A Shoreline Evolution Model with a Twin Groins Structure using Unconditionally Stable Explicit Finite Difference Techniques

Pidok Unyapoti, Nopparat Pochai

***Abstract*—Beach erosion is a natural process that occurs when conveying sediment away from the shoreline is not balanced by depositing new material on the shoreline. This is a problem that is causing beach areas to decline. To avoid beach erosion and flooding, a sea wall and groin have been built. Shoreline evolution prediction is used to investigate the beach topography in the future. There are three phenomena give a large effect to the coastal structure such as the erosion, the accretion and the water level changes. To investigate of beach erosion and beach deposition is needed qualitative understanding of idealized shoreline response to the governing process. In this research, we introduce a governing equation of a one-dimensional shoreline evolution model when a couple of groins is added. The introduced model is a transient one-line model. The manipulation of physical parameters for the model is introduced. The setting method of the initial condition and the boundary conditions techniques when a couple of groin structure effect are also proposed. The traditional forward time centered space method and the unconditionally stable Saulyev finite difference methods are employed to approximate the incremental model in each year. The proposed numerical models give practically simulation for long-term shoreline evolution investigation. The proposed simulation can be used to predict the efficiency of a groin system construction in a local beach. The model is a tool for environment impact assessment of a installing groin structure project.**

***Index Terms*—shoreline evolution, groin system, one-dimension, mathematical model, finite difference method**

I. Introduction

Beach erosion is a natural process which occurs whenever the transport of material away from the shoreline is not balanced by new material being deposited onto the shoreline. This is a problem that causes a decrease in beach areas. In order to prevent beach erosion and beach deposition so it has devised a sea wall and groin. In [9], they proposed a new approach to practical groin modeling is explained by the use of the GENESIS shoreline response model to demonstrate the action of single and multiple groins. Predictions of the study are tested in the replication of the shoreline modification found in the 15 groins of Westhampton, Long Island, New York. In [7] reported changes in beach profile due to the construction of a single zigzag type of porous groins named GROPOZAG.

Qualitative awareness of the idealized reaction of the shoreline to the governance process is required to examine beach erosion and beach deposition. Analytical solution, based on the mathematical model that explains basic physics, is the only means of understanding it. Many authors have achieved an analytical solution to the evolution of the shoreline using a basic mathematical method. Many authors have developed one-line theory, and several contributors include [3], [4], [2], [1], [6], [12], and [8] in the analytical solution of the evolution of the shoreline. Analytical solutions cannot be assumed to present quantitatively precise solutions to the problems containing complex boundary conditions and wave inputs. A numerical model of shoreline evolution would be more fitting in the actual case.

A general expression for the long-shore sand transport rate was developed by [10]. The empirical predictive formula for the amplitude of the long-shore sand transport rate presented by [5]. In [11], they have examined and presented two numerical schemes of shoreline evolution for simplified configuration beach. In [13], [14], [29], [30], [31], they have used the conditionally stable explicit finite difference methods to approximate their model solutions. In [28], [32], [33], [34], they have used the numerical methods to approximate their model solution.

In [15], they proposed the Equilibrium energy function (EEF) analytical method and the shoreline evolution model. Testing of the proposed model at Nova Icaria reveals the same capabilities with only one measurement parameter as state-of-the-art models with more than 4 free parameters. In [16], they proposed one-line model concept has been applied to achieve long-term shoreline simulation, as well as to assist and produce improved coastal engineering techniques to manage erosion. The model was applied to the two northwest coasts of Portugal: Aveiro and Figueira da Foz. The results make the qualitative evaluation of the main possible implications of continuous erosion. a general and replicable chain method was proposed, tested at fragile shorelines in Southern Italy and based on a collaborative analysis of data collected, computational methods and computational modelling. This will help to best explain the complexities of the shoreline climate, recognize normal and repeated erosion-related mechanisms, and forecast potential future trends that are useful for shoreline activity preparing. In [18], they proposed probabilistic changes to the shoreline

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are calculated by using two simulations. The first simulation is GenCade simulation, which is used to predict the long-term evolution of the shoreline induced by natural offshore waves. The second is the Monte Carlo simulation, which is used to simulate the evolution of the shoreline in response to changes in sea level. In [19], a basic coastline profile model behavioral template was proposed to be calibrated and tested against a 6-year coastline location time series derived from a shoreline imaging system on the Gold Coast, Australia. Monitoring the model on unknown data shows that it can reproduce the dominant different seasons coastline transition observed at this site and up to 77% of the degraded coastline variability. In [20], they proposed the ONELINE modelling approach and demonstrated its capabilities through concept testing and case studies. This outlines two case studies in which complicated beach structure architectures are represented. The first is a groin area at Sea Isle City, New Jersey, in the East Coast of the United States. The second is along the coast of the Nile Delta in Egypt. In [21], they proposed the comparison of analytical and numerical solutions in the idealized wave condition for four different shoreline situations.

In this research, we introduce a governing equation of a one-dimensional shoreline evolution model when a couple of groins is added.

II. Governing Equation

A. Shoreline evolution model

In a one-dimensional shoreline evolution model, while maintaining the same shape, the beach shape is supposed to move towards land and towards the sea, meaning that all the bottom outlines become parallel.

Consequently, under this premise, this is necessary to define the horizontal direction of the profile with respect to the baseline, and one contour line should be used to define changes to the design and volume of the beach plane as the beach erosions and accretes. The main premise of the model is that the sand is moved along the coast on a profile between two well-defined limit elevations. A contribution to the adjustment in volume occurs where there is a discrepancy in the rate of longshore sand transfer on the side of the segment and the related sand consistency. The principles of conservation of mass must be always adapted to the system. The following differential equation for the evolution of the shoreline is generated by considering the above concepts,

$$\frac{\partial y}{\partial t} = \frac{1}{D_B + D_C}\left(-\frac{\partial Q}{\partial x}\right), \tag{1}$$

where x is the co-ordinate on the shores (m), y is the location of the shoreline (m) and perpendicular to the x-axis, t is time (day), Q is the long-shore sand transport rate (m^3/day), D_B is the average height of the berm (m) and D_C is the average depth of closure (m).

To solve (1), it was necessary to define a term for the longshore sand transport rate Q. This quantity is assumed to have been obtained by the oblique wave occurring to the shoreline. The US Army Corp has created a generalized term for long-shore sand transport rate [11],

$$Q = Q_0 \sin(2\alpha_b), \tag{2}$$

where Q_0 is the long-shore sand transport rate amplitude. The general formula for the long-shore sand transport rate amplitude is as follows [6],

$$Q_0 = \frac{\rho}{16}\left(H_b^2 c_{gb}\right)\frac{K}{(\rho_s - \rho)(1-n)}, \tag{3}$$

The quantity α_b the angle between breaking wave crest impact angle and local shoreline, and can be written as,

$$\alpha_b = \alpha_0 - \tan^{-1}\left(\frac{\partial y}{\partial x}\right), \tag{4}$$

where α_0 is the angle between breaking wave crests impact angle and x-axis. In the case of beaches with a slight slope, it can be concluded that the angle of the wave breaking to the shoreline is minimal.

Assuming that, $\sin(2\alpha_b) \approx 2\alpha_b$, and $\tan^{-1}\left(\frac{\partial y}{\partial x}\right) \approx \left(\frac{\partial y}{\partial x}\right)$.

Substituting (4), in (2), and assuming a beach with a slight slope, we are obtaining,

$$Q = Q_0\left(2\alpha_b - 2\frac{\partial y}{\partial x}\right), \tag{5}$$

substituting (5), in (1), and ignoring the sources or sinks along the shoreline provides the following:

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2}, \tag{6}$$

for all $(x,t) \in (L,T)$, where $D = \dfrac{2Q_0}{D_B + D_C}$.

B. Physical parameters

Physical parameter of the model can be illustrated as show in Fig. 1-2. that are listed below.

α_0 is the angle between breaking wave crests impact angle and x-axis.

Q_0 is the long-shore sand transport rate amplitude.

D_B is the averaged berm height.

D_C is the averaged closure depth.

L is Alongshore.

T is Time of simulation.

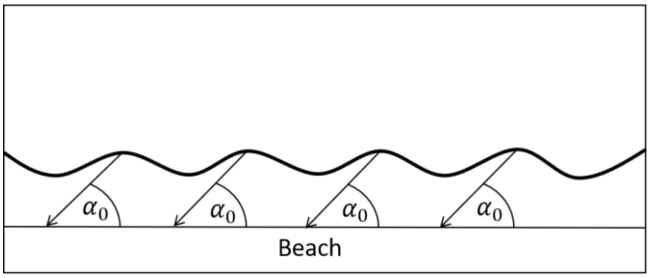

Fig. 1. Breaking wave crests impact angle

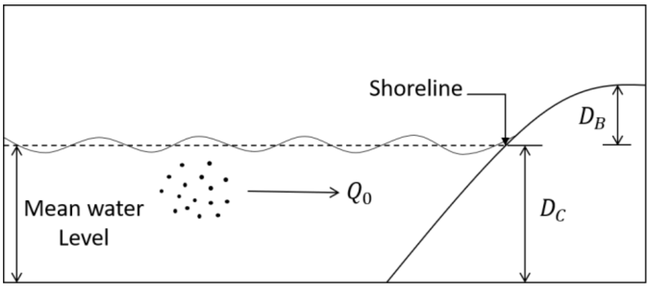

Fig. 2. Shoreline physical parameters

C. The initial and boundary conditions

Straight Impermeable groin system.

The initial shoreline is assumed to be parallel to the x-axis.

Assuming that, the angle between breaking wave crests impact angle to the shoreline is α_0 as shown in Fig. 3. It follows that the sand transport rate along the shoreline is uniform. The groin is instantaneously added at $x=0$ as shown in Fig. 3. These means that the initial condition becomes,

$$y(x,0)=0, \tag{7}$$

boundary conditions are also assumed by,

$$\frac{\partial y}{\partial x}=-\tan\alpha_0 \quad \text{at} \quad x=0, \tag{8}$$

and

$$\frac{\partial y}{\partial x}=\tan\alpha_0 \quad \text{at} \quad x=L, \tag{9}$$

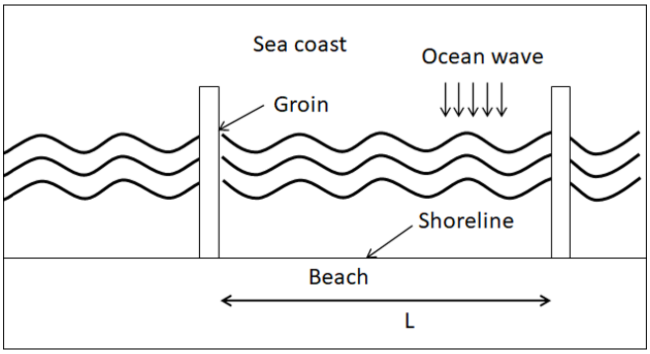

Fig. 3. Initial shoreline with configuration straight impermeable groins.

III. Numerical Techniques

A. Grid Spacing

We are discretizing (6) by splitting the interval $[0,L]$ into M subintervals such as $M\Delta x=L$ and the interval $[0,T]$ into N subintervals such as $N\Delta t=T$. We then approximate $y(x_i,t_n)$ by y_i^n, at the point $x_i=i\Delta x$ and $t_n=n\Delta t$, where $0\le i\le M$ and $0\le n\le N$ in which there are positive integers of M and N.

B. Traditional forward time centered space techniques

The forward time centered space techniques will also be used. Consequently, the finite difference approximation becomes [22],

$$y\cong y_i^n, \tag{10}$$

$$\frac{\partial y}{\partial t}\cong\frac{y_i^{n+1}-y_i^n}{\Delta t}, \tag{11}$$

$$\frac{\partial y}{\partial x}\cong\frac{y_{i+1}^n-y_{i-1}^n}{2\Delta x}, \tag{12}$$

$$\frac{\partial^2 y}{\partial x^2}\cong\frac{y_{i+1}^n-2y_i^n+y_{i-1}^n}{(\Delta x)^2}, \tag{13}$$

where $A=\dfrac{D\Delta t}{(\Delta x)^2}$.

Substituting (10) - (13), in (6), we are obtaining,

$$\frac{y_i^{n+1}-y_i^n}{\Delta t}=D\left(\frac{y_{i+1}^n-2y_i^n+y_{i-1}^n}{(\Delta x)^2}\right), \tag{14}$$

for $1\le i\le M-1$ and $0\le n\le N-1$. (14), can be written in an explicit form of finite difference as follows,

$$y_i^{n+1}=Ay_{i+1}^n+(1-2A)y_i^n+Ay_{i-1}^n, \tag{15}$$

for $1\le i\le M-1$ and $0\le n\le N-1$.

C. An unconditionally Saulyev finite difference techniques

The Saulyev finite difference techniques will also be used. We can obtain that the finite difference approximation is

$$y\cong y_i^n, \tag{16}$$

$$\frac{\partial y}{\partial t}\cong\frac{y_i^{n+1}-y_i^n}{\Delta t}, \tag{17}$$

$$\frac{\partial^2 y}{\partial x^2}\cong\frac{y_{i+1}^n-y_i^n-y_i^{n+1}+y_{i-1}^{n+1}}{(\Delta x)^2}, \tag{18}$$

where $A=\dfrac{D\Delta t}{(\Delta x)^2}$.

Substituting (16) - (18), in (6), we are obtaining,

$$\frac{y_i^{n+1}-y_i^n}{\Delta t}=D\left(\frac{y_{i+1}^n-y_i^n-y_i^{n+1}+y_{i-1}^{n+1}}{(\Delta x)^2}\right), \tag{19}$$

for $1\le i\le M-1$ and $0\le n\le N-1$. (19), can be written in an explicit form of finite difference as follows,

$$y_i^{n+1}=\frac{1}{(1+A)}\left(Ay_{i+1}^n+(1-A)\,y_i^n+Ay_{i-1}^{n+1}\right), \tag{20}$$

for $1\le i\le M-1$ and $0\le n\le N-1$.

D. The employment of traditional forward time centered space techniques to the left and the right boundary conditions

The forward time centered space techniques will also be used. Consequently, the finite difference approximation becomes,

$$y\cong y_i^n, \tag{21}$$

$$\frac{\partial y}{\partial t}\cong\frac{y_i^{n+1}-y_i^n}{\Delta t}, \tag{22}$$

$$\frac{\partial y}{\partial x}\cong\frac{y_{i+1}^n-y_{i-1}^n}{2\Delta x}, \tag{23}$$

where $A=\dfrac{D\Delta t}{(\Delta x)^2}$.

Substituting (21) - (23), in (6), we are obtaining,

$$\frac{y_i^{n+1}-y_i^n}{\Delta t}=D\left(\frac{y_{i+1}^n-2y_i^n+y_{i-1}^n}{(\Delta x)^2}\right), \tag{24}$$

For i = 0, substitution of the uncertain value of the left boundary is approximated by the method of center difference with the specified left boundary condition. We are obtaining,

$$y_{-1}^n=y_1^n-2(\Delta x)(-\tan\alpha_0), \tag{25}$$

substituting (25), in (24), we are obtaining,

$$y_i^{n+1}=(1-2A)y_i^n+2Ay_{i+1}^n-2A(\Delta x)(-\tan\alpha_0), \tag{26}$$

For i = M, substitution of the uncertain value of the right boundary is approximated by the method of center difference with the specified right boundary condition. We are obtaining,

$$y_{M+1}^n=y_{M-1}^n+2(\Delta x)(\tan\alpha_0), \tag{27}$$

substituting (27), in (24), we are obtaining,

$$y_i^{n+1}=2Ay_{i-1}^n+(1-2A)y_i^n+2A(\Delta x)(\tan\alpha_0), \tag{28}$$

(26), and (28), could be used to approximate the values y_i^n of the solution domain grid points.

IV. Sand Transport Rate Setting

Assuming that the sediment density (ρ_s) [23], the sea water's density (ρ) [24], the porosity (n) [25], the non-dimensional coefficient which is a function of particle size (K) [26], The averaged berm height (D_B) and the averaged closure depth (D_C) as listed below.

TABLE I
PARAMETERS OF SAND TRANSPORT RATE

The sediment density $(\rho_s(kg/m^3))$	1700
The sea water's density $(\rho(kg/m^3))$	1020
The porosity (n)	0.406
The non-dimensional coefficient which is a function of particle size (K)	0.375
The averaged berm height $(D_B(m))$	2
The averaged closure depth $(D_C(m))$	28

The wave group velocity (c_g) and the wave height (H) in each month along a year is measured by field data on the gulf of Thailand such that data are collected by Geo Informatics and Space Technology Development Agency (Public Organization) (GISTDA) [27] as listed below.

TABLE II
THE WAVE GROUP VELOCITY AND THE WAVE HEIGHT

Month	c_g (m/day)	H (m)
Jan 2019	8951.04	1.5
Feb 2019	6998.4	1.5
Mar 2019	5866.56	0.5
Apr 2019	6920.64	1.5
May 2019	5719.68	0.5
Jun 2019	5546.88	0.5
Jul 2018	8225.28	1.5
Aug 2018	9357.12	1.5
Sep 2018	13711.68	1.5
Oct 2018	15085.44	2.5
Nov 2018	10877.76	1.5
Dec 2018	11396.16	1.5

The long-shore sand transport rate amplitude (Q_0) are obtained by (3), and the long-shore transport rates (D) are obtained by (6), as listed below.

TABLE III
THE LONG-SHORE SAND TRANSPORT RATE AMPLITUDE AND THE LONG-SHORE TRANSPORT RATE

Month	Q_0 (m/day)	D (m/day)
Jan 2019	1191.99	79.4659
Feb 2019	931.96	62.1307
Mar 2019	86.80	5.7869
Apr 2019	921.61	61.4403
May 2019	84.63	5.6420
Jun 2019	82.07	5.4716
Jul 2018	1095.34	73.0227
Aug 2018	1246.07	83.071
Sep 2018	1825.95	121.7301
Oct 2018	5580.26	372.017
Nov 2018	1448.57	96.5710
Dec 2018	1517.60	101.1233

V. Numerical Experiment

To examine the long-term evolution of the shoreline. The numerical results of the various beach scenarios are considered and the solution to the idealized problem is introduced. Assuming, during the experiments, that the length of the shoreline considered is L = 100, 200, 300 and

400 m. The averaged berm height D_B is 2 m. The closure depth D_C is 28 m. The breaking wave impact angle α_0 is 0.02. The simulation setting is illustrated in Fig. 4.

We are going to employ the traditional forward time centered space techniques (15), and the Saulyev finite difference techniques (20), to approximate the model solution. The calculated results L = 100, 200, 300 and 400 m are shown in Fig. 5-12.

The approximated solutions of the traditional forward time centered space techniques and Saulyev finite difference techniques gives approximated solutions in Table 4-11.

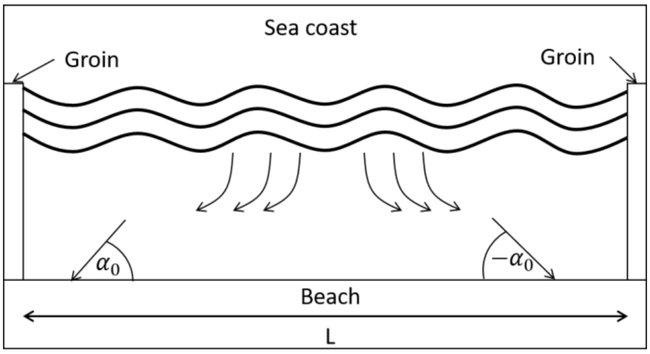

Fig. 4. Initial shoreline.

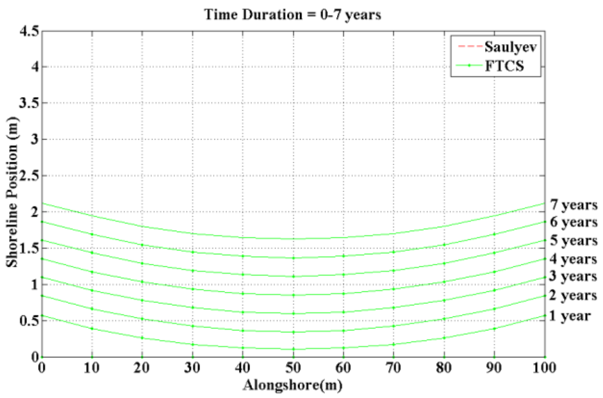

Fig. 5. Shoreline evolution with distance between groin 100 m in 0-7 years.

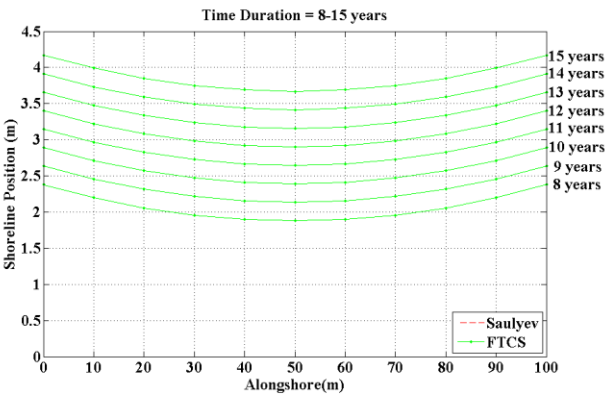

Fig. 6. Shoreline evolution with distance between groin 100 m in 8-15 years.

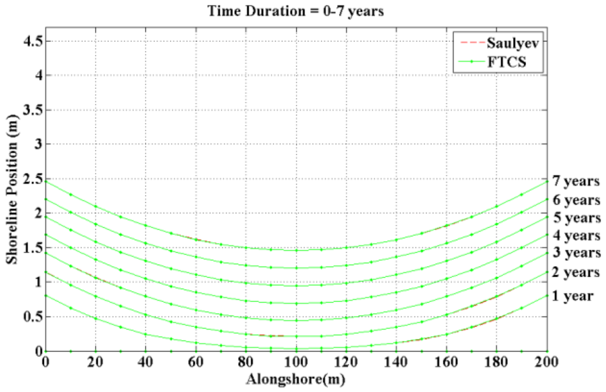

Fig. 7. Shoreline evolution with distance between groin 200 m in 0-7 years.

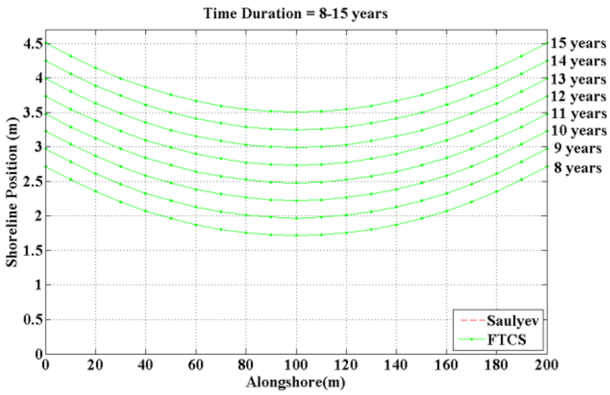

Fig. 8. Shoreline evolution with distance between groin 200 m in 8-15 years.

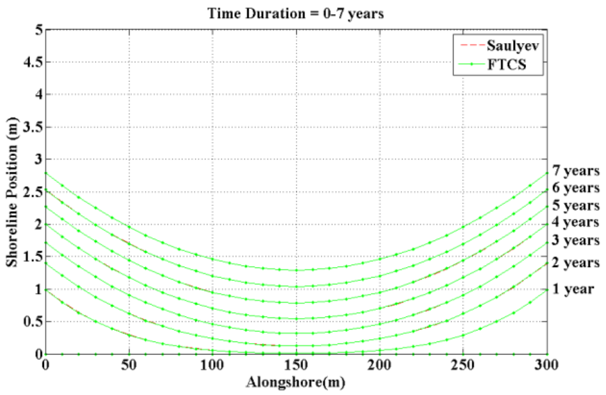

Fig. 9. Shoreline evolution with distance between groin 300 m in 0-7 years.

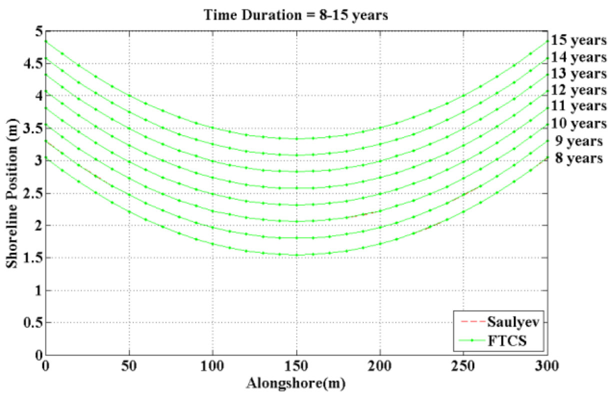

Fig. 10. Shoreline evolution with distance between groin 300 m in 8-15 years.

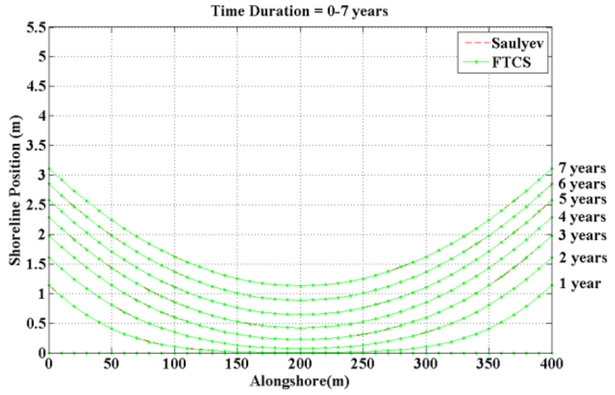

Fig. 11. Shoreline evolution with distance between groin 400 m in 0-7 years.

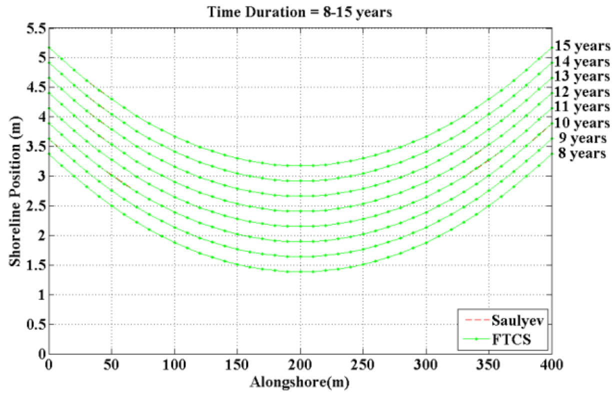

Fig. 12. Shoreline evolution with distance between groin 400 m in 8-15 years.

TABLE IV
APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES L IS 100 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	0.5679	0.2604	0.1206	0.1206	0.2604	0.5679
5	1.6102	1.2902	1.1302	1.1302	1.2902	1.6102
10	2.8905	2.5705	2.4104	2.4104	2.5705	2.8905
15	4.1708	3.8507	3.6907	3.6907	3.8507	4.1708

TABLE V
APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES L IS 100 M

Time (Years)	Distance (m)					
	0	20	40	60	80	100
1	0.5682	0.2607	0.1208	0.1206	0.2602	0.5676
5	1.6103	1.2903	1.1303	1.1302	1.2902	1.6103
10	2.8906	2.5705	2.4105	2.4105	2.5705	2.8906
15	4.1708	3.8508	3.6908	3.6908	3.8508	4.1708

TABLE VI
APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES L IS 200 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	0.8036	0.4664	0.2455	0.1176	0.0551	0.0369
5	1.9445	1.5846	1.3050	1.1054	0.9858	0.9459
10	3.2256	2.8655	2.5855	2.3854	2.2654	2.2254
15	4.5058	4.1458	3.8657	3.6657	3.5457	3.5057
Time (Years)	Distance(m)					
	120	140	160	180	200	
1	0.0551	0.1176	0.2455	0.4664	0.8036	
5	0.9858	1.1054	1.3050	1.5846	1.9445	
10	2.2654	2.3854	2.5855	2.8655	3.2256	
15	3.5457	3.6657	3.8657	4.1458	4.5058	

TABLE VII
APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES L IS 200 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	0.8043	0.4671	0.2463	0.1182	0.0555	0.0369
5	1.9449	1.5850	1.3053	1.1057	0.9859	0.9460
10	3.2257	2.8656	2.5856	2.3855	2.2655	2.2255
15	4.5059	4.1459	3.8658	3.6658	3.5458	3.5058
Time (Years)	Distance(m)					
	120	140	160	180	200	
1	0.0549	0.1172	0.2449	0.4656	0.8029	
5	0.9857	1.1053	1.3047	1.5843	1.9442	
10	2.2655	2.3855	2.5855	2.8655	3.2255	
15	3.5458	3.6658	3.8658	4.1458	4.5059	

TABLE VIII
APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES L IS 300 M

Time (Years)	Distance(m)					
	0	20	40	60	80	100
1	0.9857	0.6370	0.3858	0.2179	0.1145	0.0560
5	2.2700	1.8974	1.5796	1.3162	1.1067	0.9503
10	3.5593	3.1860	2.8660	2.5993	2.3860	2.2260
15	4.8398	4.4664	4.1463	3.8796	3.6663	3.5062
Time (Years)	Distance(m)					
	120	140	160	180	200	220
1	0.0143	0.0143	0.0264	0.0560	0.1145	0.0143
5	0.7947	0.7947	0.8464	0.9503	1.1067	0.7947
10	2.0661	2.0661	2.1194	2.2260	2.3860	2.0661
15	3.3462	3.3462	3.3996	3.5062	3.6663	3.3462
Time (Years)	Distance(m)					
	240	260	280	300		
1	0.2179	0.3858	0.6370	0.9857		
5	1.3162	1.5796	1.8974	2.2700		
10	2.5993	2.8660	3.1860	3.5593		
15	3.8796	4.1463	4.4664	4.8398		

TABLE IX

APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES L IS 300 M

Time (Years)	Distance(m) 0	20	40	60	80	100
1	0.9869	0.6382	0.3869	0.2190	0.1153	0.0567
5	2.2709	1.8983	1.5805	1.3170	1.1073	0.9508
10	3.5597	3.1863	2.8663	2.5996	2.3863	2.2263
15	4.8399	4.4665	4.1465	3.8798	3.6664	3.5064

Time (Years)	Distance(m) 120	140	160	180	200	220
1	0.0268	0.0145	0.0143	0.0261	0.0556	0.1138
5	0.8468	0.7948	0.7946	0.8462	0.9499	1.1062
10	2.1196	2.0662	2.0661	2.1194	2.2259	2.3859
15	3.3997	3.3463	3.3463	3.3996	3.5062	3.6663

Time (Years)	Distance(m) 240	260	280	300
1	0.2170	0.3847	0.6359	0.9846
5	1.3156	1.5789	1.8967	2.2692
10	2.5991	2.8657	3.1857	3.5591
15	3.8796	4.1463	4.4663	4.8397

TABLE X

APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING THE TRADITIONAL FORWARD TIME CENTERED SPACE TECHNIQUES L IS 400 M

Time (Years)	Distance(m) 0	20	40	60	80	100
1	1.1392	0.7836	0.5136	0.3198	0.1886	0.1051
5	2.5781	2.1998	1.8647	1.5723	1.3220	1.1127
10	3.8917	3.5117	3.1718	2.8721	2.6125	2.3929
15	5.1733	4.7933	4.4533	4.1532	3.8932	3.6732

Time (Years)	Distance(m) 120	140	160	180	200	220
1	0.0553	0.0274	0.0130	0.0064	0.0046	0.0064
5	0.9434	0.8130	0.7207	0.6656	0.6473	0.6656
10	2.2134	2.0738	1.9741	1.9143	1.8944	1.9143
15	3.4932	3.3532	3.2532	3.1932	3.1732	3.1932

Time (Years)	Distance(m) 240	260	280	300	320	340
1	0.0130	0.0274	0.0553	0.1051	0.1886	0.3198
5	0.7207	0.8130	0.9434	1.1127	1.3220	1.5723
10	1.9741	2.0738	2.2134	2.3929	2.6125	2.8721
15	3.2532	3.3532	3.4932	3.6732	3.8932	4.1532

Time (Years)	Distance(m) 360	380	400
1	0.5136	0.7836	1.1392
5	1.8647	2.1998	2.5781
10	3.1718	3.5117	3.8917
15	4.4533	4.7933	5.1733

TABLE XI

APPROXIMATED SHORELINE EVOLUTION ALONG 15 YEARS USING THE SAULYEV FINITE DIFFERENCE TECHNIQUES L IS 400 M

Time (Years)	Distance(m) 0	20	40	60	80	100
1	1.1407	0.7852	0.5152	0.3212	0.1899	0.1061
5	2.5796	2.2012	1.8661	1.5737	1.3232	1.1138
10	3.8924	3.5124	3.1726	2.8728	2.6131	2.3935
15	5.1737	4.7937	4.4536	4.1536	3.8935	3.6735

Time (Years)	Distance(m) 120	140	160	180	200	220
1	0.0560	0.0280	0.0134	0.0066	0.0046	0.0064
5	0.9443	0.8138	0.7213	0.6659	0.6474	0.6655
10	2.2138	2.0741	1.9744	1.9145	1.8944	1.9142
15	3.4935	3.3534	3.2534	3.1933	3.1733	3.1932

Time (Years)	Distance(m) 240	260	280	300	320	340
1	0.0128	0.0270	0.0546	0.1042	0.1874	0.3184
5	0.7203	0.8124	0.9425	1.1117	1.3208	1.5711
10	1.9739	2.0735	2.2130	2.3925	2.6120	2.8716
15	3.2532	3.3531	3.4931	3.6730	3.8930	4.1530

Time (Years)	Distance(m) 360	380	400
1	0.5121	0.7821	1.1376
5	1.8634	2.1984	2.5768
10	3.1712	3.5110	3.8910
15	4.4530	4.7930	5.1731

VI. DISCUSSION

In this paper, we measure the long-shore transport rates (D) in each month along a year by field data. We obtain the long-shore transport rates (D) from (6), $D = \frac{2Q_0}{D_B + D_C}$. The averaged berm height (D_B). The averaged closure depth (D_C). The long-shore sand transport rate amplitude (Q_0). We obtain the long-shore sand transport rate amplitude (Q_0) from (3), $Q_0 = \frac{\rho}{16}\left(H_b^2 c_{gb}\right)\frac{K}{(\rho_s - \rho)(1-n)}$. The sediment density ($\rho_s$), the sea water's density (ρ), the porosity (n) and the non-dimensional coefficient which is a function of particle size (K) as shown in Table 1. The wave group velocity (c_g) and the wave height (H) in each month along a year is measured by field data as shown in Table 2. The long-shore sand transport rate amplitude (Q_0) and the long-shore transport rates (D) as shown in Table 3.

The shoreline evolution in each year can be obtained by the traditional forward time centered space techniques and the Saulyev finite difference techniques.

The length of the considered shoreline is 100 m as shown in Tables 4, 5 and Fig. 5, 6. The distance from the farthest shoreline evolution is 4.1708 m. The shortest distance from the shoreline evolution is 3.6707 m.

The length of the considered shoreline is 200 m as shown in Tables 6, 7 and Fig. 7, 8. The distance from the farthest shoreline evolution is 4.5059 m. The shortest distance from the shoreline evolution is 3.5707 m.

The length of the considered shoreline is 300 m as shown in Tables 8, 9 and Fig. 9, 10. The distance from the farthest shoreline evolution is 4.8398 m. The shortest distance from the shoreline evolution is 3.3396 m.

The length of the considered shoreline is 400 m as shown in Tables 10, 11 and Fig. 11, 12. The distance from the farthest shoreline evolution is 5.1737 m. The shortest distance from the shoreline evolution is 3.1732 m.

Approximate shoreline evolutions of all numerical approaches in 4 lengths of the considered shoreline are compatible.

VII. CONCLUSION

In this study, when a couple of groins are installed, we introduce a governing equation of a one-dimensional model of shoreline evolution. The implemented model is a one-line model, which is transient. The modification is implemented of physical parameters for the model. The initial condition setting approach and the boundary conditions techniques while also proposing some groin structure effect. The classical forward-time centered-space method and the unconditionally stable Saulyev finite differential methods are used to measure the incremental model in each year. The proposed numerical models give practical simulation for long-term shoreline evolution investigation. The simulation proposed can be used to predict the efficiency of constructing a groin network at a local beach. The classical forward-time centered-space method provides more precise solutions than the solutions approximated by the Saulyev. For many cases, when the time increment for the classical forward-time centered-space method has increased, solution cannot be handled. However, the Saulyev method can still handle numerical solutions for any case because the stability condition is not limited. This means the Saulyev method proposed is a functional computational approach for the concept of shoreline evolution. The model is a tool for environment impact assessment of an installing groin structure project. It can be used to evaluate the groin system in a focused area.

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