Cross-correlation Analysis Method on Explosion Seismic Waves Data

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Abstract—In this paper, we propose a novel data pre-processing method for conducting explosion, which is to resolve the complex, aperiodic, and stochastic seismic waves generated by underground explosion. The method can estimate the level of similarity and data effectiveness. Based on the theoretical analysis on cross-correlation influence factors, three critical parameters, i.e., the proportionality coefficient mean, the proportionality coefficient deviation weighted mean, and the proportionality coefficient deviation weighted variance, were put forward to establish the relationship between cross-correlation coefficient and data similarity. The verification results show that the cross-correlation coefficient can well represent the level of similarity and effectively figure out the similarity on explosion seismic waves, and develop necessary technical support with the following seismic effect analysis.

Index Terms—Explosion seismic wave, cross-correlation analysis, data pre-processing, weighting

I. INTRODUCTION

Taking the underground explosion as the research background, the seismic waves data needs to be processed to obtain effective characteristic parameters and analyze the seismic effects under different explosion conditions[1]-[2]. Explosion seismic wave is a kind of complex non-periodic and transient random wave, containing complex high-low-frequency vibration signal and interference noise [3], which leads to the regular variation of waveform characteristics in the time domain. Therefore, when comparing seismic effects under different explosion conditions, the frequency spectral analysis method is generally adopted. In particular, when the seismic waveform is affected by random interference and the dispersion degree increases [4]-[7], the confidence and accuracy of data processing are greatly reduced and it is difficult to effectively characterize the changes of seismic effect. Therefore, it is necessary to solve the problem of data validity determination through pre-processing in seismic wave data processing. However, there are few studies addressing this problem thus far and it is highly desired to carry out the research on the method of seismic wave data pre-processing.

This paper proposes a method based on cross-correlation analysis to analyze the explosion seismic waves under different conditions, which can quantitatively determine the similarity of data, identify the influence of random interference in the comparative data, and solve the problem of data validity. Through the innovative analysis of the influencing factors of the cross-correlation coefficient, the relationship between the cross-correlation coefficient and similarity is established, and the rationality and effectiveness of the pre-processing method are confirmed.

II. CROSS CORRELATION ANALYSIS THEORY

Seismic waves generated in underground explosions are mainly affected by the detonation source conditions, characteristics of propagation media, distance from the detonation center, topography and geology, etc. [8]-[9], which result in a certain randomness of seismic waves generated under different explosion conditions. In order to study the influence of one factor on explosive seismic waves, it is necessary to try to control the change of other factors and reduce the joint effect of compound factors, especially select the same location for measurement to make the interference coming from the propagation path to be systematic different and improve the consistency of seismic wave data at the same measuring point effectively. As shown in Fig. 1, the vertical seismic waveforms of two explosions with the same equivalent, the same measuring point and different detonation source conditions are compared. It shows that the waveforms of the two explosions have good consistency, but there are certain differences in amplitude and it is difficult to effectively determine the degree of difference between the two only by peak value or duration.

![Fig. 1. Explosion seismic comparison waveforms](image-url)
The characteristics of explosive seismic waves under different conditions are studied with the cross-correlation method [10]-[11]. The similarity degree of data can be quantitatively determined by using the cross-correlation coefficient, and the influence of random interference can be identified. Since the test data is a series of finite discrete values, the normalized cross-correlation function is adopted to quantitatively analyze the degree of cross-correlation [12]:

\[
R_{xy}(m) = \frac{\sum_{i=0}^{N-1} x(i) \ast y(i+m)}{\sqrt{\sum_{i=0}^{N-1} x(i)^2 \ast \sum_{i=0}^{N-1} y(i)^2}}
\]

(1)

Where, \(x\) and \(y\) are seismic wave data sampled at equal intervals, \(N\) is the length of data sequence, \(m\) is the offset of the correlation function and \(m=-N+1,\ldots,1,0,\ldots,N-1\), \(R_{xx}(0)\) and \(R_{yy}(0)\) are the autocorrelation function values of signals \(x\) and \(y\) when \(m=0\), respectively.

![Cross-correlation function of explosion seismic waves](image)

Fig. 2. Cross-correlation function of explosion seismic waves

Fig. 2 is the cross-correlation function of the two groups of experimental data in Fig. 1, and the maximum value of the cross-correlation function in Fig. 2 is 0.954 \((m=0)\), indicating that the linear correlation between them is very high and there is no delay, which is consistent with the waveform comparison in Fig. 1. Therefore, the cross-correlation function can be used to quantitatively analyze the similarity of different experimental seismic wave data.

III. INFLUENCE FACTORS ANALYSIS OF CROSS-CORRELATION

To study the rationality of the similarity analysis of explosive seismic wave data by the cross-correlation theory, this section reveals the corresponding relationship between the key influence factors of the cross-correlation and the similar characteristics of the waveform through theoretical analysis method, and determines the correlation characteristics between the cross-correlation coefficient and the similarity.

A. Cross-correlation Coefficient Calculation

In this paper, the similarity between two explosion seismic waves is mainly considered. It is assumed that there is a certain linear correlation in the time domain, and thus Eq. (1) is transformed into

\[
R_{xy}(m) = \frac{\sum_{i=0}^{N-1} x(i) \ast y(i+m)}{\sqrt{\sum_{i=0}^{N-1} x(i)^2 \ast \sum_{i=0}^{N-1} y(i)^2}}
\]

(2)

Where, \(R_{xy}\) are the proportional coefficients of the corresponding moment amplitude of the two explosion seismic waves and the mean of \(R_{xx}\) to \(R_{yy}\) is very close to \(r(>0)\).

According to Eq. (2), when \(x\) and \(y\) have a certain linear relation and no time delay, its maximum value is the value of \(m=0\), which can reflect and judge the similarity degree of two groups of signals, and is determined as the cross-correlation coefficient \(R_{xy}\) of two waves.

\[
R_{xy} = \frac{\sum_{i=0}^{N-1} R_{xy}(i)^2}{\sqrt{\sum_{i=0}^{N-1} R_{x}(i)^2} \ast \sqrt{\sum_{i=0}^{N-1} R_{y}(i)^2}}
\]

(3)

When there is a certain delay between the two signals, the maximum value is intended to be the cross-correlation coefficient \(R_{xy}\) and the corresponding offset \(m_d\) is the delay.

B. Theory Analysis of Influence Factors

In fact, proportional coefficients of any two explosive seismic waves are non-equal. Even if the explosive signals at the same place and in the same experiment have certain deviation, Eq. (3) can be converted into:

\[
R_{xy} = \frac{\sum_{i=0}^{N-1} R_{xy}(i)^2}{\sqrt{\sum_{i=0}^{N-1} R_{x}(i)^2} \ast \sqrt{\sum_{i=0}^{N-1} R_{y}(i)^2}}
\]

(4)

Where, \(w_i\) is the weight of energy at each moment and \(w_1+w_2+\ldots+w_{N-1}=1\), \(r\) is the mean of \(R_{xy}\), \(\Delta R_{xy}\) is the deviation value of proportionality coefficient at the corresponding moment.

In the actual data processing of explosion seismic waves, if \(\Delta R_{xy}\) is too large to be judged to be a gross error or abnormal point and the point will be abandoned. Therefore, it can be assumed that \(\Delta R_{xy}\) is far less than \(r\), and \(\Delta R\) can be defined as the proportionality coefficient deviation weighted mean.

\[
\Delta R = \frac{\sum_{i=0}^{N-1} \Delta R_{xy} w_i}{\sum_{i=0}^{N-1} w_i}
\]

(5)

Then, Eq. (4) can be transformed into:


\[
R_{\text{yef}} = \frac{r + \Delta R}{\sqrt{(r + \Delta R)^2 + \sum_{i=0}^{N-1} \Delta R_i^2 w_i - \Delta R^2}}
\]

\[
\Delta R_y = \sum_{i=0}^{N-1} \Delta R_i^2 w_i - \Delta R^2
\]

Where, \(\Delta R_y\) is the proportional coefficient deviation weighted variance, which is the difference between the weighted mean squared of the proportional coefficient deviation and the weighted mean of the proportionality coefficient deviation squared.

From Eq. (6), there are three factors affecting the cross-correlation coefficient \(R_{\text{yef}}\), the proportionality coefficient mean \(r\), proportionality coefficient deviation weighted mean \(\Delta \bar{R}\), and proportional coefficient deviation weighted variance \(\Delta R_y\). The analysis of definition and interrelationship of the three factors are conducted to obtain the influence mode of waveform similarity.

Proportionality Coefficient Mean

According to the definition of the proportionality coefficient mean \(r\), its value mainly represents the expansion degree of the two explosive seismic waves and the proportion relation of the overall waveform. From Eq. (6), when the dispersion degree of proportionality coefficient \(R\) is similar, the greater the proportionality coefficient mean \(r\) is, the weaker the influence of the proportionality coefficient deviation weighted mean \(\Delta \bar{R}\) on \((r + \Delta \bar{R})\) is.

Proportionality Coefficient Deviation Weighted Mean

On basis of the definition of proportional coefficient deviation weighted mean \(\Delta \bar{R}\), its value is not only related to the value of deviation \(\Delta R\), but also affected by the energy weight \(w\), which is related to the amplitude \(A(i)\), that is, the larger the amplitude is, the greater the weight will be. Therefore, \(\Delta \bar{R}\) is mainly determined by the deviation of the proportional coefficient corresponding to the main signal of the explosion seismic wave and indicates the deviation and similarity degree of the main signal of two explosion seismic waves.

In the particular explosion seismic waveform, it is difficult to effectively and accurately judge the similarity between the two explosion seismic waves by directly using the waveform curves, but it can be simply determined by the number of different peak points. The more different peak points, the greater the deviation value, that is, they are the points that are obviously inconsistent with the comparison characteristics of waveform peaks at other time periods. Taking the two explosion seismic waves shown in Fig. 3 as the example, within the time range from 0 to 0.5s, there are eight obvious abnormal points of peak contrast waveforms, but the amplitude is relatively small and the weight is relatively small, so the influence on the cross-correlation coefficient is small. The cross-correlation coefficient of the two is 0.954 shown in Fig. 2, which verifies the correctness of the above analysis.

![Disparate points of explosion seismic waves peaks](image)

**Proportional Coefficient Deviation Weighted Variance**

According to the definition of variance,

\[
\sigma^2 = \sum x^2(i) - (\sum x)^2
\]

\(\Delta R_y\) is defined as the weighted variance of the proportionality coefficient deviation,

\[
\Delta R_y = \sum_{i=0}^{N-1} \Delta R_i^2 w_i - (\sum_{i=0}^{N-1} \Delta R_i w_i)^2
\]

Equation (8) and (7) have the same definition form and calculation method. The difference lies in that \(\Delta R_y\) represents the degree of dispersion under the weighted condition, that is, \(\Delta R_y\) is not only related to the size of the deviation value, but also its weight. The greater the weight is, the greater the influence on the degree of dispersion will be. Therefore, the dispersion degree of the proportional coefficients corresponding to the main signals of the two explosive seismic waves is indicated by \(\Delta R_y\).

**IV. TEST VERIFICATION**

**A. Explosion Seismic Waveforms Calculation and Results**

To verify the correctness of the cross-correlation influence factor analysis, correlation analysis is carried out by using seismic wave data of underground explosion. Six groups of explosion seismic data with different proportional coefficients are selected, as shown in Fig. 4.

According to Eq. (6), the proportionality coefficient mean \(r\), the proportionality coefficient deviation weighted mean \(\Delta \bar{R}\), the proportionality coefficient deviation weighted variance \(\Delta R_y\), and the cross-correlation coefficient \(R_{\text{yef}}\) of each group of seismic wave data are calculated respectively, as shown in Table I and Fig. 5.
Fig. 4. Explosion seismic waves data of different proportionality coefficient

Fig. 5. Comparison of cross-correlation influence factors
B. Comparison of Factors

Table I and Fig. 5 show that:

1) The more the proportional coefficient mean $r$ of the two explosion seismic waves approaches to 1, the larger the cross-correlation coefficient $R_{xy}$ is and the closer to 1, that is, the better the correlation is. Affected by the degree of data dispersion, the change trend under the condition $r<1$ tends to be gentle, which is basically similar to that under the condition $r>1$.

2) The absolute value of the proportional coefficient deviation weighted mean $|\Delta R|$ increases with the increase of $r$ deviates from 1, and especially increases rapidly when $r<1$, indicating that the similarity of the explosion seismic wave peak becomes worse. It can be observed from the waveform comparison in Fig. 4 that the difference between the waveform peaks of group (f) is the largest, and the relationship between the two waves is very different. group (a) is followed by group (d), and group (c) is the best, which is consistent with the change trend of $\Delta R$ in the table.

3) The proportional coefficient deviation weighted variance $\Delta R_{xy}$ is consistent with $\Delta R$, and it increases with the increase of $r$ deviates from 1, and the increase rate of $r>1$ is much higher than that of $r<1$, which declares that the dispersion degree of the main waveform of explosion seismic wave increases. As shown in Fig. 4, the data waveforms of group (e) and group (f) have the largest difference, group (a) and group (b) are the next, group (c) and group (d) are the best, which are consistent with $\Delta R_{xy}$ in the Table I.

4) Among the influence of the three factors, i.e. $r$, $\Delta R$, and $\Delta R_{xy}$, they are not completely independent and $\Delta R$ and $\Delta R_{xy}$ have already contained changes of $r$, that is, $r$ has an impact on $R_{xy}$ through the change of $\Delta R$ and $\Delta R_{xy}$. Compared to $\Delta R$, $\Delta R_{xy}$ has a significant influence on $R_{xy}$ and they have a good linear relationship. From Table I, the data correlation shown in Fig. 6 and the expression (9) is obtained, whose fitting correlation coefficient (r-square) is 0.94 and residual standard deviation is 0.008, which declare that the fitting degree is very ideal and can well represent the relationship between $\Delta R_{xy}$ and $R_{xy}$.

\[ R_{xy} = -0.2816 \times \Delta R_{xy} + 0.9884 \]  

C. Discussion

The analysis of the six groups of explosion seismic waves indicate that the cross-correlation coefficient $R_{xy}$ is mainly influenced by the proportionality coefficient mean $r$ that represents the expansion degree of the two waves, the proportionality coefficient deviation weighted mean $\Delta R$ that represents the similarity of two waves peaks and the proportionality coefficient deviation weighted variance $\Delta R_{xy}$ that represents the dispersion degree of the two waves. The relationship between cross-correlation coefficient and the similarity of explosion seismic waves is established to validate the rationality of the cross-correlation analysis. In addition, $\Delta R$ and $\Delta R_{xy}$ contains the influence of $r$, and $\Delta R_{xy}$ has more significant influence on $R_{xy}$ than $\Delta R$ and they have a good linear relationship, which provides effective technical support for the subsequent similarity analysis.

V. Conclusion

In view of complexity, non-periodicity and transient randomness of explosive seismic waves, a similarity analysis method based on cross-correlation theory is proposed to preprocess seismic wave data and quantitatively determine the degree of similarity of data. It can be concluded that:

The cross-correlation theory can effectively and reasonably analyze the similarity of explosive seismic waves, identify the influence of random interference in the comparative data, realize data pre-processing, and provide necessary data support for subsequent seismic effect analysis.

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similarity of two waves peaks and the proportionality coefficient deviation weighted variance $\Delta R_{xy}$ that represents the dispersion degree of the two waves. The relationship between cross-correlation coefficient and the similarity of explosion seismic waves is established to validate the rationality of the cross-correlation analysis. In addition, $\Delta R$ and $\Delta R_{xy}$ contain the influence of $r$, and $\Delta R_{xy}$ has more significant influence on $R_{xycf}$ than $\Delta R$. $\Delta R_{xy}$ and $R_{xycf}$ have a good linear relationship, which is obtained from fitting curve and can provide effective technical support for the subsequent data processing.

REFERENCES


