Optimal Reinsurance and Investment Strategy Under CEV Model with Fractional Power Utility Function

Maulana Malik*, *Member, IAENG,* Siti Sabariah Abas, Mustafa Mamat, Sukono, *Member, IAENG*, and Agung Prabowo

***Abstract*—This paper studies the optimal reinsurance and investment problem for insurance companies (insurers) with a fractional power utility function. Assuming that the insurer surplus process is approximated by Brownian motion with drift, the insurer may purchase reinsurance and invest the capital in a financial market consisting of risk-free asset and risk asset whose price is modeled by constant elasticity variance (CEV) model. The insurer's objective is to maximize the expected fractional power utility from terminal wealth. The explicit expressions for optimal reinsurance-investment strategy and value function are determined by the stochastic approach, which uses the equations of Hamilton-Jacobi-Bellman. Finally, the numerical simulations are presented to show the effects of model parameters on the insurer's optimal reinsurance and investment strategies.**

***Index Terms*—Constant elasticity variance, fractional power utility, Hamilton-Jacobi-Bellman equation, reinsurance, insurer, surplus process, stochastic approach.**

I. Introduction

THE reinsurance and investment problem in the insurance business have recently become more relevant, attracting great interest. For example, Li [1] studied the optimal reinsurance and investment problem of the maximum expected two exponential utility function whose claim process modeled as Brownian motion with drift. Mwanakatwe [2] used an investment model that followed the Hull and White SV model and obtained a reinsurance and investment strategy to maximize expected utility function. Li and Gu [3] discussed the problem of maximizing expected exponential utility function to both proportional reinsurance and investment. Lin and Li [4], and Wang et al. [5] studied an insurer investment strategy of exponential utility maximation with the jump-diffusion process. Gu [6], Li and Gu [7] focused the optimal Excess-of-Loss reinsurance and investment with maximizing exponential utility. Sheng [8] and Lhedioha [9] considered the optimal reinsurance and investment problem of maximizing the expected power utility function. Deng [10] studied constructs a reinsurance-investment optimization problem with bounded memory. Li et al. in [11] and [12] discussed the problem of ruin probability minimization, the ruin probability for the insurer, and maximizing the exponential utility function. Gao [13] discussed the optimal investment strategy for annuity contracts with CRRA and CARA utility functions. Chunxiang et al. [14] studied optimal excess of loss reinsurance and investment problem with delay and jump-diffusion risk. Li et al. [15] discussed time consistent reinsuranceinvestment strategy for an insurer and a reinsurer with mean-variance criterion. Xiao J et al. [16] used the Legendre transformdual solution for annuity contracts, and Wang et al. [17] focused on the optimal investment problem with consumption.

The CEV model with stochastic volatility is more practical than the Black-Scholes (BS) model. The CEV model proposed by Cox and Ross [18] is a natural extension of the BS model. We focus on an optimal problem for the insurer. In this paper, we construct again the model basic claim process which assumed based on Brownian motion with drift, and the insurer can purchase a proportional reinsurance contract from the reinsurer. The insurer is permitted to invest a risk asset and a risk-free asset in the financial market whose price process follows the CEV model. The objective is to maximize the expected utility fractional power utility function of terminal wealth. Using stochastic control theory, we establish the corresponding Hamilton Jacobi Bellman (HJB) equation, optimal reinsurance-investment strategies, and the value function of the optimization. Moreover, the novelty of this paper is different from those of [1]-[8], [11], and [13]-[17], in which the CEV model is used to study the optimal investment strategy with a fractional power utility function. For good references for studies about the optimal investment can be seen in [19] and [20] .

This paper is organized as follows. We introduce the model formulation for the insurer's wealth process model with reinsurance proportion and investment in section II. In section III, we propose an optimal strategy for the insurer and using the HJB equation to find explicit solution proportional reinsurance and investment with maximizing expected fractional power utility function. In section IV, the numerical experimets are presented to illustrate our results. Conclusions are given in section V.

II. Model Formulation

In this section, we derived the insurance risk model with reinsurance and investment. The model surplus process via the Classical Cramer-Lundberg model, and an insurer can

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invest in some risky assets by the CEV model. We finish this section by presenting the wealth process model.

A. The Surplus Process

In this subsection, we formulate a form of the surplus process of the insurer. In insurance, the surplus process is the process of accumulation of wealth. To derive the surplus process, we need the claim process. Following the framework of [12] and [21], we model the claim process $C(t)$ matching to a Brownian motion with drift as

$$dC(t) = adt - bdW(t) \tag{1}$$

where a and b are positive constant, and $W(t)$ is a standard Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$. According to the expected value principle [22], the premium rate of insurer is $c = (1+\theta)a$ and $\theta > 0$ is the safety loading of insurer. In this paper, we assume a classical Cramer-Lundberg model for surplus process [22] as

$$R(t) = x_0 + ct - C(t), \quad t \geq 0$$

where $R(t)$ and x_0 are the insurers capital at time t and initial capital $R(0) = x_0$, respectively. According to (1), the surplus process for the insurer is given as

$$dR(t) = cdt - dC(t) = a\theta dt + bdW(t).$$

Furthermore, the insurer can buy reinsurance contract to reduce risk. Suppose the insurer pays reinsurance premium continuously at rate $c_1 = (1+\eta)a$, where $\eta > \theta > 0$ is the safety loading of the reinsurer. So, the surplus process $R_1(t)$ associated with reinsurance of the insurer follows

$$\begin{aligned} R_1(t) &= cdt - (1-p(t))dC(t) - c_1 p(t)dt \\ &= cdt - (1-p(t))(adt - bdW(t)) - \\ &\quad (1+\eta)ap(t)dt \\ &= (\theta - \eta p(t))adt + b(1-p(t))dW(t) \end{aligned} \tag{2}$$

where $p(t)$ is proportion reinsurance at time t.

B. The Financial Market

In practice, an insurer can invest part of its capital in a financial market. We assume that the financial market consists of a risk-free asset and risk asset. Example risk free asset like an obligation and a bank account, risky asset like a stock and option. Suppose insurer invest some part of its capital into risk-free asset at time t with prices $S_0(t)$ satisfying

$$dS_0(t) = rS_0(t)dt \tag{3}$$

where $r > 0$ is interest rate for free risk asset. Assume that the price of risky asset $S(t)$ is described by the CEV ([1], [2], [4]-[8], [11], and [13]-[17]) model as

$$\begin{aligned} dS(t) &= \mu S(t)dt + \sigma S(t)^{\beta+1} dW_1(t) \\ &= S(t)(\mu dt + \sigma S(t)^{\beta} dW_1(t)) \end{aligned} \tag{4}$$

where $\mu > 0$, $\sigma S(t)^{\beta}$,$r > \mu$ and $\beta < 0$ are the expected return rate, the instantaneous volatility and the elasticity parameters of the risky asset, respectively. $W_1(t)$ is a standard Brownian motion independent of $W(t)$ defined on the complete probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$.

C. The Wealth Process

The amount of wealth of insurer invested on risk asset at time t denoted by $\pi(t)$ and $X(t)$ represents the wealth of the insurer. So, the remainder $X(t) - \pi(t)$ invested in risk-free asset. Therefore, from (2), (3), and (4), the wealth processes model of the insurer to follow stochastic differential equation as

$$\begin{aligned} dX(t) &= dR_1(t) + \pi(t)\frac{dS(t)}{dt} + (X(t) - \pi(t))\frac{dS_0(t)}{dt} \\ &= (\theta - \eta p(t))adt + b(1-p(t))dW(t) + \\ &\quad \pi(t)[\mu dt + \sigma S(t)^{\beta} dW_1(t)] + (X(t) - \pi(t))rdt \\ &= [rX(t) + \pi(t)(\mu - r) + (\theta - \eta p(t))a]dt \\ &\quad + b(1-p(t))dW(t) + \pi(t)\sigma S(t)^{\beta} dW_1(t). \end{aligned} \tag{5}$$

Based on the explanation above for the insurer, then reinsurance-investment strategy γ is described by a pair $\gamma = (p(t), \pi(t))$. The pair γ called admissible if it is $(\mathcal{F}_t)$-progressively measurable and satisfies $0 \leq p(t) \leq 1$, $\left[\int_0^T \pi^2(t)dt < \infty\right]$ with $t \in [0, T]$.

Suppose the insurer has a fractional power utility function is defined as [8]

$$U(x) = x^{\alpha} \quad ,0 < \alpha < 1 \tag{6}$$

where x is the wealth level of an insurer and α is constant relative risk aversion.

In this paper, we will find the optimal reinsurance and investment strategy by maximizing expected fractional power utility (constant relative risk aversion) function of terminal wealth. The objective of the problem can be written as:

$$\max_{\gamma} E[U(X(T))] \tag{7}$$

with constraint (5).

III. Optimal Strategy for the Insurer

In this section, we will find the optimal strategy $\gamma^* = (p^*, \pi^*)$ for the optimization problem (7), where p^* is called the optimal reinsurance strategy and π^* is called the optimal investment strategy. For strategy γ, the value function at the time t is defined as

$$M(t, x, s) = \sup_{\gamma} E[U(X(T))|X(t) = x, S(t) = s]$$

with boundary condition $M(T, x, s) = U(x)$.

We propose the optimal reinsurance and investment strategy of the insurer who aims to maximize the expected fractional power utility of terminal wealth in the following theorem.

Theorem 1. *For the optimal problem (7), assume that the objective is to maximize (6) of terminal wealth, at the fixed terminal time T then the optimal reinsurance strategy is given by*

$$p^* = 1 + \frac{\eta a x}{b^2(\alpha - 1)}$$

and the optimal investment strategy is

$$\pi^* = \frac{x(-\alpha r(T-t)\sigma^2 + (r-\mu)s^{-2\beta})}{\sigma^2(\alpha - 1)}.$$

The optimal value function is

$$M(t,x,s) = \exp\Bigg[-\frac{1}{6x\sigma^2 b^2(\alpha-1)}\Bigg(\Bigg[3b^2(\alpha r^2 - 2\mu a(r-\frac{\mu}{2}))xs^{-2\beta} + (\alpha\sigma^2 b^2 r^2 x(T-t)^2 s^{2\beta} + (3b^2r^2 s(T-t)\alpha - 3b^2 s\mu(T-t) + 3a^2\eta^2)x - 6b^2 a((\theta-\eta)\alpha - \theta - \eta))\sigma^2\alpha\Bigg](T-t)\Bigg) - \alpha r(T-t)\Bigg]x^\alpha$$

Proof: To get the optimal strategy (p^*, π^*), we use the Hamilton Jacobi Bellman equation. The Hamilton Jacobi Bellman equation associated with the problem is

$$M_t + \sup_\gamma([rx + \pi(\mu-r) + (\theta-\eta p)a]M_x + \mu s M_s + \frac{1}{2}(\pi^2\sigma^2 s^{2\beta} + b^2(1-p)^2)M_{xx} + \frac{1}{2}\sigma^2 s^{2\beta+2}M_{ss} + \pi\sigma^2 s^{2\beta+1}M_{xs}) = 0. \quad (8)$$

Differentiating (8) with respect to π, we have

$$\begin{aligned}&(\mu-r)M_x + \pi\sigma^2 s^{2\beta}M_{xx} + \sigma^2 s^{2\beta+1}M_{xs} = 0\\ \iff\ &\pi\sigma^2 s^{2\beta}M_{xx} = -((\mu-r)M_x + \sigma^2 s^{2\beta+1}M_{xs})\\ \iff\ &\pi^* = -\frac{(\mu-r)M_x + \sigma^2 s^{2\beta+1}M_{xs}}{\sigma^2 s^{2\beta}M_{xx}}.\end{aligned}$$

Putting π^* to (8), we obtain

$$M_t + \Bigg(rx - \frac{\left((\mu-r)M_x + \sigma^2 s^{2\beta+1}M_{xs}\right)(\mu-r)}{\sigma^2 s^{2\beta}M_{xx}} + (\theta-\eta p)a\Bigg)M_x + \mu s M_s + \frac{1}{2}\Bigg(\frac{((\mu-r)M_x + \sigma^2 s^{2\beta+1}M_{xs})^2}{\sigma^2 s^{2\beta}M_{xx}} + b^2(1-p)^2\Bigg)M_{xx} + \frac{1}{2}(\sigma^2 s^{2\beta+2})M_{ss} - \frac{1}{M_{xx}}((\mu-r)M_x + \sigma^2 s^{2\beta+1}M_{xs})sM_{xs} = 0.$$

Differentiating equation above respect to p, we have

$$\begin{aligned}&-\eta a M_x - b^2(1-p)M_{xx} = 0\\ \iff\ &(-b^2 + b^2 p)M_{xx} - \eta a M_x = 0\\ \iff\ &b^2 p M_{xx} = \eta a M_x + b^2 M_{xx}\\ &p^* = \frac{\eta a M_x + b^2 M_{xx}}{b^2 M_{xx}} = 1 + \frac{\eta a M_x}{b^2 M_{xx}}\end{aligned}$$

Substitution π^* and p^* into (8), we obtain

$$M_t + \Bigg[rx - \frac{\left((\mu-r)M_x + \sigma^2 s^{2\beta+1}M_{xs}\right)(\mu-r)}{\sigma^2 s^{2\beta}M_{xx}} + \Bigg(\theta - \Bigg(1 + \frac{\eta a M_x}{b^2 M_{xx}}\Bigg)\eta\Bigg)a\Bigg]M_x + \mu s M_s + \frac{1}{2}\Bigg[\frac{\left((\mu-r)M_x + \sigma^2 s^{2\beta+1}M_{xs}\right)^2}{\sigma^2 s^{2\beta}M_{xx}^2} + \frac{\eta^2 a^2 M_x^2}{b^2 M_{xx}^2}\Bigg]M_{xx} + \frac{1}{2}\sigma^2 s^{2\beta+2}M_{ss} - \frac{\left((\mu-r)M_x + \sigma^2 s^{2\beta+1}M_{xs}\right)s^{2\beta+1}M_{xs}}{s^{2\beta}M_{xx}} = 0. \quad (9)$$

According to the fractional power utility function described by equation (6), we try to find the solution (9). Suppose value function in this problem by (see [8])

$$M(t,x,s) = f(t)e^{h(t)s}x^\alpha$$

with $f(T) = 1$ and $h(T) = 0$. The derivatives of the $M(t,x,s)$ with respect to t, x, s, respectively,

$$\begin{aligned}M_x &= \alpha x^{\alpha-1}e^{h(t)s}f(t)\\ M_{xx} &= \alpha(\alpha-1)x^{\alpha-2}e^{h(t)s}f(t)\\ M_t &= x^\alpha(f(t)h'(t)e^{h(t)s}s + e^{h(t)s}f'(t))\\ M_s &= f(t)h(t)e^{h(t)s}x^\alpha\\ M_{xs} &= \alpha x^{a\alpha-1}e^{h(t)s}f(t)h(t)\\ M_{ss} &= f(t)h(t)^2 e^{h(t)s}x^\alpha.\end{aligned}$$

Plugging these derivatives into the HJB equation (9), we obtain

$$\begin{aligned}&-\frac{(e^{h(t)s}\sigma^2 s^{2\beta+2}f(t)(h(t))^2x^\alpha)}{2(\alpha-1)} - \frac{e^{h(t)s}a\alpha^2 f(t)x^\alpha\eta}{x(\alpha-1)} + \frac{e^{h(t)s}a\alpha^2 f(t)x^\alpha\theta}{x(\alpha-1)} + \frac{e^{h(t)s}a\alpha f(t)x^\alpha\eta}{x(\alpha-1} - \frac{e^{h(t)s}a\alpha f(t)x^\alpha\theta}{x(\alpha-1} - \frac{e^{h(t)s}\alpha f(t)x^\alpha\mu^2}{2\sigma^2(\alpha-1)s^{2\beta}} + \frac{e^{h(t)s}\alpha f(t)x^\alpha\mu r}{\sigma^2(\alpha-1)s^{2\beta}}\\ &- \frac{e^{h(t)s}\alpha f(t)x^\alpha r^2}{2\sigma^2(\alpha-1)s^{2\beta}} + \frac{e^{h(t)s}x^\alpha\alpha f(t)h(t)rs}{\alpha-1} - \frac{e^{h(t)s}x^\alpha\alpha^2\eta^2 f(t)}{2(\alpha-1)b^2} + \frac{e^{h(t)s}x^\alpha\alpha^2 f(t)r}{\alpha-1} + \frac{e^{h(t)s}x^\alpha a\alpha f(t)h'(t)s}{\alpha-1} - \frac{e^{h(t)s}x^\alpha f(t)h(t)\mu s}{\alpha-1}\\ &- \frac{e^{h(t)s}x^\alpha\alpha f(t)r}{\alpha-1} - \frac{e^{h(t)s}x^\alpha f(t)h'(t)s}{\alpha-1} + \frac{e^{h(t)s}\alpha x^\alpha f'(t)}{\alpha-1} - \frac{e^{h(t)s}x^\alpha f'(t)}{\alpha-1} = 0\\ \iff &\Bigg(\frac{a\alpha^2\eta}{x} + \frac{a\alpha^2\theta}{x} + \frac{a\alpha\eta}{x} - \frac{a\alpha\theta}{x} - \frac{\alpha\mu^2}{2\sigma^2 s^{2\beta}} + \frac{\alpha\mu r}{\sigma^2 s^{2\beta}} - \frac{\alpha r}{2\sigma^2 s^{2\beta}} - \frac{a^2\alpha\eta^2}{2b^2} + \alpha^2 r - \alpha r\Bigg)f(t)\\ &+ (\alpha-1)f'(t) + (\alpha s - s)f(t)h'(t) + (-\mu s + \alpha r s)f(t)h(t) - \frac{1}{2}\sigma^2 s^{2\beta+2}f(t)h^2(t) = 0.\end{aligned}$$

Splitting into two equations (see [8])

$$\begin{aligned}&(\alpha^2 r - \alpha r)f(t) + (\alpha s - s)f(t)h'(t) = 0\\ \iff\ &\alpha r(\alpha-1)f(t) + s(\alpha-1)f(t)h'(t) = 0\\ \iff\ &(\alpha-1)f(t)(\alpha r + sh'(t)) = 0,\end{aligned}$$

i.e.,

$$\alpha r + sh'(t) = 0,\ \ h(T) = 0 \quad (10)$$

and

$$\left[-\frac{a\alpha^2\eta}{x}+\frac{a\alpha^2\theta}{x}+\frac{a\alpha\eta}{x}-\frac{a\alpha\theta}{x}-\frac{\alpha\mu^2}{2\sigma^2 s^{2\beta}}+\frac{\alpha\mu r}{\sigma^2 s^{2\beta}}-\frac{\alpha r}{2\sigma^2 s^{2\beta}}-\frac{a^2\alpha\eta^2}{2b^2}\right]f(t)+(\alpha-1)f'(t)+(-\mu s+\alpha rs)f(t)h(t)-\frac{1}{2}\sigma^2 s^{2\beta+2}f(t)h^2(t)=0 \quad ,f(T)=1. \tag{11}$$

The solution of (10) is

$$h(t)=-\frac{\alpha r}{s}(T-t).$$

Substitution $h(t)$ above into (11), we have

$$\left[-\frac{a\alpha^2\eta}{x}+\frac{a\alpha^2\theta}{x}+\frac{a\alpha\eta}{x}-\frac{a\alpha\theta}{x}-\frac{\alpha\mu^2}{2\sigma^2 s^{2\beta}}+\frac{\alpha\mu r}{\sigma^2 s^{2\beta}}-\frac{\alpha r}{2\sigma^2 s^{2\beta}}-\frac{a^2\alpha\eta^2}{2b^2}\right]f(t)+(\alpha-1)f'(t)-(-\mu s+\alpha rs)\frac{\alpha r}{s}(T-t)f(t)-\frac{\sigma^2 s^{2\beta+2}\alpha^2 r^2(T-t)^2}{2s^2}f(t)=0 \;\; f(T)=1.$$

$$\Longleftrightarrow f'(t)+\frac{1}{\alpha-1}\left[-\frac{a\alpha^2\eta}{x}+\frac{a\alpha^2\theta}{x}+\frac{a\alpha\eta}{x}-\frac{a\alpha\theta}{x}-\frac{\alpha\mu^2}{2\sigma^2 s^{2\beta}}+\frac{\alpha\mu r}{\sigma^2 s^{2\beta}}-\frac{\alpha r}{2\sigma^2 s^{2\beta}}-\frac{a^2\alpha\eta^2}{2b^2}+(\mu-\alpha r)(T-t)\alpha r\frac{1}{2}(\sigma s^{\beta}\alpha(T-t))^2\right]f(t)=0 \;\; ,f(T)=1.$$

Solution of the above ordinary differential equation is

$$f(t)=\exp\left[-\frac{1}{6x\sigma^2 b^2(\alpha-1)}\left(\left[3b^2(\alpha r^2-2\mu a(r-\frac{\mu}{2}))xs^{-2\beta}+(\alpha\sigma^2 b^2 r^2 x(T-t)^2 s^{2\beta}+(3b^2r^2 s(T-t)\alpha-3b^2 s\mu(T-t)+3a^2\eta^2)x-6b^2a((\theta-\eta)\alpha-\theta-\eta))\sigma^2\alpha\right](T-t)\right)\right].$$

We obtain the value function as follows

$$M(t,x,s)=\exp\left[-\frac{1}{6x\sigma^2 b^2(\alpha-1)}\left(\left[3b^2(\alpha r^2-2\mu a(r-\frac{\mu}{2}))xs^{-2\beta}+(\alpha\sigma^2 b^2 r^2 x(T-t)^2 s^{2\beta}+(3b^2r^2 s(T-t)\alpha-3b^2 s\mu(T-t)+3a^2\eta^2)x-6b^2a((\theta-\eta)\alpha-\theta-\eta))\sigma^2\alpha\right](T-t)\right)-\alpha r(T-t)\right]x^{\alpha}$$

and optimal strategy $\gamma^*=(p^*,\pi^*)$ is

$$p^*=1+\frac{\eta a x}{b^2(\alpha-1)}$$

and

$$\pi^*=\frac{x(-\alpha r(T-t)\sigma^2+(r-\mu)s^{-2\beta})}{\sigma^2(\alpha-1)}.$$

■

IV. Numerical Experiments

In this section, we present some numerical experiments to show the relationship between the optimal reinsurance and investment strategies with the parameters in our model. The basic parameters for the numerical analysis are as follows:

$$\eta=2,\mu=0.5,b=1,T=10,\sigma=1,a=1.5,$$
$$r=0.3,s=10,\alpha=0.5,x=0.1.$$

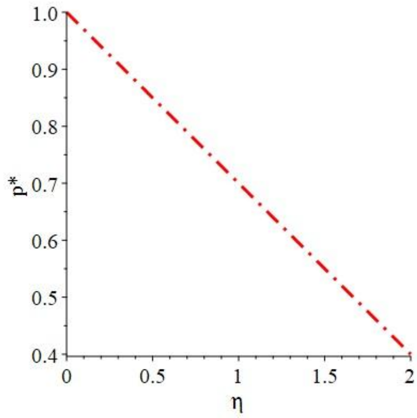

Fig. 1: The effect η of on the optimal reinsurance proportion p^*

From Figure 1, we see the effect between safety loading η and reinsurance proportion p^*, the greater the value of the reinsurer safety loading η will yields a smaller of the p^*. So, to maintain a stable income, the insurer would prefer buying less reinsurance.

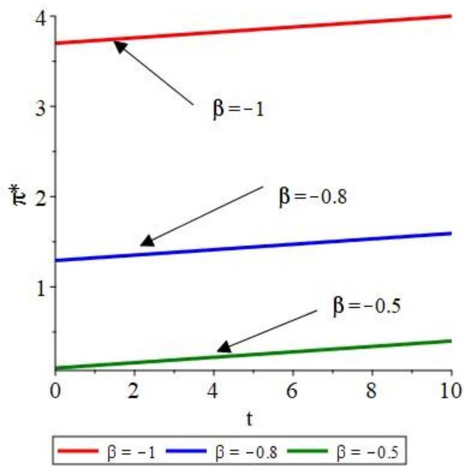

Fig. 2: The effect β of on the optimal reinsurance proportion π^*

From Figure 2, we see the effect of the elasticity coefficient β on the investment strategy π^*. There is a positive relation between β and π^*. This can be interpreted as a more negative β leads to a decrease in greater volatility and the possibility of an increase in bad movements on the price of risky assets. So, the insurer will invest less in risk assets because β is reduced.

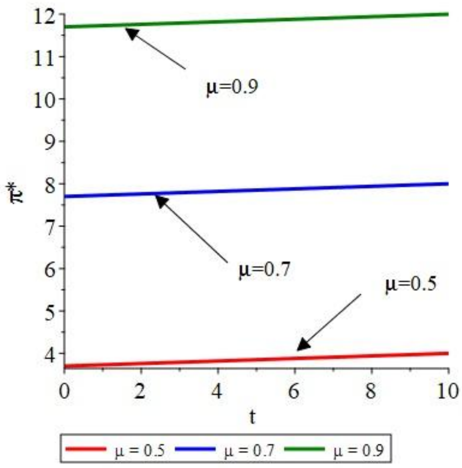

Fig. 3: The effect μ of on the optimal reinsurance proportion π^*

Figure 3, illustrates the effect of the risky assets return μ on the investment strategy π^*. There is a positive relation between μ and π^*. This implies that risky assets will be more attractive in the future. Therefore, the optimal amount invested in risk assets is increasing at this time.

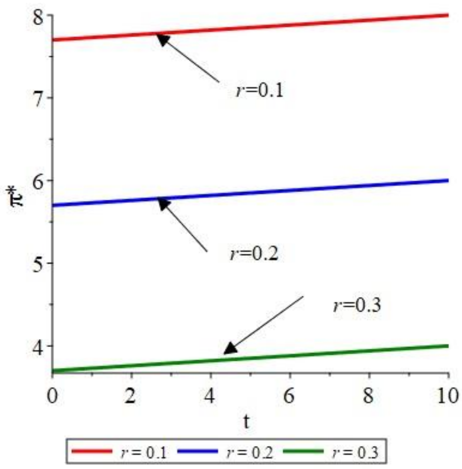

Fig. 4: The effect r of on the optimal reinsurance proportion π^*

Figure 4, illustrates the effect of interest rate risk-free asset r on the investment strategy π^*. There is a negative relation between r and π^*. This implies that risk-free assets will not be attractive in the future. Therefore, the optimal amount invested in risk-free assets is decreasing at this time.

V. Conclusion

In this paper, we focus on the optimal investment problem for insurance companies. The basic claim process is assumed to follow Brown's motion with drift, and the insurance company can buy proportional reinsurance. Insurers are allowed to invest in risk-free assets and risk assets, whose prices are described as a constant elasticity variance (CEV) model. The value function and explicit solutions to maximize expected fractional power utility function are obtained by solving the corresponding Hamilton-Jacobi-Bellman (HJB) equation. Finally, the numerical experiments are presented to show the effects of model parameters on the insurer's optimal reinsurance and investment strategies.

References

[1] D. Li, X. Rong, and H. Zhao, "Optimal reinsurance-investment problem for maximizing the product of the insurer's and the reinsurer's utilities under a CEV model," Journal of Computational and Applied Mathematics, vol. 255, pp. 671-683, 2014.
[2] P. K. Mwanakatwe, X. Wang, and Y. Su, "Optimal Investment and Risk Control Strategies for an Insurance Fund in Stochastic Framework," Journal of Mathematical Finance, vol. 9, no. 3, pp. 254-265, 2019.
[3] Q. C. Li, "Optimal reinsurance and investment policies with the CEV stock market, Acta Mathematicae Applicatae Sinica, English Series, vol. 32, no. 3, pp. 647-658, 2016.
[4] X. Lin, and Y. Li, "Optimal Reinsurance and Investment for a Jump Diffusion Risk Process under the CEV Model," North American Actuarial Journal, vol. 15, no. 3, pp. 417-431, 2011.
[5] Y. Wang, X. Rong, and H. Zhao, "Optimal investment strategies for an insurer and a reinsurer with a jump diffusion risk process under the CEV model," Journal of Computational and Applied Mathematics, vol. 328, pp. 414-431, 2018.
[6] A. Gu, X. Guo, Z. Li, and Y. Zeng, "Optimal control of excess-of-loss reinsurance and investment for insurers under a CEV model," Insurance: Mathematics and Economics, vol. 51, no. 3, pp. 674-684, 2012.
[7] Q. Li, and M. Gu, "Optimization Problems of Excess-of-Loss Reinsurance and Investment under the CEV Model," ISRN Mathematical Analysis, vol. 2013, 2013.
[8] D. Sheng, "Explicit Solution of the Optimal Reinsurance-Investment Problem with Promotion Budget," Journal of Systems Science and Information, vol. 4, no. 2, pp. 131-148, 2016.
[9] S. A. Ihedioha, "Optimal Portfolios of an Insurer and a Reinsurer under Proportional Reinsurance and Power Utility Preference," Open Access Library Journal, vol. 2, no. 12, p. 1, 2015.
[10] C. Deng, W. Bian, and B. Wu, "Optimal reinsurance and investment problem with default risk and bounded memory," International Journal of Control, pp. 113, 2019.
[11] D. Li, X. Rong, and H. Zhao, "Optimal investment problem with taxes, dividends and transaction costs under the constant elasticity of variance (CEV) model," WSEAS Transactions on Mathematics, vol. 12, no. 3, pp. 243-245, 2013.
[12] D. Li, X. Rong, and H. Zhao, "Optimal investment problem for an insurer and a reinsurer," Journal of Systems Science and Complexity, vol. 28, no. 6, pp. 1326-1343, 2015.
[13] J. Gao, Optimal investment strategy for annuity contracts under the constant elasticity of variance (CEV) model, Insur. Math. Econ., vol. 45, no. 1, pp. 918, 2009.
[14] A. Chunxiang, Y. Lai, and Y. Shao, "Optimal excess-of-loss reinsurance and investment problem with delay and jumpdiffusion risk process under the CEV model," Journal of Computational and Applied Mathematics, vol. 342, pp. 317-336, 2018.
[15] D. Li, X. Rong, and H. Zhao, "Time-consistent reinsurance-investment strategy for an insurer and a reinsurer with mean-variance criterion under the CEV model," Journal of Computational and Applied Mathematics, vol. 283, pp. 142-162, 2015.
[16] J. Xiao, Z. Hong, and C. Qin, "The constant elasticity of variance (CEV) model and the Legendre transform-dual solution for annuity contracts," Insurance: Mathematics and Economics, vol. 40, no. 2, pp. 302-310, 2007.
[17] A. Wang, L. Yong, Y. Wang, and X. Luo, "The CEV model and its application in a study of optimal investment strategy," Mathematical Problems in Engineering, vol. 2014, 2014.
[18] J. C. Cox, and S. A. Ross, "The valuation of options for alternative stochastics processes," Journal of financial economics, vol. 3, no. 1-2, pp. 145-166, 1976.
[19] Charles I. Nkeki, and Chukwuma R. Nwozo, "Optimal Investment under Inflation Protection and Optimal Portfolios with Stochastic Cash Flows Strategy," IAENG International Journal of Applied Mathematics, 43:2, pp54-63, 2013.
[20] Lingyan Cao, and Zheng-Feng Guo, "Optimal Variance Swaps Investments," IAENG International Journal of Applied Mathematics, 41:4, pp334-338, 2011.
[21] Y. Cao, and N. Wan, "Optimal proportional reinsurance and investment based on Hamilton-Jacobi-Bellman equation," Insurance: Mathematics and Economics, vol. 45, no. 2, pp. 157-162, 2009.

[22] A. E. Nozadi, “Optimal Constrained Investment and Reinsurance in Lunberg insurance models, Doctoral dissertation,” Verlag nicht ermittelbar, 2014.

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