Spatial Autoregressive in Ecological Studies: A Comparison of the SAR and CAR Models

IGNM. Jaya* and A. Chadidjah

Abstract— **Spatial autoregressive in ecological studies are often modeled using the simultaneous autoregressive (SAR) and conditional autoregressive (CAR) models. Both models are known as network-based or graphical models. SAR and CAR models have been developed to analyse spatially autocorrelated data based on neighborhood proximity. The models have different conceptual concepts with the equivalent objectives. In practice, selecting which model should be used becomes a crucial issue since there are no standard criteria for comparing SAR and CAR models. We evaluate the similarity and differences between SAR and CAR modes based on the Monte Carlo simulation study and real application on diarrhea data. The evaluations of both models are essential in regression modelling to get more reliable result. In general, the smallest of bias parameter estimates and the smallest differences in estimated spatial autoregressive parameters between SAR and CAR models were found for weak and strong spatial dependencies. For medium spatial dependence, the differences estimated spatial autoregressive between SAR and CAR model relatively large.**

Index Terms— **Bandung city, diarrhea, conditional autoregressive, Monte Carlo, Moran's I, simultaneous autoregressive, spatial**

I. Introduction

Spatial autoregressive models are the regression models that consider spatial dependency among spatial units. The spatial dependency occurs if spatial units that are close together to have more similar values than those farther away [1]. The existence of spatial dependence or autocorrelation in the data causes a violation of the independent assumption of the random error and classical statistical models inappropriate [2]. Spatial autoregressive models have been used often in modeling ecological data [1]. The two spatial autoregressive models that most commonly used are simultaneous autoregressive (SAR) and conditional autoregressive (CAR) models ([3], [4]). Both the autoregressive models have been utilized in many fields, including diseases mapping, agriculture, image analysis, econometrics, and ecology [1].

SAR and CAR models have different conceptual models with the equivalent main objectives. The SAR model uses a regression on the values from the other areas to account for the spatial dependence [5] while CAR model depends on the conditional distribution of the spatial errors. Because SAR and CAR models are widely used for lattice data, it is natural to compare and contrast them in order to get the best fitted model for real data. In practice, selecting which model should be used becomes a crucial issue since there is no standard criteria for comparing SAR and CAR models [1]. SAR model is commonly used in econometrics modeling [6] and CAR model is often used in disease modelling and mapping studies ([7],[8]). Both models can be used to handle ecological data which is commonly measured over spatial units [1]. Our objective is to investigate the potential difference of the CAR and SAR models using simulation and practical approaches, so that their potential may be more fully realized and used by researchers, and we begin with an overview of their statistical property [9]. The diarrhea data set were used to review the differences between parameter estimates for SAR and CAR models in a practical approach.

II. Spatial Autoregressive Models

Spatial autoregressive models including SAR and CAR models consider the error spatial dependence [10]. Both of them based on a graphical model or network structures which are known as the Gaussian Markov random field [9]. Let $\mathbf{y}$ denote vector of observed response variable with dimension ($n \times 1$) with n is number of spatial units. Let consider the spatial regression model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z} + \boldsymbol{\varepsilon} \tag{1}$$

where $\mathbf{X}$ is $n \times p$ design matrix with $p = K+1$ and K denotes the number of predictor variables. $\boldsymbol{\beta}$ is an $p\times 1$ vector regression coefficients including the intercept. The purpose of model (1) is to model a mean structure E[$\mathbf{y}|\mathbf{X,Z}$] includes fixed effect $\mathbf{X}\boldsymbol{\beta}$ and a latent spatial random error $\mathbf{Z}$, where $\mathbf{Z} \sim N(\mathbf{0}, \boldsymbol{\Sigma})$, and independent error $\boldsymbol{\varepsilon} \sim N(\mathbf{0}, \sigma_\varepsilon^2\mathbf{I})$. The component $\mathbf{Z}$ is unobserved and has to be estimated through statistical model such as frequentist or Bayesian approach [1]. The spatial dependence structured is accommodated in covariance matrix $\boldsymbol{\Sigma}$. The difference between SAR and CAR is in the covariance matrix $\boldsymbol{\Sigma}$ specifications [4, 11]. For SAR model $\boldsymbol{\Sigma} \sim \boldsymbol{\sigma}_z^2(\mathbf{I}-\mathbf{B})(\mathbf{I}-\mathbf{B}')^{-1}$ and for CAR is $\boldsymbol{\Sigma} \sim \boldsymbol{\sigma}_n^2(\mathbf{I}-\mathbf{C})^{-1}\mathbf{M}$ Matrix $\mathbf{B}$ and $\mathbf{C}$ are used to model spatial dependence between $\mathbf{Z}_i$ and $\mathbf{Z}_j$ based on SAR and CAR models respectively. $\mathbf{B} = \{b_{ij}\}$ and $\mathbf{C} = \{c_{ij}\}$ with $b_{ii} = c_{ij} = 0$. Matrix $\mathbf{M} = \{m_{ij}\}$ denotes a diagonal matrix with all of off-diagonal elements are $\mathbf{0}$ and the main diagonal m_{ii} is proportional to the conditional variance of $\mathbf{Z}_i$ given all of its neighbors. The spatial dependence structure matrix $\mathbf{B}$ are commonly defined as $\mathbf{B} = \lambda\mathbf{W}$ and $\mathbf{C}=\lambda\mathbf{W}$, where is a spatial weight matrix. The queen structured was commonly chosen

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for $\mathbf{W} = \{w_{ij}\}$, with $w_{ij} = 1$ if unit spatial i and j are neighbourhood and $w_{ij} = 0$ otherwise. λ is the autoregressive parameter that controls the strength of dependence. SAR model is specified as [5]:

$$z_i = \sum_{i=1}^{n} b_{ij} z_j + v_i \tag{2}$$

with $v_i \sim N(0, \sigma_v^2)$ and the CAR model is specified as [5]:

$$Z_i \mid \mathbf{z_{-i}} \sim N\left(\sum_{\forall c_{ij} \neq 0} c_{ij} z_j, m_{ii}\right)$$
$$\frac{c_{ij}}{m_{ii}} = \frac{c_{ji}}{m_{jj}}, \forall i, j \tag{3}$$
$$Z_i \sim N\left(\sum_{j \in \Omega_i} z_j / |\Omega_i|, \sigma^2 / |\Omega_i|\right)$$

where Ω_i is the total number neighbourhood of spatial unit i-th and $\mathbf{z}_{-i}$ is a vector latent effect without observation i-th. maximum likelihood (ML) estimation was often applied to estimate the model parameters.

III. Monte Carlo Simulation Study

To evaluate the relationships between SAR and CAR model we developed a Monte Carlo (MC) simulation study. MC simulation has been applied widely [12]. The Monte Carlo simulation study was developed by setting some parameters as following n = {9, 25, 100, 400}, λ = {0.1, 0.3, 0.5, 0.7, 0.9} and $\beta_0 = \beta_1 = \beta_2 = 1$. The spatial weight matrices were developed based on grid approach. The examples of spatial weight matrix $\mathbf{W}$ for $n = 9$ and $n = 100$ are presented below. The red colors indicate the observations of the areal units.

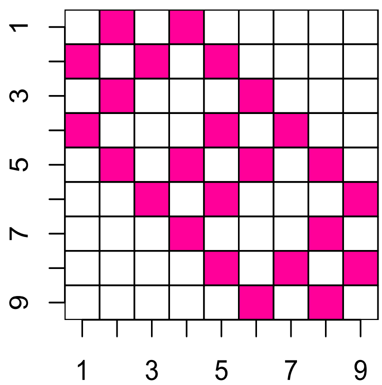

Fig.1 Spatial weight matrix for n=9

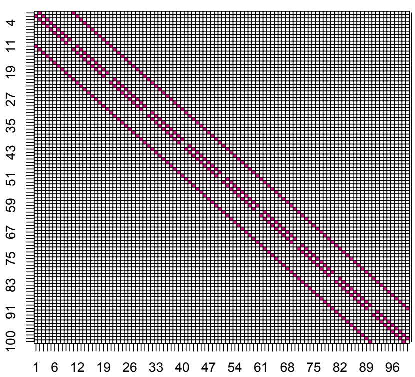

Fig.2 Spatial weight matrix for n=100

The data were generated based on SAR model. The simulations were done with 500 iterations. Here we evaluate the BIAS and the differences between SAR and CAR parameter estimates. The simulation results are presented in Tables I-II.

TABLE I
MONTE CARLO SIMULATION RESULT OF SAR MODEL

n	λ	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$
9	0.1	0.998	0.998	0.997	-0.191
	0.3	1.002	0.996	0.999	-0.045
	0.5	0.997	1.000	1.002	0.027
	0.7	0.999	1.001	1.005	0.221
	0.9	1.005	1.002	1.002	0.272
25	0.1	0.999	1.002	1.000	0.008
	0.3	1.002	0.999	1.000	0.192
	0.5	0.995	0.999	1.000	0.373
	0.7	1.001	1.001	1.000	0.576
	0.9	1.019	1.000	0.999	0.775
100	0.1	1.000	0.999	1.000	0.074
	0.3	1.000	1.000	1.001	0.271
	0.5	1.000	1.000	1.000	0.477
	0.7	1.002	0.999	1.000	0.673
	0.9	0.998	0.999	0.999	0.872
400	0.1	0.999	1.000	1.000	0.092
	0.3	1.000	1.000	1.000	0.292
	0.5	1.000	1.000	1.000	0.492
	0.7	0.999	1.000	1.000	0.695
	0.9	1.001	1.000	1.000	0.893

TABLE II
MONTE CARLO SIMULATION RESULT OF CAR MODEL

n	λ	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\lambda}$
9	0.1	0.998	0.999	0.998	-0.019
	0.3	1.002	0.998	0.999	-0.012
	0.5	0.998	1.001	1.001	0.008
	0.7	0.998	1.003	1.003	0.032
	0.9	1.005	0.999	1.000	0.046
25	0.1	0.999	1.002	1.000	-0.002
	0.3	1.002	0.999	1.000	0.102
	0.5	0.996	0.999	1.000	0.219
	0.7	1.001	1.000	1.001	0.360
	0.9	0.997	0.999	1.000	0.537
100	0.1	1.000	0.999	1.000	0.102
	0.3	1.000	1.000	1.001	0.366
	0.5	1.000	1.000	1.000	0.608
	0.7	1.001	1.000	1.000	0.780
	0.9	0.889	0.998	0.999	0.909
400	0.1	0.999	1.000	1.000	0.172
	0.3	1.000	1.000	1.000	0.512
	0.5	0.999	1.000	1.001	0.765
	0.7	0.997	1.000	1.000	0.911
	0.9	0.808	1.000	1.000	0.974

Tables I-II present the parameter estimates of the SAR and CAR model respectively. The estimated coefficients of the covariates in the SAR and CAR models are similar. But it is different in the estimated autoregressive coefficient. To evaluate the bias estimates of autoregressive parameters, we present the estimated bias of autoregressive parameters for SAR and CAR models in Figure 3-4.

Figure 3-4 show the Bias estimate of the SAR model close to zero except for a small sample size n = 9. It is not surprising since the data were generated based on the SAR model. The CAR model has a larger bias for a small sample and the bias was reduced for a larger sample size. The high bias for large sample size was found for medium spatial dependency. To evaluate the different results between SAR and CAR models, we presented the different of the estimated parameters covariates and spatial autoregressive.

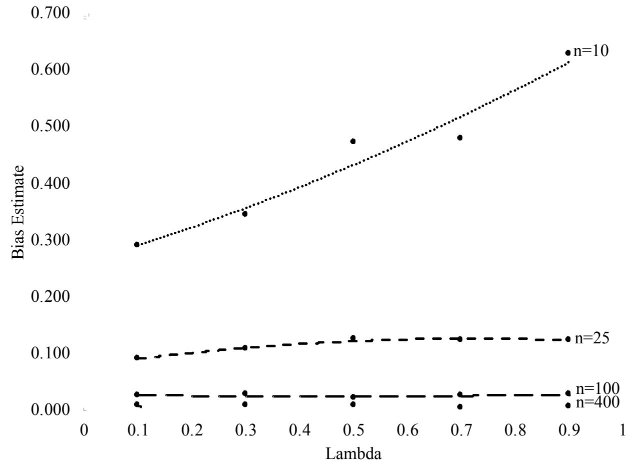

Fig.3 Estimated bias of spatial autoregressive based on SAR model

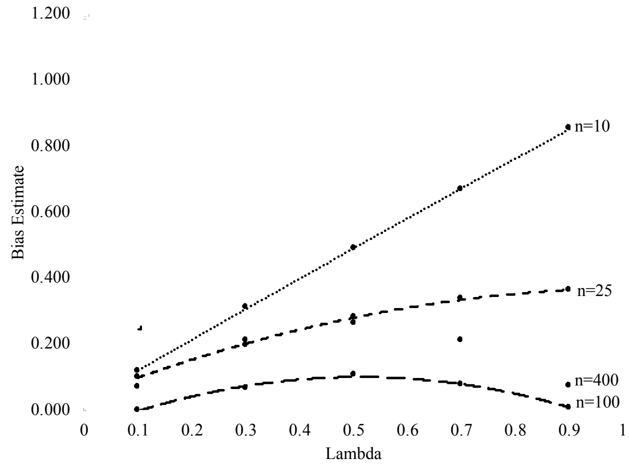

Fig.4 Estimated bias of spatial autoregressive based on CAR model

TABLE III
DIFFERENCESS OF THE PARAMETER ESTIMATES BETWEEN SAR AND CAR MODELS

n	λ	$\beta_0^{SAR}-\beta_0^{CAR}$	$\beta_1^{SAR}-\beta_1^{CAR}$	$\beta_2^{SAR}-\beta_2^{CAR}$	$\lambda^{SAR}-\lambda^{CAR}$
9	0.1	0.001	-0.001	-0.001	-0.172
	0.3	0.000	-0.002	0.001	-0.033
	0.5	-0.002	0.000	0.001	0.019
	0.7	0.001	-0.002	0.002	0.189
	0.9	0.001	0.002	0.002	0.226
25	0.1	0.000	0.000	0.000	0.010
	0.3	0.000	0.000	0.000	0.090
	0.5	0.000	0.001	0.000	0.155
	0.7	0.000	0.001	-0.001	0.216
	0.9	0.022	0.001	-0.001	0.239
100	0.1	0.000	0.000	0.000	-0.028
	0.3	0.000	0.000	0.000	-0.094
	0.5	0.000	0.000	0.000	-0.131
	0.7	0.001	0.000	0.000	-0.107
	0.9	0.109	0.001	0.000	-0.037
400	0.1	0.000	0.000	0.000	-0.080
	0.3	0.000	0.000	0.000	-0.221
	0.5	0.001	0.000	0.000	-0.273
	0.7	0.002	0.000	0.000	-0.216
	0.9	0.193	0.000	0.001	-0.081

Based on the differences of the parameter estimates between SAR and CAR models in Table III, the differences regression parameters of covariates close to zero except for spatial autoregressive parameters. There are high differences in spatial autoregressive parameters between SAR and CAR, particularly for high sample size with medium spatial dependency and small sample size with a strong spatial dependency. This finding is presented in Figure 5.

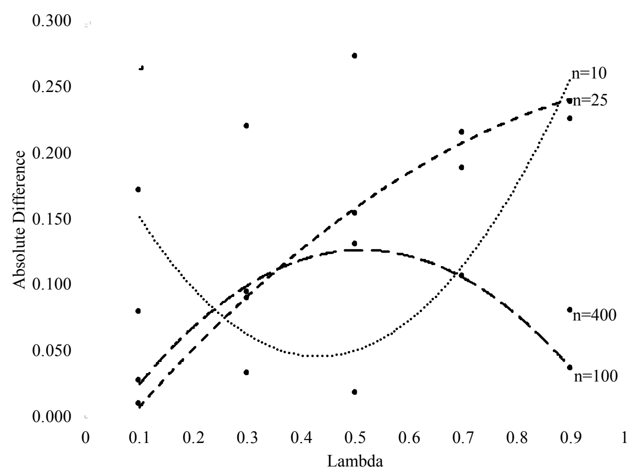

Fig.5 Estimated different of spatial autoregressive parameters between SAR and CAR models

In general, small bias and the differences of estimated spatial autoregressive parameters between SAR and CAR models occur in weak spatial dependency. It makes sense because of for weak spatial dependency the spatial regression models become ordinary regression models. For strong spatial dependency, the estimated parameter or spatial autoregressive will be very close since SAR and CAR models will have similar covariance structure.

For a large sample size (i.e., $n > 100$), the SAR and CAR models provide good results with small biases. However, for small sample size, the bias increased, particularly for $n = 9$.

The difference of spatial autoregressive parameters estimate between SAR and CAR models are relatively small for small and large true parameters spatial autoregressive (λ) particularly for medium and large sample size. It indicates that SAR and CAR models are sensitive to capture the small and strong spatial dependencies.

The high difference between SAR and CAR models occurs for a large sample size with medium spatial dependency (see Fig. 5). It describes that the SAR and CAR models are not too sensitive to capture the medium spatial dependencies. This condition makes it difficult to choose which model should be used. Because in general, spatial dependence is at a moderate level. In this condition, the researcher must have a clear concept which model should be used in addition to conducting an evaluation based on the measure of the goodness of the model such as Akaike Information Criterion (AIK), and determination coefficient.

IV. DIARRHEA MODELING

Diarrhea is a viral gastrointestinal infection disease. It is caused by several pathogenic microorganisms such as E. coli, Rotavirus, and Salmonella spp. [13,14] [15]. Diarrhea is one of the top five diseases causes of death in low- and middle-income countries [16]. Children are more susceptible to be infected by pathogenic microorganisms but the chances of an adult still high [17]. Acute diarrhea in adults is a common problem with the most common etiology is viral gastroenteritis caused by bacteria. Diarrhea in children under five years are more often occurs since of the age between birth and age 5 of a child's life is a sensitive period for growing up [15]. In total 1.5 million children worldwide death was caused by diarrhea disease and there are about two billion cases of diarrhea every year [16].

A total of 59,511 diarrhea incidences in 30 sub-districts in Bandung city in 2018 were used in the study. The healthy behaviours and population density were considered as risk factors. The data were obtained from www.data.bandung.go.id. The unit analysis in this study is sub-district. Figure 1-3 show the spatial distributions of the variables' interest.

In 2018 the average number incidence of diarrhea in Bandung reaches 1,984 per sub-district with an incidence rate IR=2,373 per 100,000 inhabitants. Note that, the IR was very high. The highest incidences were found in the southwest region and the lowest incidence in southeast regions. It is thought to be related to healthy behaviors and population density. The healthy behaviour indices were high in the southeast region and population densities were high in southwest regions.

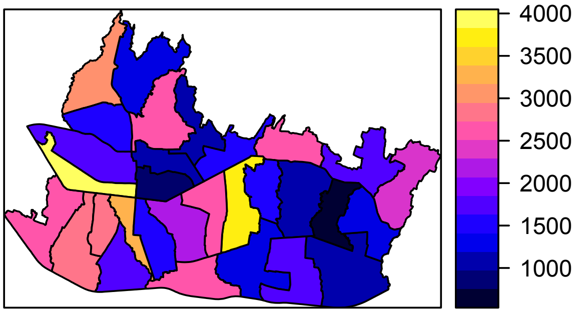

Fig.6 Spatial distribution of number of diarrhea incidences in Bandung, 2018

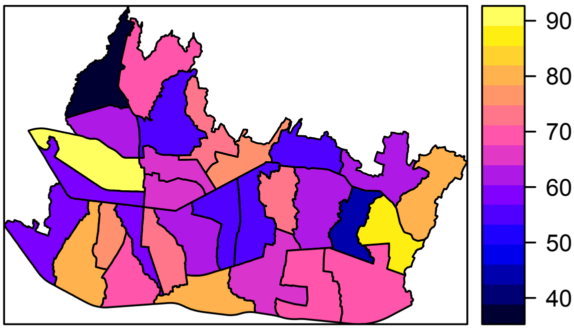

Fig.7 Spatial distribution of healthy behaviours, 2018

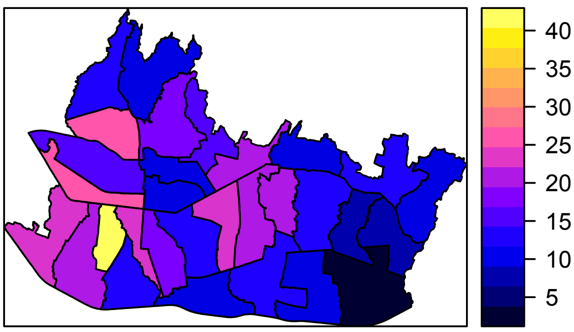

Fig.8 Spatial distribution of population density in Bandung, 2018

In order to evaluate the effects of the risk factors, we estimated the spatial econometrics models. Previously we checked the spatial dependence of diarrhea incidence through Moran's Index.

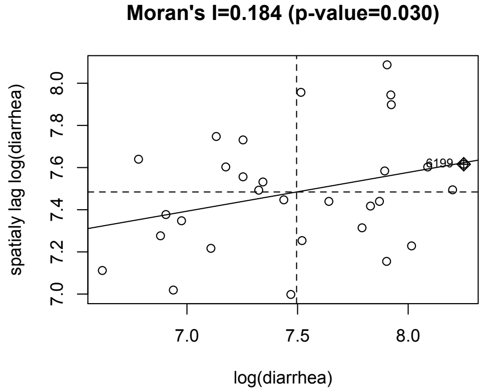

Fig.9 Moran's I plot

Figure 9 shows the Moran's I plot and its coefficient. We found there is a significant spatial autocorrelation with global Moran's I coefficient 0.184 and p-value 0.030. This fact supports that the spatial econometrics models are the appropriate models in evaluating the effect of the risk factors. To achieve a more objective evaluation, we compared two different models that commonly used in ecological studies are SAR and CAR models. The frequentist approach through ML estimation was used to estimate the risk factors' effects. The parameter estimates of SAR and CAR models are given in Tables IV and V respectively.

TABLE IV
PARAMETER ESTIMATION OF SAR MODEL

Parameters	SAR			
	Estimate	Std. Error	z	Pr(>lzl)
(Intercept)	7.338	0.421	17.423	0.000
Health Behaviour	-0.006	0.006	-1.012	0.312
Pop. Density	0.034	0.008	4.077	0.000

TABLE V
PARAMETER ESTIMATION OF CAR MODEL

Parameters	CAR			
	Estimate	Std. Error	z	Pr(>lzl)
(Intercept)	7.415	0.423	17.532	0.000
Health Behaviour	-0.006	0.006	-1.077	0.281
Pop. Density	0.031	0.009	3.502	0.000

Tables IV and V show the parameter estimates of healthy behaviour and population density based on the SAR and CAR model respectively. Both models produce almost similar results which provide more reliable information on the effects of risk factors. We found the healthy behaviours have a negative effect on diarrhea incidences and population density provides positive effects. Given the p-value with a threshold of 5%, we found only population density is significant. Table VI present the spatial autocorrelation based on SAR and CAR model.

TABLE VI
SPATIAL AUTOCORRELATION ESTIMATES

Lambda	Estimate	LR test	p-value
SAR	-0.169	0.179	0.672
CAR	-0.055	0.030	0.862

This result supports the simulation study where if the spatial dependence is weak, the estimated of the spatial autoregressive parameters between SAR and CAR models are relatively close. In order to evaluate the model validity, we checked the normality assumption and white noise residuals.

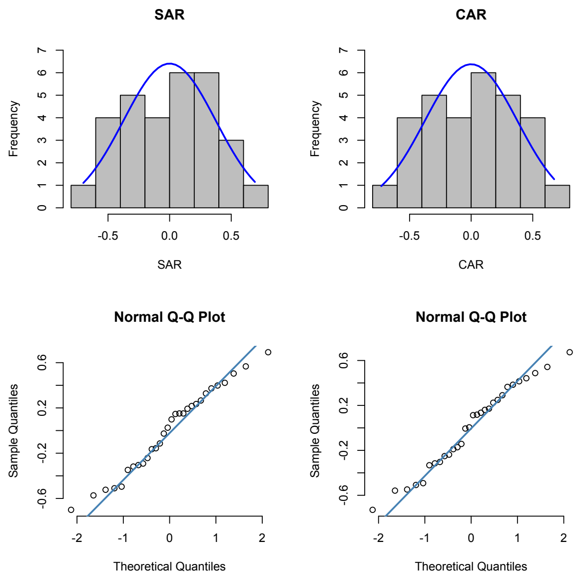

Fig. 10. Normality assumption

TABLE VII
TESTING OF THE NORMALITY ASSUMPTIONS

Model	Shapiro Wilk	p-value
SAR	0.971	0.572
CAR	0.970	0.529

Figure 10 provides the normality plot of the residuals and Table VII provides the Shapiro Wilk test of normality. Both of them support that the residuals of SAR and CAR models satisfy the normality assumption.

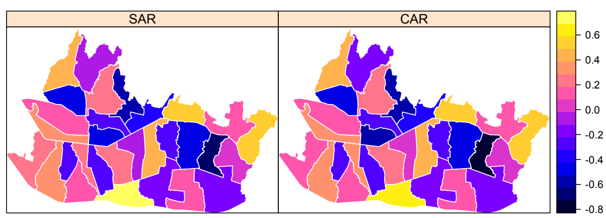

Fig. 11. Residuals values of SAR and CAR model

Figure 11 shows the spatial distribution of the residuals of SAR and CAR models. Both models provide a random pattern. It is supported by the insignificant of global Moran's I of SAR and CAR models -0.01 (p-value = 0.456) and -0.019 (p-value = 0.449), respectively.

SAR and CAR models also have similar fitted values of diarrhea incidences with the Pearson correlation coefficient is 0.998. The similarity shows in Figure 12. It indicates that SAR and CAR models provide similar predicted values.

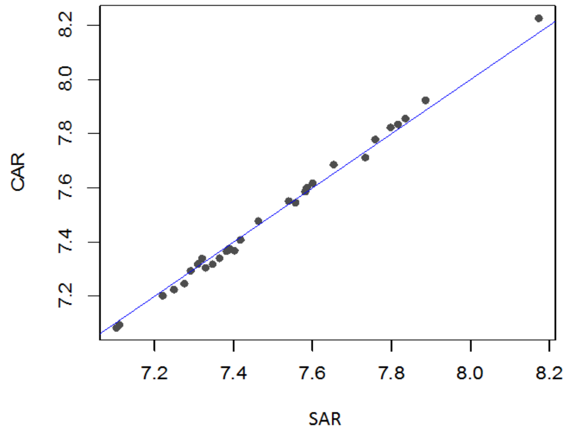

Fig. 12. Association between SAR and CAR models

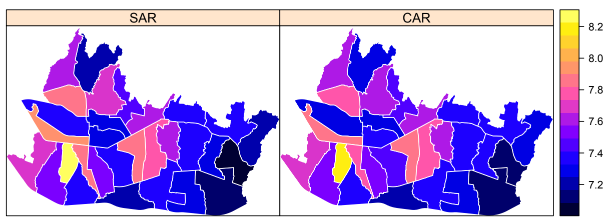

Fig. 13. Fitted values of SAR and CAR model

Finally, Figure 13 shows the fitted values of SAR and CAR models that can be used to identify the high-risk regions. The high-risk regions were found in the majority sub-districts of western and central regions of Bandung. The low-risk regions were found in eastern regions. Both method provide a similar result.

V. Discussion And Conclusion

Modelling spatial interaction in ecological data are commonly done via the spatial autoregressive models by considering spatial dependence structure into the covariance matrix. Two standard autoregressive models have been used often are the conditional autoregressive model (CAR) and the simultaneously autoregressive model (SAR) [18]. SAR and CAR models have different conceptual models with the equivalent main objectives. In practice, both models may produce similar results, particularly in the regression coefficient. However, due to the different concepts in defining the spatial structure, the estimated spatial autoregressive parameters relatively different which is supported by Monte Carlo simulation and practical example. The regression coefficients based on SAR and CAR are very similar and the predicted values are also very similar. If the objective to evaluate the regression effects of covariates or risk factors, we can choose one of the SAR or CAR models. However, if the objective to evaluate spatial dependency, we have to be more careful particularly if the prior information on the effect of the spatial dependency is at the medium level. However, if the prior information of spatial dependency in small or high effects, SAR and CAR models provide very similar estimates of the spatial autoregressive parameter. Model SAR model might be more interpretable because the model specifies in a simple way however, CAR models more preferable for cases with symmetric dependency structure [4]. SAR and CAR models should be selected by considering the application. CAR is more often used to smoothing estimates commonly in disease modelling and mapping and the SAR model is more often for specification testing purposes.

We apply SAR and CAR models on diarrhea data in Bandung city to provide a clear review of SAR and CAR models in practice. Given the healthy behaviours and population density as the risk factors, both models presented the population density has a significant effect on diarrhea in Bandung, Indonesia. Both models also have similar predicted values with a very high correlation. We believe, based on SAR and CAR models, increasing population density tends to increase diarrhea cases. Although the healthy behaviours is not gives a significant effect in describing the diarrhea variation in Bandung due to the small sample size, the good sanitation and hygiene life will help in a great control of diarrhea cases. Population density is difficult to control due to the population size increases every year while the area was not increased. Therefore, healthy behaviours become an important aspect of daily life to avoid the disease burden. In addition, Bandung government should develop an early system program to early detection of the high-risk area and allocate more resources to handle the diarrhea problem in high-risk areas. Both models produce a negative spatial autoregressive which supports the unobserved variables that are not interacted by space. The identification of high-risk areas will assist in the planning of control strategies in the specific areas rather than plan for the whole city. Fitted values of the SAR or CAR model provide information about the predicted number of diarrhea incidence that can be used to detect more reliable high-risk areas after separate the noises.

In practice, modelling ecological data using both approaches and select the best model based on the ability of the model in describing the empirical phenomenon could be a good solution to get more reliable result.

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