2}	21.4427	11.8594	14.4261	4.3847	0.0252	1.0918	0.4235	0.1621

TABLE V

ABSOLUTE ERROR OF ONE DAY-AVERAGED SALINITY MEASUREMENT BY THE FTCS MATHEMATICAL MODEL AND THE ACTUAL MEASUREMENT DATA

u_w	Q	k	Approximation salinity (g/ l)							
(m/ s)	(m^3/ s)	(m/ s)	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0449	0.3751	0.1152	0.8309	0.2849	0.0370	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0458	0.3763	0.1182	0.8317	0.2846	0.0364	0.0021
	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0467	0.3774	0.1212	0.8325	0.2843	0.0359	0.0021
0.30	0.55×10^{-6}	0.25×10^{-2}	0.0673	0.0476	0.3786	0.1242	0.8333	0.2839	0.0353	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0469	0.3772	0.1173	0.8329	0.2828	0.0349	0.0021
	0.60×10^{-6}	0.15×10^{-2}	0.0673	0.0479	0.3783	0.1203	0.8337	0.2825	0.0343	0.0021
	0.65×10^{-6}	0.02×10^{-2}	0.0673	0.0474	0.3775	0.1145	0.8337	0.2812	0.0337	0.0021
	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0450	0.3753	0.1156	0.8310	0.2848	0.0369	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0460	0.3765	0.1188	0.8318	0.2845	0.0363	0.0021
	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0469	0.3777	0.1220	0.8327	0.2842	0.0357	0.0021
0.32	0.55×10^{-6}	0.25×10^{-2}	0.0673	0.0479	0.3790	0.1252	0.8336	0.2838	0.0351	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0470	0.3773	0.1177	0.8330	0.2828	0.0348	0.0021
	0.60×10^{-6}	0.13×10^{-2}	0.0673	0.0476	0.3781	0.1196	0.8336	0.2826	0.0345	0.0021
	0.65×10^{-6}	0.02×10^{-2}	0.0673	0.0475	0.3775	0.1145	0.8337	0.2812	0.0337	0.0021
	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0451	0.3754	0.1160	0.8311	0.2848	0.0368	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0462	0.3767	0.1194	0.8320	0.2844	0.0362	0.0021
	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0472	0.3780	0.1228	0.8329	0.2841	0.0356	0.0021
0.34	0.55×10^{-6}	0.25×10^{-2}	0.0673	0.0482	0.3794	0.1262	0.8338	0.2837	0.0349	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0472	0.3775	0.1181	0.8331	0.2827	0.0348	0.0021
	0.60×10^{-6}	0.13×10^{-2}	0.0673	0.0478	0.3783	0.1201	0.8337	0.2825	0.0344	0.0021
	0.65×10^{-6}	0.02×10^{-2}	0.0673	0.0475	0.3775	0.1146	0.8337	0.2812	0.0337	0.0021
	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0452	0.3756	0.1164	0.8312	0.2847	0.0368	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0463	0.3769	0.1200	0.8322	0.2844	0.0361	0.0021
	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0474	0.3784	0.1236	0.8331	0.2840	0.0354	0.0021
0.36	0.55×10^{-6}	0.23×10^{-2}	0.0673	0.0480	0.3792	0.1258	0.8337	0.2838	0.0350	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0473	0.3776	0.1185	0.8333	0.2827	0.0347	0.0021
	0.60×10^{-6}	0.12×10^{-2}	0.0673	0.0477	0.3782	0.1199	0.8336	0.2825	0.0344	0.0021
	0.65×10^{-6}	0.01×10^{-2}	0.0673	0.0473	0.3773	0.1140	0.8335	0.2812	0.0338	0.0021
	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0454	0.3757	0.1168	0.8313	0.2847	0.0367	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0465	0.3772	0.1206	0.8323	0.2843	0.0360	0.0021
	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0476	0.3787	0.1244	0.8334	0.2839	0.0353	0.0021
0.38	0.55×10^{-6}	0.22×10^{-2}	0.0673	0.0481	0.3793	0.1260	0.8338	0.2838	0.0350	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0474	0.3778	0.1189	0.8334	0.2826	0.0346	0.0021
	0.60×10^{-6}	0.11×10^{-2}	0.0673	0.0477	0.3781	0.1196	0.8336	0.2826	0.0345	0.0021
	0.65×10^{-6}	0.01×10^{-2}	0.0673	0.0473	0.3773	0.1140	0.8335	0.2812	0.0338	0.0021
	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0455	0.3759	0.1172	0.8314	0.2847	0.0366	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0467	0.3774	0.1212	0.8325	0.2843	0.0359	0.0021
0.40	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0479	0.3790	0.1252	0.8336	0.2838	0.0351	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0475	0.3779	0.1193	0.8335	0.2826	0.0345	0.0021
	0.65×10^{-6}	0.01×10^{-2}	0.0673	0.0473	0.3773	0.1141	0.8335	0.2812	0.0338	0.0021

TABLE VI

ABSOLUTE ERROR OF ONE DAY-AVERAGED SALINITY MEASUREMENT BY THE NEW FOURTH-ORDER METHOD MATHEMATICAL MODEL AND THE ACTUAL MEASUREMENT DATA

u_w	Q	k	Approximation salinity (g/ l)							
(m/ s)	(m^3/ s)	(m/ s)	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0482	0.3717	0.1164	0.8321	0.2855	0.0367	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0491	0.3729	0.1194	0.8329	0.2852	0.0361	0.0021
	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0500	0.3740	0.1224	0.8337	0.2848	0.0356	0.0021
0.30	0.55×10^{-6}	0.25×10^{-2}	0.0673	0.0508	0.3753	0.1254	0.8345	0.2845	0.0350	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0502	0.3738	0.1185	0.8341	0.2834	0.0346	0.0021
	0.60×10^{-6}	0.15×10^{-2}	0.0673	0.0512	0.3749	0.1215	0.8349	0.2831	0.0341	0.0021
	0.65×10^{-6}	0.02×10^{-2}	0.0673	0.0508	0.3741	0.1157	0.8349	0.2818	0.0334	0.0021
	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0483	0.3719	0.1168	0.8322	0.2854	0.0366	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0493	0.3731	0.1200	0.8330	0.2851	0.0360	0.0021
	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0511	0.3757	0.1264	0.8347	0.2844	0.0348	0.0021
0.32	0.55×10^{-6}	0.25×10^{-2}	0.0673	0.0513	0.3759	0.1271	0.8349	0.2843	0.0347	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0504	0.3739	0.1189	0.8342	0.2833	0.0345	0.0021
	0.60×10^{-6}	0.13×10^{-2}	0.0673	0.0509	0.3747	0.1208	0.8348	0.2832	0.0342	0.0021
	0.65×10^{-6}	0.02×10^{-2}	0.0673	0.0508	0.3741	0.1158	0.8349	0.2818	0.0334	0.0021
0.34	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0484	0.3720	0.1172	0.8323	0.2854	0.0365	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0494	0.3733	0.1206	0.8332	0.2850	0.0359	0.0021
	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0504	0.3747	0.1240	0.8341	0.2847	0.0353	0.0021
	0.55×10^{-6}	0.25×10^{-2}	0.0673	0.0514	0.3761	0.1274	0.8350	0.2843	0.0346	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0505	0.3741	0.1193	0.8343	0.2833	0.0345	0.0021
	0.60×10^{-6}	0.13×10^{-2}	0.0673	0.0511	0.3749	0.1213	0.8349	0.2831	0.0341	0.0021
	0.65×10^{-6}	0.02×10^{-2}	0.0673	0.0508	0.3741	0.1159	0.8349	0.2818	0.0334	0.0021
	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0485	0.3722	0.1176	0.8324	0.2853	0.0365	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0496	0.3736	0.1212	0.8334	0.2850	0.0358	0.0021
	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0507	0.3750	0.1248	0.8343	0.2846	0.0351	0.0021
0.36	0.55×10^{-6}	0.23×10^{-2}	0.0673	0.0513	0.3759	0.1270	0.8349	0.2844	0.0347	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0506	0.3742	0.1197	0.8345	0.2833	0.0344	0.0021
	0.60×10^{-6}	0.12×10^{-2}	0.0673	0.0510	0.3748	0.1211	0.8348	0.2831	0.0341	0.0021
	0.65×10^{-6}	0.01×10^{-2}	0.0673	0.0506	0.3739	0.1152	0.8347	0.2818	0.0335	0.0021
	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0487	0.3723	0.1180	0.8325	0.2853	0.0364	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0498	0.3738	0.1218	0.8335	0.2849	0.0357	0.0021
	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0509	0.3753	0.1256	0.8345	0.2845	0.0350	0.0021
0.38	0.55×10^{-6}	0.22×10^{-2}	0.0673	0.0513	0.3760	0.1271	0.8349	0.2843	0.0347	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0507	0.3744	0.1201	0.8346	0.2832	0.0343	0.0021
	0.60×10^{-6}	0.11×10^{-2}	0.0673	0.0510	0.3747	0.1208	0.8348	0.2831	0.0342	0.0021
	0.65×10^{-6}	0.01×10^{-2}	0.0673	0.0506	0.3739	0.1153	0.8347	0.2818	0.0335	0.0021
	0.55×10^{-6}	0.10×10^{-2}	0.0673	0.0488	0.3725	0.1184	0.8326	0.2853	0.0363	0.0021
	0.55×10^{-6}	0.15×10^{-2}	0.0673	0.0500	0.3740	0.1224	0.8337	0.2848	0.0356	0.0021
0.40	0.55×10^{-6}	0.20×10^{-2}	0.0673	0.0511	0.3757	0.1264	0.8347	0.2844	0.0348	0.0021
	0.60×10^{-6}	0.10×10^{-2}	0.0673	0.0508	0.3745	0.1205	0.8347	0.2832	0.0342	0.0021
	0.65×10^{-6}	0.01×10^{-2}	0.0673	0.0506	0.3739	0.1153	0.8348	0.2818	0.0335	0.0021