A New Method for Multi-Attribute Decision-Making Based on Single-Valued Neutrosophic Sets

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***Abstract*—A single-valued neutrosophic set is a special case of neutrosophic set, which can describe a great deal of imprecise, uncertain and inconsistent information in the real world. Aiming at the problem of multi-attribute decision-making with the decision information presented as the form of single-valued neutrosophic sets (SVNSs). In this paper, a new method for multi-attribute decision-making (MADM) under single-valued neutrosophic environment is proposed. Firstly, the concepts of neutrosophic sets and single-valued neutrosophic sets are introduced, a distance formula based on single-valued neutrosophic sets is given. Secondly, the weighted comprehensive decision matrix is constructed. In addition, TOPSIS method is extended to single-valued neutrosophic environment, the relative closeness coefficient is used to sort the alternatives, the optimal decision-making scheme is obtained. Finally, a numerical example is given to illustrate the feasibility and effectiveness of the proposed method.**

***Index Terms*—neutrosophic sets, single-valued neutrosophic sets, TOPSIS method, decision matrix, multi-attribute decision-making.**

I. Introduction

With the continuous development of society, along with the diversity, complexity and uncertainty of a large amount of information, the multi-attribute decision-making (MADM) problem is becoming more and more complicated. MADM is an important part of modern decision-making science, which has important applications in economy, management, design, military and other fields. However, in the real world, due to the ambiguity and complexity of decision-making problems, it is difficult for people to give accurate data. After Zadeh [1] put forward the concept of fuzzy set, the problem of fuzzy multi-criterion decision-making has been studied extensively, but fuzzy sets can only represent fuzzy information by a dimension of membership degree, which still can not solve many uncertain problems in real life. Atanassov [2] came up with intuitionistic fuzzy sets, which used the information of membership, non membership and hesitation to represent uncertain information, and solved many decision-making problems. Then, Atanassov [3] extended the intuitionistic fuzzy set to interval intuitionistic fuzzy set. It is more convenient to deal with uncertain fuzzy information. At present, intuitionistic fuzzy set has been widely used in decision science [4, 5, 6, 7]. Wang et al. [8] introduced three memberships of neutrosophic sets (NSs) to standard units subinterval and also proposed SVNSs, SVNS is a subclass of NS, which can be easily applied to practical science and engineering field, in order to deal with incomplete, inconsistent and inaccurate fuzzy information, just a few short years time, SVNSs have a wide range of applications in the field of science and technology.

Ye [9, 10] proposed a weighted correlation coefficient of SVNSs and cross entropy of SVNSs, and studied its application in MADM. At the same time, Ye [11, 12, 13] expanded some classical operators to aggregate the intelligence set information, proposed the aggregation operator of SVNSs, studied the similarity measure between SVNSs, and explored their application in clustering analysis and decision-making. Li et al. [14] proposed several Heronian mean operators of SVNSs and applied them to MADM problems. Pramanik et al. [15] introduced a series of mixed vector similarity measures of SVNSs in the MADM analysis and also applied them to MADM problems, which further expanded the application of SVNSs. Abdel-Basset [16, 17, 18] applied SVNSs and rough set in the information system of smart city construction, then combined the NSs with DEMATEL method to establish the criteria of supplier selection, at the same time, NSs and analytic hierarchy process are applied to strategic planning and decision-making. For the case where the interval neutrosophic sets and attribute weight are unknown, Broumi [19] proposed an improved TOPSIS decision-making method. Li and Lin [20] gave a new grey hesitant fuzzy set, which extended the hesitant fuzzy set to the field of grey set. On this basis, the TOPSIS decision-making method of grey relation was put forward. Based on the cross-entropy, Tian and Zhang [21] proposed a decision-making method of interval neutrosophic sets. Wang and Li [22] defined the expected value of multi-valued neutrosophic sets and Hamming distance. Combined with TODIM method, multi-criteria decision-making method based on multi-valued neutrosophic sets was put forward.

The main purpose of this paper is to extend the TOPSIS method to SVNSs environment and apply it to MADM problems. Firstly, the concept of NSs is given, the generalized distance formula of SVNSs is defined. Then the corresponding relationship between language variables and SVNSs are given to determine the attributes of decision-making experts. According to expert ratings of each at-

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tribute, the weights of the attributes are calculated. Next, the weighted comprehensive decision matrix is constructed. Furthermore, the distance between all alternatives and the positive and negative ideal solution is calculated. The relative closeness degree function is used to comprehensively sort the alternatives, and then the optimal decision scheme is obtained. Finally, an illustrative example for optimization of investment decision scheme is used to verify its effectiveness and feasibility.

II. Preliminaries

A. Neutrosophic Set

Definition 1 [23] Let X be a universal space of points (objects), with a generic element in X denoted by x. A neutrosophic set(NS) A $\subset X$ is characterized by truth-membership function $T_{A(x)}$, indeterminacy-membership function $I_{A(x)}$ and falsity-membership function $F_{A(x)}$, where $T_{A(x)}$, $I_{A(x)}$, $F_{A(x)}$ are real standard or nonstandard subsets of $[^-0, 1^+]$, so that it means $T_{A(x)}$: X$\rightarrow$ $[^-0, 1^+]$, $I_{A(x)}$: X$\rightarrow$ $[^-0, 1^+]$, $F_{A(x)}$: X$\rightarrow$ $[^-0, 1^+]$.

That is to say, A NS can be expressed as

$$\text{A} = \left\{ \left[\langle x, T_{A(x)}, I_{A(x)}, F_{A(x)} \rangle \right] | x \in X \right\}$$

The sum of three independent membership degrees $T_{A(x)}$, $I_{A(x)}$ and $F_{A(x)}$ are related as follows:

$$^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$$

Definition 2 [24] The complement of a NS A is denoted by A^C and is defined as

$$T_{A^C}(x) = \{1^+\} \ominus T_A(x), I_{A^C}(x) = \{1^+\} \ominus I_A(x),$$
$$F_{A^C}(x) = \{1^+\} \ominus F_A(x)$$

for all $x \in X$.

Definition 3 [25] A NS A is contained in the other NS B, $A \subseteq B$ if and only if

$$\inf T_A(x) \le \inf T_B(x), \sup T_A(x) \le \sup T_B(x),$$
$$\inf I_A(x) \ge \inf I_B(x), \sup I_A(x) \ge \sup I_B(x),$$
$$\inf F_A(x) \ge \inf F_B(x), \sup F_A(x) \ge \sup F_B(x)$$

for all $x \in X$.

B. Single-Valued Neutrosophic Set

Definition 4 [8] Let X be a universal space of points (objects), with a generic element in X denoted by x. A single-valued neutrosophic set (SVNS) A $\subset X$ is characterized by truth-membership function $T_{A(x)}$, indeterminacy-membership function $I_{A(x)}$ and falsity-membership function $F_{A(x)}$. A SVNS can be expressed as

$$\text{A} = \left\{ \left[\langle x, T_{A(x)}, I_{A(x)}, F_{A(x)} \rangle \right] | x \in X \right\}$$

where $T_{A(x)}$, $I_{A(x)}$, $F_{A(x)}$ are real standard or nonstandard subsets of $[0, 1]$, so that it means $T_{A(x)}$: X$\rightarrow$ $[0, 1]$, $I_{A(x)}$: X$\rightarrow$ $[0, 1]$, $F_{A(x)}$: X$\rightarrow$ $[0, 1]$, with the condition of $0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3$, for all $x \in X$.

When X is continuous, a SVNS A can be written as

$$\text{A} = \int_X \langle T_{A(x)}, I_{A(x)}, F_{A(x)} \rangle / x, \quad x \in X;$$

When X is discrete, a SVNS A can be written as

$$\text{A} = \sum_{i=1}^{n} \langle T_{A(x_i)}, I_{A(x_i)}, F_{A(x_i)} \rangle / x_i, \quad x_i \in X.$$

Definition 5 [26, 27, 28, 35, 36, 37] Let A and B be two SVNSs, A $= \langle T_{A(x)}, I_{A(x)}, F_{A(x)} \rangle$, B $= \langle T_{B(x)}, I_{B(x)}, F_{B(x)} \rangle$, then $\forall\ x \in X$, $\lambda \in R$ and $\lambda > 0$, there is

(1)A$\oplus$B $= \langle T_{A(x)} + T_{B(x)} - T_{A(x)} \cdot T_{B(x)}, I_{A(x)} \cdot I_{B(x)}, F_{A(x)} \cdot F_{B(x)} \rangle$

(2)A$\otimes$B $= \langle T_{A(x)} \cdot T_{B(x)}, I_{A(x)} + I_{B(x)} - I_{A(x)} \cdot I_{B(x)}, F_{A(x)} + F_{B(x)} - F_{A(x)} \cdot F_{B(x)} \rangle$

(3)λA $= (1 - (1 - T_{A(x)})^\lambda, {I_{A(x)}}^\lambda, {F_{A(x)}}^\lambda)$

(4)$A^\lambda = ({T_{A(x)}}^\lambda, 1 - (1 - I_{A(x)})^\lambda, 1 - (1 - F_{A(x)})^\lambda)$

theorem 1 Let A $= \langle T_{A(x)}, I_{A(x)}, F_{A(x)} \rangle$, B $= \langle T_{B(x)}, I_{B(x)}, F_{B(x)} \rangle$ and C $= \langle T_{C(x)}, I_{C(x)}, F_{C(x)} \rangle$ be three SVNSs, Peng, Wang et al. [28] proposed some properties as follows:

(1)A$\oplus$B = B$\oplus$A

(2)A$\otimes$B = B$\otimes$A

(3)$\lambda(A \oplus B) = \lambda A \oplus \lambda B, \lambda > 0$

(4)$(A \otimes B)^\lambda = A^\lambda \otimes B^\lambda, \lambda > 0$

(5)$\lambda_1 A \oplus \lambda_2 A = (\lambda_1 + \lambda_2) A, \lambda_1 > 0, \lambda_2 > 0$

(6)$A^{\lambda_1} \otimes A^{\lambda_2} = A^{\lambda_1 + \lambda_2}, \lambda_1 > 0, \lambda_2 > 0$

(7)$(A \oplus B) \oplus C = A \oplus (B \oplus C)$

(8)$(A \otimes B) \otimes C = A \otimes (B \otimes C)$

It can be proved that the theorem is valid, and the proof process is omitted.

Definition 6 [8] Let A and B be two SVNSs, then$\forall\ x \in X$, operations can be defined as follows:

(1)A SVNS A is contained in the other SVNS B, denoted as A $\subseteq$ B, iff, $T_A(x) \le T_B(x), I_A(x) \ge I_B(x), F_A(x) \ge F_B(x)$, for all $x \in X$.

(2)Two SVNSs A and B are equal, denoted as A = B, iff, $A \subseteq B$ and $B \subseteq A$, for all $x \in X$.

(3)The complement of a SVNS A is denoted by A^c and is defined by $T_{A^c}(x) = F_A(x)$, $I_{A^c}(x) = 1 - I_A(x)$, $F_{A^c}(x) = T_A(x)$, for all $x \in X$.

(4)A $\bigcup$ B$=\langle max(T_{A(x)}, T_{B(x)}), min(I_{A(x)}, I_{B(x)}), min(F_{A(x)}, F_{B(x)}) \rangle$, for all $x \in X$.

(5)A $\bigcap$ B$=\langle min(T_{A(x)}, T_{B(x)}), max(I_{A(x)}, I_{B(x)}), max(F_{A(x)}, F_{B(x)}) \rangle$, for all $x \in X$.

C. Distance Between Two SVNSs

Majumder [26] and Broumi [29] studied similarity and entropy measures by incorporating euclidean distances of neutrosophic sets. In this paper, we extend the concept of the distance of a SVNS and give the generalized distance formula of a SVNS.

Definition 7 (Hamming distance) Let $A = \sum_{i=1}^{n} \langle T_{A(x_i)}, I_{A(x_i)}, F_{A(x_i)} \rangle$ and $B = \sum_{i=1}^{n} \langle T_{B(x_i)}, I_{B(x_i)}, F_{B(x_i)} \rangle$ be two SVNSs for $x_i \in X (i = 1, 2, \ldots, n)$, then the Hamming distance between two SVNSs A and B can be defined as follows:

$$\begin{aligned} D_{Hamm}(A, B) = \sum_{i=1}^{n} \{ & |T_{A(x_i)} - T_{B(x_i)}| \\ & + |I_{A(x_i)} - I_{B(x_i)}| + |F_{A(x_i)} - F_{B(x_i)}| \} \end{aligned} \tag{1}$$

and the normalized Hamming distance between two SVNSs A and B can be defined as follows:

$$D_{Hamm}^{N}(A,B)=\frac{1}{3n}\sum_{i=1}^{n}\{|T_{A(x_i)}-T_{B(x_i)}| + |I_{A(x_i)}-I_{B(x_i)}|+|F_{A(x_i)}-F_{B(x_i)}|\} \tag{2}$$

where $0<D_{Hamm}^{N}(A,B)<1$.

Definition 8 (Euclidean distance) Let $A=\sum_{i=1}^{n}\langle T_{A(x_i)}, I_{A(x_i)}, F_{A(x_i)}\rangle$ and $B=\sum_{i=1}^{n}\langle T_{B(x_i)}, I_{B(x_i)}, F_{B(x_i)}\rangle$ be two SVNSs for $x_i\in X(i=1,2,\ldots,n)$, then the Euclidean distance between two SVNSs A and B can be defined as follows:

$$D_{Eucl}(A,B)=\{\sum_{i=1}^{n}[(T_{A(x_i)}-T_{B(x_i)})^2 + (I_{A(x_i)}-I_{B(x_i)})^2+(F_{A(x_i)}-F_{B(x_i)})^2]\}^{\frac{1}{2}} \tag{3}$$

and the normalized Euclidean distance between two SVNSs A and B can be defined as follows:

$$D_{Eucl}^{N}(A,B)=\{\frac{1}{3n}\sum_{i=1}^{n}[(T_{A(x_i)}-T_{B(x_i)})^2 + (I_{A(x_i)}-I_{B(x_i)})^2+(F_{A(x_i)}-F_{B(x_i)})^2]\}^{\frac{1}{2}} \tag{4}$$

where $0<D_{Eucl}^{N}(A,B)<1$.

Definition 9 (Weighted Hamming(Euclidean)distance) Let $A=\sum_{i=1}^{n}\langle T_{A(x_i)}, I_{A(x_i)}, F_{A(x_i)}\rangle$ and $B=\sum_{i=1}^{n}\langle T_{B(x_i)}, I_{B(x_i)}, F_{B(x_i)}\rangle$ be two SVNSs for $x_i\in X(i=1,2,\ldots,n)$, the weight of x_i is w_i, then the normalized weighted Hamming distance between two SVNSs A and B can be defined as follows:

$$D_{Hamm}^{NW}(A,B)=\frac{1}{3n}\sum_{i=1}^{n}\{\omega_i[|T_{A(x_i)}-T_{B(x_i)}| + |I_{A(x_i)}-I_{B(x_i)}|+|F_{A(x_i)}-F_{B(x_i)}|]\} \tag{5}$$

and the normalized weighted Euclidean distance between two SVNSs A and B can be defined as follows:

$$D_{Eucl}^{NW}(A,B)=\{\frac{1}{3n}\sum_{i=1}^{n}\{\omega_i[(T_{A(x_i)}-T_{B(x_i)})^2 + (I_{A(x_i)}-I_{B(x_i)})^2+(F_{A(x_i)}-F_{B(x_i)})^2]\}\}^{\frac{1}{2}} \tag{6}$$

where $0<D_{Hamm}^{NW}(A,B)<1$, $0<D_{Eucl}^{NW}(A,B)<1$.

The Hamming distance and Euclidean distance both take into account truth-membership degree, indeterminacy membership degree and falsity-membership degree, which can objectively reflect the distance between two SVNSs. On this basis, this paper extends the distance formula of two SVNSs and gives the generalized distance formula of two SVNSs as follows:

Definition 10 (Generalized distance) Let $A=\sum_{i=1}^{n}\langle T_{A(x_i)}, I_{A(x_i)}, F_{A(x_i)}\rangle$ and $B=\sum_{i=1}^{n}\langle T_{B(x_i)}, I_{B(x_i)}, F_{B(x_i)}\rangle$ be two SVNSs for $x_i\in X(i=1,2,\ldots,n)$, P is any nonzero positive real number, the weight of x_i is w_i, then the generalized distance between two SVNSs A and B can be defined as follows:

$$D_1^{G}(A,B)=\{\sum_{i=1}^{n}(|T_{A(x_i)}-T_{B(x_i)}|^p+|I_{A(x_i)}-I_{B(x_i)}|^p + |F_{A(x_i)}-F_{B(x_i)}|^p)\}^{\frac{1}{p}} \tag{7}$$

and the generalized normalized distance between two SVNSs A and B can be defined as follows:

$$D_2^{G}(A,B)=\{\frac{1}{3n}\sum_{i=1}^{n}(|T_{A(x_i)}-T_{B(x_i)}|^p + |I_{A(x_i)}-I_{B(x_i)}|^p+|F_{A(x_i)}-F_{B(x_i)}|^p)\}^{\frac{1}{p}} \tag{8}$$

and the generalized normalized weighed distance between two SVNSs A and B can be defined as follows:

$$D_3^{G}(A,B)=\{\frac{1}{3n}\sum_{i=1}^{n}[\omega_i(|T_{A(x_i)}-T_{B(x_i)}|^p + |I_{A(x_i)}-I_{B(x_i)}|^p+|F_{A(x_i)}-F_{B(x_i)}|^p)]\}^{\frac{1}{p}} \tag{9}$$

If $p=1$, the formula (8) is degraded as formula (2), the formula (9) is degraded as formula (5). If $p=2$, the formula (8) is degraded as formula (4), the formula (9) is degraded as formula (6). Considering the weight of elements at the same time, the distance between two SVNSs can be objectively reflected , which is closer to the truth value in the actual decision problem.

The generalized distance formula $D_i^{G}(A,B)(i=1,2,3)$ of the two SVNSs defined above obviously satisfies the following distance criterion:

Standard 1 (Non-negativity): $D_i^{G}(A,B)\geq 0$

Standard 2 (Polarity): $D_i^{G}(A,B)=0\Leftrightarrow A=B$

Standard 3 (Symmetry): $D_i^{G}(A,B)=D_i^{G}(B,A)$

Standard 4 (Monotonicity): if$A\subseteq B\subseteq C$, then $D_i^{G}(A,C)\geq max\{D_i^{G}(A,B),D_i^{G}(B,C)\}$

D. Entropy of SVNSs

Entropy [30] is an important research topic in fuzzy set theory, it is a degree of uncertainty. For entropy of SVNSs, we should take into account the following three factors: truth-membership degree, indeterminacy-membership degree and falsity-membership degree.

Definition 11 Let $D=(a_{ij})_{m\times n}$ be a decision matrix, if $C_j(j=1,2,\ldots,n)$ be the beneficent attribute, then the decision matrix remains unchanged. Otherwise, $D=(a_{ij})_{m\times n}$ is standardized as follows to obtain the standard SVN decision matrix $\tilde{D}=(\tilde{a}_{ij})_{m\times n}$. Where, if C_j is the beneficent attribute, $\tilde{a}_{ij}=a_{ij}$. If C_j is the cost attribute, $\tilde{a}_{ij}=a_{ij}^{c}$.

Definition 12 Let $\tilde{a}_{ij}=\langle\alpha_1,\alpha_2,\alpha_3\rangle$ is a single-valued neutrosophic number, trigonometric functions are used to construct the following information entropy measure formula:

$$E_1(\tilde{a}_{ij})=\frac{1}{3(\sqrt{2}-1)}\sum_{t=1}^{3}(\sin\frac{\alpha_t-{\alpha_t}^c+1}{4}\pi + \cos\frac{\alpha_t-{\alpha_t}^c+1}{4}\pi-1) \tag{10}$$

$$E(C_j)=\frac{1}{m}\sum_{i=1}^{m}E_1(\tilde{a}_{ij}), j=1,2,\ldots,n. \tag{11}$$

In the multi-attribute decision-making problems, the properties of each alternative are often complex and ambiguous, different attributes have different degrees of importance in the decision-making process. Based on the above considerations, according to the lagrange multiplier method, the weight ω_j calculation formula of each attribute C_j can be obtained, as follows:

$$\omega_j = \frac{(E(C_j))^{-1}}{\sum_{j=1}^{n}(E(C_j))^{-1}} \tag{12}$$

III. Problem Description

When investors choose enterprises for investment, they need to consider more and more aspects, such as enterprise output value index, enterprise sales volume index, environmental pollution index, employees' education level and so on. Most of these properties are real numbers, only by correctly evaluating the investment objectives, we can choose the appropriate investment objects and maximize the profits. Before investment, investors will investigate several attribute values of each enterprise, and then choose the most satisfied enterprise for investment. In this paper, we assume that investors want to invest in a business. At this time, we can choose m enterprises to invest. Before investment, investors will investigate the n attribute values of each enterprise and select the most satisfied enterprise for investment. In a nutshell, let $A = \{A_1, A_2, \ldots, A_n\}$ is a finite set of schemes($n \geq 2$), where A_i represents the ith alternative. $C = \{C_1, C_2, \ldots, C_m\}$ is a collection of attributes ($m \geq 2$), where C_j represents the jth attribute. $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of the attribute, where ω_j represents the importance of the attribute C_j and meets the condition of $\omega_j \geq 0$. Based on the above description, investors who want to choose the best enterprises from many investments will analyze the different attributes of different enterprises and prioritize these schemes according to the known theories, so as to make the best decision.

IV. TOPSIS Method for MADM based on SVNSs

TOPSIS [31] is short for Technique for Order Preference by Similarity to Ideal Solution, this is, a evaluation method for order preference by similarity to ideal solution, it was proposed by Hwang C. L and Yoon K in 1981. The solution principle of TOPSIS is based on the concept of positive and negative ideal solutions. In other words, in the alternative scheme, it is found that the solution which is closest to the positive ideal solution, but not necessarily far away from the negative ideal solution. Therefore, the concept of closeness degree is put forward by combining the two indexes. Then, according to the value of closeness, the alternatives are sorted and optimized.

In the MADM problem, there are usually decision experts, alternatives, and scheme attributes. Next, we consider that there are t experts who choose the best scheme from m alternatives by analyzing n attributes of m alternatives in the SVNSs environment. Let $A = \{A_1, A_2, \ldots, A_m\}$ is a discrete set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ is a set of attributes, the ratings provided by decision makers describe the performance of alternative A_i relative to attribute C_j, the weight of C_j is ω_j, $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ is a set of weight vectors, where $0 \leq \omega_j \leq 1$, $\sum_{j=1}^{n} \omega_j = 1$, $1 \leq j \leq n$. $D = \{D_1, D_2, \ldots, D_t\}$ is a set of decision experts, the weight of each expert is e_k, $e = \{e_1, e_2, \ldots, e_t\}$ is a set of expert weights, where $0 \leq e_k \leq 1$, $\sum_{k=1}^{t} e_k = 1$. The values related to the scheme of the MADM problem can be expressed in the following decision matrix:

$$D = (d_{ij})_{m\times n} = \begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mn} \end{pmatrix} \end{array} \tag{13}$$

In the decision matrix, d_{ij} is the attribute value of the ith scheme under the jth attribute, where $d_{ij} \geq 0$, $i \in \{1, 2, \ldots, m\}$, $j \in \{1, 2, \ldots, n\}$.

The following gives the complete TOPSIS method (Figure 1).

Step 1: Determining the weights of decision makers;

In the MADM problem, the optimization of investment decision scheme is very important. In general, there are t experts to investigate n attributes of m alternatives and then give the optimal alternative. Due to the differences in the knowledge structure and qualification level ofeach decision maker, the importance of each decision maker should be rated, which is generally expressed by language variables. A language variable is a variable whose value is replaced by a descriptive language. Here, we consider the language variables in the SVNSs environment, the corresponding relationship given in reference [32] is adopted, so their decision rights are treated as the linguistic terms shown in table I.

TABLE I
LINGUISTIC TERMS FOR RATING OF ATTRIBUTES AND DECISION MAKERS

Linguistic	SVNNs
Very good/very important (VG/VI)	$< 0.90, 0.10, 0.10 >$
Good/important (G/I)	$< 0.80, 0.20, 0.15 >$
Fair/medium (F/M)	$< 0.50, 0.40, 0.45 >$
Bad/unimportant (B/UI)	$< 0.35, 0.60, 0.70 >$
Very bad/very unimportant (VB/VUI)	$< 0.10, 0.80, 0.90 >$

The linguistic term along with single-valued neutrosophic numbers (SVNNs) is defined in Table II to rate n attributes of m alternatives.

TABLE II
LINGUISTIC TERMS FOR RATING THE CANDIDATES WITH SVNNs

Linguistic terms	SVNNs
Extremely good/high (EG/EH)	$< 1.00, 0.00, 0.00 >$
Very good/high (VG/VH)	$< 0.90, 0.10, 0.05 >$
Good/high (G/H)	$< 0.80, 0.20, 0.15 >$
Medium good/high (MG/MH)	$< 0.65, 0.35, 0.30 >$
Medium/fair (M/F)	$< 0.50, 0.50, 0.45 >$
Medium bad/medium law (MB/ML)	$< 0.35, 0.65, 0.60 >$
Bad/law (B/L)	$< 0.20, 0.75, 0.80 >$
Very bad/low (VB/VL)	$< 0.10, 0.85, 0.90 >$
Very very bad/low (VVB/VVL)	$< 0.05, 0.90, 0.95 >$

We assume that the importance of each decision maker is treated as a linguistic variable, which corresponds to a

SVNN. Let $E_k = \langle T_k, I_k, F_k\rangle$ be a SVNN defined for the rating of kth decision maker, where T_k is the degree of truth, I_k is the indeterminacy degree and F_k is the falsity degree. Of course, we want to know as much information as possible, the determination part of preference information contained in indeterminacy degree I_k should also be taken into account, so that the evaluation is more objective and closer to the reality. Based on this consideration, the weight of the kth decision maker can be defined as follows [33]:

$$e_k = \frac{\mu_k}{\sum\limits_{k=1}^{t}\mu_k} = \frac{T_k + I_k\cdot(\frac{T_k}{T_k+F_k})}{\sum\limits_{k=1}^{t}[T_k + I_k\cdot(\frac{T_k}{T_k+F_k})]} \tag{14}$$

$$\sum_{k=1}^{t} e_k = 1, \quad 0 \le e_k \le 1.$$

Then, the weight vector of t decision makers is obtained as follows:

$$e = \{e_1, e_2, \dots, e_t\} \tag{15}$$

Step 2: Constructing the aggregated SVN decision matrix based on the assessments from each decision maker;

Let $D^k = (d_{ij}^k)_{m\times n}$ is the SVN decision matrix of the kth decision maker, $e = \{e_1, e_2, \dots, e_t\}$ is the weight vector for decision makers. In the group decision making process, all individual evaluations need to consider the weight vector, so as to generate the NS decision matrix that aggregates the views of each decision maker. The decision matrix can be expressed as [34] $D = (d_{ij})_{m\times n}$, where

$$d_{ij} = \langle 1 - \prod_{k=1}^{t}(1 - T_{ij}^t)^{e_k}, \prod_{k=1}^{t}(I_{ij}^t)^{e_k}, \prod_{k=1}^{t}(F_{ij}^t)^{e_k}\rangle \tag{16}$$

So,

$$D = (d_{ij})_{m\times n} = \langle T_{ij}, I_{ij}, F_{ij}\rangle_{m\times n} =$$

$$\begin{array}{c c} & \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{pmatrix} \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \cdots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \cdots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ \vdots & \vdots & \cdots & \vdots \\ \langle T_{m1}, I_{m1}, F_{m1}\rangle & \langle T_{m2}, I_{m2}, F_{m2}\rangle & \cdots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{pmatrix} \end{array} \tag{17}$$

for $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$.

Step 3: Determining the attribute weights;

In the decision-making process, the decision maker may think that the attributes in the evaluation scheme are not equally important, so each decision maker has its own opinion on the selection of attribute values. Because the attribute weight is completely unknown, it conforms to the entropy property of fuzzy theory. Therefore, we use Equ.(11) and Equ.(12) to determine the weights of n attributes $\omega = \{\omega_1, \omega_2, \dots, \omega_n\}$, where ω_j is the weight of the nth attribute.

Step 4: Determining the positive ideal scheme and the negative ideal scheme for SVNSs;

Let J_1 is the benefit-type attribute, J_2 is the cost-type attribute, A^+ is the positive ideal scheme and A^- is the negative ideal scheme. Then A^+ is defined as

$$\begin{aligned} A^+ &= [d_1^{w+}, d_2^{w+}, \dots, d_n^{w+}] \\ &= [\langle T_1^{w+}, I_1^{w+}, F_1^{w+}\rangle, \langle T_2^{w+}, I_2^{w+}, F_2^{w+}\rangle, \\ &\quad \dots, \langle T_n^{w+}, I_n^{w+}, F_n^{w+}\rangle] \end{aligned} \tag{18}$$

where

$$\begin{aligned} T_j^{w+} &= \{(\max_i\{T_{ij}^{\omega_j}\} \mid j\in J_1), (\min_i\{T_{ij}^{\omega_j}\} \mid j\in J_2)\} \\ I_j^{w+} &= \{(\min_i\{I_{ij}^{\omega_j}\} \mid j\in J_1), (\max_i\{I_{ij}^{\omega_j}\} \mid j\in J_2)\} \\ F_j^{w+} &= \{(\min_i\{F_{ij}^{\omega_j}\} \mid j\in J_1), (\max_i\{F_{ij}^{\omega_j}\} \mid j\in J_2)\} \end{aligned} \tag{19}$$

and A^- is defined as

$$\begin{aligned} A^- &= [d_1^{w-}, d_2^{w-}, \dots, d_n^{w-}] \\ &= [\langle T_1^{w-}, I_1^{w-}, F_1^{w-}\rangle, \langle T_2^{w-}, I_2^{w-}, F_2^{w-}\rangle, \\ &\quad \dots, \langle T_n^{w-}, I_n^{w-}, F_n^{w-}\rangle] \end{aligned} \tag{20}$$

where

$$\begin{aligned} T_j^{w-} &= \{(\min_i\{T_{ij}^{\omega_j}\} \mid j\in J_1), (\max_i\{T_{ij}^{\omega_j}\} \mid j\in J_2)\} \\ I_j^{w-} &= \{(\max_i\{I_{ij}^{\omega_j}\} \mid j\in J_1), (\min_i\{I_{ij}^{\omega_j}\} \mid j\in J_2)\} \\ F_j^{w-} &= \{(\max_i\{F_{ij}^{\omega_j}\} \mid j\in J_1), (\min_i\{F_{ij}^{\omega_j}\} \mid j\in J_2)\} \end{aligned} \tag{21}$$

Step 5: Calculating the distance of each alternative from the positive ideal scheme and the negative ideal scheme for SVNSs;

Let $D_i^+(d_{ij}^{\omega_j}, d_j^{w+})$ is the distance measure of each alternative $\langle T_{ij}^{\omega_j}, I_{ij}^{\omega_j}, F_{ij}^{\omega_j}\rangle$ from the positive ideal scheme $\langle T_j^{w+}, I_j^{w+}, F_j^{w+}\rangle$ and $D_i^-(d_{ij}^{\omega_j}, d_j^{w-})$ is the distance measure of each alternative $\langle T_{ij}^{\omega_j}, I_{ij}^{\omega_j}, F_{ij}^{\omega_j}\rangle$ from the negative ideal scheme $\langle T_j^{w-}, I_j^{w-}, F_j^{w-}\rangle$. Similar to Equ.(9), $D_i^+(d_{ij}^{\omega_j}, d_j^{w+})$ can be written as

$$\begin{aligned} D_i^+(d_{ij}^{\omega_j}, d_j^{w+}) &= \{\frac{1}{3n}\sum_{i=1}^{n}[\omega_i(|T_{ij}^{\omega_j}(x_j) - T_j^{w+}(x_j)|^p \\ &+ |I_{ij}^{\omega_j}(x_j) - I_j^{w+}(x_j)|^p + |F_{ij}^{\omega_j}(x_j) - F_j^{w+}(x_j)|^p)]\}^{\frac{1}{p}} \end{aligned} \tag{22}$$

Similarly, $D_i^-(d_{ij}^{\omega_j}, d_j^{w-})$ can be written as

$$\begin{aligned} D_i^-(d_{ij}^{\omega_j}, d_j^{w-}) &= \{\frac{1}{3n}\sum_{i=1}^{n}[\omega_i(|T_{ij}^{\omega_j}(x_j) - T_j^{w-}(x_j)|^p \\ &+ |I_{ij}^{\omega_j}(x_j) - I_j^{w-}(x_j)|^p + |F_{ij}^{\omega_j}(x_j) - F_j^{w-}(x_j)|^p)]\}^{\frac{1}{p}} \end{aligned} \tag{23}$$

By changing the value of positive number p, the accuracy of operation can be adjusted, but the final result will not be affected.

Step 6: Calculating the relative closeness coefficient;

The relative closeness coefficient of each alternative A_i relative to positive ideal scheme A^+ is defined as follows:

$$C_i^* = \frac{D_i^-(d_{ij}^{\omega_j}, d_j^{w-})}{D_i^+(d_{ij}^{\omega_j}, d_j^{w+}) + D_i^-(d_{ij}^{\omega_j}, d_j^{w-})} \tag{24}$$

where $0 \le C_i^* \le 1$.

Step 7: Ranking the alternatives;

According to the relative closeness coefficient values, we can sort the schemes and find out the best scheme.

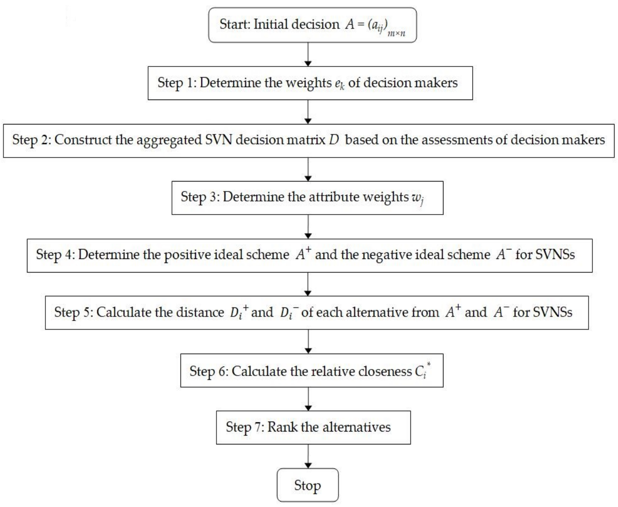

Fig. 1. TOPSIS method

V. Practical Example

We assume that an e-commerce enterprise decides to cooperate with a third-party logistics service provider to carry out its logistics distribution business, after the preliminary investigation, there are four logistics service providers (A_1, A_2, A_3, A_4) that can choose from the qualified alternatives, four decision makers (DM_1, DM_2, DM_3, DM_4) are hired from the relevant fields, through the scientific analysis of the five attributes of quality, service, wisdom, cost and greenness $(C_1, C_2, C_3, C_4, C_5)$, the best logistics service provider is selected. According to the TOPSIS method we discussed in section IV, this problem considered can be solved by the following steps:

Step 1: Determining the weights of decision makers;

In a selection committee, the positions of the four decision makers may not be equal. Their decision rights are treated as the linguistic terms shown in table I. The importance of each decision maker is expressed in the linguistic terms and their corresponding SVNN ,which is shown in table III. According to Equ.(14), the weight of each decision maker can be calculated as follows:

$$\begin{aligned} e_1 =& (0.9+0.1\cdot\frac{0.9}{0.9+0.1})/[(0.9+0.1\cdot\frac{0.9}{0.9+0.1}) \\ &+(0.8+0.2\cdot\frac{0.8}{0.8+0.15})+(0.5+0.4\cdot\frac{0.5}{0.5+0.45}) \\ &+(0.8+0.2\cdot\frac{0.8}{0.8+0.15})]=0.2722 \end{aligned} \tag{25}$$

Similarly, the weights of the other three decision makers can also be calculated as $e_2 = 0.2662$, $e_3 = 0.1953$, $e_4 = 0.2662$. Thus, the weight vector of four decision makers is

$$e = (0.2722, 0.2662, 0.1953, 0.2662) \tag{26}$$

Step 2: Constructing the aggregated SVN decision matrix based on the assessments from each decision maker;

Linguistic terms for rating the candidates with SVNNs is defined in table II. The evaluation values for each alternative $A_i(i = 1, 2, 3, 4)$ relative to each attribute $C_j(j = 1, 2, 3, 4, 5)$ provided by the four decision makers $DM_i(i = 1, 2, 3, 4)$are shown in Table IV.

According to Equ.(16), here, we only give the detailed calculation process of T_{11}, I_{11} and F_{11} as following:

$$\begin{aligned} T_{11} =& 1-[(1-0.8)^{0.2722}\cdot(1-0.8)^{0.2662}\cdot(1-0.9)^{0.1953} \\ &\cdot(1-0.5)^{0.2662}]=0.7770 \end{aligned}$$

$$\begin{aligned} I_{11} =& (0.2)^{0.2722}\cdot(0.2)^{0.2662}\cdot(0.1)^{0.1953}\cdot(0.5)^{0.2662} \\ =& 0.2230 \end{aligned}$$

$$\begin{aligned} F_{11} =& (0.15)^{0.2722}\cdot(0.15)^{0.2662}\cdot(0.05)^{0.1953}\cdot(0.45)^{0.2662} \\ =& 0.1622 \end{aligned}$$

In the same way, the aggregated SVN decision matrix can be obtained in table V.

Step 3: Determining the attribute weights;

Each decision maker thinks the importance of attributes in the scheme is different, so the attribute weight evaluation is given in Table VI.

Since C_4 is a cost-type indicator, according to the table VI and the transformation definition 12, the standard SVN decision matrix is calculated as follows:

$$\tilde{D} = (\tilde{a}_{ij})_{4\times 5} = \begin{pmatrix} \langle 0.80, 0.20, 0.15\rangle & \langle 0.90, 0.10, 0.10\rangle & \langle 0.80, 0.20, 0.15\rangle & \langle 0.20, 0.80, 0.85\rangle & \langle 0.50, 0.40, 0.45\rangle \\ \langle 0.90, 0.10, 0.10\rangle & \langle 0.80, 0.20, 0.15\rangle & \langle 0.50, 0.40, 0.45\rangle & \langle 0.10, 0.90, 0.90\rangle & \langle 0.80, 0.20, 0.15\rangle \\ \langle 0.50, 0.40, 0.45\rangle & \langle 0.80, 0.20, 0.15\rangle & \langle 0.80, 0.20, 0.15\rangle & \langle 0.50, 0.60, 0.55\rangle & \langle 0.90, 0.10, 0.10\rangle \\ \langle 0.80, 0.20, 0.15\rangle & \langle 0.90, 0.10, 0.10\rangle & \langle 0.80, 0.20, 0.15\rangle & \langle 0.20, 0.80, 0.85\rangle & \langle 0.90, 0.10, 0.10\rangle \end{pmatrix} \tag{27}$$

According to Equ.(11), we can figure out SVNSs Entropy $E(C_j) = \frac{1}{4}\sum_{i=1}^{4} E_1(\tilde{\alpha}_{ij})(j = 1, 2, 3, 4, 5)$ of each attribute $C_j(j = 1, 2, 3, 4, 5)$. As shown in Table VI, we can get the SVNSs entropy of five attributes. Namely, $E(C_1) = 3.1051$, $E(C_2) = 2.3166$, $E(C_3) = 3.3986$, $E(C_4) = 3.1051$, $E(C_5) = 2.8116$.

According to Equ.(12), the weight $\omega_j = \frac{(E(C_j))^{-1}}{\sum_{j=1}^{n}(E(C_j))^{-1}}(j = 1, 2, 3, 4, 5)$ of each attribute $C_j(j = 1, 2, 3, 4, 5)$ can be determined. Thus, the weight vectors of the five attributes are as follows:

$$\omega = (0.1866, 0.2501, 0.1705, 0.1866, 0.2061) \tag{28}$$

Step 4: Determining the positive ideal scheme and the negative ideal scheme for SVNSs;

According to Equ.(18) and Equ.(20), the positive ideal scheme A^+ and the negative ideal scheme A^- are determined as follows:

$$A^+ = \begin{bmatrix} \langle 0.880, 0.120, 0.067\rangle \\ \langle 0.862, 0.138, 0.083\rangle \\ \langle 0.825, 0.175, 0.121\rangle \\ \langle 0.815, 0.186, 0.128\rangle \\ \langle 0.862, 0.138, 0.083\rangle \end{bmatrix}^T$$

$$A^- = \begin{bmatrix} \langle 0.672, 0.328, 0.271\rangle \\ \langle 0.788, 0.212, 0.180\rangle \\ \langle 0.669, 0.331, 0.262\rangle \\ \langle 0.787, 0.213, 0.151\rangle \\ \langle 0.674, 0.326, 0.269\rangle \end{bmatrix}^T$$

Step 5: Calculating the distance of each alternative from the positive ideal scheme and the negative ideal scheme for SVNSs and relative closeness coefficient;

Equ.(22) and Equ.(23) are used to calculate the distances of each alternative from the positive ideal scheme and the negative ideal scheme. In order to make the calculation result

TABLE III
IMPORTANCE OF DECISION MAKERS EXPRESSED WITH SVNNS

	DM_1	DM_2	DM_3	DM_4
Linguistic terms	VI	I	M	I
SVNNs	$\langle 0.90, 0.10, 0.10\rangle$	$\langle 0.80, 0.20, 0.15\rangle$	$\langle 0.50, 0.40, 0.45\rangle$	$\langle 0.80, 0.20, 0.15\rangle$

TABLE IV
ASSESSMENTS OF ALTERNATIVES GIVEN BY FIVE DECISION MAKERS

Alternatives(A_i)	Decision makers	C_1	C_2	C_3	C_4	C_5
A_1	DM_1	G	VG	M	G	G
	DM_2	G	VG	MG	G	M
	DM_3	VG	G	G	MG	MG
	DM_4	M	G	G	VG	G
A_2	DM_1	VG	G	G	M	G
	DM_2	G	G	G	VG	M
	DM_3	VG	VG	VG	G	G
	DM_4	VG	MG	G	G	M
A_3	DM_1	VG	G	M	G	VG
	DM_2	G	VG	M	G	VG
	DM_3	G	G	VG	M	MG
	DM_4	VG	M	G	VG	G
A_4	DM_1	M	G	MG	G	VG
	DM_2	M	VG	M	M	VG
	DM_3	G	M	VG	G	G
	DM_4	G	G	M	VG	G

TABLE V
THE AGGREGATED SVN DECISION MATRIX

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.777, 0.223, 0.162\rangle$	$\langle 0.862, 0.138, 0.083\rangle$	$\langle 0.702, 0.298, 0.243\rangle$	$\langle 0.815, 0.186, 0.128\rangle$	$\langle 0.715, 0.285, 0.230\rangle$
A_2	$\langle 0.880, 0.120, 0.067\rangle$	$\langle 0.797, 0.203, 0.180\rangle$	$\langle 0.825, 0.175, 0.121\rangle$	$\langle 0.787, 0.213, 0.151\rangle$	$\langle 0.674, 0.326, 0.269\rangle$
A_3	$\langle 0.862, 0.138, 0.083\rangle$	$\langle 0.788, 0.212, 0.150\rangle$	$\langle 0.714, 0.286, 0.219\rangle$	$\langle 0.801, 0.199, 0.139\rangle$	$\langle 0.847, 0.154, 0.095\rangle$
A_4	$\langle 0.672, 0.328, 0.271\rangle$	$\langle 0.801, 0.199, 0.139\rangle$	$\langle 0.669, 0.331, 0.262\rangle$	$\langle 0.788, 0.212, 0.150\rangle$	$\langle 0.862, 0.138, 0.083\rangle$

TABLE VI
EVALUATION OF THE ATTRIBUTE WEIGHTS BY FIVE DECISION MAKERS

Weights	Decision makers	C_1	C_2	C_3	C_4	C_5
ω_1	DM_1	I	VI	I	I	M
ω_2	DM_2	VI	I	M	VI	I
ω_3	DM_3	M	I	I	M	VI
ω_4	DM_4	I	VI	I	I	VI

more direct and simple, let's say that $p = 2$. In fact, the value of p does not affect the result of sorting, but the precision of operation can be adjusted. Based on these distances, relative closeness coefficient can be obtained by using Equ.(24). These results are shown in Table VII.

TABLE VII
DISTANCE MEASURE AND RELATIVE CLOSENESS COEFFICIENT OF EACH ALTERNATIVE

$Alternatives(A_i)$	D_i^+	D_i^-	D_i^*
A_1	0.0032	0.0006	0.1578
A_2	0.0028	0.0009	0.2432
A_3	0.0040	0.0011	0.2156
A_4	0.0051	0.0012	0.1904

Step 6: Ranking the alternatives;

According to the relative closeness of each alternative in Table VII, the ranking order of the four alternatives is

$$A_2 > A_3 > A_4 > A_1$$

Thus, A_2 is the best logistics service providers.

Compared with the approach proposed by Xu [35], the difference is that this paper proposes a new entropy measure between SVNSs, but our ranking results and optimal supplier have the same values to calculate the same decision problem as that of Xu [35], which is able to show our approach is practical and effective.

VI. CONCLUSION

In this paper, a new TOPSIS method is proposed to solve the MADM problem under simplified SVN environment. Firstly, the weights of decision makers and attribute values are assigned by considering evaluation opinions of different decision makers, and then the SVN entropy is used to determine the attribute weights. In the evaluation process, the single-valued weighted average operator is used to aggregate the opinions of decision makers. The positive and negative ideal schemes are defined from the aggregate weighted decision matrix. Secondly, the generalized distance formula is used to determine the distance between each scheme and positive and negative ideal schemes as well as the relative closeness of each alternative. Finally, an example is given to show the validity and rationality of the method. However, we hope that the theory and method proposed in this paper can expand the application of SVNSs in the field of decision-making. In future studies, we propose to further extend the TOPSIS method and adopt other aggregation techniques, such as unified single-valued weighted mean operator, induction variable and improved SVNSs theory.

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