The Generalized Riemann Problem for the Simplified Model in Magnetogasdynamics with Combustion

Wenhua Sun and Yujin Liu

Abstract—**We consider the generalized Riemann problem for the simplified combustible model in magnetogasdynamics in a neighborhood of the origin $(t>0)$ in the (x,t) plane. In the light of the different Riemann solutions situations, we obtain the unique solutions after perturbation. We observe that there are essential differences between the above two cases. We obtain that the transition between the detonation wave and the deflagration wave occurs. Although the combustion wave does not appear in the corresponding Riemann problem, the perturbed combustion wave may appear, which shows that the unburnt gas is unstable.**

Index Terms—**Generalized Riemann problem, Hyperbolic conservation laws, Detonation wave, Deflagration wave, Magnetogasdynamics.**

I. Introduction

MAGNETOGASDYNAMICS is very important in studying engineering physics ([1], [2], [3], [4], [5]). It is difficult to investigate the governing equations of Magnetogasdynamics flows, the corresponding results are less than the conventional gas dynamics. When the velocity field and the magnetic field are everywhere orthogonal, the magnetogasdynamics flow is still important.

In [2], Helliwell discussed the non-conducting inviscid gas at rest, the author found that the stucture of the combustion wave is similar with the known conventional gas dynamics model.

In [3], Hu and Sheng constructed the unique Riemann solution of the one-dimensional inviscid flow

$$\begin{cases} \tau_t - u_x = 0, \\ u_t + (p + \frac{B^2}{2\mu})_x = 0, \\ (E + \frac{B^2\tau}{2\mu})_t + (pu + \frac{B^2 u}{2\mu})_x = 0, \end{cases} \tag{1}$$

under the assumption $B = k\rho$, where $\tau > 0$, $p \geq 0$, u, $B \geq 0$ and μ are respectively the specific volume, pressure, velocity, transverse magnetic field and magnetic permeability. The specific total energy is $E = e + \frac{u^2}{2}$. and e is the specific internal energy. Many authors ([6], [7], [8], [9], [10], etc.) studied the Riemann problem with combustion for the conventional gas dynamics models. Zhang and Zheng [7] studied the Riemann problem of the conventional gas dynamics flow of combustible ideal gases

$$\begin{cases} u_t + p_x = 0, \\ \tau_t - u_x = 0, \\ E_t + (up)_x = 0, \end{cases} \tag{2}$$

with an infinite rate of reaction which is described by

$$q(x,t) = \begin{cases} 0, & \text{if } \sup\limits_{0\le y\le t} T(x,y) > T_i; \\ q(x,0), & \text{otherwise.} \end{cases} \tag{3}$$

Under the proposed global entropy conditions, they constructed uniquely the Riemann solutions by the characteristic analysis.

In [10], we obtained uniquely the Riemann solutions for the Chapman-Jouguet (CJ) combustion model (1) and (3) with the following initial data

$$(\tau, p, u, q)(x, 0) = (\tau^{\pm}, p^{\pm}, u^{\pm}, q^{\pm}), \quad \pm x > 0, \tag{4}$$

where $\tau^{\pm} > 0, p^{\pm}, u^{\pm}$ are arbitrary constants,

$$q^{\pm} = \begin{cases} 0, & \text{if } T^{\pm} > T_i, \\ 0 \text{ or } q_0, & \text{if } T^{\pm} \le T_i, \end{cases}$$

and $q_0 > 0$ is a constant. The specific total energy is given by $E = e + \frac{u^2}{2} + q$, and q is the chemical binding energy. T is the temperature which satisfies the Boyle and Gay-Lussac's law: $p\tau = RT$. T_i is the ignition temperature. For the polytropic gases, it follows that $e = e(T)$ and $E = \frac{u^2}{2} + \frac{p\tau}{\gamma - 1} + q$, here $\gamma > 1$ is the adiabatic exponent. For simplicity, it is assumed that during the reaction R and γ remain unchanged. It is also assumed that the combustion process is exothermic [6].

Many works ([8], [11], [12], [13], [14], [15], [16], [17], [18]) have been done for the hyperbolic system for conservation laws.

In [8], we considered the generalized Riemann problem for the conventional gas dynamics (2) with combustion and obtained the unique perturbed Riemann solution. It was found that after perturbation the strong detonation wave can be transformed into the weak deflagration wave coalescing with the pre-compression shock wave.

In [17] and [18], the authors studied the generalized Riemann problem for the scalar convex and nonconvex Chapman-Jouguet combustion model respectively

$$\begin{cases} (u+q)_t + f(u)_x = 0, \\ q(x,t) = \begin{cases} q(x,0), & \sup\limits_{0\le y\le t} u(x,y) \le u_i, \\ 0, & \text{otherwise,} \end{cases} \end{cases} \tag{5}$$

where u and u_i is respectively the lumped quality representing density and the ignition temperature. Under the pointwise

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and global entropy conditions, they obtained constructively the perturbed Riemann solution. They especially found that the perturbation can transform the Chapman-Jouguet detonation wave into the strong detonation wave and the strong detonation wave into the weak deflagration wave followed by the shock wave.

In the present paper, we mainly consider the generalized Riemann problem of the CJ model (1) and (3) with the initial data

$$(\tau, p, u, q)(x, 0) = (\tau_0^{\pm}, p_0^{\pm}, u_0^{\pm}, q_0^{\pm})(x), \quad \pm x > 0, \quad (6)$$

where $q_0^{\pm}(x) = q^{\pm}$, and $\tau_0^{\pm}(x)$, $p_0^{\pm}(x)$, $u_0^{\pm}(x)$ are arbitrary smooth functions satisfying

$$\lim_{x\to 0\pm}(\tau_0^{\pm}, p_0^{\pm}, u_0^{\pm})(x) = (\tau^{\pm}, p^{\pm}, u^{\pm}).$$

We regard the problem (1)(3) and (6) as a small perturbation for our corresponding Riemann problem (1)(3) and (4). We want to know that whether the solutions of (1)(3) and (6) are similar with the corresponding Riemann solutions of (1)(3) with (4) in the neighborhood of the origin. We find the structures of the Riemann solutions keep their forms after perturbation for most of the cases, but for some cases, the structure of the Riemann solutions change essentially. It follows that the perturbation can make the combustion wave extinguished. The transition between the detonation wave and the deflagration wave can be found after perturbation. It is also found that although there is no combustion wave, the combustion wave may appear after perturbation which shows the instability of the unburnt gas.

This paper is arranged as follows. We give briefly the results of the Riemann problem for the CJ model (1)(3) with the initial values (4) in Section II. In Section III, according to the different cases of the corresponding Riemann solutions, the perturbed Riemann solutions are constructed under the modified global entropy conditions. Our main conclusion is given in Section IV.

II. Preliminaries

As a preparation, we study the Riemann problem for the CJ model (1), (3) with the initial data (4) and we refer the detailed discussions to [3], [10].

There are three eigenvalues of (1) which are $\sigma_1 = -(\frac{p-e_p\frac{BB_\tau}{\mu}+e_\tau}{e_p})^{\frac{1}{2}}$, $\sigma_2 = 0$ and $\sigma_3 = (\frac{p-e_p\frac{BB_\tau}{\mu}+e_\tau}{e_p})^{\frac{1}{2}}$. If $e_p > 0$ and $e_\tau + p > 0$, (1) is strictly hyperbolic. The characteristic fields $\sigma_{1,3}$ are genuinely nonlinear, and σ_2 is linearly degenerate.

Considering the self-similar solution $(\tau, p, u)(\zeta)(\zeta = \frac{x}{t})$, for any smooth solution we have

$$\begin{cases} \zeta \mathrm{d}\tau = -\mathrm{d}u, \\ \zeta \mathrm{d}u = \mathrm{d}(p + \frac{B^2 u}{2\mu}), \\ \zeta \mathrm{d}(E + \frac{B^2 u}{2\mu}\tau) = \mathrm{d}(up + \frac{B^2 u}{2\mu}u). \end{cases} \quad (7)$$

The forward or backward rarefaction waves $\overleftrightarrow{R}$ passing through the point (τ_0, p_0, u_0) are

$$\begin{cases} p\tau^\gamma = p_0\tau_0^\gamma, \\ u = u_0 \pm \int_{p_0}^{p} \frac{\sqrt{\gamma p\tau + \frac{B^2\tau}{\mu}}}{\gamma p}\mathrm{d}p. \end{cases} \quad (8)$$

The Rankine-Hugoniot jump conditions at $\zeta = \sigma$ are as follows

$$\begin{cases} \sigma[\tau] = -[u], \\ \sigma[u] = [p + \frac{B^2 u}{2\mu}], \\ \sigma[E + \frac{B^2 u}{2\mu}\tau] = [up + \frac{B^2 u}{2\mu}u], \end{cases} \quad (9)$$

where $[\tau] = \tau_r - \tau_l$, etc.

The contact discontinuity J is given by

$$[u] = [p + \frac{B^2}{2\mu}] = 0. \quad (10)$$

It follows that J is a curve in the space (τ, p, u), the projection of it on the plane (p, u) is a straight line which is parallel with the p axis. Denote J by $\overset{<}{J}$ when $p_l < p_r,\ \tau_l < \tau_r$, and $\overset{>}{J}$ when $p_l > p_r,\ \tau_l > \tau_r$.

If $[q] = 0$ in (9), we get the forward or backward shock waves $\overleftrightarrow{S}$ passing through the point (τ_0, p_0, u_0)

$$\begin{cases} (p + \theta^2 p_0 + \theta^2(\frac{3B^2}{2\mu} + \frac{B_0^2}{2\mu}))\tau \\ = (p_0 + \theta^2 p + \theta^2(\frac{3B_0^2}{2\mu} + \frac{B^2}{2\mu}))\tau_0, \\ u = u_0 \pm (p + \frac{B^2}{2\mu} - p_0 - \frac{B_0^2}{2\mu})\sqrt{-\frac{\tau - \tau_0}{p + \frac{B^2}{2\mu} - p_0 - \frac{B_0^2}{2\mu}}}, \end{cases} \quad (11)$$

where $\theta^2 = \frac{\gamma-1}{\gamma+1}$ and $B_0 = \frac{k}{\tau_0}$.

If $[q] \neq 0$ in (9), we obtain the combustion wave curve in the (τ, p) plane

$$D_u(0): \quad (\tau - \theta^2\tau_0)(p + \theta^2(p_0 + \frac{B_0^2}{2\mu} + \frac{3B^2}{2\mu})) \\ = (1-\theta^4)\tau_0 p_0 + \frac{\theta^2\tau_0}{\mu}[B_0^2(3-\theta^2) + B^2(1-3\theta^2)] + 2\theta^2 q_0. \quad (12)$$

In the (τ, p)-plane we have

$$R_u(l): \quad p\tau^\gamma = p_-\tau_-^\gamma, \quad (0 < p < p_l),$$

$$S_u(l): \quad (\tau - \theta^2\tau_l)(p + \theta^2(p_l + \frac{B_l^2}{2\mu} + \frac{3B^2}{2\mu})) \\ = (1-\theta^4)\tau_l p_l + \frac{\theta^2\tau_l}{\mu}[B_l^2(3-\theta^2) + B^2(1-3\theta^2)], \quad (13) \\ (p > p_l),$$

$$SDT(l): \quad (\tau - \theta^2\tau_l)(p + \theta^2(p_l + \frac{B_l^2}{2\mu} + \frac{3B^2}{2\mu})) \\ = (1-\theta^4)\tau_l p_l + \frac{\theta^2\tau_l}{\mu}[B_l^2(3-\theta^2) + B^2(1-3\theta^2)] + 2\theta^2 q_0, \\ (p > p_A), \quad (14)$$

$$WDF(i): \quad (\tau - \theta^2\tau_i)(p + \theta^2(p_i + \frac{B_i^2}{2\mu} + \frac{3B^2}{2\mu})) \\ = (1-\theta^4)\tau_i p_i + \frac{\theta^2\tau_i}{\mu}[B_i^2(3-\theta^2) + B^2(1-3\theta^2)] + 2\theta^2 q_0, \\ ((p_D)_i < p < p_i), \quad (15)$$

$$R(CJDT(l)): \quad p\tau^\gamma = p_A\tau_C^\gamma, \quad (p < p_A),$$

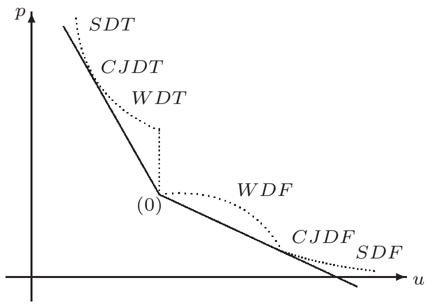
$$R(CJDF(l)): \quad p\tau^\gamma = (p_B)_i(\tau_B)_i^\gamma, \quad (p < (p_B)_i).$$

The combustion wave curves in the (u, p) plane is as follows (Fig. 2.1.)

$$\overleftarrow{D}_\tau(0):\quad u=u_0-\sqrt{(p+\frac{B^2}{2\mu}-p_0-\frac{B_0^2}{2\mu})}\cdot\sqrt{\frac{(1-\theta^2)\tau_0(p-p_0)+\frac{\theta^2\tau_0}{\mu}(B^2-B_0^2)-2\theta^2 q_0}{p+\theta^2(p_0+\frac{B_0^2}{2\mu}+\frac{3B^2}{2\mu})}}. \tag{16}$$

Now denote the backward DF and DT wave curve by $\overleftarrow{W}_{DF}(l)$ and $\overleftarrow{W}_{DT}(l)$, respectively, where

$$\overleftarrow{W}_{DF}(l) := \overleftarrow{WDF}(i_s) \cup \overleftarrow{CJDF}(i_s) \cup \overleftarrow{R}(\overleftarrow{CJDF}(i_s)),$$
$$\overleftarrow{W}_{DT}(l) := \overleftarrow{SDT}(l) \cup \overleftarrow{CJDT}(l) \cup \overleftarrow{R}(\overleftarrow{CJDT}(l)).$$



combustion wave curve

Fig. 2.1. The combustion wave curve in the plane (u, p).

Denote the backward combustion wave curve $\overleftarrow{W}(l)$ which can be linked to the state $(l) = (\tau_l, p_l, u_l, q_l)$, we have

$$\overleftarrow{W}(l) := \overleftarrow{W}_S(l) \cup \overleftarrow{W}_D(l),$$

where

$$\overleftarrow{W}_S(l) := (\overleftarrow{W}_S(l), q_l = 0) \ or \ (\overleftarrow{W}_S(l), q_l > 0),$$
$$\overleftarrow{W}_D(l) := \overleftarrow{W}_{DF}(l) \cup \overleftarrow{W}_{DT}(l).$$

Similarly, we can construct the forward wave curve $\overrightarrow{W}(r)$ which can be linked to $(r) = (\tau_r, p_r, u_r, q_r)$.

Since the image of J in (τ, p, u) is a straight line which parallels with the τ axis and the projection on the plane (u, p) is a point, it follows that J is a plane curve in the space (τ, p, u), and it's projection in (u, p) is a straight line which parallels with the p axis. Thus the Riemann prblem for (1) is much more complicated than that of the conventional gas dynamics.

If $q_l = q_r = 0$, the gas on both sides are burnt, no combustion wave occurs.

If q_l and q_r are not both zero, there may exist more than one intersection points of $\overleftarrow{W}(l)$ and $\overrightarrow{W}(r)$. Each intersection point corresponds to a unique Riemann solution. When the intersection point is unique, the solution is also unique; otherwise, for the unique solution we put forward the following *modified global entropy conditions [8]:*

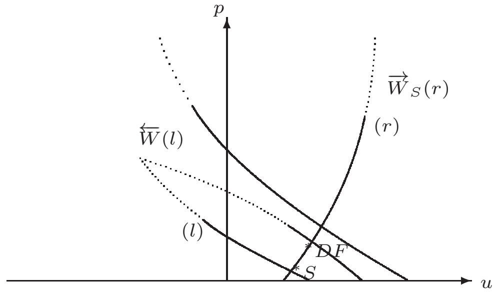
we pick out the unique solution from nine (at the most) intersection points of $\overleftarrow{W}(l)$ and $\overrightarrow{W}(r)$ in the following given order:

(A). the propagating speed of the combustion wave solution is as low as possible;

(B). the solution with the parameter β as small as possible, where β is defined as oscillation frequency of $T(\xi)$ between the set $\{\xi \in R^1 : T(\xi) \le T_i\}$ and the set $\{\xi \in R^1 : T(\xi) > T_i\}$;

(C). the solution containing as many combustion wave as possible.

Case 1. $q_l > 0,\ q_r = 0$. The gas is unburnt on the left side for this case, while the gas is burnt on the right side, thus we know that $\overleftarrow{W}(l) = \overleftarrow{W}_S(l) \cup \overleftarrow{W}_{DF}(l) \cup \overleftarrow{W}_{DT}(l)$, $\overrightarrow{W}(r) = \overrightarrow{W}_S(r)$. If there exists only one intersection point of $\overleftarrow{W}(l)$ and $\overrightarrow{W}(r)$, we obtain the unique solution is a detonation wave solution $\overleftarrow{DT} + \overrightarrow{R} \ or \ \overrightarrow{S}$ if $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, or $\overleftarrow{DT} + J + \overrightarrow{R} \ or \ \overrightarrow{S}$ if $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$.



Case 1 $q_l > 0,\ q_r = 0$

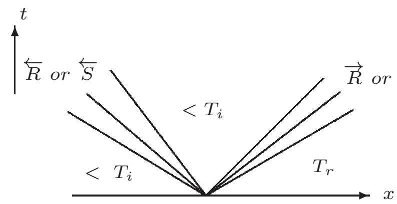
Fig. 2.2. There are there interactions.

If there are three intersection points of $\overleftarrow{W}(l)$ and $\overrightarrow{W}_S(r)$ (Fig. 2.2.), from the modified global entropy condition *(A)*, we discard the intersection point of $\overrightarrow{W}_S(r)$ and $\overleftarrow{W}_{DT}(l)$. Denote the intersection point of $\overrightarrow{W}_S(r)$ and $\overleftarrow{W}_S(l)$ by $*_S$ and the intersection point of $\overrightarrow{W}_S(r)$ and $\overleftarrow{W}_{DF}(l)$ by $*_{DF}$. Denote the temperature at the point $*_S$, $*_{DF}$ on $\overrightarrow{W}_S(r)$ by T_S, T_{DF}, respectively. The temperature at $*_{DF}$ on $\overleftarrow{W}_{DF}(l)$ is greater than T_i since the combustion process is exothermic.

Subcase 1.1. $p_l\tau_l^\gamma = p_r\tau_r^\gamma$.

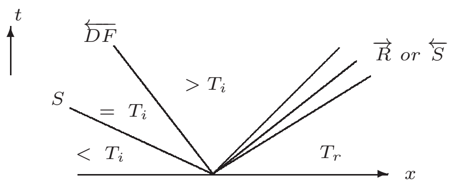
If $T_r \le T_i$, then $\beta(*_S) = 0$, $\beta(*_{DF}) = 2$, from the condition *(B)*, we pick out $*_S$, and we get the noncombustion wave solution $\overleftarrow{R} \ or \ \overleftarrow{S} + \overrightarrow{R} \ or \ \overrightarrow{S}$ (Fig. 2.3.).

If $T_r > T_i$, then $\beta(*_S) = 1$, $\beta(*_{DF}) = 1$, from the condition *(C)*, we pick out $*_{DF}$, and we get the combustion wave solution $\overleftarrow{DF} + \overrightarrow{R} \ or \ \overrightarrow{S}$ (Fig. 2.4.).



noncombustion wave

Fig. 2.3. Riemann solutions in Subcase 1.1.

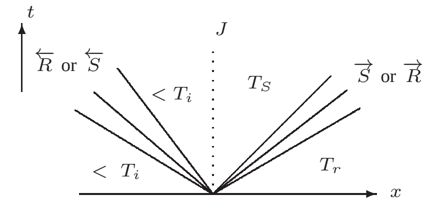


combustion wave *DF*

Fig. 2.4. Riemann solutions in Subcase 1.1.

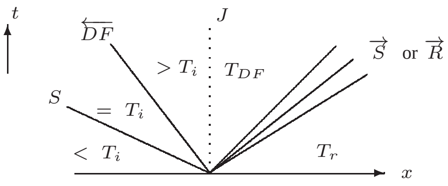
Subcase 1.2. $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$.

From the condition *(A)*, the possible detonation DT wave solution is discarded, and it is found that the possible Riemann solution is $\overleftarrow{R} \ or \ \overleftarrow{S} + J + \overrightarrow{R} \ or \ \overrightarrow{S}$ or $\overleftarrow{DF} + J + \overrightarrow{R} \ or \ \overrightarrow{S}$. From the modified global entropy conditions, the unique Riemann solution is constructed as follows.



noncombustion wave

Fig. 2.5. Riemann solutions in Subcase 1.2.



combustion wave *DF*

Fig. 2.6. Riemann solutions in Subcase 1.2.

a) If $T_r > T_i$, $T_{DF} \leq T_i (\Rightarrow T_S \leq T_i)$, then $\beta(*_S) = 1$, $\beta(*_{DF}) = 3$, from the condition *(B)*, we pick out $*_S$ and obtain a noncombustion wave solution (Fig. 2.5.).

b) If $T_r > T_i$, $T_{DF} > T_i$, then $\beta(*_S) = 1$, $\beta(*_{DF}) = 1$, from the condition *(C)*, we pick out $*_{DF}$ and obtain a combustion wave solution containing a DF (Fig. 2.6.).

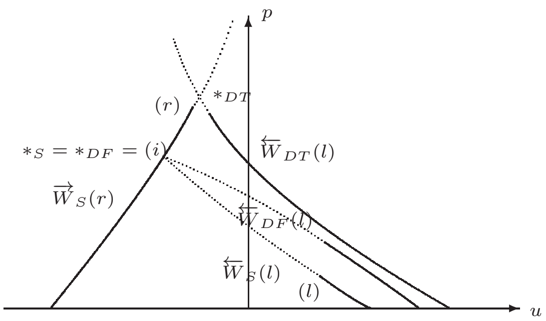
c) If $T_r \leq T_i$, $T_S \leq T_i$, then $\beta(*_S) = 0$, $\beta(*_{DF}) = 2$, from the condition *(B)*, we pick out $*_S$ and obtain the noncombustion wave solution (Fig. 2.5.).

d) If $T_r \leq T_i$, $T_S > T_i (\Rightarrow T_{DF} > T_i)$, then $\beta(*_S) = 2$, $\beta(*_{DF}) = 2$, from the condition *(C)*, we select $*_{DF}$ and obtain the combustion wave solution containing a DF (Fig. 2.6.).

Case 2. $q_l > 0$, $q_r = 0$ and there are two intersection points of $\overleftarrow{W}(l)$ and $\overrightarrow{W}_S(r)$ (Fig. 2.7.).

Subcase 2.1. $p_l \tau_l^\gamma = p_r \tau_r^\gamma$.

In this case, we select the point $*_S$ or $*_{DT}$ and obtain the possible solutions $\overleftarrow{S} + \overrightarrow{R}$ *or* $\overrightarrow{S}$ or $\overleftarrow{DT} + \overrightarrow{R}$ *or* $\overrightarrow{S}$. Now we select the unique Riemann solution as follows.

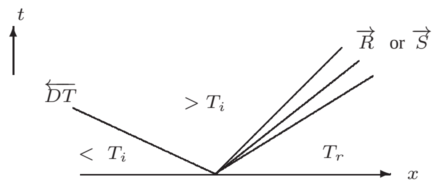


Case 2 $q_l > 0, \ q_r = 0$

Fig. 2.7. There are two interactions.

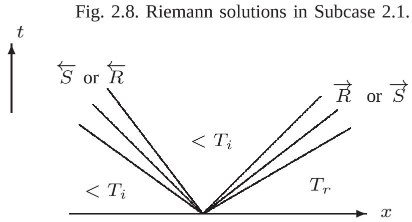
If $T_r > T_i$, then $\beta(*_S) = 1$, $\beta(*_{DT}) = 1$, from the condition *(C)*, we pick out $*_{DT}$ and obtain the combustion wave solution $\overleftarrow{DT} + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.8.).

If $T_r \leq T_i$, then $\beta(*_S) = 0$, $\beta(*_{DT}) = 2$, from the condition *(B)*, we pick out $*_S$ and obtain the noncombustion wave solution $\overleftarrow{S} + \overrightarrow{R}$ *or* $\overrightarrow{S}$ (Fig. 2.9.).



combustion wave *DT*

Fig. 2.8. Riemann solutions in Subcase 2.1.



noncombustion wave

Fig. 2.9. Riemann solutions in Subcase 2.1.

Subcase 2.2. $p_l \tau_l^\gamma \neq p_r \tau_r^\gamma$.

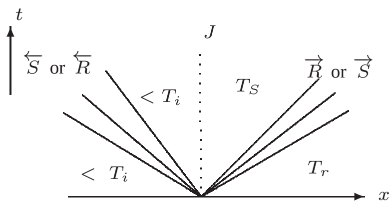
We know that one possibility is that there is uniquely intersection point of $\overleftarrow{W}(l)$ and $\overrightarrow{W}(r)$, it follows that the unique Riemann solution is $\overleftarrow{DT} + \overset{>}{J} + \overrightarrow{R}$ *or* $\overrightarrow{S}$. The other possibility is that there are three possible solutions which are the noncombution wave solution $\overleftarrow{S}$ *or* $\overleftarrow{R} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$, or the DF combustion wave solution $\overleftarrow{DF} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$, or the DT combustion wave solution $\overleftarrow{DT} + \overset{<}{J} + \overrightarrow{R}$ *or* $\overrightarrow{S}$. From the global entropy condition *(A)*, we discard the DT combustion wave solution.

a) If $T_r > T_i$, $T_{DF} \leq T_i (\Rightarrow T_S \leq T_i)$, then $\beta(*_S) = 1$, $\beta(*_{DF}) = 3$, from the condition *(B)*, we pick out $*_S$ and obtain the noncombustion wave solution (Fig. 2.10.).

b) If $T_r > T_i$, $T_{DF} > T_i$, then $\beta(*_S) = 1$, $\beta(*_{DF}) = 1$, from the condition *(C)*, we pick out $*_{DF}$ and obtain the combustion wave solution containing a DF (Fig. 2.11.).

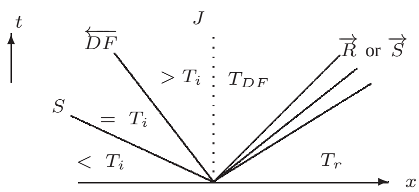
c) If $T_r \leq T_i$, $T_S \leq T_i$, then $\beta(*_S) = 0$, $\beta(*_{DF}) = 2$, from the condition *(B)*, we pick out $*_S$ and obtain the noncombustion wave solution (Fig. 2.10.).

d) If $T_r \leq T_i$, $T_S > T_i (\Rightarrow T_{DF} > T_i)$, then $\beta(*_S) = 2$, $\beta(*_{DF}) = 2$, from the condition *(C)*, we pick out $*_{DF}$ and obtain the combustion wave solution containing a DF (Fig. 2.11.).



noncombustion wave

Fig. 2.10. Riemann solutions in Subcase 2.2.



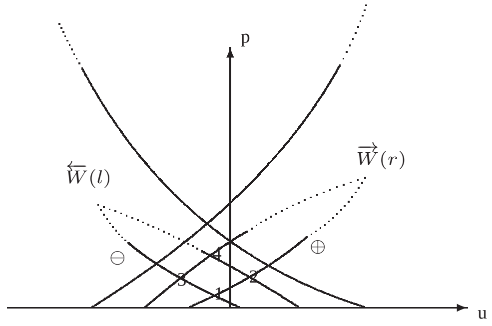
combustion wave *DF*

Fig. 2.11. Riemann solutions in Subcase 2.2.

Case 3. $q_l > 0, \ q_r > 0$, for this case the gas are unburnt on the both sides. For this case, we have $\overleftarrow{W}(l) = \overleftarrow{W}_S(l) \cup$

$\overleftarrow{W}_{DF}(l) \cup \overleftarrow{W}_{DT}(l)$, $\overrightarrow{W}(r) = \overrightarrow{W}_S(r) \cup \overrightarrow{W}_{DF}(r) \cup \overrightarrow{W}_{DT}(r)$.

If the intersection point of $\overleftarrow{W}(l)$ and $\overrightarrow{W}(r)$ is unique, the solution is $\overleftarrow{DT} + \overrightarrow{DT}$ if $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, or $\overleftarrow{DT} + J + \overrightarrow{DT}$ if $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$. Otherwise, there are two possible subcases: one is that there is an intersection point of $\overleftarrow{W}_S(l)$ and $\overrightarrow{W}_S(r)$, the other is that there is no intersection point of $\overleftarrow{W}_S(l)$ and $\overrightarrow{W}_S(r)$.

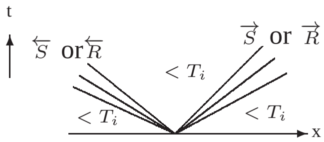


Case 3.1
Fig. 2.12. There is an intersection point of $\overleftarrow{W}_S(l)$ and $\overrightarrow{W}_S(r)$.

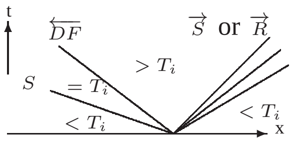
Case 3.1. For the former subcase (Fig. 2.12.), we discuss it in the following two subcases.

Subcase 3.1.1. $p_l\tau_l^\gamma = p_r\tau_r^\gamma$.

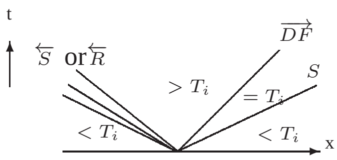
From the condition *A*, we consider the intersection points 1, 2, 3, 4. We should select the unique solution from the four possible solutions (Fig. 2.13.-Fig. 2.16.).



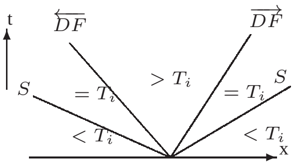
intersection point 1
Fig. 2.13. Subcase 3.1.1. (i)



intersection point 2
Fig. 2.14. Subcase 3.1.1. (ii)



intersection point 3
Fig. 2.15. Subcase 3.1.1. (iii)

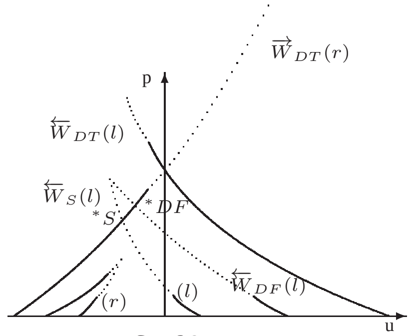


intersection point 4
Fig. 2.16. Subcase 3.1.1. (iv)

It is obvious that $\beta = 0$ for (i), and it holds that $\beta = 2$ for (ii), (iii) and (iv). From the condition *(B)*, we pick out the intersection point 1 and obtain the unique noncombustion wave solution $\overleftarrow{R}$ *or* $\overleftarrow{S} + \overrightarrow{R}$ *or* $\overrightarrow{S}$.

Subcase 3.1.2. $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$. Similarly, we obtain the unique Riemann solution is still the noncombustion wave solution $\overleftarrow{R}$ *or* $\overleftarrow{S} + J + \overrightarrow{R}$ *or* $\overrightarrow{S}$. The only difference is that here the contact discontinuity appears.

Case 3.2. In the latter subcase, there are only two possibilities: $\overleftarrow{W}(l)$ intersects $\overrightarrow{W}_{DT}(r)$ only or $\overrightarrow{W}(+)$ intersects $\overleftarrow{W}_{DT}(l)$ only. We just need to consider the former. If the intersection point is unique, the solution is $\overleftarrow{DT} + \overrightarrow{DT}$ if $p_-\tau_-^\gamma = p_+\tau_+^\gamma$, or $\overleftarrow{DT} + J + \overrightarrow{DT}$ if $p_-\tau_-^\gamma \neq p_+\tau_+^\gamma$, otherwise, there are at most three intersection points (Fig. 2.17.).



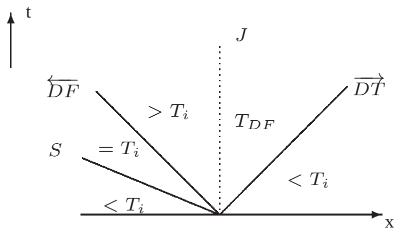
Case 3.2
Fig. 2.17. $\overleftarrow{W}(-)$ intersects $\overrightarrow{W}_{DT}(r)$ only.

Subcase 3.2.1. $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$.

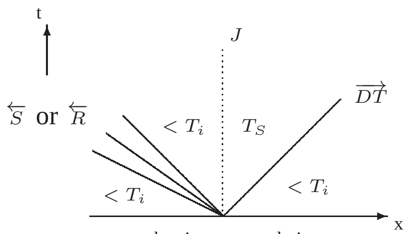
From the condition *(A)*, we discard the intersection point of $\overleftarrow{W}_{DT}(l)$ and $\overrightarrow{W}_{DT}(r)$. Denote the intersection point of $\overrightarrow{W}_{DT}(r)$ and $\overleftarrow{W}_S(l)$ by $^*S$ and denote the intersection point of $\overrightarrow{W}_{DT}(+)$ and $\overleftarrow{W}_{DF}(l)$ by $^*DF$, respectively. We denote the temperature at the point $^*S$, $^*DF$ on $\overrightarrow{W}_{DT}(r)$ by T_S, T_{DF}, respectively (Fig. 2.18. and Fig. 2.19.).

Since $T_S > T_i$ we have $T_{DF} > T_i$, then $\beta(^*S) = 2$, $\beta(^*DF) = 2$. From the condition *(C)*, we pick out $^*DF$, it follows that the combustion wave solution is $\overleftarrow{DF}$ + $\overrightarrow{DT}$.

Subcase 3.2.2. $p_l\tau_l^\gamma = p_r\tau_r^\gamma$. Similarly with Subcase 3.2.1., we obtain the unique Riemann solution is the combustion wave solution $\overleftarrow{DF}$ + $\overrightarrow{DT}$. The only difference is that there exists the contact discontinuity in Subcase 3.2.1.



combustion wave solution
Fig. 2.18. Riemann solutions in Subcase 3.2.1.



combustion wave solution
Fig. 2.19. Riemann solutions in Subcase 3.2.1.

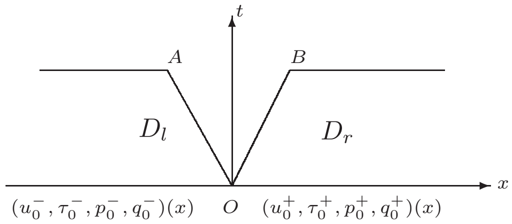
Based on the above analysis, we have the following result.

Theorem 2.1 There exists uniquely solution of the Riemann problem (1)(3) with the initial values (4) under the given modified global entropy conditions.

III. Solutions of the Generalized Riemann Problem (1) and (3)

Now we investigate the solutions for the initial value problem (1) and (3) with (6) in a neighborhood of the origin $(t > 0)$ on the (x, t) plane. From the results in [16], the classical solution $(u_l, \tau_l, p_l, q_l)(x, t)$ $((u_r, \tau_r, p_r, q_r)(x, t))$ can be defined in a strip domain $D_l(D_r)$ for a local time. The right boundary of D_l has characteristic $OA : x = \lambda_- t$, and the left boundary of D_r has characteristic $OB : x = \lambda_+ t$ (Fig. 3.1).

According to the different cases of the corresponding Riemann solutions of (1) and (3) with (4), we construct the solutions under the modified entropy conditions case by case for (1) and (3) with (6). For simplicity, we only consider some interesting cases. Similar discussion can be carried out for the other cases. For simplicity, we use the same symbols after perturbation.



the region

Fig. 3.1. The discussed local region.

Case 1. When $q_l > 0,\ q_r = 0$, and we know that there are three intersection points of $\overleftarrow{W}(l)$ and $\overrightarrow{W}_S(r)$ (Fig. 2.2).

Subcase 1.1. The solution of the corresponding Riemann problem is the noncombustion wave solution.

Subcase 1.1.1 $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ and $T_r \le T_i$ (Fig. 2.3.).

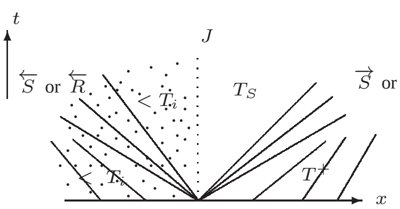
After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$, $T_r \le T_i$ or $T_r > T_i$, $T_{DF} \le T_i$ or $T_{DF} > T_i$, $T_S \le T_i$ or $T_S > T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ and $T_r \le T_i$, then $\beta(*_S) = 0$, $\beta(*_{DF}) = 2$, from the condition *(B)*, we pick out $*_S$ and obtain the perturbed solution is the noncombustion wave solution (Fig. 3.2. and there is no contact discontinuity).

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ and $T_r > T_i$, then $\beta(*_S) = 1$, $\beta(*_{DF}) = 1$, from the condition *(C)*, we pick out $*_{DF}$ and obtain the combustion wave solution containing a DF (Fig. 3.3. and there is no contact discontinuity).

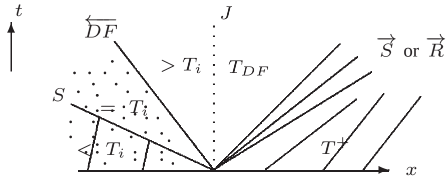
If $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$ and $T_r \le T_i$, as $T_S \le T_i$ then $\beta(*_S) = 0$, $\beta(*_{DF}) = 2$, from the condition *(B)*, we pick out $*_S$ and obtain the noncombustion wave solution (Fig. 3.2.); as $T_S > T_i$ then $\beta(*_S) = 2$, $\beta(*_{DF}) = 2$, from the condition *(C)*, we pick out $*_{DF}$ and obtain the combustion wave solution containing a DF (Fig. 3.3.).

If $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$ and $T_r > T_i$, as $T_{DF} > T_i$ then $\beta(*_S) = 1$, $\beta(*_{DF}) = 1$, from the condition *(C)*, we pick out $*_{DF}$ and obtain the combustion wave solution containing a DF (Fig. 3.3.); as $T_{DF} \le T_i$ then $\beta(*_S) = 1$, $\beta(*_{DF}) = 3$, from the condition *(B)*, we pick out $*_S$ and obtain the noncombustion wave solution (Fig. 3.2.).



a perturbed noncombustion solution

Fig. 3.2. The perturbed solutions in Subcase 1.1.



a perturbed combustion solution

Fig. 3.3. The perturbed solutions in Subcase 1.1.

Subcase 1.1.2 $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$ and $T_r > T_i$, $T_{DF} \le T_i$, $T_S \le T_i$ (Fig. 2.5.).

After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$, $T_r > T_i$, $T_{DF} \le T_i$ or $T_{DF} > T_i$, $T_S \le T_i$ or $T_S > T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ and $T_r > T_i$, then we have combustion wave solution containing a DF (Fig. 3.3. and there is no contact discontinuity).

If $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$ and $T_r > T_i$, as $T_{DF} > T_i$ then we have combustion wave solution containing a DF (Fig. 3.3.); as $T_{DF} \le T_i$, we obtain a noncombustion wave solution (Fig. 3.2.).

Subcase 1.1.3 $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$ and $T_r \le T_i$, $T_S \le T_i$ (Fig. 2.5.).

After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$, $T_r \le T_i$ or $T_r > T_i$, $T_{DF} \le T_i$ or $T_{DF} > T_i$, $T_S \le T_i$ or $T_S > T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, as $T_r \le T_i$, then we have a noncombustion wave solution (Fig. 3.2.); as $T_r > T_i$, then we have combustion wave solution containing a DF (Fig. 3.3.). And notice that there is no contact discontinuity in the perturbed Riemann solutions.

If $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$ and $T_r > T_i$, as $T_{DF} > T_i$ then we obtain combustion wave solution containing a DF (Fig. 3.3.); as $T_{DF} \le T_i$ then we have a noncombustion wave solution (Fig. 3.2.).

If $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$ and $T_r \le T_i$, as $T_S \le T_i$ then we have a noncombustion wave solution (Fig. 3.2.); as $T_S > T_i$ then we obtain combustion wave solution containing a DF (Fig. 3.3.).

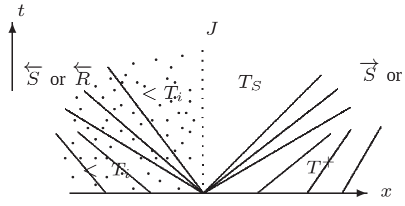
Subcase 1.2. The solution of the corresponding Riemann problem is the combustion wave solution containing a DF.

Subcase 1.2.1 $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ and $T_r > T_i$ (Fig. 2.4.).

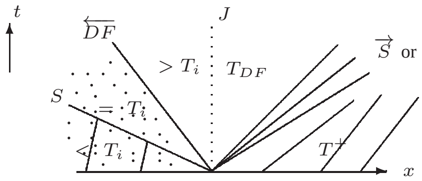
After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$, $T_r > T_i$, $T_{DF} \le T_i$ or $T_{DF} > T_i$, $T_S \le T_i$ or $T_S > T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ and $T_r > T_i$, then $\beta(*_S) = 1$, $\beta(*_{DF}) = 1$, from the condition *(C)*, we pick out $*_{DF}$ and obtain the combustion wave solution containing a DF (Fig. 3.5. and there is no contact discontinuity).

If $p_l\tau_l^\gamma \ne p_r\tau_r^\gamma$ and $T_r > T_i$, as $T_{DF} > T_i$ then $\beta(*_S) = 1$, $\beta(*_{DF}) = 1$, from the condition *(C)*, we pick out $*_{DF}$ and obtain the combustion wave solution containing a DF (Fig. 3.5.); as $T_{DF} \le T_i$ then $\beta(*_S) = 1$, $\beta(*_{DF}) = 3$, from the condition *(B)*, we pick out $*_S$ and obtain the noncombustion wave solution (Fig. 3.4.).



a perturbed noncombustion solution
Fig. 3.4. The perturbed solutions in Subcase 1.2.



a perturbed combustion solution
Fig. 3.5. The perturbed solutions in Subcase 1.2.

Subcase 1.2.2 $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ and $T_r > T_i$, $T_{DF} > T_i$ (Fig. 2.6.).

After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$, $T_{DF} > T_i$, $T_S \leq T_i$ or $T_S > T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ and $T_r > T_i$, then we obtain combustion wave solution containing a DF (Fig. 3.5. and there is no contact discontinuity).

If $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$ and $T_r > T_i$, $T_{DF} > T_i$ then we obtain combustion wave solution containing a DF (Fig. 3.5.).

Subcase 1.2.3 $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ and $T_r \leq T_i$, $T_S > T_i$ (Fig. 2.6.).

After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r \leq T_i$ or $T_r > T_i$, $T_S > T_i$, $T_{DF} > T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, as $T_r > T_i$ then we obtain combustion wave solution containing a DF (Fig. 3.5.); as $T_r \leq T_i$ then we obtain a noncombustion wave solution (Fig. 3.4.). Notice that there is no contact discontinuity in the perturbed solutions.

If $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$ and $T_r > T_i$ or $T_r \leq T_i$, $T_S > T_i$, $T_{DF} > T_i$ then we obtain combustion wave solution containing a DF (Fig. 3.5.).

Theorem 3.1 Although there is no combustion wave of the corresponding Riemann for this case, after perturbation the combustion wave can appear. It follows that the unburnt gas is unstable. And we observe that the combustion wave solution may be extinguished under such local perturbation.

Case 2. Assume that $q_l > 0$, $q_r = 0$ and there are two intersection points of $\overleftarrow{W}(l)$ and $\overrightarrow{W}_S(r)$ (Fig. 2.7).

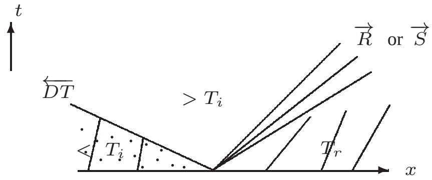
After perturbation there are two possibilities: one is that there is only one intersection point of $\overleftarrow{W}(-)$ and $\overrightarrow{W}_S(+)$, and we get the unique perturbed solution is combustion wave solution containing DT; the other case is that there are three intersection points of $\overleftarrow{W}(-)$ and $\overrightarrow{W}_S(+)$.

Subcase 2.1 When the corresponding Riemann problem have the noncombustion wave solution.

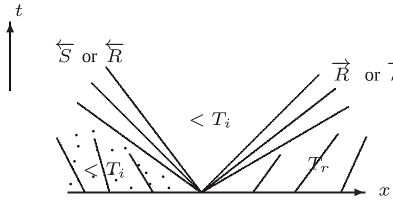
Subcase 2.1.1 $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ and $T_r \leq T_i$ (Fig. 2.9.).

After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$ or $T_r \leq T_i$, $T_S > T_i$ or $T_S \leq T_i$, $T_{DF} > T_i$ or $T_{DF} \leq T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, as $T_r > T_i$, then $\beta(*_S) = 1$, $\beta(*_{DT}) = 1$, from the condition *(C)*, we pick out $*_{DT}$ and obtain the combustion wave solution containing DT (Fig. 3.6.); as $T_r \leq T_i$, then $\beta(*_S) = 0$, $\beta(*_{DT}) = 2$, from the condition *(B)*, we pick out $*_S$ and obtain the noncombustion wave solution (Fig. 3.7.). For this subcase, we can see that there are no contact discontinuity.



combustion wave DT
Fig. 3.6. The perturbed solutions in Case 2. without J



noncombustion wave
Fig. 3.7. The perturbed solutions in Case 2. without J

If $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$ and $T_r > T_i$, as $T_{DF} > T_i$ then $\beta(*_S) = 1$, $\beta(*_{DF}) = 1$, from the condition *(C)*, we pick out $*_{DF}$ and get the combustion wave solution containing a DF (Fig. 3.9.); as $T_{DF} \leq T_i$ then $\beta(*_S) = 1$, $\beta(*_{DF}) = 3$, from the condition *(B)*, we pick out $*_S$ and get the noncombustion wave solution (Fig. 3.8.).

If $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$ and $T_r \leq T_i$, as $T_S > T_i$ then $\beta(*_S) = 2$, $\beta(*_{DF}) = 2$, from the condition *(C)*, we pick out $*_{DF}$ and have the combustion wave solution containing a DF (Fig. 3.9.); as $T_S \leq T_i$ then $\beta(*_S) = 0$, $\beta(*_{DF}) = 2$, from the condition *(B)*, we pick out $*_S$ and have the noncombustion wave solution (Fig. 3.8.).

Subcase 2.1.2 $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$ and $T_r > T_i$, $T_S \leq T_i$, $T_{DF} \leq T_i$ (Fig. 2.10.).

After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$, $T_S > T_i$ or $T_S \leq T_i$, $T_{DF} > T_i$ or $T_{DF} \leq T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, $T_r > T_i$ then we get combustion wave solution containing a DT (Fig. 3.6.). Notice that there is no contact discontinuity for this subcase.

If $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$, as $T_{DF} > T_i$ then we get combustion wave solution containing a DF (Fig. 3.9.); as $T_{DF} \leq T_i$ then we obtain a noncombustion wave solution (Fig. 3.8.).

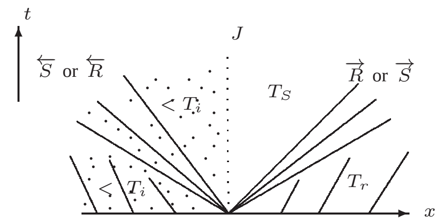
Subcase 2.1.3 $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$ and $T_r \leq T_i$, $T_S \leq T_i$ (Fig. 2.10.).

After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$ or $T_r \leq T_i$, $T_S > T_i$ or $T_S \leq T_i$, $T_{DF} > T_i$ or $T_{DF} \leq T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, as $T_r > T_i$ then we get combustion wave solution containing a DT (Fig. 3.6.); as $T_r \leq T_i$ then we obtain a noncombustion wave solution (Fig. 3.7.). Notice that there are no contact discontinuity here.

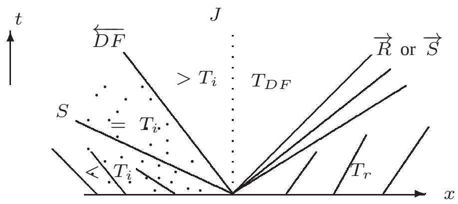
If $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$, as $T_{DF} > T_i$ then we get combustion wave solution containing a DF (Fig. 3.9.); as $T_{DF} \leq T_i$ then we obtain a noncombustion wave solution (Fig. 3.8.).

If $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r < T_i$, as $T_S > T_i$ then we get combustion wave solution containing a DF (Fig. 3.9.); as $T_S \leq T_i$ then we obtain a noncombustion wave solution (Fig. 3.8.).



noncombustion wave

Fig. 3.8. The perturbed solutions in Case 2. with J



combustion wave *DF*

Fig. 3.9. The perturbed solutions in Case 2. with J

Subcase 2.2 The solution of the corresponding Riemann problem is the combustion wave solution containing a DT. For this subcase, we have $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ and $T_r > T_i$ (Fig. 2.8.).

After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$, $T_S > T_i$ or $T_S \leq T_i$, $T_{DF} > T_i$ or $T_{DF} \leq T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, $T_r > T_i$ then we get combustion wave solution containing a DT (Fig. 3.6.). Notice that there are no contact discontinuity here.

If $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$, as $T_{DF} > T_i$ then we get combustion wave solution containing a DF (Fig. 3.9.); as $T_{DF} \leq T_i$ then we obtain a noncombustion wave solution (Fig. 3.8.).

Subcase 2.3 The solution of the corresponding Riemann problem is the combustion wave solution containing a DF.

Subcase 2.3.1 $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$ and $T_r > T_i$, $T_{DF} > T_i$ (Fig. 2.11.).

After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$, $T_{DF} > T_i$, $T_S > T_i$ or $T_S \leq T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, $T_r > T_i$ then we get combustion wave solution containing a DT (Fig. 3.6.). Notice that there are no contact discontinuity here.

If $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$, $T_{DF} > T_i$ then we get combustion wave solution containing a DF (Fig. 3.9.).

Subcase 2.3.2 $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$ and $T_r \leq T_i$, $T_S > T_i$, $T_{DF} > T_i$ (Fig. 2.11.).

After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$ or $T_r \leq T_i$, $T_S > T_i$, $T_{DF} > T_i$ or $T_{DF} \leq T_i$ and construct the perturbed solution as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, as $T_r > T_i$, then we get combustion wave solution containing a DT (Fig. 3.6.); as $T_r \leq T_i$, then we obtain a noncombustion wave solution (Fig. 3.7.). Notice that there are no contact discontinuity here.

If $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$ or $T_r \leq T_i$, $T_S > T_i$, $T_{DF} > T_i$ then we get combustion wave solution containing a DF (Fig. 3.9.).

Theorem 3.2 For this case, the transition between the detonation wave and the deflagration wave may occur after perturbation. The combustion wave solution (detonation wave or deflagration wave) of the corresponding Riemann may be extinguished. Although the corresponding Riemann problem has no combustion wave, after perturbation the combustion wave can appear, and it follows that the unburnt gas is unstable.

Case 3. Assume that $q_l > 0$, $q_r > 0$ and there are three intersection points of $\overleftarrow{W}(l)$ and $\overrightarrow{W}(r)$ (Fig. 2.17). There is a combustion wave solution $\overleftarrow{DF}$ + $\overrightarrow{DT}$ for the corresponding Riemann problem, and $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_S > T_i$, $T_{DF} > T_i$.

After perturbation we obtain $p_l\tau_l^\gamma = p_r\tau_r^\gamma$ or $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_S > T_i$, $T_{DF} > T_i$, the perturbed solution is given as follows.

If $p_l\tau_l^\gamma = p_r\tau_r^\gamma$, $T_r > T_i$ or $T_r \leq T_i$, $T_S > T_i$, $T_{DF} > T_i$ then we get combustion wave solution $\overleftarrow{DF}$ + $\overrightarrow{DT}$.

If $p_l\tau_l^\gamma \neq p_r\tau_r^\gamma$, $T_r > T_i$ or $T_r \leq T_i$, $T_S > T_i$, $T_{DF} > T_i$ then we get still combustion wave solution $\overleftarrow{DF}$ + $\overrightarrow{DT}$. The only difference is that there exists the contact discontinuity for the latter subcase.

Theorem 3.3 For this case, the combustion wave solution of the corresponding Riemann is stable under such small perturbation.

IV. Conclusion

We generalize our main results as follows. The corresponding Riemann solutions are stable for most of cases. However we find that after the small perturbation there are fundamental changes of the corresponding Riemann solutions. The perturbation may transform a detonation wave into a deflagration wave or a deflagration wave into a detonation wave in the neighborhood of the origin. Furthermore, the perturbation can extinguish the combustion wave. When the corresponding Riemann problem has no combustion wave solution, the combustion wave appears after perturbation. It follows that the unburnt gas is unstable. Notice that there are much more richer structure of the perturbed Riemann solutions of our combustible model (1) and (3) in magnetogasdynamics than that of the conventional combustible gas dynamics model (2) and (3).

Based on the above analysis, we have the following main result.

Theorem 4.1 Under the modified global entropy conditions, the generalized Riemann problem (1)(3) with the initial data (6) has unique solution in a neighborhood of the origin $(t > 0)$ on the (x, t) plane.

Our model is very important in many applications. Considering the reaction rate of this model is infinite and it is an idealized hypothesis, we will study the initial value problem for the self-similar Zeldovich-von Neumann-Döring (ZND) magnetogasdynamic combustion model with finite reaction rate in our coming works.

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