Solving the Telecommunication Network Problem using Vague Graph

Lihua Lin, Meichun Zhang, Li Ma

***Abstract*—Vague graph plays a major role in our day to day life for example in communication networks, web browsing, transportation networks and biological networks. It helps to model the uncertainties and imprecision of those problem. Telecommunication is one of the most significant aspects of our society and culture. The service provider of telecommunication companies generally uses crisp graph to model the telecommunication network. In this manuscript, we introduce a vague telecommunication network using vague graph theory and a vague graph model to find a best health care centre within a city. We use sign vague graph to model those two problems. In this work, we introduce certain ideas on vague graph in dombi notion. We introduce the concepts of vague dombi graphs and investigate certain properties and its attributes. We also analyze the concepts of complement of vague dombi graphs and obtain some important perceptions over it. We also define frustration index, frustration number of vague dombi graph. Dombi vague graphs can be used to model many real life problems in communication networks, web browsing, decision making problem, transportation networks and biological networks..**

***Index Terms*—Vague graph; Telecommunication; dombi graph; frustration index degrees-of-freedom.**

I. Introduction

In 1965, Zadeh [1] introduced the concepts of fuzzy sets which has a vast usage in science and technology. In 1975, Rosenfeld [2] discussed the concept of fuzzy graphs whose ideas are implemented by Kauffman [3]. The relationship between two different fuzzy sets described by Rosenfeld. He presented the concept of graph theory in fuzzy environment and also develop many theorems and proofs of fuzzy graph. Bhattacharya [4] gave some remarks of fuzzy graphs. The complement of fuzzy graphs was introduced by Mordeson [5]. S.N.Mishra and A. Pal [6] introduced the concepts of product of interval-valued intuitionistic fuzzy graph. The concept of dombi graph was introduced by [7]. Cartwright and Harary connected with the problems in social psychology, where each node represents a person and the corresponding edges respresent symmetric dyadic relations among them. Later many other scientists established sequential results. Nirmala and Prabhavathi [8] introduced the idea of dombi fuzzy and further defined many terms. Gau and Buehrer [9] developed the idea of vague set by changing the element in a set within the sub interval $[0, 1]$. In [10], the introduced the concept of maximal vague graphs and analyzed some of its properties in the year 2020.

There are several efforts in vague graph [11], [12], [13], [14], [15], [16], [17], [18]. Several basic ideas to vague graphs, such as the isomorphism of vague graphs have been proposed by Talebi et al. [19] and Borzooei and Rashmanlou [20] presented idea of the Laplacian matrix and spectrum in vague graph. Borzooei et al. [21] described the properties of homomorphism in vague graphs. In [22], Samanta et al. presented the behavior of different types of vague graphs, and introduced the concept to find the strength of any vague graphs. Their research work was further studied in [23]. Rashmanlou et al. [24] presented the properly of vague h-morphism. In [25], Darabian et al. defined the regular and irregular vague graph. They used those graphs to model road transport networks, fullerene molecules and wireless multihop networks. Borzooei and Rashmanlou [26] described different product operations between two vague graphs and studied the correctness of these idea. In [27], Borzooei et al. presented the idea to determine the strong domination numbers of a vague graph and they have used this concept in different real life application.

It is used in the extensive study of modeling, a kind of socio psychological processes and also it has various interesting connections with many classical mathematical analysis. In this manuscript, we introduce a vague telecommunication network using vague graph theory and a vague graph model to find a best health care centre within a city. We use sign vague graph to model those two problems. In this work, we introduce certain ideas on vague graph in dombi notion. We introduce the concepts of vague dombi graphs and investigate certain properties and its attributes. We also analyze the concepts of complement of vague dombi graphs and obtain some important perceptions over it. We also define frustration index, frustration number of vague dombi graph. dombi vague graphs can be used to model many real life problems in communication networks, web browsing, decision making problem, transportation networks and biological networks.

II. Preliminaries

Definition 2.1: A (crisp) set is a collection of distinct objects. Let A be a crisp set. If the elements of a set A are

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subset of X, then set A can be represented by the elements $x \in X$ following the characteristic function (1).

$$C_A(x) = \begin{cases} 1 & \text{if } x \in X \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Thus in classical set theory, belonging of x in A, i.e., $C_A(x)$ has only one of two values 0 and 1. Fuzzy set is an extension of crisp set, where the members (elements) have varying degrees of membership within the interval [0,1]. The classical set, based on two truth values, true (1) and false (0), is unable to describe human thoughts. Instead of considering only those two truth values, fuzzy set uses the interval between 0 and 1. The membership degree of a fuzzy set is not same as probability, where the uncertainty arises due to randomness. Let x be an element of X then a fuzzy set A in X a set of ordered pairs as defined as below in (2).

$$A = \{(x, C_A(x)) \,|x \in X\} \quad (2)$$

Here, X be the universe of discourse and $C_A(x)$ is known as membership function of x in A and $C_A(x) \rightarrow$ [0,1].

Definition 2.2: Let a graph $\tilde{G}$=(V,σ_1,C) be a fuzzy graph. Here, V represents the set of nodes, σ_1 describes a fuzzy subset of nodes and C presents a membership degree on σ such that $C(p,q) \leq \sigma_1(p) \wedge \sigma_1(q)$ for every p,q $\in$ V.

Definition 2.3: The complement fuzzy graph $\tilde{G} = (\sigma_1, C_1)$ is an another fuzzy graph. A complement fuzzy graph is described as $\overline{G} = (\overline{\sigma_1}, \overline{C_1})$. Here $\overline{\sigma_1}$ and σ_1 are equal.

$$\overline{C_1}(p,q) = \sigma_1(p) \wedge \sigma_1(q) - C(p,q) \quad (3)$$

Definition 2.4: A fuzzy graph $\tilde{G} = (\sigma_1, C)$ is called strong fuzzy graph if

$$C_1(p,q) = (\sigma_1(p) \cap \sigma_1(q)), \forall p, q \in C \quad (4)$$

Definition 2.5: A fuzzy graph $\tilde{G} = (\sigma_1, C)$ is called complete fuzzy graph if

$$C_1(p,q) = (\sigma_1(p) \cap \sigma_1(q)), \forall p, \forall q \in \sigma_1 \quad (5)$$

Definition 2.6: **(Dombi Fuzzy Graph)** Let $\tilde{G}' = (V_G, E_G)$ be a crips undirected graph contain no self-loop and parallel edges. Also, let $C : V_G \rightarrow [0,1]$ membership degree on V_G and $D : V_G \times V_G \rightarrow [0,1]$ be the membership degree on the symmetric fuzzy relation $E \subset V_G \times V_G$. Then $\tilde{G} = (V_G, C, D)$, is said to be a dombi graph if

$$D(pq) \leq \frac{C(p)C(q)}{C(p) + C(q) - C(p)C(q)}, \forall (pq) \in E_G.$$

We can consider a vague set as an extension of classical fuzzy set. It is developed by two different membership degree: truth membership, i.e., $t_v(i)$ and false membership, i.e., $f_v(i)$. The truth membership degree describes the lower bound of the grade of membership function of i. The main difference, i.e., $1 - t_v(i) - f_v(i)$ describes the uncertainties in the set. If the difference value is small the knowledge is precisely relative and if it is large the knowledge is little. The boundedness of this vague set is represented by $t_v(i) \leq C_v(i) \leq 1 - f_v(i)$ where $t_v(i) + f_v(i) \leq 1$

The Example shows the vague set $X[t_B(x), 1 - f_B(x)] = [0.6, 0.2]$. This describes the membership degree of element x belonging to the vague set B is 0.6 and the degree of x not belonging to the set B is 0.2. 0.2 is the degree representing the neutral position. This is a interval valued set on a vague relation.

Example 2.1: In a remote sensor network, suppose in a testing region we have the sensor database application consists of set of ten sensors each capturing a record of sensor data say $S = \{S_1, S_2, S_3, ..., S_{10}\}$. We obtain the corresponding measurements of these ten sensors say $S = \{18, 20, 18, 19, 18, 18, -, 18, -, 18\}$ at any time t. Here we see that "-" refers to the sensor data which is not reachable at any time t. ie., we consider the set with six values as 18, one value 19, one value 20 and two missing values. We now obtain the vague set p_t for these sensor data. The occurence of 18 is counted six times, but we see that two values 19 and 20 are against it. There are also two missing values in the set which are considered as neutral. Thus the truth membership value κ is 0.6 and the false membership value λ is 0.2. Hence we say for vague set $1 - \lambda$ = 0.8. We obtain the vague membership values as [0.6 , 0.8] for the sensor data 18 occured six times. Similarly we could obtain the vague membership value for 19 as [0.1, 0.3] and for the sensor data 20 as [0.1, 0.3]. Hence we see that the combination of all these results leads ot the vague set p_t as $p_t = [0.6, 0.8]/18 + [0.1, 0.3]/19 + [0.1, 0.3]/20$. This Example shows that the vagueness data captured follows the truth membership value $\kappa_V(p)$.

Definition 2.7: A vague graph $\tilde{G}$ is said to be

1. The $\tilde{G}$ is a self complement vague graph if $\tilde{G}$=$\tilde{G}^c$.
2. The $\tilde{G}$ is a self weak complement vague graph if $\tilde{G}$ is nothing but weak isomorphic to $\tilde{G}^c$.

Definition 2.8: Let $\tilde{G} = (C, D)$ is vague graph. The vague open degree of a vague vertex u of $\tilde{G}$ is described as $deg(p) = (d_t(p), d_f(p))$

$$d_t(p) = \sum_{\substack{p \in V \\ u \neq v}} t_D(p,q),$$

$$d_f(p) = \sum_{\substack{p \in V \\ p \neq q}} f_D(p,q), \quad \forall p, q \in E \quad (6)$$

Let $\tilde{G} = (C, D)$ is vague graph. The vague order of $\tilde{G}$ is defined as

$$O(G) = \sum_{p \in V} t_D(p), \sum_{p \in V} f_D(p) \quad (7)$$

The size of a vague graph is defined as $S(G) = (S_t(G), S_f(G))$ where

$$S_t(G) = \sum_{\substack{p \in V \\ p \neq q}} t_D(p,q), S_f(G) = \sum_{\substack{p \in V \\ p \neq q}} f_D(p,q) \quad (8)$$

Definition 2.9: Let $\tilde{G} = (C, D)$ is a vague graph. Then the busy value of any node node $u \in V$ is described by $D(q) = (D_t(q), D_f(q))$, where

$D_t(p) = \sum t_D(p) \wedge t_D(u_i)$, $D_f(p) = \sum f_D(p) \vee f_D(u_i)$

Here, u_i are adjacent nodes of u.

Definition 2.10: Let $\tilde{G} = (C, D)$ be a vague graph. A vertex u of $\tilde{G}$ is said to be busy vertex if

$$t_A(p) \leq d_t(p), f_A(p) \geq d_t(p) \quad (9)$$

If it is not a busy node then it is said to be free vertex.

Definition 2.11: Let $\tilde{G} = (C, D)$ be a vague graph. Then a vague arc p, q is as effective vague arc if

$t_D(p,q) = t_C(p) \wedge t_C(q)$, $f_D(u,v) = f_C(p) \vee f_C(q)$

Definition 2.12: A dombi vague graph $\tilde{G} = (C, D)$ is a regular vague graph if

$$\sum_{\substack{u \\ p \neq q}} t_D(p,q) = constant,$$
$$\sum_{\substack{u \\ p \neq q}} f_D(p,q) = constant, \forall p, q \in E \quad (10)$$

Definition 2.13: A dombi vague graph $\tilde{G} = (C, D)$ is defined as complete dombi vague graph if

$$t_D(p,q) = t_C(p) \wedge t_C(q),$$
$$f_D(p,q) = f_C(p) \vee f_C(q) \quad \forall p, q \in V \quad (11)$$

Definition 2.14: A dombi vague graph $\tilde{G} = (C, D)$ is defined as strong dombi vague graph if

$$t_D(p,q) = t_C(p) \wedge t_C(q),$$
$$f_D(p,q) = f_C(p) \vee f_C(q) \quad \forall p, q \in E \quad (12)$$

Definition 2.15: Let $\tilde{G} = (C, D)$ is a dombi vague graph. Then, $\tilde{G}$ is defined to be connected vague graph if all the vertices $u, p \in V$, $t_D^{\infty}(p,q) > 0$ or $f_D^{\infty}(p,q) < 1$.

Definition 2.16: Let $\tilde{G} = (C, D)$ is a vague graph. A vague path p in a $\tilde{G}$ and p is a sequence of different vertices $p_0, p_1, p_2, ..., p_m$ such that

$$(t_D(p_{i-1}p_i)\, f_D(p_{i-1}p_i)) > 0, i = 1, 2, ..., m. \quad (13)$$

Here, m represents the path length.

Definition 2.17: The complement of a vague graph $\tilde{G} = (C, D)$ is a vague graph $\tilde{G}^c = (C^c, D^c)$ if follows the (14).

$$t_C^c = t_C, f_C^c = f_C$$
$$t_D^c(p,q) = t_C(p) \wedge t_C(q) - t_D(p,q),$$
$$f_D^c(p,q) = f_C(p) \vee f_C(q) - f_D(p,q) \quad \forall p, q \in V \quad (14)$$

Definition 2.18: A dombi vague graph $\tilde{G}$ is said to be

1. The $\tilde{G}$ is self complement dombi vague graph if $\tilde{G} = \tilde{G}^c$.
2. The $\tilde{G}$ is self weak complement dombi vague graph if $\tilde{G}$ weak isomorphic vague graph to $\tilde{G}^c$.

Definition 2.19: Let $\tilde{G} = (C, D)$ is vague graph. The vague open degree of a vague vertex u of $\tilde{G}$ is described as $deg(p) = (d_t(p), d_f(p))$

$$d_t(p) = \sum_{\substack{p \in V \\ p \neq q}} t_D(p,q),$$
$$d_f(p) = \sum_{\substack{p \in V \\ p \neq q}} f_D(p,q), \quad \forall p, q \in E \quad (15)$$

Let $\tilde{G} = (C, D)$ is vague graph. The vague order of $\tilde{G}$ is defined as

$$O(G) = \sum_{p \in V} t_D(p), \sum_{p \in V} f_D(p) \quad (16)$$

The size of a vague graph is defined as $S(G) = (S_t(G), S_f(G))$ where

$$S_t(G) = \sum_{\substack{p \in V \\ p \neq q}} t_D(p,q), S_f(G) = \sum_{\substack{p \in V \\ p \neq q}} f_D(p,q) \quad (17)$$

Definition 2.20: Let $\tilde{G} = (C, D)$ is a vague graph. Then the busy value of any node node $u \in V$ is described by $D(q) = (D_t(q), D_f(q))$, where

$D_t(p) = \sum t_D(p) \wedge t_D(u_i)$, $D_f(p) = \sum f_D(p) \vee f_D(u_i)$

Here, u_i are adjacent nodes of u.

Definition 2.21: Let $\tilde{G} = (C, D)$ is a vague graph. A vertex p of $\tilde{G}$ is known as busy vertex if

$$t_A(p) \leq d_t(p), f_A(p) \geq d_t(p) \quad (18)$$

If it is not a busy node then it is said to be free vertex.

Definition 2.22: Let $\tilde{G} = (C, D)$ be a vague graph. Then a vague arc p, q is as effective vague arc if

$t_D(p,q) = t_C(p) \wedge t_C(q)$, $f_D(u,v) = f_C(p) \vee f_C(q)$

III. Dombi Vague graphs and its properties

Definition 3.1: Let $\tilde{G}' = (V_G, E_G)$ be a simple graph. Also, let $C : V_G \to [0,1]$ membership degree on V_G and $D : V_G \times V_G \to [0,1]$ be the membership degree on the symmetric fuzzy relation $E \subset V_G \times V_G$. Then $\tilde{G} = (V_G, C, D)$, is said to be a dombi graph if

$$D(pq) \leq \frac{C(p)C(q)}{C(p) + C(q) - C(p)C(q)}, \forall (pq) \in E_G.$$

Definition 3.2: Let $\tilde{G}' = (V_G, E_G)$ be a simple graph. Also, let $C = (t_C, f_C)$ s.t., $t_C : V_G \to [0,1]$, and $f_C : V_G \to [0,1]$ be the truth membership function and falsity function respectively on the dombi vague graph (DVG) V_G. The $D = (t_D, f_D)$ s.t., $t_D : V_G \times V_G \to [0,1]$, and $f_D : V_G \times V_G \to [0,1]$ are assumed as the truth membership function and falsity membership function respectively in the symmetric DVG $E_G \subset V_G \times V_G$. A vague graph $\tilde{G} = (V_G, C, D)$ is a DVG if $\forall (pq) \in E_G$

$$t_D(pq) \leq \frac{t_C(p)t_C(q)}{t_C(p) + t_C(q) - t_C(p)t_C(q)};$$

$$f_D(pq) \geq \frac{f_{C_G}(p) + f_{C_H}(q) - 2f_{C_G}(p)f_{C_H}(q)}{1 - f_{C_G}(p)f_{C_H}(q)}.$$

where $t_D(pq) + f_D(pq) \leqslant 1, \forall pq \in E_G (i, j = 1, 2, ...n)$.

Definition 3.3: Let $\tilde{G} = (V_G, C, D)$ be a DVG. Then the degree of its vertex i.e p_i in $G' = (A, B)$ is the summation of degree its Truth membership,indeterminacy membership and Falsity membership of all those edges which are incident on vertex p_i and it is denoted by:

$d(p_i) = (d_T(p_i), d_I(p_i), d_F(p_i))$;

1) $d_T(p_i) = \sum_{p_i \neq p_j} t_D(p_i p_j)$ denotes the degree of truth membership value.
2) $d_F(p_i) = \sum_{p_i \neq p_j} f_D(p_i p_j)$ denotes the degree of falsity membership value.

Definition 3.4: Let $\tilde{G}' = (V_G, E_G)$ be a simple graph. Also, let $C = (t_C, f_C)$ s.t., $t_C : V_G \rightarrow [0,1]$ and $f_C : V_G \rightarrow [0,1]$ be the truth membership degree and falsity membership degree respectively on V_G. The $D = (t_D, f_D)$ s.t., $t_D : V_G \times V_G \rightarrow [0,1]$ and $f_D : V_G \times V_G \rightarrow [0,1]$ are considered as the truth membership degree, and false membership degree respectively on $E_G \subset V_G \times V_G$. Then $\tilde{G} = (V_G, C, D)$ is defined as a strong DVG if $(\forall (pq) \in E_G)$

$$t_D(pq) = \frac{t_C(p)t_C(q)}{t_C(p) + t_C(q) - t_C(p)t_C(q)};$$

$$f_D(pq) = \frac{f_{C_G}(p) + f_{C_H}(q) - 2f_{C_G}(p)f_{C_H}(q)}{1 - f_{C_G}(p)f_{C_H}(q)},$$

where $t_D(pq) + f_D(pq) \leqslant 1, \forall pq \in E_G$

Definition 3.5: Let $\tilde{G}' = (V_G, E_G)$ be a simple graph. Also, let $C = (t_C, f_C)$, s.t., $t_C : V_G \rightarrow [0,1]$, and $f_C : V_G \rightarrow [0,1]$ be the truth membership degree and false membership degree respectively on V_G. The vague set $D = (t_D, f_D)$ s.t., $t_D : V_G \times V_G \rightarrow [0,1], and f_D : V_G \times V_G \rightarrow [0,1]$ as the truth membership degree, indeterministic membership degree and falsicity membership degree respectively, on $E_G \subset V_G \times V_G$. Then $\tilde{G} = (V_G, C, D)$, is said to be a complete DVG if $(\forall u,\ v \in V_G)$

$$t_D(pq) = \frac{t_C(p)t_C(q)}{t_C(p) + t_C(q) - t_C(p)t_C(q)};$$

$$f_D(pq) = \frac{f_{C_G}(p) + f_{C_H}(q) - 2f_{C_G}(p)f_{C_H}(q)}{1 - f_{C_G}(p)f_{C_H}(q)},$$

where $0 \leqslant t_D(pq) + I_D(pq) + f_D(pq) \leqslant 3, \forall u,\ v \in V_G$

Definition 3.6: Let $\tilde{G} = (V_G, C, D)$ be a DVG. Then the degree of its vertex v in V_G is defined as
$d(q) = (d_T(q), d_F(q))$, where for each $u \in V_G$

$d_T(q) = \sum_{v \neq u} t_D(pq)$ denotes the degree of truth membership degree,

$d_F(q) = \sum_{v \neq u} f_D(pq)$ denotes the degree of falsity membership degree.

The G is called **regular** DVG if for each $p, q \in V_G$

$$\sum_{u \neq v} t_D(pq) = \text{Constant},$$
$$\sum_{u \neq v} f_D(pq) = \text{Constant}.$$

Definition 3.7: The union of two DVG $\tilde{G} = (V_G, C, D)$ and $H = (V_G, C, D)$ of the graphs $\tilde{G}' = (V_G, E_G)$ and $H' = (V_H, E_H)$ respectively, is denoted by $G \cup H$ and is defined as $(V_G \cup V_H, C_G \cup C_H, D_G \cup D_H)$, where $C_G \cup C_H = (t_{C_G \cup C_H}, I_{C_G \cup C_H}, f_{C_G \cup C_H})$ and $D_G \cup D_H = (t_{D_G \cup D_H}, I_{D_G \cup D_H}, f_{D_G \cup D_H})$ s.t.,

1) $$(t_{C_G \cup C_H})(\phi) = \begin{cases} t_{C_G}(\phi),\ if\ \phi \in V_G - V_H \\ t_{C_H}(\phi),\ if\ \phi \in V_H - V_G \\ \dfrac{t_{C_G}(\phi) + t_{C_H}(\phi) - 2t_{C_G}(\phi)t_{C_H}(\phi)}{1 - t_{C_G}(\phi)t_{C_H}(\phi)}, \\ \qquad if\ \phi \in V_G \cap V_H. \end{cases}$$

$$(f_{C_G \cup C_H})(\phi) = \begin{cases} f_{C_G}(\phi),\ if\ \phi \in V_G - V_H \\ f_{C_H}(\phi),\ if\ \phi \in V_H - V_G \\ \dfrac{f_{C_G}(\phi)f_{C_H}(\phi)}{f_{C_G}(\phi) + f_{C_H}(\phi) - f_{C_G}(\phi)f_{C_H}(\phi)}, \\ \qquad if\ \phi \in V_G \cap V_H. \end{cases}$$

2) $$(t_{D_G \cup D_H})(pq) = \begin{cases} t_{D_G}(pq),\ if\ pq \in E_G - E_H \\ t_{D_H}(pq),\ if\ pq \in E_H - E_G \\ \dfrac{t_{D_G}(pq) + t_{D_H}(pq) - 2t_{D_G}(pq)t_{D_H}(pq)}{1 - t_{D_G}(pq)t_{D_H}(pq)}, \\ \qquad if\ pq \in E_G \cap E_H \end{cases}$$

$$(f_{D_G \cup D_H})(pq) = \begin{cases} f_{D_G}(pq),\ if\ pq \in E_G - E_H \\ f_{D_H}(pq),\ if\ pq \in E_H - E_G \\ \dfrac{f_{D_G}(pq)f_{D_H}(pq)}{f_{D_G}(pq) + f_{D_H}(pq) - f_{D_G}(pq)f_{D_H}(pq)}, \\ \qquad if\ pq \in E_G \cap E_H. \end{cases}$$

Definition 3.8: The Intersection of two DVG $\tilde{G} = (V_G, C_G, D_G)$ and $H = (V_H, C_H, D_H)$ of the graphs $\tilde{G}' = (V_G, E_G)$ and $H' = (V_H, E_H)$ respectively, is denoted by $G \cap H$ and is defined as $(V_G \cap V_H, C_G \cap C_H, D_G \cap D_H)$, where $C_G \cap C_H = (t_{C_G \cap C_H}, f_{C_G \cap C_H})$ and $D_G \cap D_H = (t_{D_G \cap D_H}, f_{D_G \cap D_H})$ s.t.,

1) $$(t_{C_G \cap C_H})(\phi) = \frac{t_{C_G}(\phi) + t_{C_H}(\phi) - 2t_{C_G}(\phi)t_{C_H}(\phi)}{1 - t_{C_G}(\phi)t_{C_H}(\phi)},\ if\ \phi \in V_G \cap V_H.$$

$$(f_{C_G \cap C_H})(\phi) = \frac{f_{C_G}(\phi)f_{C_H}(\phi)}{f_{C_G}(\phi) + f_{C_H}(\phi) - f_{C_G}(\phi)f_{C_H}(\phi)},\ if\ \phi \in V_G \cap V_H.$$

2) $$(t_{D_G \cap D_H})(pq) = \frac{t_{D_G}(pq) + t_{D_H}(pq) - 2t_{D_G}(pq)t_{D_H}(pq)}{1 - t_{D_G}(pq)t_{D_H}(pq)},\ if\ (pq) \in E_G \cap E_H.$$

$$(f_{D_G \cap D_H})(pq) = \frac{f_{D_G}(pq)f_{D_H}(pq)}{f_{D_G}(pq) + f_{D_H}(pq) - f_{D_G}(pq)f_{D_H}(pq)},\ if\ (pq) \in E_G \cap E_H.$$

Definition 3.9: The Cartesian product of two DVG $\tilde{G} = (V_G, C_G, D_G)$ and $H = (V_H, C_H, D_H)$ of the graphs $\tilde{G}' = (V_G, E_G)$ and $H' = (V_H, E_H)$ respectively, is denoted by $\tilde{G} \times H$ and is defined as $(V_G \times V_H, E, C_G \times C_H, D_G \times D_H)$, where $C_G \times C_H = (t_{C_G \times C_H}, f_{C_G \times C_H})$, $D_G \times D_H = (t_{D_G \times D_H}, f_{D_G \times D_H})$ and $E = \{((s_0, t_0)(s_0, t_1)) : s_0 \in V_G, (t_0 t_1) \in E_H\} \cup \{((s_0, t_0)(s_1, t_0)) : (s_0 s_1) \in E_G, t_0 \in$

$V_H\} \cup \{((s_0,t_0)(s_1,t_1)) : (s_0s_1) \in E_G, t_0 \neq t_1\}$ s.t.,

(i) $\forall(\kappa,\lambda) \in V_G \times V_H$,

a) $(t_{C_G \times C_H})(\kappa,\lambda)$ $= \dfrac{t_{C_G}(\kappa)t_{C_H}(\lambda)}{t_{C_G}(\kappa)+t_{C_H}(\lambda)-t_{C_G}(\kappa)t_{C_H}(\lambda)}$

b) $(f_{C_G \times C_H})(\kappa,\lambda)$ $= \dfrac{f_{C_G}(\kappa)+f_{C_H}(\lambda)-2f_{C_G}(\kappa)f_{C_H}(\lambda)}{1-f_{C_G}(\kappa)f_{C_H}(\lambda)}$

(ii) $\forall D \in V_G$ and $\forall(\kappa,\lambda) \in E_H$,

a) $(t_{D_G \times D_H})((D,\kappa)(D,\lambda))$ $= \dfrac{t_{C_G}(D)t_{D_H}(\kappa\lambda)}{t_{C_G}(D)+t_{D_H}(\kappa\lambda)-t_{C_G}(D)t_{D_H}(\kappa\lambda)}$;

b) $(f_{D_G \times D_H})((D,\kappa)(D,\lambda))$ $= \dfrac{f_{C_G}(D)+f_{D_H}(\kappa\lambda)-2f_{C_G}(D)f_{D_H}(\kappa\lambda)}{1-f_{C_G}(D)f_{D_H}(\kappa\lambda)}$;

(iii) $\forall D \in V_H$ and $\forall(\kappa,\lambda) \in E_G$,

a) $(t_{D_G \times D_H})((\kappa,D)(\lambda,D))$ $= \dfrac{t_{C_H}(D)t_{D_G}(\kappa\lambda)}{t_{C_H}(D)+t_{D_G}(\kappa\lambda)-t_{C_H}(D)t_{D_G}(\kappa\lambda)}$;

b) $(f_{D_G \times D_H})((\kappa,D)(\lambda,D))$ $= \dfrac{f_{C_H}(D)+f_{D_G}(\kappa\lambda)-2f_{C_H}(D)f_{D_G}(\kappa\lambda)}{1-f_{C_H}(D)f_{\Gamma_G}(\kappa\lambda)}$;

(iv) $\forall(\kappa,\lambda)(D,\alpha) \in (V_G \widetilde{\times} V_H)^2 - E$,

a) $(t_{D_G \times D_H})((\kappa,\lambda)(D,\alpha)) = 0$;

b) $(f_{D_G \times D_H})((\kappa,\lambda)(D,\alpha)) = 0$.

IV. Application of dombi vague graph in telecommunication

There exists several networks which are considered in our daily life. The graph theory acts a significant role in the modeling parts of these real life networks. Many of those networks are projected for distributing service purpose. Internet, telecommunication, power grid, and road transportation networks, railway are some basic examples of those real life networks. In the recent years, a phenomenal change occurs in the area of network research from a very small graphical model to a large scale networks.

In real world scenarios, ambiguity and vagueness may exist in almost every the networks/structures and they are not properly defined. The fuzzy graph is a very popular idea of graph theory to model the uncertain network. However, hesitation may present in the network in which the vertex/edge may or may not be belonged to the fuzzy set. The classical fuzzy graph is not properly model this kind of uncertainty (hesitation) only considering the single membership grade. Vague graph is more efficient to design this type of network when compared to the neutrosophic graph or fuzzy graph. We can design many real life networks which consist of relationship between entities using graph. The entities/objects are described by vertices/nodes and relationship among the node are described by arcs. If any network consists the uncertainties due to vague data about the entities (node) or relationships (arc) or both, then vague graph networking model is very much popular and useful to model such real life networks. vague graphs are very much widespread and significant in the field of artificial intelligence, social networks, mathematical modeling, optimization problem, pattern recognition, operation research, relational mapping, information systems and biological structure. It is easily to use the artificial intelligence with vague logic in inference system. These networks are considered in several different idea of data structures in different application fields internally. However, the primary mathematical model of those network modeling is the vague graph.

It is very to hard to the expert to modeling all those information carrying real life networks to a vague graph model considering different kinds of data from all possible existing levels. Expert needs to consider a multilevel method where all possible information are considered and analyzed mathematically in the time of modeling. Vague graphs have more real life applications associated to computer science such that these vague graphs are applied to illustrate chip design, wireless communication, data optimization, machine learning and much more. As graphs plays a major role in our everyday life for example in communication networks, web browsing, transportation networks and biological networks. Here in this section we are going to discuss some applications of vague graph in communication networks. We use vague graph to model a telephone network. The algorithm is described in the steps given as.

Algorithm 1 Pseudocode of vague telecommunication network

1: Setup the set of vertices $\tilde{V} = \{p_i, i = 1,2,3,\ldots,n\}$. In our case each $p_i = (C_{1i}, \gamma_{1i})$ represent a telephone number where C_{1i} is the tendency of calling while γ_{1i} is the tendency of not calling to other numbers.

2: Establish the set of edges $\tilde{E} = \{(C_{2ij}, \gamma_{2ij})\ for\ all\ i,j = 1,2,3,4,\ldots n.\}$ and calculate the degree of membership and non-membership for all (C_{2i}, γ_{2i}) using the relation $C_2(p_i,p_j) \leq \Lambda[C_1(p_i), C_1(p_j)]$ and $\gamma_2(p_i,p_j) \leq \vee[\gamma_1(p_i), \gamma_1(p_j)]$

3: Obtain a PF directed graph $\tilde{G} = \left\langle V, \tilde{E} \right\rangle$

Example 1: Consider $\tilde{V} = p_1 = (0.7,0.2), p_2 = (0.8,0.2), p_3 = (0.2,0.6), p_4 = (0.7,0.2), p_5 = (0.6,0.4)$, the set of 5 telephone numbers. $\tilde{E} = (p_1,p_2), (p_1,p_3), (p_2,p_3), (p_2,p_5), (p_4,p_3), (p_4,p_5), (p_5,p_1), (p_5,p_3)$ be the set of edges whose values are given in the Table I. It is shown in Figure 1.

The analysis is clear and it shows that the calling ratio from p_2 to p_5 is maximum while p_3 didnt make any call according to our analysis. Overall the maximum number of calls were made by p_5 as its degree is the highest i.e. 0.9.

V. Conclusion

In this paper we introduced the ideas of vague dombi graph and investigated on some of its properties. Complement of vague signed graph is analyzed. Also we defined the concept of frustration index and frustration number. We also focus on the areas of vague signed graph applications in real time

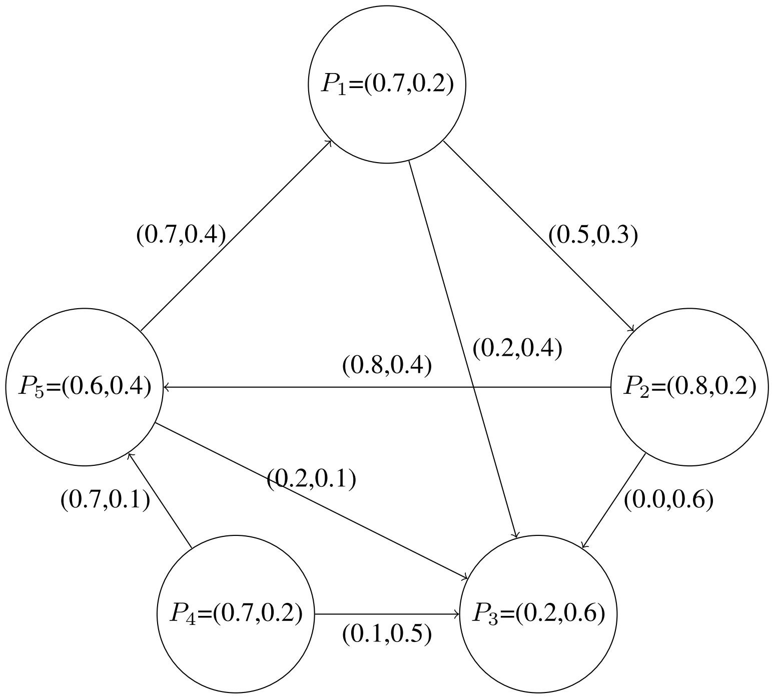

Fig. 1: An example of vague graph in telecommunication

TABLE I: Value of the true and false membership.

Edge	$TrueMembership$	$FalseMembership$
(p_1, p_2)	0.5	0.3
(p_1, p_3)	0.2	0.4
(p_2, p_3)	0.0	0.6
(p_2, p_5)	0.6	0.4
(p_4, p_3)	0.1	0.5
(p_4, p_5)	0.7	0.1
(p_5, p_1)	0.7	0.4
(p_5, p_3)	0.2	0.0

situations. In future one can develop the notions of signed graph with any other uncertain graphs and extend the results established in this paper.

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