A Mathematical Model for the Risk Analysis of Airborne Infectious Disease in an Outpatient Room with Personal Classification Factor

Kewalee Suebyat, Pravitra Oyjinda, Sureerat A. Konglok, and Nopparat Pochai

Abstract— **Every day, a large number of people will use a hospital, creating a main air quality problem which may mean the risk of airborne infectious disease contamination in outpatient rooms, and affects human health. TB, COVID-19, MERS, and SARS are a hazardous communicable disease which are spread from person to person through the air or the aerosol in different ways, such as through coughing, spitting, sneezing, speaking, or through wounds. US scientists in the laboratory have shown that the virus can live in an aerosol and remain infectious for at least 3 hours. A new human coronavirus now known as the serious acute respiratory syndrome coronavirus 2 (SARS-CoV-2) (formerly known as HCoV-19) emerged in late 2019 in Wuhan, China, and is now triggering a pandemic. COVID-19, TB, MERS and SARS - threats and opportunities progress against deadly infection make more people sick in the hospital. Therefore, we should be aware of the care and control of these diseases. Consequently, good air quality management is required to control and reduce possible infected air, such as carbon dioxide (CO_2) concentration. In this research, a mathematical model for the risk analysis of airborne infectious disease in an outpatient room is proposed. Not only considering one type of person but also in this research, people are considered according to personal classifications. There are 4 types - patient, relative, worker, and outsider, staying in an outpatient room, which is in accordance with the real world. Air quality control manipulations are simulated using the inlet and outlet ventilation rates adjustment under the condition of a number of surrounding people with a personal classified factor. The fourth-order Runge-Kutta (RK4) is used to approximate the model solution. The proposed numerical model can be used to describe the dynamical dispersion of airborne infectious disease in an outpatient room. The results of the model are satisfactory, and it will be able to control airborne disease in more complicated structures.**

Index Terms—**Airborne, Infectious Disease, Carbon Dioxide, Ventilation Rate, Personal Classification**

I. INTRODUCTION

Tuberculosis (TB), Coronavirus Disease Starting in 2019 (COVID-19), Middle East Respiratory Syndrome (MERS), and Severe Acute Respiratory Syndrome (SARS) are a hazardous communicable disease which are spread from person to person through the air or the aerosol in different ways, such as through coughing, spitting, sneezing, speaking, or through wounds. In [1], US scientists in the laboratory have shown that the virus can live in an aerosol and remain infectious for at least 3 hours. Tuberculosis (commonly known as TB), this communicable disease is caused by Mycobacterium Tuberculosis, which most often affects the lungs. At present, we have an effective TB disinfectant. TB can be treated, but recovery takes a long time. If the treatment is not continued, or is incomplete, death may result. Therefore, TB is an important public health issue in Thailand. In [2], a new procedure was developed to study the distribution of epidemics for predicting the possibility of airborne infectious diseases in high-density urban areas. It can analyze the chance of spread in sub-transportation, and it can also help understand dispersion of airborne diseases in public transportation in China. In [3], the researchers studied the behaviors of Korean TB infection. TB transmission dynamic was proposed by using mathematical TB model with exogenous reinfection. Then, the least squares method was used to approximate the considered parameters. From the results, the most significant factor was the case finding effort, which led to a decrease of active TB patients.

In [4], the researchers developed an infectious diseases model of SARS by using two methods for estimating both small-scale SARS outbreak parameter at the Amoy Gardens, Hong Kong and large-scale outbreak parameter in the entire Hong Kong Special Administrative Region. In [5], the inpatient nursing records from EMR of the University of Miyazaki Hospital were analyzed by using a text data mining technique. This result indicated that vocabulary related to appropriate treatment methods. In [6], airflow and the airborne spread of infectious agents from an indoor environment was focused on. From this, it was confirmed that infected individuals and susceptible individuals should use masks, and also should use personalized ventilation for a short-range airborne route. In [7], the researchers predicted an airborne disease transmission from infected patients in high-rise hospitals. This simulation could analyze the ventilation system by using multi-zone airflow simulation and tracer (CFD) simulation. In [8], basic epidemiological and multivariate state-space models are proposed to predict optimal control measure strategies. This approach can be used for various diffusion diseases include Ebola, MERS. In

Manuscript received May 13, 2020; revised Sep 14, 2020. This work was supported in part by the Centre of Excellence in Mathematics (CEM), Thailand.

Kewalee Suebyat, Pravitra Oyjinda, and Sureerat A. Konglok are with School of Educational Studies, Sukhothai Thammathirat Open University, Muang Thong Thani, Chaengwattana Rd., Bangpood, Pakkret, Nonthaburi 11120, Thailand, Email: kew26_tan@hotmail.com, pravitra.o@gmail.com, and sakonglok@yahoo.com.

Nopparat Pochai is with Department of Mathematics, Faculty of Science, King Mongkuts Institute of Technology Ladkrabang, Bangkok 10520, Thailand. Email: nop_math@yahoo.com.

[9],[10], a vaccination strategy for the SEIR model was designed. It was oriented towards the measurement and used for the infectious population to epidemic models for design the general time-varying, the vaccination control rule. In [11], a new discrete-time SEIR epidemic model was presented by using the Forward-Euler difference method. Besides, the numerical simulations were presented to compare the continuous-time epidemic and discrete-time system.

In [12], the possibility of patients developing secondary infections was examined. As a result, the disease was found to be capable of spreading from the general patient space to the anteroom entry at concentrations greater than ambient. Consequently, isolation rooms should be built to be completely remote from neighboring patient rooms. In [13], the researchers improved DOT quality to create and develop an accurate logistic regression model for predicting the probability of high-risk patients to fail in TB treatment course completion. The DOT may also be used to determine the level of patients' supervision and support. In [14], the Wells–Riley equation was used to estimate the risk of indoor airborne infection transmission estimated from carbon dioxide (CO_2) concentration. The results showed that CO_2 concentration could be used as a marker for exhaled-breath exposure. In [15], a mathematical model was simulated and developed to predict the risk of airborne infectious disease. CO_2 was used for considering TB transmission in a room. From the experiment, the exhalation of infected patients influenced a limited space. Then, the TB transmission probability increased when the number of infected patients and amount of airborne infectious disease increased.

Nowadays, an increase in the number of people using hospital services has caused the risk of airborne infectious disease. Therefore, if we know the CO_2 concentration value, we might be able to control the concentration of CO_2 in an outpatient room so as to not exceed the standard. In this research, a mathematical model for the risk analysis of airborne infectious disease in an outpatient room is proposed. The personal classification of people who stay in an outpatient room is also considered. The air quality control manipulations are simulated using the inlet and outlet ventilation rates adjustment under the condition of a number of surrounding people with a personal classified factor.

II. Governing Equation

The basis for the description of the relationship between the mass or concentration of a gaseous substance in space as a function of time is the mass balance equation. Thus, the generalized tracer mass balance equation can be presented as the following first-order differential equation [16]:

$$V\frac{dC}{dt} = F + QC_e - QC, \tag{1}$$

where C is the indoor exhaled air concentration (ppm), V is the volume of the classroom (m^3), and F is an emission of tracer gas into space by a tracer gas source (mass per time unit). Furthermore, QC_e is the transport of tracer gas from the outside air into the room air (mass per time unit), and QC is the transport of tracer gas from the room air to the outside (mass per time unit).

The basic equation for the exhaled air accumulation rate in an atmospheric carbon dioxide (CO2) space, which is then occupied, is equal to the rate of exhaled air produced by the occupants plus the ambient rate of CO2, minus the exhaled air eliminated by the rate of ventilation. Moreover, if we consider the term F of the mass balance equation in (1), we find the rate of exhaled air generated by occupants is the production rate of tracer by all sources within the enclosure, i.e., n are the number of people (person), p is the breathing rate for each person in the room (L/s), and C_a is the CO_2 fraction containing inbreathed air. Thus, the fundamental equation for exhaled air accumulation rate in the room with environmental CO_2 can be formulated as the following:

$$V\frac{dC}{dt} = npC_a + QC_e - QC. \tag{2}$$

We assume that people in the room will contribute equally to the generation of CO_2 as a marker of exhaled air. At the start of the day, environmental CO_2 concentration is C_e (ppm); it becomes occupied by $n.$ This implies that the level of exhaled air concentration that might contain airborne infectious particles given the presence of infectors will start to increase in the room depending on the ventilation rate Q (L/s) and $n.$

We consider the equation for exhaled air accumulation rate in the room with environmental CO_2 in (2), and we divide the ventilation rate into inlet ventilation rate and outlet ventilation rate, which can be written as:

$$V\frac{dC}{dt} = npC_a + Q_{in}C_e - Q_{out}C, \tag{3}$$

where Q_{in} and Q_{out} are the inlet ventilation rate and outlet ventilation rate, respectively. After dividing both sides of (3) by the volume, we obtain an ordinary differential equation which describes the concentration change of the indoor exhaled air per time unit:

$$\frac{dC}{dt} = \frac{npC_a + Q_{in}C_e - Q_{out}C}{V}. \tag{4}$$

In this paper, we are interested in the amount of air pollution that leads to tuberculosis. Using the same initial equation as the above equation, the equation is used to describe the CO_2 concentration in the outpatient room of the hospital.

III. Numerical Techniques for Solution of Governing Equation

A. *Fourth-Order Runge-Kutta Method*

The approximation of solution for a first order differential equation can be written as:

$$\frac{dC}{dt} = f\left(t, C\right). \tag{5}$$

The fourth-order Runge-Kutta (RK4) is a well-known method. This method is a reasonable and good general method for the numerical solution of first-order differential equation with an intelligent adaptive step-size. Thus, we use the RK4 formula for approximating the solution of (5):

$$C_{i+1} = C_i + \frac{1}{6}\left[F_1 + 2F_2 + 2F_3 + F_4\right], \tag{6}$$

where

$$F_1 = hf\left(t_i, C_i\right),$$
$$F_2 = hf\left(t_i + \frac{h}{2}, C_i + \frac{F_1}{2}\right),$$
$$F_3 = hf\left(t_i + \frac{h}{2}, C_i + \frac{F_2}{2}\right),$$
$$F_4 = hf\left(t_i + h, C_i + F_3\right).$$

We consider that (4) and (5) can be simplified to:

$$f\left(t, C\right) = \frac{npC_a + Q_{in}C_e - Q_{out}C}{V}. \tag{7}$$

The exhaled air contains about 40,000 ppm of carbon dioxide (CO_2) compared with almost 400 ppm of CO_2 in the environmental air [14],[17],[18]. Thus, the initial value problem and the initial condition for number of people are obtained, such that:

$$IVP: \frac{dC}{dt} = \frac{npC_a + Q_{in}C_e - Q_{out}C}{V}, \quad IC: C\left(0\right) = 400. \tag{8}$$

B. The Fifth-Order Polynomial Interpolation

A well-designed hospital environment maximizes the effectiveness of clinical care delivery and enhances the well-being of patients and hospital staff. In every year, however, a percentage of patients develop infections during a hospital stay. Though protection from infection is an important measure of a hospital's safety, it is still important for a hospital to provide services to other patients of varied conditions. An increase in the number of people using hospital services can be observed. The average number of people using hospital services in Phramongkutklao hospital, Thailand was approximately 636 people/day [19]. The area with the most patients using Phramongkutklao hospital are shown in Table I.

TABLE I
THE NUMBER OF PEOPLE USING HOSPITAL SERVICES IN THE PHRAMONGKUTKLAO HOSPITAL EACH PERIOD PER DAY

Time	The number of people using hospital services				Total
	Patient	Relative	Worker	Outsider	
Lan Sai	80 (32.9%)	51 (21.0%)	72 (29.6%)	40 (16.5%)	243 (100%)
Roman garden	29 (39.2%)	8 (10.8%)	19 (25.7%)	18 (24.3%)	74 (100%)
Health garden	25 (22.9%)	11 (10.1%)	42 (38.5%)	31 (28.4%)	109 (100%)
Around Udom Vanaporn Throne Hall	48 (32.0%)	45 (30.0%)	34 (22.7%)	23 (15.3%)	150 (100%)
In front of Phayathai Palace	11 (18.3%)	12 (20.0%)	28 (30.0%)	19 (31.7%)	60 (100%)
Total	193 (30.3%)	127 (20.0%)	185 (29.1%)	131 (20.6%)	636 (100%)

From Table I, we conclude that the area with the most patients using the hospital is the Lan Sai area. Therefore, this area is one of great interest, and we take this area into consideration. The number of people using hospital services in the Lan Sai each period per day are shown in Table II.

TABLE II
THE NUMBER OF PEOPLE USING HOSPITAL SERVICES IN THE LAN SAI AREA EACH PERIOD PER DAY

Time	The number of people using hospital services				Total
	Patient	Relative	Worker	Outsider	
6.00 – 9.00 a.m.	28 (45.2%)	12 (19.4%)	8 (12.9%)	14 (22.6%)	62 (25.5%)
9.00 – 12.00 a.m.	16 (31.4%)	19 (37.3%)	12 (23.5%)	4 (7.8%)	51 (21.0%)
12.00 – 15.00 p.m.	18 (27.7%)	16 (24.6%)	24 (36.9%)	7 (10.8%)	65 (26.7%)
15.00 – 18.00 p.m.	18 (27.7%)	4 (6.2%)	28 (43.1%)	15 (23.1%)	65 (26.7%)
Total	80 (32.9%)	51 (21.0%)	72 (29.6%)	40 (16.5%)	243 (100%)

From Table II, we can divide the number of people using the services of the hospital into 2 parts. In the first part, we will not classify persons who use hospital services, but we will focus only on the total number of all persons in each period. The second part, we focus on personal classification; that is, we separate this part into 4 types - patient, relative, worker, and outsider.

The data in Table II shows the number of people using hospital services in various areas of the hospital, not actual data on the number of people using the services in the outpatient room. However, the above data is consistent with the data that we are interested in. Then, we compare each area as each outpatient room. So, we assume that the data in Table II is like the data of the number of people using the services in the outpatient room. Accordingly, by relying on the information in the above table, the amount of air pollution control that leads to Tuberculosis (TB) in the outpatient room of the hospital was studied in this paper.

We define the function $n(t)$ as the function of the total number of all persons by using the polynomial degree 5 interpolation. Thus:

$$n(t) = \left(5.07\times10^{-12}\right)t^5 - \left(1.43\times10^{-8}\right)t^4 + \left(1.41\times10^{-5}\right)t^3 - \left(5.91\times10^{-3}\right)t^2 + 1.03t + 7.19\times10^{-13}. \tag{9}$$

Therefore, the initial value problem and the initial condition for the function of the total number of all persons are obtained, such that:

$$IVP: \frac{dC}{dt} = \frac{n(t)\,pC_a + Q_{in}C_e - Q_{out}C}{V}, \quad IC: C\left(0\right) = 400. \tag{10}$$

Furthermore, we define the function $n(t)$ as a function of the personal classification by using the polynomial degree 5 interpolation. From Table I, we classify the data into 4 functions of each person, i.e., patient, relative, worker, and outsider, as per the following:

The function of patient:

$$n_p(t) = \left(3.09\times10^{-12}\right)t^5 - \left(8.34\times10^{-9}\right)t^4 + \left(8.04\times10^{-6}\right)t^3 - \left(3.34\times10^{-3}\right)t^2 + 0.54t + 4.28\times10^{-13}. \quad (11)$$

The function of relative:

$$n_r(t) = \left(4.41\times10^{-13}\right)t^5 - \left(5.56\times10^{-10}\right)t^4 + \left(1\times10^{-7}\right)t^3 - \left(4.37\times10^{-5}\right)t^2 + \left(7.41\times10^{-2}\right)t + 3.1\times10^{-14}. \quad (12)$$

The function of worker:

$$n_w(t) = \left(8.82\times10^{-13}\right)t^5 - \left(2.69\times10^{-9}\right)t^4 + \left(2.54\times10^{-6}\right)t^3 - \left(9\times10^{-4}\right)t^2 + \left(1.39\times10^{-1}\right)t + 1.37\times10^{-13}. \quad (13)$$

The function of outsider:

$$n_o(t) = \left(1.10\times10^{-12}\right)t^5 - \left(3.77\times10^{-9}\right)t^4 + \left(4.24\times10^{-6}\right)t^3 - \left(1.89\times10^{-3}\right)t^2 + \left(3.03\times10^{-1}\right)t + 1.82\times10^{-13}. \quad (14)$$

So, the initial value problem and the initial condition for the function of the personal classification are obtained, such that:

$$IVP: \frac{dC}{dt} = \frac{\left(n_p + n_r + n_w + n_o\right)pC_a + Q_{in}C_e - Q_{out}C}{V}, \quad (15)$$
$$IC: C(0) = 400.$$

IV. Numerical Experiments

In this section, we distinguish two parts of the released concentration of carbon dioxide (CO_2) by using the fourth-order Runge-Kutta (RK4) for approximating the solution of concentration. In the first part, we consider the dynamic number of people. This can be divided into 3 cases - Case I: static number of people, Case II: dynamic number of people, and Case III: dynamic number of people with personal classification into 4 types. In the second part, we consider the ventilation rate effect. This can be divided into 4 scenarios - Scenario A: equal inlet and outlet ventilation rates, Scenario B: high inlet ventilation rate, Scenario C: high outlet ventilation rate, and Scenario D: lowest inlet ventilation rate.

We assume that $V = 150\ m^3$, $p = 0.12\ L/s$, $C_a = 400$ and $C_e = 400\ ppm$, $Q_{in} = Q_{out} = 3$ or 4, $h = 1$. Time duration is assumed to be 120 min.

A. Dynamic Number of People

For simplicity, we define n as a constant. That is, the number of people remains the same at all times, but in reality, the number of people entering and exiting the room each time is different. Consequently, the number of people is something that we must consider. In this part, we consider 3 cases.

Case I: Static Number of People

Consider n is a constant. That is, when the time after pass, the number of people is still the same. We assume $n = 50$. The results of Case I after 120 min pass are shown in Fig. 1 and Table III.

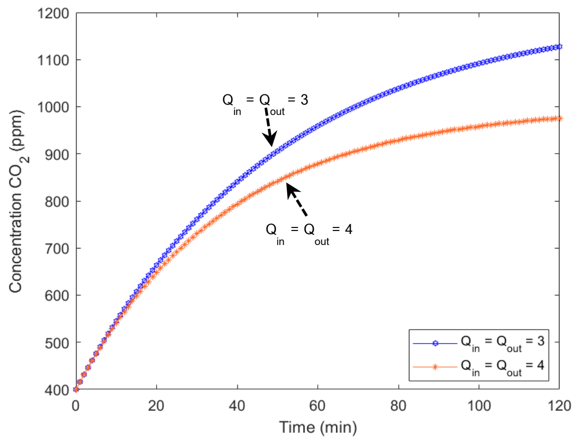

Fig. 1. The concentration of carbon dioxide for Case I

Case II: Dynamic Number of People

Consider $n(t)$ is the function of all persons depending on time, i.e., over time, the number of people will change according to that time. We assume $n(t)$ as (9); then, the results in this case are shown in Fig. 2 and Table IV.

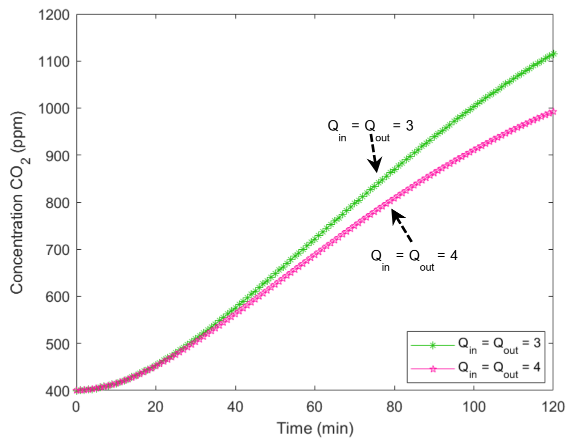

Fig. 2. The concentration of carbon dioxide for Case II

Case III: Dynamic Number of People with Personal Classification into 4 Types

Consider $n_p + n_r + n_w + n_o$ is a function of the personal classification. It is classified into 4 types, i.e., patient, relative, worker, and outsider. The functions of each type are satisfied by (10) - (13). Therefore, the results of Case III are shown in Fig. 3 and Table V.

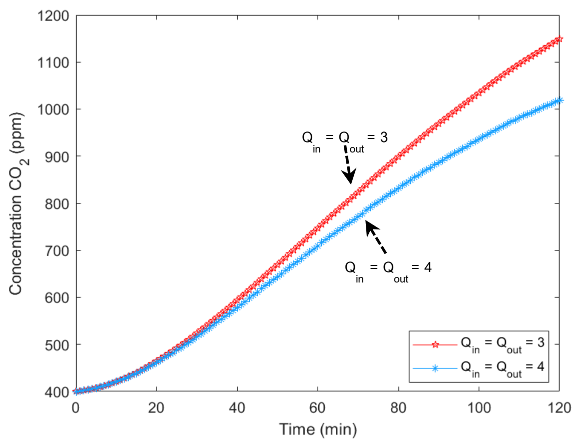

Fig. 3. The concentration of carbon dioxide for Case III

The results obtained from the curves of the CO_2 concentration depicted in Fig. 1 - Fig. 3 and Table III - Table V were approximated using the RK4. The calculated concentration of CO_2 ranges from 400 to 1150 ppm.

B. *Ventilation Rate Effect*

We divided ventilation rate into inlet ventilation rate (Q_{in}) and outlet ventilation rate (Q_{out}). In the basic case, the fundamental principle is that Q_{in} is always equal to Q_{out}, which means that there is equal suction and suction throughout. However, when considering the real problem, the rate of Q_{in} and Q_{out} should be different. Thus, the ventilation rate affects the indoor exhaled air concentration. In this part, we distinguish 4 scenarios and consider them as follows:

Scenario A: Equal Inlet and Outlet Ventilation Rates

We assume $Q_{in} = Q_{out} = 3$; the results of Scenario A are shown in Fig. 4 and Table VI. After 120 minutes pass, as the inlet and outlet ventilation rates are equal to 3 L/s, the average concentration of CO_2 for n, $n(t)$, and $n_p + n_r + n_w + n_o$ is 1130.604 ppm.

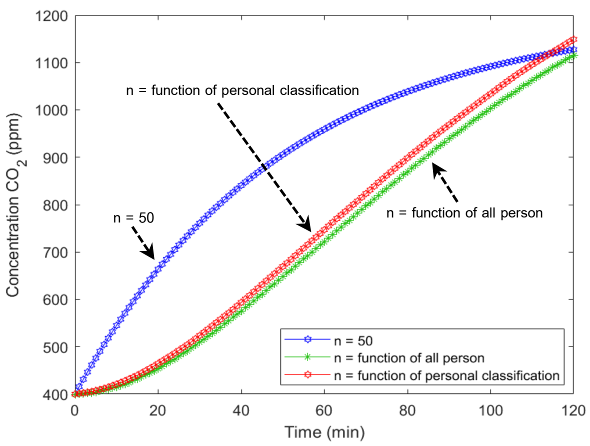

Fig. 4. The concentration of carbon dioxide for Scenario A

Scenario B: High Inlet Ventilation Rate

We defend $Q_{in} = 6$, $Q_{out} = 3$. Fig. 5 and Table VII show the results of Scenario B. As inlet ventilation rate increases from 3 to 6 L/s, the average concentration of CO_2 for n, $n(t)$, and $n_p + n_r + n_w + n_o$ increases from 1130.604 to 1494.317 ppm when 120 minutes pass.

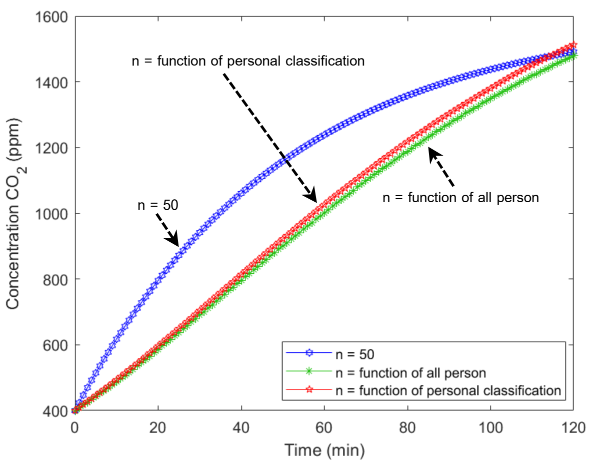

Fig. 5. The concentration of carbon dioxide for Scenario B

Scenario C: High Outlet Ventilation Rate

Assume the $Q_{in} = 3$, $Q_{out} = 6$. The results of Scenario C are shown in Fig. 6 and Table VIII. After 120 minutes pass, the outlet ventilation rate increases from 3 to 6 L/s, and the average concentration of CO_2 for n, $n(t)$, and $n_p + n_r + n_w + n_o$ decreases from 1130.604 to 628.852 ppm.

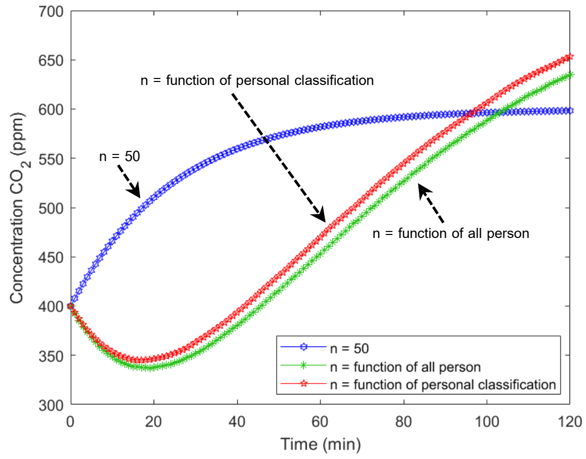

Fig. 6. The concentration of carbon dioxide for Scenario C

Scenario D: Lowest Inlet Ventilation Rate

We assume $Q_{in} = 1$, $Q_{out} = 5$. Fig. 7 and Table IX show the results of Scenario D. The inlet ventilation rate decreases from 3 to 1 L/s and the outlet ventilation rate increases from 3 to 5 L/s, and average CO_2 concentration for n, $n(t)$, and $n_p + n_r + n_w + n_o$ decreases from 1130.604 to 584.750 ppm.

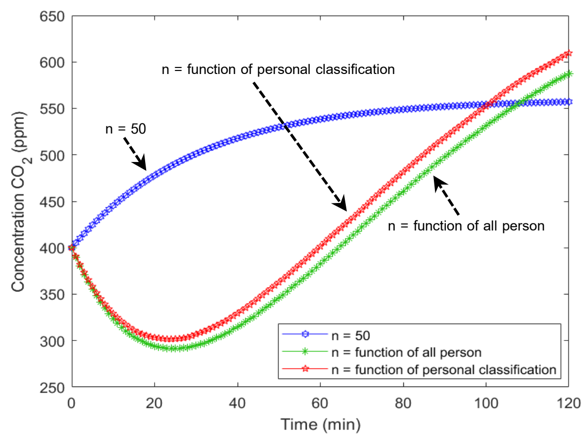

Fig. 7. The concentration of carbon dioxide for Scenario D

V. Discussion and Conclusion

Cases I, II and III demonstrate the effect of changes in the dynamic number of people. If comparing between cases where the number of people is a constant and function is dependent on time, this results in the concentration of CO_2 in the function case being less than the constant case. Moreover, if we compare the function of all persons and personal classification, the CO_2 concentration of personal classification is more than all persons. Although the concentration of CO_2 is high, it is more realistic for the patient, relative, worker, and outsider.

Scenarios A, B, C and D, demonstrate the effect of changes in ventilation rate. The inlet ventilation rate is equal to the outlet ventilation rate. The concentration of CO_2 depends on the number of people. As the inlet ventilation rate increases, the CO_2 concentration increases too. On the other hand, the outlet ventilation rate increases, the concentration of CO_2 decreases. Moreover, if the outlet ventilation rate increases and the inlet ventilation rate decreases a lot, the CO_2 concentration is greatly decreased.

The concentration of CO_2 not only depends on the number of people using hospital services but also on the inlet and outlet ventilation rates. However, no exceeding concentration can occur without the environmental CO_2 rate and exhaled air removed by ventilation rate. For this reason, the number of people using hospital services and ventilation rate should be given priority in the first order. To avoid air pollutants, from the concentration of CO_2, appropriate remedial measures are necessary.

This relationship could be confirmed by our investigations. Improvement of ventilation, i.e., increasing the outlet ventilation rate and decreasing the inlet ventilation rate, is an efficient measure to reduce the concentration of CO_2 in an outpatient room. Therefore, to control indoor pollutants, we consider a combination of both measures, that is, the number of people using hospital services and the ventilation rate are important. However, suitable indoor pollution control depends on many factors and the context of each area. Consequently, in the future studies will focus on using more factors and areas to improve the results for the risk analysis of airborne infectious disease.

REFERENCES

[1] N.V. Doremalen, T. Bushmaker, and D.H. Morris, “Aerosol and Surface Stability of SARS-CoV-2 as Compared with SARS-CoV-1,” *The New England Journal of Medicine*, vol. 382, no. 16, pp. 1564-1567, 2020.

[2] M. Shan, X. Zhou, Y. Zhu, Z. Zu, T. Zheng, B. Alexander, and S. Peter, “Simulating city-level airborne infectious diseases, Computers,” *Environment and Urban Systems*, vol. 51, pp. 97–105, 2011.

[3] K. Sara, C. Seoyoon, K. Junseong, N.Sanga, S. Yeon, and L. Sunmi, “What does a mathematical model tell about the impact of reinfection in Korean tuberculosis infection?,” *Osong Public Health and Research Perspectives*, vol. 5. no. 1, pp. 40-45, 2014.

[4] T. Mkhatshwa and A. Mummert, “Modeling super-spreading events for infectious diseases: case study SARS,” *IAENG International Journal of Applied Mathematics*, vol. 41, no. 2, pp82-88, 2011.

[5] M. Kushima, K. Araki, M. Suzuki, S. Araki, and T. Nikama, “Text data mining of in-patient nursing records within electronic medical records using KeyGraph,” *IAENG International Journal of Computer Science*, vol. 38, no. 3, pp215-224, 2011.

[6] W. Jianjian and L. Yuguo, “Airborne spread of infectious agents in the indoor environment,” *American Journal of Infection Control*, vol. 44, pp. S102-S108, 2016.

[7] L. Taesub, C. Jinkyun, and K. Byungseon, “The predictions of infection risk of indoor airborne transmission of diseases in high-rise hospitals: tracer gas simulation,” *Energy and Buildings*, vol. 42, pp. 1172-1181, 2010.

[8] K.D. Gebreyesus and C.H. Chang, "Infectious diseases dynamics and complexity: multicompartment and multivariate state-space modeling," *Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2015*, 21-23 October, 2015, San Francisco, USA, pp552-555.

[9] M. De la Sen, S. Alonso-Quesada, and A. Ibeas, “A SEIR epidemic model with infectious population measurement,” *Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2011*, 6-8 July, 2011, London, U.K., pp2685-2689.

[10] M. De la Sen, S. Alonso-Quesada, A. Ibeas, and R. Nistal, "Analysis of an SEIR epidemic model with a general feedback vaccination law," *Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering 2015*, 1-3 July, 2015, London, U.K., pp571-576.

[11] W.J. Du, S. Qin, J.G. Zhang, and J.N. Yu, "Dynamical behavior and bifurcation analysis of SEIR epidemic model and its discretization," *IAENG International Journal of Applied Mathematics*, vol. 47, no. 1, pp1-8, 2017.

[12] M. Ehsan and G. Kevin, “Directional airflow and ventilation in hospitals: a case study of secondary airborne infection,” *Energy Procedia*, vol. 78, pp. 1201-1206, 2015.

[13] S.R.N. Kalhori, M. Nasehi, and X.J. Zeng, “A logistic regression model to predict high risk patients to fail in tuberculosis treatment course completion,” *IAENG International Journal of Applied Mathematics*, vol. 40, no. 2, pp1-6, 2010.

[14] S.N. Rudnick and D.K. Milton, “Risk of indoor airborne infection transmission estimated from carbon dioxide concentration,” *Indoor Air*, vol. 13, no. 3, pp. 237–245, 2003.

[15] C.M. Issarowa, N. Mulder, and R. Wood, “Modelling the risk of airborne infectious disease using exhaled air,” *Journal of Theoretical Biology*, vol. 372, pp. 100–106, 2015.

[16] F.D. Heidt and H. Werner, “Microcomputer-aided measurement of air change rates,” *Energy and Buildings*, vol. 9, no. 4, pp. 313-320, 1986.

[17] S.J. Emmerich and A.K. Persily, “State-of-the-art review of carbon dioxide demand controlled ventilation technology and application,” *NISTIR*, pp. 6729, 2001.

[18] E.T. Richardson, C.D. Morrow, D.B. Kalil, and L.G. Bekker, “Shared air a renewed focus on ventilation for the prevention of tuberculosis transmission,” *PloS One*, vol. 9, no. 5, 2014.

[19] P. Sirisali, “Main green spaces development in health promoting hospital: a case study of Phramongkutklao hospital,” M.S. thesis, Dept. Landscape architecture program, Fac. Architecture, Chulalongkorn univ, Thailand, 2008.

[20] J.J. Swiegers, “Inlet and outlet shape design of natural circulation building ventilation systems,” M.S. thesis, Dept. Engineering (Mechanical), Fac. Engineering, Stellenbosch univ, 2015.

[21] D. Laussmann and D. Helm, “Air change measurements using tracer gases, Chemistry, Emission Control, Radioactive Pollution and Indoor Air Quality,” Robert Koch Institute, Germany, 365-406.

[22] H. Zhang, D. Li, L. Xie, and Y. Xiao, “Documentary research of human respiratory droplet characteristics,” *Procedia Engineering*, vol. 121, pp. 1365-1374, 2015.

[23] J.R. Cash and A.H., Karp, “A variable order Runge-Kutta method for initial value problems with rapidly varying right-hand sides,” *ACM Transactions on Mathematical Software*, vol. 16, no. 3, pp. 201-222, 1990.

TABLE III
THE CONCENTRATION OF CARBON DIOXIDE FOR CASE I: STATIC NUMBER OF PEOPLE

Ventilation		The concentration of carbon dioxide (ppm)									
Inlet	Outlet	0 min	10 min	20 min	30 min	40 min	60 min	80 min	100 min	110 min	120 min
3	3	400	545.015	663.744	760.950	840.536	959.044	1038.482	1091.731	1111.357	1127.425
4	4	400	540.443	648.012	730.402	793.507	878.862	928.934	958.309	968.068	975.542

TABLE IV
THE CONCENTRATION OF CARBON DIOXIDE FOR CASE II:
DYNAMIC NUMBER OF PEOPLE

Ventilation		The concentration of carbon dioxide (ppm)									
Inlet	Outlet	0 min	10 min	20 min	30 min	40 min	60 min	80 min	100 min	110 min	120 min
3	3	400	415.199	454.045	509.586	575.601	722.086	869.658	1002.971	1062.285	1115.442
4	4	400	414.851	451.798	503.071	562.220	687.916	807.945	910.571	954.394	992.392

TABLE V
THE CONCENTRATION OF CARBON DIOXIDE FOR CASE III:
DYNAMIC NUMBER OF PEOPLE WITH PERSONAL CLASSIFICATIONINTO 4 TYPES

Ventilation		The concentration of carbon dioxide (ppm)									
Inlet	Outlet	0 min	10 min	20 min	30 min	40 min	60 min	80 min	100 min	110 min	120 min
3	3	400	421.002	464.650	523.836	593.662	746.923	898.526	1033.939	1095.680	1148.947
4	4	400	420.472	461.790	516.135	578.424	709.417	832.077	935.657	981.367	1019.046

TABLE VI
THE CONCENTRATION OF CARBON DIOXIDE FOR SCENARIO A:
EQUAL INLET AND OUTLET VENTILATION RATES

$Q_{in}=3$, $Q_{out}=3$	The concentration of carbon dioxide (ppm)									
Number of People	0 min	10 min	20 min	30 min	40 min	60 min	80 min	100 min	110 min	120 min
50	400	545.015	663.744	760.950	840.536	959.044	1038.482	1091.731	1111.357	1127.425
$n(t)$	400	415.199	454.045	509.586	575.601	722.086	869.658	1002.971	1062.285	1115.442
$n_p+n_r+n_w+n_o$	400	421.002	464.650	523.836	593.662	746.923	898.526	1033.939	1095.680	1148.947

TABLE VII
THE CONCENTRATION OF CARBON DIOXIDE FOR SCENARIO B:
HIGH INLET VENTILATION RATE

$Q_{in}=6$, $Q_{out}=3$	The concentration of carbon dioxide (ppm)									
Number of People	0 min	10 min	20 min	30 min	40 min	60 min	80 min	100 min	110 min	120 min
50	400	617.523	795.615	941.426	1060.805	1238.566	1357.724	1437.597	1467.036	1491.138
$n(t)$	400	487.707	585.917	690.062	795.870	1001.608	1188.899	1348.836	1417.964	1479.155
$n_p+n_r+n_w+n_o$	400	493.509	596.522	704.311	813.930	1026.445	1217.768	1379.805	1451.358	1512.660

TABLE VIII
THE CONCENTRATION OF CARBON DIOXIDE FOR SCENARIO C:
HIGH OUTLET VENTILATION RATE

$Q_{in}=3$, $Q_{out}=6$	The concentration of carbon dioxide (ppm)									
Number of People	0 min	10 min	20 min	30 min	40 min	60 min	80 min	100 min	110 min	120 min
50	400	465.935	510.134	539.761	559.620	581.856	591.847	596.336	597.544	598.354
$n(t)$	400	348.255	337.584	351.978	380.258	453.173	526.790	588.252	613.682	634.862
$n_p+n_r+n_w+n_o$	400	353.536	346.499	363.070	393.524	469.829	544.555	605.951	632.782	653.341

TABLE IX
THE CONCENTRATION OF CARBON DIOXIDE FOR SCENARIO D:
LOWEST INLET VENTILATION RATE

$Q_{in}=1$, $Q_{out}=5$	The concentration of carbon dioxide (ppm)									
Number of People	0 min	10 min	20 min	30 min	40 min	60 min	80 min	100 min	110 min	120 min
50	400	445.354	477.853	501.139	517.824	538.346	548.882	554.292	555.910	557.069
$n(t)$	400	323.805	293.986	294.865	314.735	382.562	460.841	531.246	561.606	587.640
$n_p+n_r+n_w+n_o$	400	329.252	303.417	306.882	329.356	401.387	481.381	552.089	584.032	609.541