Improved Grey FMEA Evaluation with Interval Uncertain Linguistic Variables and TOPSIS

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Abstract—**There are certain defects in the conventional effective failure mode and effect analysis (FMEA) model. Thus, a more effective FMEA method is proposed in this paper by incorporating the optimization method with interval grey uncertain language variables, the Technique of Order Preference Similarity to the Ideal Solution (TOPSIS) decision method, and the grey target decision method. The paper introduces the related concepts and rules of grey interval linguistic variables. On the advantage of examining the fuzzy attributes alongside grey characteristics of expert assessment, internal grey uncertain linguistic variables are applied in the proposed method of optimization. The optimization method is anchored on the weights of the experts and the weights of the three attributes of detectivity, occurrence, and severity in FMEA. The final risk level is determined through incorporation of the grey target decision and TOPSIS method. A practical example is implemented to illustrate the application and validity of the proposed method.**

Index Terms—**FMEA, grey target, grey uncertain language variable, optimization method, TOPSIS**

I. Introduction

With the advancement of society, systems become increasingly complex and interrelated, and a failure at one point can result in a disaster for the entire system, which would bring tremendous loss to enterprises. To avoid this outcome, successful evaluation of the risk of failure is thus a hot topic in the practical and research field. Failure mode and effect analysis (FMEA) is a well-known methodology for risk evaluation that identifies potential failure modes through bottom-up analysis [1]. Discrimination of the failure risks in the 1960s led to the official application of FMEA in the aerospace industry. At that time, the quick usefulness of FMEA along with its easy implementation in a significant proportion of other industries, such as medical and health care, energy, automotive, engineering, and maintenance industries, enabled the extensive use of FMEA in practice [2].

In traditional FMEA, risk is evaluated in accordance with the multiplication of three factors: occurrence (O), severity (S), and detection (D). Experts subjectively rate the three factors in the range of 1 to 10. The multiplication result is known as the risk priority number (RPN). According to the final RPN number of each failure mode, preventive measures can be taken to reduce the risk. Although it has achieved tremendous success, traditional FMEA still suffers from shortcomings [3]. The major ones are the following [4]: (1) The problem of priority. FMEA does not account for differences in the factors and experts. Different weights may be appropriate for different situations and various experts, depending on the importance of the factors and experience and professionalism of the experts. (2) The crispy number problem. The traditional FMEA method utilizes a certain number. However, in practice, it is complicated to evaluate the failure mode accurately. (3) Different values of O, S, and D may yield same RPN results. This implies that the risk of the failure modes may not be discerned and identified clearly.

To handle these limitations of the traditional FMEA method, many scholars have tried to improve it. On the crispy number problem, given the linguistic characteristics of the expert evaluations, a large body of researches utilize the fuzzy mathematical method, for instance, fuzzy language and fuzzy linguistic variables. Recently, the popular fuzzy numbers used are triangular and trapezoidal numbers. The risk evaluation procedure is also a process of multi-criteria decision making (MCDM) [5]. Thus, to overcome the weight problem, an MCDM evaluation is always integrated into the traditional FMEA method. The following methods have been extensively used to prioritize the risks of failure modes: Grey relation analysis [2] and grey relation projection [6], Technique of order preference by similarity to the ideal solution (TOPSIS) [7], Multi-criteria optimization and compromise solution (VIKOR) [8], Elimination and choice translating reality (ELECTRE) [9], Interactive and multi-criteria decision making (TODIM) [10], Multi-attributive border approximation area comparison (MABAC) [11], Multiple multi-objective optimization by ratio analysis (MULTIMOORA) [12], Adaptive multi-objective optimization artificial immune algorithm (AMOAIA) [13].

Nevertheless, there is a scarcity of studies utilizing grey language in FMEA research. Moreover, a grey system, in which part of the information is known and part is unknown, is an essential, convenient, and effective method for measurement and decision making under uncertainty. This paper aims to improve the traditional FMEA method by combining the interval grey uncertain linguistic variables method and the grey target TOPSIS decision.

The remainder of the paper is arranged as follows. Section II reviews and summarizes the main improvements in FMEA. Section III explains the basic concepts related to the interval

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uncertain linguistic variables method and grey target TOPSIS decision. Section IV presents the enhanced grey risk assessment approach with the interval uncertain linguistic and TOPSIS methods. Section V provides an illustrative example and comparative analysis to demonstrate the application and validity of the proposed FMEA framework. Section VI discusses the theoretical and practical significance of the proposed framework. Section VII provides conclusions and suggests future research directions.

II. Literature Review

Journal articles in the Science Citation Index and the Social Science Citation Index are representatives of high-quality research. Thus, we selected articles with the words “FMEA” or “failure mode effect analysis” in the title from index database limited to journal articles. The FMEA literature is basically twofold: one part concerns enhancement of the FMEA method, and the other part is on the application of FMEA in practice for discriminating crises and risks. We obtained a total of 251 articles from the past five years. Here, we only focus on articles on improvement of the FMEA method.

A. FMEA Improvement Trend over the Past Five Years

In the selected literature, scholars have tried various methods to overcome the shortcomings of the traditional FMEA. The most common and popular way to tackle the crispy number problem is through the use of fuzzy theory and linguistic variables. The analytic hierarchy process (AHP) and entropy are the methods that are most widely used to overcome the weight problem. The researchers always do not simply compare experts’ average scores, instead, they use MCDM methods to enhance the comparison for the final decision problem. With the advancement of big data, machine learning and artificial intelligence are significantly applied in several fields; therefore, since 2018, researchers have started improving the traditional FMEA by utilizing machine learning.

Here we provide a general summary of the development of FMEA method in the literature. In 2015 and 2016, researches mainly incorporated FMEA and fuzzy theory and the MCDM method, for instance, the fuzzy TOPSIS [14], fuzzy DEMATEL [15], and fuzzy inference system [16]. In 2016, more papers simultaneously used two methods to enhance traditional FMEA than in 2015. Additional researches in 2016 introduced linguistic variables or linguistic fuzzy sets [17] and the Dempster-Shafer theory [18]. In 2017, more MCDM methods were brought in, for instance, MULTIMOORA [19], and the improvement procedure was relatively more complicated than that in 2016. Nonetheless, several papers in 2017 conducted further research on the research findings from 2016. Typical examples are the D-number, an extension of the Dempster-Shafer (D-S) theory [20]; interval value intuitionistic fuzzy sets, an extension of intuitionistic fuzzy sets [19]; interval fuzzy inference system, an extension of the fuzzy inference system [21]; and linguistic distribution, an extension of linguistic variables [10]. In general, the methods of improvement in 2018 resembled those in 2017, with different combinations of weight calculations and MCDM. Some of the papers in 2018 adopted the MOORA method [22] and the Z-number method [23]. In 2019 and 2020, there was a significant increase in FMEA research, and much more diversified methods were adopted. These methods included the interval-valued Pythagorean fuzzy number [24], Bonferroni mean operator [24], improved weighted arithmetic averaging operator of generalized trapezoidal fuzzy number [25], best-worst [26], potentially all pairwise rankings of all possible alternatives [27], and rough sets [28]. Notably, more literatures in 2019 and 2020 implemented machine learning techniques to enhance the traditional FMEA, such as fuzzy Bayesian [29], fuzzy inference [30], logistic regression [31], Petri net [32], support vector machine [33, 34], fuzzy evidential reasoning rules [35], and others.

B. Main Improvements of the FMEA Method

The improvements are mainly on three aspects from the perspective of the entire FMEA process: (1) the first rating procedure by experts—improvements on crispy numbers by a linguistic variable and corresponding mapping method; (2) the second aggregation procedure—assembling experts’ ratings by a specific operator; and (3) the final decision process—the multi-criteria group decision to enhance the prioritization of failure modes in FMEA.

First Rating Procedure by Experts: Improvements on Crispy Numbers by a Linguistic Variable and Corresponding Mapping Method

To a large extent, for the traditional FMEA, the values of S, O, and D, which are integers between 1 and 10, are scored by eminent experts on the problem. Nevertheless, under real-world circumstances, experts find it challenging to assess these values using precise numbers. Hence, the researchers usually initially attempt to apply uncertain variables as substitutes for the traditional crispy numbers. The most commonly used method is to map the linguistic variables to the uncertain numbers.

Experts are usually more comfortable in expressing their opinions in a linguistic way. Thus, in this improvement approach, experts evaluate S, O, and D simply using natural language, which is a linguistic variable. In most cases, the linguistic terms for ratings are always five-term, seven-term, and nine-term [22, 36, 37]. The five-term set is {high (H), medium high (MH), medium (M), medium low (ML), low (L)}. The seven-term set is {very high (VH), high (H), medium high (MH), medium (M), medium low (ML), low (L), very low (VL)}. And the nine-term set is {extremely high (EH), very high (VH), high (H), medium high (MH), medium (M), medium low (ML), low (L), very low (VL), extremely low (EL)}.

Then the linguistic variables are mapped to uncertain numbers, with various authors applying different mapping methods. The most prominent methods are fuzzy number [22, 36, 37], grey number [17], D-number [20], interval number [38], intuitionistic fuzzy sets [3], and hesitant fuzzy sets [39]. Furthermore, combinations of these methods are commonly used, for instance, fuzzy interval number [40], interval value intuitionistic fuzzy sets [19] or interval value intuitionistic fuzzy number [41], hesitant fuzzy linguistic sets [39, 42], probabilistic linguistic term sets [41], and fuzzy D-number [43]. Recently, some new mapping methods have emerged, including Z-number [23, 44], Z-soft fuzzy rough sets [45], cloud number [46], and Pythagorean fuzzy number [47]. All these mapping numbers have similar features that denote uncertainty, implying that none of these numbers is certain.

Second Aggregation Procedure: Assembling Experts' Ratings by a Specific Operator

In the transformation of the corresponding uncertain numbers, the experts' ratings are aggregated after the natural language evaluation. Several operators are employed to conduct the aggregation, and various numbers require diverse operators.

The traditional FMEA method adopts the simple arithmetic averaging operator, which does not include expert weights. Therefore, to overcome this limitation, numerous operators considering the experts' weights are introduced. The weighted averaging operator and its extended forms have been applied extensively by researchers, particularly the order weighted averaging operator [27]. Several scholars further use extended weighted averaging operators to aggregate numbers, such as the hesitant fuzzy linguistic weighted average operator [48] or interval-valued intuitionistic fuzzy weighted averaging. While the widely used D-S theory has a specific aggregation method. Furthermore, using the simple arithmetic weighted operator for aggregation requires first determining the weights by the analytic hierarchy process, entropy, cosine similarity, or another approach. A well-known operator that is used to deal with fuzzy numbers, no matter triangular fuzzy numbers or trapezoid fuzzy numbers, is the Choquet integral operator [49]. Some new research implements the Bonferroni mean operator [24] for aggregating the fuzzy numbers. For aggregating interval type-2 fuzzy numbers, the interval type-2 fuzzy ordered weighted averaging operator [31] is usually considered. In addition, the use of fuzzy numbers is an innovative way to integrate the ratings through the application of a fuzzy inference system such as Mamdani along with the Takagi–Sugeno operator [50], interval cloud ordered weighted averaging operator [46], or two-dimensional uncertain linguistic weighted generalized Heronian mean operator [51]. For the rough numbers, some scholars use the simple average operator, while others adopt the best-worst method [52].

Final Decision Process: The Multi-Criteria Group Decision to Enhance the Prioritization of Failure Modes in FMEA

For the final process of making the decision, MCDM methods are often utilized for improvement. The popular methods are grey relational analysis [2], grey relational projection [6], TOPSIS [7], VIKOR [8], ELECTRE [9], TODIM [10], MABAC [53], and MULTIMOORA [12].

D-S theory is also commonly used in FMEA research. A systematic method for researchers from the beginning process of mapping to the last process of decision making is provided by D-S theory. As a result, D-S theory has become widely known. Therefore, some researchers have extended D-S theory to decision-making theory, extending it by a series of proofs [54]. Moreover, to improve the traditional FMEA methods, researchers have presented several other strategies to help in decision making, including DEA (Data Envelopment Analysis) [55], consensus decision theory [48] and prospect theory [56]. Since a final decision can be made through reasoning or inference [55], and Petri net [31] is a useful tool for conducting inference, some scholars use Petri net in decision making.

III. Preliminaries

A. Interval Uncertain Linguistic Variables

Definition 1. Grey interval data [57]**.** Grey data with upper and lower limits are referred to as grey interval data, for instance, $a(\otimes)\in[\underline{a},\overline{a}]$ for $\underline{a}\le\overline{a}$. The difference between the upper limit value and the lower limit value is called the interval length of the grey data, for instance, $l_a=\overline{a}-\underline{a}$.

Definition 2. Linguistic variable set [58]**.** The set S = {s_0, s_1, s_2,..., s_{l-1}} denotes a set of evaluation linguistic variables. The elements in the set are finite and arranged sequentially, and l is an odd number. In practice, the l value is usually set as 3, 5, 7, or 9. The number of items in the set, l, is adopted as 7 in this paper, so S = {s_0, s_1, s_2, s_3, s_4, s_5, s_6} = {very high (VH), high (H), medium high (MH), medium (M), medium low (ML), low (L), very low (VL)}. The linguistic variable set S = {s_0, s_1, s_2,..., s_{l-1}}, the subscript i, and the element value s_i are expressed in purely monotonically incremental order. That is, let s_i be a function of the subscript i, f: $s_i = f(i)$. If $i < j$, then $s_i < s_j$, and vice versa. To incorporate more information, the discrete linguistic variable set S = {s_0, s_1, s_2,..., s_{l-1}} is usually mapped and extended to the continuous variable set $\overline{S}=\{s_\alpha\mid\alpha\in R\}$.

The operation rules of linguistic variables are listed in expression (1).

$$\begin{aligned}&\beta s_i = s_{\beta\times i}\\&s_i\oplus s_j = s_{i+j}\\&s_i\oplus s_j = s_j\oplus s_i\\&\lambda(s_i\oplus s_j)=\lambda s_i\oplus\lambda s_j\\&(\lambda_1+\lambda_2)s_i=\lambda_1 s_i+\lambda_2 s_i\end{aligned}\tag{1}$$

Definition 3. Linguistic variable distance [3]**.** The distance between linguistic variable s_α and linguistic variable s_β is defined by equation (2).

$$d(s_\alpha,s_\beta)=|\alpha-\beta|\big/(l-1)\tag{2}$$

Definition 4. Uncertain linguistic variable [9]**.** Assuming that $\tilde{S}=[s_a,s_b]$, $s_a,s_b\in\overline{S}$, and $a\le b$, s_a and s_b are upper and lower limits of $\tilde{S}$, respectively. $\tilde{s}$ is an uncertain linguistic variable and $\tilde{S}$ is an uncertain linguistic variable set.

Assuming that $\tilde{s}_1=[s_{a1},s_{b1}]$ and $\tilde{s}_2=[s_{a2},s_{b2}]$, the operation rules of uncertain linguistic variables are listed in equation (3).

$$\begin{aligned}
&\tilde{s}_1 \oplus \tilde{s}_2 = [s_{a1}, s_{b1}] \oplus [s_{a2}, s_{b2}] = [s_{a1+a2}, s_{b1+b2}] \\
&\tilde{s}_1 \otimes \tilde{s}_2 = [s_{a1}, s_{b1}] \otimes [s_{a2}, s_{b2}] = [s_{a1\times a2}, s_{b1\times b2}] \\
&\tilde{s}_1 / \tilde{s}_2 = [s_{a1}, s_{b1}] / [s_{a2}, s_{b2}] = [s_{a1/b2}, s_{b1/a2}] \\
&if\ \ a2 \neq 0, b2 \neq 0 \\
&\lambda\tilde{s}_1 = [s_{\lambda * a1}, s_{\lambda * b1}] \\
&\lambda(\tilde{s}_1 \oplus \tilde{s}_2) = \lambda\tilde{s}_1 \oplus \lambda\tilde{s}_2 \\
&(\lambda_1 + \lambda_2)\tilde{s}_1 = \lambda_1\tilde{s}_1 + \lambda_2\tilde{s}_1
\end{aligned} \tag{3}$$

Definition 5. Interval grey uncertain linguistic variables [59]. $\underset{\otimes}{\tilde{A}} = (\tilde{A}, \underset{\otimes}{A})$ is referred to as grey fuzzy data. Its fuzzy part is the uncertain linguistic variable $\tilde{S} = [s_a, s_b]$. $s_a, s_b \in \bar{S}$ and its grey part $\underset{\otimes}{A}$ denotes interval value $[g_a^L, g_b^U]$. $\underset{\otimes}{\tilde{A}}$ is called a grey uncertain linguistic variable.

$\underset{\otimes}{\tilde{A}} = ([s_{a1}, s_{a2}],[g_a^L, g_a^U])$, $\underset{\otimes}{\tilde{B}} = ([s_{b1}, s_{b2}],[g_b^L, g_b^U])$, and $\underset{\otimes}{\tilde{C}} = ([s_{c1}, s_{c2}],[g_c^L, g_c^U])$ are known as uncertain linguistic variables. The relevant operations are listed in equation (4).

$$\begin{aligned}
&\underset{\otimes}{\tilde{A}} + \underset{\otimes}{\tilde{B}} = ([s_{a1+b1}, s_{a2+b2}],[\max(g_a^L, g_b^L), \max(g_a^U, g_b^U)]) \\
&\underset{\otimes}{\tilde{A}} \times \underset{\otimes}{\tilde{B}} = ([s_{a1\times b1}, s_{a2\times b2}],[\max(g_a^L, g_b^L), \max(g_a^U, g_b^U)]) \\
&\underset{\otimes}{\tilde{A}} / \underset{\otimes}{\tilde{B}} = ([s_{a1/b1}, s_{a2/b2}],[\max(g_a^L, g_b^L), \max(g_a^U, g_b^U)]), \\
&\quad and\ \ b1, b2 \neq 0 \\
&k\underset{\otimes}{\tilde{A}} = ([s_{k\times a1}, s_{k\times a2}],[g_a^L, g_a^U]) \\
&(\underset{\otimes}{\tilde{A}})^k = ([s_{a1^k}, s_{a2^k}],[g_b^L, g_b^U])
\end{aligned} \tag{4}$$

The operation rules are listed in equation (5).

$$\begin{aligned}
&\underset{\otimes}{\tilde{A}} + \underset{\otimes}{\tilde{B}} = \underset{\otimes}{\tilde{B}} + \underset{\otimes}{\tilde{A}} \\
&\underset{\otimes}{\tilde{A}} \times \underset{\otimes}{\tilde{B}} = \underset{\otimes}{\tilde{B}} \times \underset{\otimes}{\tilde{A}} \\
&\underset{\otimes}{\tilde{A}} + \underset{\otimes}{\tilde{B}} + \underset{\otimes}{\tilde{C}} = \underset{\otimes}{\tilde{A}} + (\underset{\otimes}{\tilde{B}} + \underset{\otimes}{\tilde{C}}) \\
&\underset{\otimes}{\tilde{A}} \times \underset{\otimes}{\tilde{B}} \times \underset{\otimes}{\tilde{C}} = \underset{\otimes}{\tilde{A}} \times (\underset{\otimes}{\tilde{B}} \times \underset{\otimes}{\tilde{C}}) \\
&(\lambda_1 + \lambda_2)\underset{\otimes}{\tilde{A}} = \lambda_1 \underset{\otimes}{\tilde{A}} + \lambda_2 \underset{\otimes}{\tilde{A}}
\end{aligned} \tag{5}$$

Definition 6. Rank of interval uncertain linguistic vector (or vector length) [38]**.** The vector rank (or vector length) is defined by equation (6), when the interval uncertain linguistic vector is given.

$$|A| = \sqrt{\sum_{j=1}^{n}(((1-g_{aj}^L)^2 + (1-g_{aj}^U)^2) \times (a1_j^2 + a2_j^2))} \tag{6}$$

Definition 7. Cosine of two interval uncertain linguistic vectors [49]**.** For two given interval uncertain linguistic vectors A and B, the computation of their cosine is defined by equation (7).

$$\begin{aligned}
&\cos(A,B) = \\
&(\sum_{j=1}^{n}(((1-g_{aj}^L)(1-g_{bj}^L) + (1-g_{aj}^U)(1-g_{bj}^U))(a_{1j}b_{1j} + a_{2j}b_{2j}))) \\
&* \frac{1}{\sqrt{\sum_{j=1}^{n}(((1-g_{aj}^L)^2 + (1-g_{aj}^U)^2) \times (a1_j^2 + a2_j^2)}} \\
&* \frac{1}{\sqrt{\sum_{j=1}^{n}(((1-g_{bj}^L)^2 + (1-g_{bj}^U)^2) \times (b1_j^2 + b2_j^2)}}
\end{aligned} \tag{7}$$

where A and B are defined by equation (8).

$$\begin{aligned}
&A = (\underset{\otimes j}{\tilde{A}}) = (([s_{a11}, s_{a21}],[g_{a1}^L, g_{a1}^U]),([s_{a12}, s_{a22}],[g_{a2}^L, g_{a2}^U]), \\
&\ldots,([s_{a1n}, s_{a2n}],[g_{an}^L, g_{an}^U])) \\
&B = (\underset{\otimes j}{\tilde{B}}) = (([s_{b11}, s_{b21}],[g_{b1}^L, g_{b1}^U]),([s_{b12}, s_{b22}],[g_{b2}^L, g_{b2}^U]), \\
&\ldots,([s_{b1n}, s_{b2n}],[g_{bn}^L, g_{bn}^U]))
\end{aligned} \tag{8}$$

It is readily observable that the larger the cosine value is, the closer the A and B vectors are.

Definition 8. Projection of the interval uncertain linguistic vector [60]**.** For the two given interval uncertain linguistic vectors A and B, the projection of vector B on vector A is defined by equation (9).

$$\mathrm{Prj}_A B = \frac{\sum_{j=1}^{n}(((1-g_{aj}^L)(1-g_{bj}^L) + (1-g_{aj}^U)(1-g_{bj}^U))(a_{1j}b_{1j} + a_{2j}b_{2j}))}{\sqrt{\sum_{j=1}^{n}(((1-g_{aj}^L)^2 + (1-g_{aj}^U)^2) \times (a1_j^2 + a2_j^2)}} \tag{9}$$

It can be inferred that the larger is the value of $\mathrm{Prj}_A B$, the closer the vector A is to B.

Definition 9. Distance of interval uncertain linguistic vectors [61]**.** For any two given interval uncertain linguistic vectors A and B, their distance is defined by equation (10).

$$d(A,B) = |A-B| = \sqrt{\begin{aligned}&\sum_{j=1}^{n}(((1-\max(g_{aj}^L, g_{bj}^L))^2 + \\ &(1-\max(g_{aj}^U, g_{bj}^U))^2) \times ((a1_j - b1_j)^2 + (a2_j - b2_j)^2)\end{aligned}} \tag{10}$$

Definition 10. Expectation of the interval uncertain linguistic variable [61]**.** The expectation for a given interval uncertain linguistic variable $\underset{\otimes}{\tilde{A}} = ([s_{a1}, s_{a2}],[g_a^L, g_a^U])$ is defined by equation (11).

$$E(\underset{\otimes}{\tilde{A}}) = \frac{s_{a1} + s_{a2}}{2} \times (1 - \frac{g_A^L + g_A^U}{2}) = S_{(1-\frac{g_A^L + g_A^U}{2}) \times (a1+a2)/2} \tag{11}$$

Definition 11. Comparison of two interval uncertain linguistic variables [60]. For given $\underset{\otimes}{\tilde{A}} = ([s_{a1}, s_{a2}],[g_a^L, g_a^U])$ and $\underset{\otimes}{\tilde{B}} = ([s_{b1}, s_{b2}],[g_b^L, g_b^U])$, if $E(\underset{\otimes}{\tilde{A}}) \geq E(\underset{\otimes}{\tilde{B}})$, then $\underset{\otimes}{\tilde{A}} \geq \underset{\otimes}{\tilde{B}}$, and vice versa.

B. Grey Target TOPSIS Decision

Definition 12. Negative target distance [62]**.** For a given interval uncertain linguistic vector A and the negative target vector TN, the negative target distance is defined by equation (12).

$$d^- = |A - TN|$$
$$A = (\underset{\otimes j}{\tilde{A}}) = (([s_{a11}, s_{a21}],[g_{a1}^L, g_{a1}^U]),([s_{a12}, s_{a22}],[g_{a2}^L, g_{a2}^U]), ..., ([s_{a1n}, s_{a2n}],[g_{an}^L, g_{an}^U]))$$
$$TN = (\underset{\otimes j}{\tilde{T}\tilde{N}}) = (([s_{tn11}, s_{tn21}],[g_{tn1}^L, g_{tn1}^U]),([s_{tn12}, s_{tn22}],[g_{tn2}^L, g_{tn2}^U]), ..., ([s_{tn1n}, s_{tn2n}],[g_{tnn}^L, g_{tnn}^U])) \quad (12)$$

Therefore, the closer a vector is to the negative target, the worse it becomes.

Definition 13. Positive target distance [62]**.** For a given interval uncertain linguistic vector A and the positive target vector TO, the positive target distance is defined by equation (13).

$$d^+ = |A - TO|$$
$$A = (\underset{\otimes j}{\tilde{A}}) = (([s_{a11}, s_{a21}],[g_{a1}^L, g_{a1}^U]),([s_{a12}, s_{a22}],[g_{a2}^L, g_{a2}^U]), ..., ([s_{a1n}, s_{a2n}],[g_{an}^L, g_{an}^U]))$$
$$TO = (\underset{\otimes j}{\tilde{T}\tilde{O}}) = (([s_{to11}, s_{to21}],[g_{to1}^L, g_{to1}^U]),([s_{t012}, s_{t022}],[g_{to2}^L, g_{to2}^U]), ..., ([s_{to1n}, s_{to2n}],[g_{ton}^L, g_{ton}^U])) \quad (13)$$

Hence, the closer a vector is to the positive target, the more appropriate it is.

Definition 14. Integrated target distance [4]**.** Based on the TOPSIS principle, the integrated target distance is defined by equation (14).

$$d = \frac{d^-}{d^- + d^+} \quad (14)$$

IV. Proposed Improved Grey FMEA Evaluation Method

A. Optimal Weights Determination

There are p experts E = {E_1, E_2, E_3, ..., E_p}, m failure modes A = {A_1, A_2, A_3, ..., A_m}, and n evaluation criteria C = {C_1, C_2, ..., C_n}. The evaluation parameters, namely S, O, and D, are available in FMEA, such that the value n is 3 (i.e., n = 3). So, for failure mode A_j, the risk value of criterion C_k rated by expert E_i using a grey uncertain linguistic variable is $\underset{\otimes ijk}{\tilde{A}} = ([s_{a1ijk}, s_{a2ijk}],[g_{aijk}^L, g_{aijk}^U])$. Therefore, based on the ratings on all the n criteria given by expert E_k, an evaluation matrix will be formed, that is, $\underset{\otimes k}{\tilde{A}} = [\underset{\otimes ijk}{\tilde{A}}]_{m\times n}$. The ratings given by expert E_k include both the fuzzy part and the grey part.

The evaluation weights of the n criteria are $w = (w_1, w_2, ..., w_n)$, where $\sum_{j=1}^{n} w_j = 1$ (n=3). The weights of the p experts are $\lambda = (\lambda_1, \lambda_2, \lambda_3, ..., \lambda_p)$, where $\sum_{k=1}^{p} \lambda_k = 1$.

Determination of the Weights of the Experts

Following the aggregation of the ratings of the p experts, the group evaluation matrix is $\underset{\otimes}{\tilde{X}} = [\underset{\otimes ij}{\tilde{X}}]_{m\times n}$, where

$$\underset{\otimes ijk}{\tilde{X}} = ([s_{a1_{ij}^X}, s_{a2_{ij}^X}],[g_{ij}^L, g_{ij}^U]) = \sum_{k=1}^{p} (\lambda_k \underset{\otimes ijk}{\tilde{A}}),$$

$s_{a1_{ij}^X} = \sum_{k=1}^{p} (\lambda_k s_{a1ijk})$, and

$[g_{ij}^L, g_{ij}^U] = [\max(g_{ijk}^L \mid k = 1,2,3..., p), \max(g_{ijk}^U \mid k = 1,2,3..., p)]$.

The rating vector of expert E_k for failure mode A_i is A_{ik} and the vector length of A_{ik} is defined by equation (15).

$$|A_{ik}| = \sqrt{\sum_{j=1}^{n} (((1-g_{aijk}^L)^2 + (1-g_{aijk}^U)^2) \times (a1_{ijk}^2 + a2_{ijk}^2))} \quad (15)$$

The projection of X_i on A_{ik} is revealed in equation (16).

$$\mathrm{Prj}_{Aik} X_i = \frac{\sum_{j=1}^{n} (g_{ijk}(a1_{ijk}(\sum_{k=1}^{p} \lambda_k a1_{ijk}) + a2_{ijk}(\sum_{k=1}^{p} \lambda_k a2_{ijk})))}{\sqrt{\sum_{j=1}^{n} (((1-g_{aijk}^L)^2 + (1-g_{aijk}^U)^2) \times (a1_{ijk}^2 + a2_{ijk}^2))}} \quad (16)$$

where

$$g_{ijk} = (1-g_{aijk}^L)(1-\max(g_{ijk}^L \mid k = 1,2,3..., p)) + (1-g_{aijk}^U)(1-\max(g_{ijk}^U \mid k = 1,2,3..., p))$$

Experts' weights are computed using the optimization problem shown in equation (17).

$$\max \sum_{i=1}^{m} \sum_{k=1}^{p} \mathrm{Prj}_{A_{ik}} X_i$$
$$s.t. \begin{cases} \sum_{k=1}^{p} \lambda_k = 1 \\ \lambda_k \geq 0, k = 1,2,3,..., p \end{cases} \quad (17)$$

Determination of the Weights of the Evaluation Criteria

For the j[th] evaluation criterion, C_j (j = 1,2,3), the difference between failure mode A_i and the rest of the failure modes is $D_{ij}(w_j) = \sum_{l=1}^{m} d(\underset{\otimes ij}{\tilde{X}}, \underset{\otimes lj}{\tilde{X}}) w_j$. Thus, for all the n evaluation criteria (n = 3), the difference between A_i and the other modes is $D(w_j) = \sum_{j=1}^{n} \sum_{i=1}^{m} D_{ij}(w_j) = \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{l=1}^{m} d(\underset{\otimes ij}{\tilde{X}}, \underset{\otimes lj}{\tilde{X}}) w_j$.

The weights of the evaluation criteria are determined through the following optimization problem (18).

$$\max D(w_j)=\sum_{j=1}^{n}\sum_{i=1}^{m}D_{ij}(w_j)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{l=1}^{m}d(\tilde{X}_{\otimes ij},\tilde{X}_{\otimes lj})w_j$$
$$s.t.\begin{cases}\sum_{j=1}^{n}w_j^2=1\\ w_j\geq 0, j=1,2,3,...,n\end{cases} \quad (18)$$

B. Proposed Model

Figure 1 describes the framework of the proposed FMEA approach, inclusive of the following three phases:

(1) Rating and mapping. The assessments of the three evaluation criteria (S, O, and D) are rated by the linguistic variables and the linguistic evaluation is mapped to interval grey uncertain numbers.

(2) Aggregation. The weights of the experts along with the evaluation criteria are determined using the optimization method, and then the experts' evaluations are aggregated through the arithmetic weighted operator.

(3) Final decision. The failure modes are ranked using the grey target TOPSIS method.

The following five steps are used to establish the procedure for the overall FMEA model:

Step 1. Experts rate S, O, and D for each failure mode. Based on the definition in equation (2), the interval for the grey uncertain linguistic decision matrix is established.

Step 2. Each expert is weighted and the weights of S, O, and D are determined by equations (15) to (18).

Step 3. A decision matrix is established in accordance with each expert's weight and the weights of S, O, and D according to equations (4) and (5).

Step 4. The positive and negative target values are determined based on equation (11) and definition 11. The maximum value of each expert evaluation is taken by the negative target TN, and the minimum value of each expert evaluation is taken by the positive target TO. The subscript i denotes the score of the item, which depends on the experts' weights and the weights of S, O, and D in the ith failure mode, that is, in the ith row of the evaluation matrix, which is obtained in step 3. The subscripts s, o, and d depict S, O, and D, respectively.

For instance, $([s_{s1i},s_{s2i}],[g_{si}^L,g_{si}^U])$ is the ith row S value of the evaluation matrix, which is obtained from step 3. And $\min([s_{s1i},s_{s2i}],[g_{si}^L,g_{si}^U])$ essentially implies the minimum value in the severity column in the evaluation matrix derived from step 3, that is,

$$TO=(\tilde{T}\tilde{O}_{\otimes})=((\min([s_{s1i},s_{s2i}],[g_{si}^L,g_{si}^U])),$$
$$(\min([s_{o1i},s_{o2i}],[g_{oi}^L,g_{oi}^U])),(\min([s_{d1i},s_{d2i}],[g_{di}^L,g_{di}^U])))$$

$$TN=(\tilde{T}\tilde{N}_{\otimes})=((\max([s_{s1i},s_{s2i}],[g_{si}^L,g_{si}^U])),$$
$$(\max([s_{o1i},s_{o2i}],[g_{oi}^L,g_{oi}^U])),(\max([s_{d1i},s_{d2i}],[g_{di}^L,g_{di}^U])))$$

Step 5. The risk of each failure mode is computed and ranked according to equations (12) to (14). The larger the integrated target distance is, the higher the risk of the failure mode becomes.

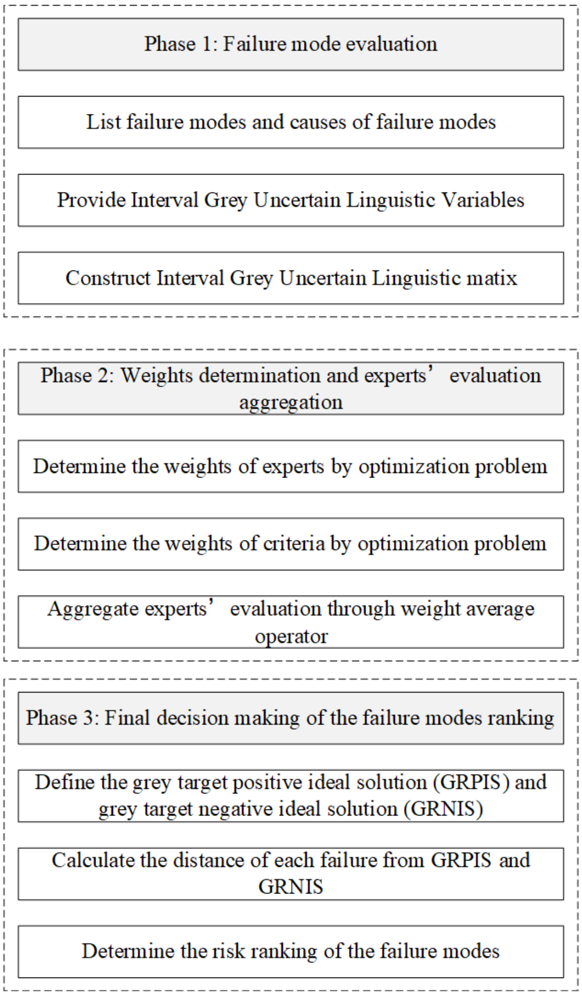

Fig. 1. Framework of the proposed FMEA

V. CASE STUDY

In this case, there are four failure modes and three experts. The objective of the case study is to evaluate the three experts' evaluations and rank the failure modes. The interval grey possibilities of all the statuses are shown in table I.

TABLE I
GREY STATUS AND THE CORRESPONDING VALUE

Status	Very certain	Certain	Medium	Not certain	Very not certain
Interval grey value $[g^L,g^U]$	[0.0, 0.2]	[0.2, 0.4]	[0.4, 0.6]	[0.6, 0.8]	[0.8, 1.0]

A. Expert Evaluation Score

First, the experts' ratings are collected. The evaluations of S, O, and D by the experts are presented in tables II to IV.

TABLE II
EXPERT E1 EVALUATION

Failure mode	S	O	D
F1	([s2,s3],[0.1,0.2])	([s1,s2],[0.3,0.4])	([s3,s4], [0.2,0.3])
F2	([s1,s2],[0.2,0.3])	([s3,s4],[0.2,0.3])	([s4,s5], [0.4,0.5])
F3	([s3,s4],[0.4,0.5])	([s2,s3],[0.2,0.3])	([s5,s6], [0.3,0.4])
F4	([s4,s5],[0.3,0.4])	([s5,s6],[0.1,0.2])	([s3,s4], [0.2,0.3])

TABLE III
EXPERT E2 EVALUATION

Failure mode	S	O	D
F1	([s1,s2],[0.1,0.2])	([s2,s3],[0.2,0.3])	([s2,s3], [0.1,0.2])
F2	([s2,s3],[0.2,0.3])	([s4,s5],[0.3,0.4])	([s3,s4], [0.3,0.4])
F3	([s4,s5],[0.3,0.4])	([s3,s4],[0.3,0.4])	([s4,s5], [0.2,0.3])
F4	([s3,s4],[0.4,0.5])	([s4,s5],[0.2,0.3])	([s3,s4], [0.3,0.4])

TABLE IV
EXPERT E3 EVALUATION

Failure mode	S	O	D
F1	([s2,s3],[0.2,0.3])	([s2,s3],[0.3,0.4])	([s3,s4], [0.1,0.2])
F2	([s2,s3],[0.3,0.4])	([s3,s4],[0.3,0.4])	([s3,s4], [0.4,0.5])
F3	([s3,s4],[0.3,0.4])	([s2,s3],[0.2,0.3])	([s5,s6], [0.3,0.4])
F4	([s3,s4],[0.2,0.3])	([s4,s5],[0.1,0.2])	([s3,s4], [0.2,0.3])

B. Weights Determination

Second, the weights of the experts and the weights of S, O, and D are determined. To determine the expert weights, a Lagrange equation for equation (17) is established and solved. We have

$$\lambda_1 = 0.327843, \lambda_2 = 0.348343, \lambda_3 = 0.323814$$

Based on the expert weights (0.327843, 0.348343, and 0.323814), tables II to IV can be combined into a group decision matrix. The evaluation matrixes corresponding to tables II to IV are $\underset{\otimes}{\tilde{X}}\tilde{1}$, $\underset{\otimes}{\tilde{X}}\tilde{2}$, and $\underset{\otimes}{\tilde{X}}\tilde{3}$. Thus, the group decision matrix based on the expert weights is

$$\underset{\otimes}{\tilde{X}} = 0.327843\,\underset{\otimes}{\tilde{X}}\tilde{1} + 0.348343\,\underset{\otimes}{\tilde{X}}\tilde{2} + 0.323814\,\underset{\otimes}{\tilde{X}}\tilde{3}.$$

Therefore, we have

$$\underset{\otimes}{\tilde{X}} = \begin{bmatrix} ((s_{1.65}, s_{2.65}),[0.2,0.3]) & ((s_{1.67}, s_{2.67}),[0.3,0.4]) & ((s_{2.65}, s_{3.65}),[0.2,0.3]) \\ ((s_{1.67}, s_{2.67}),[0.3,0.4]) & ((s_{3.35}, s_{4.35}),[0.3,0.4]) & ((s_{3.33}, s_{4.33}),[0.4,0.5]) \\ ((s_{3.35}, s_{4.35}),[0.4,0.5]) & ((s_{2.35}, s_{3.35}),[0.3,0.4]) & ((s_{4.65}, s_{5.65}),[0.3,0.4]) \\ ((s_{3.35}, s_{2.35}),[0.4,0.5]) & ((s_{4.33}, s_{5.33}),[0.3,0.4]) & ((s_{3.00}, s_{4.00}),[0.3,0.4]) \end{bmatrix} \tag{19}$$

Based on the expert decision-making group decision result in (19), the weights of S, O, and D are obtained. To get these weights, a Lagrange equation for equation (18) is established and solved. We then obtain the following results:

$$w_1 = 0.2903, w_2 = 0.4114, w_3 = 0.2983$$

Thus, the decision matrix based on the weights of S, O, and D is updated as follows:

$$\underset{\otimes}{\tilde{X}} = \begin{bmatrix} ((s_{0.48}, s_{0.77}),[0.2,0.3]) & ((s_{0.69}, s_{1.10}),[0.3,0.4]) & ((s_{0.79}, s_{1.09}),[0.2,0.3]) \\ ((s_{0.49}, s_{0.78}),[0.3,0.4]) & ((s_{1.38}, s_{1.79}),[0.3,0.4]) & ((s_{0.99}, s_{1.29}),[0.4,0.5]) \\ ((s_{0.98}, s_{1.27}),[0.4,0.5]) & ((s_{0.97}, s_{1.38}),[0.3,0.4]) & ((s_{1.39}, s_{1.69}),[0.3,0.4]) \\ ((s_{0.96}, s_{1.26}),[0.4,0.5]) & ((s_{1.78}, s_{2.19}),[0.3,0.4]) & ((s_{0.89}, s_{1.19}),[0.3,0.4]) \end{bmatrix} \tag{20}$$

C. Ranking the Results

The concluding step is to obtain the final ranking based on grey target TOPSIS. According to the decision matrix (20), equation (11), and definition 11, the positive and negative target values are obtained as follows:

$$TO = (\underset{\otimes}{\tilde{T}\tilde{O}}) = (((s_{0.49}, s_{0.78}),[0.3,0.4]), ((s_{0.69}, s_{1.10}),[0.3,0.4]), ((s_{0.99}, s_{1.29}),[0.4,0.5]))$$

$$TN = (\underset{\otimes}{\tilde{T}\tilde{N}}) = (((s_{0.97}, s_{1.26}),[0.4,0.5]), ((s_{1.78}, s_{2.19}),[0.3,0.4]), ((s_{1.39}, s_{1.69}),[0.3,0.4]))$$

Therefore, the positive target distance, negative target distance, and integrated target distance are computed as follows:

d^+ =[0.016563, 0.000021, 0.013798, 0.003251]

d^- =[0.017976, 0.013798, 0.000213, 0.000121]

d =[0.520462, 0.998481, 0.015201, 0.035882]

Based on the final result d, the risk ranking is F2 > F1 > F4 > F3. That is, the risk of F2 is the highest; thus, the leaders and employees in the company should pay special attention to F2, and according to the analysis of F2, all the potential factors that can cause F2 should be tackled.

D. Comparison

A comparative analysis of three popular, related FMEA methods, the traditional FMEA, fuzzy FMEA, and fuzzy TOPSIS, was conducted to explain the effectiveness and improvements of the proposed FMEA method. The original data are presented in table V, and the final comparison results are presented in table VI, which depicts that the proposed FMEA and the other three methods do not yield consistent risk ranks of the failure modes.

From table VI, it is readily observable that the ranking of the traditional FMEA conforms with the ranking of the fuzzy FMEA method. Nevertheless, the result of the proposed method differs slightly from the traditional FMEA, and the result of the proposed method is concordant with the fuzzy TOPSIS method, which demonstrates that the proposed method is valid and tends to deal with fuzzy evaluation information soundly. The TOPSIS method considers the positive and negative ideal solutions concurrently, thereby resulting in a different rank from a method that only considers one aspect.

Although the proposed method is fully in agreement with the fuzzy TOPSIS method in this case, there are only four failure modes in this instance. Therefore, the results of the two techniques could be dissimilar if more failure modes are present. The comparison shows that the proposed method corroborates the other methods, although there are some differences. We surmise that the proposed method is useful and practical. And the proposed method can address fuzzy information and grey information. The proposed method sheds light on the way to handle uncertain information. And the proposed method gains weight in the optimization solution, which is considerably more precise and objective. Likewise, the proposed method obtains the final ranking by the grey target and TOPSIS methods, which comprehensively take all the data and information into consideration.

TABLE V
ORIGINAL DATA

FM	Severity	Occurrence	Detectivity
FM1	7	7	5
FM2	7	5	5
FM3	5	6	2
FM4	5	3	5

TABLE VI
RANKING COMPARISON

	T FMEA		F FMEA		F TOPSIS		Proposed	
FM	S	R	S	R	S	R	S	R
FM1	245	1	0.48	1	0.54	2	0.52	2
FM2	175	2	0.26	2	0.99	1	0.99	1
FM3	60	4	0.12	4	0.21	4	0.15	4
FM4	75	3	0.18	3	0.39	3	0.36	3

T FMEA: Traditional FMEA; F FMEA: Fuzzy FMEA; F TOPSIS: Fuzzy TOPOSIS; Proposed: the proposed improved FMEA in this paper. S: final risk value; R: the rank of the failure mode.

VI. DISCUSSION

A. Theoretical Significance

The enhanced method that is proposed in this study adopts the fuzzy TOPSIS method and grey theory and concurrently considers linguistic attributes. As people use natural language to examine risk, we find that it is natural to use linguistics as our proposed method to obtain a solution, rather than the traditional method of numbers. Furthermore, our proposed method incorporates fuzzy theory and grey theory, the two classic theories on uncertainty. Fuzzy theory is especially appropriate for studying cognitive uncertainties, and grey theory is a good fit for decision making. Furthermore, grey theory can address problems with little data and information.

One of the critical limitations of the traditional FMEA method is that it requires crispy numbers, which can result in an inaccurate evaluation. In contrast, the method proposed in this paper, based on linguistic fuzzy numbers, considers the uncertain attribute and the linguistic attribute of the expert evaluation process. Furthermore, the paper takes advantage of interval uncertainty linguistic variables, which are easier and more utilitarian, while keeping the improvement of crispy numbers.

Furthermore, when inviting experts to evaluate risk, the number of experts is usually limited. Therefore, our adoption of grey theory in this paper is appropriate when the amount of data is insufficient. On the limitation of not considering the relevance of experts, the paper also makes an improvement in that the guidance of the expert who possesses the greatest level of support is most relevant. That is, if several experts hold the same opinion, the rest of the experts will have lower weights, as fewer other experts concur with them. In general, this weight allocation method contrasts with the traditional philosophy of the minority that is in support of the majority. In a nutshell, improvements are made on various limitations of the traditional FMEA method based on the method we propose.

B. Practical Significance

Evaluation in many areas can be executed with the adoption of the proposed method. With the use of fuzzy linguistic phrases, the proposed method is more practicable for experts to examine and more comfortable for use in business practice. In addition, in our proposed method, the situation where the opinions of experts carry different weights corresponds more to reality. The proposed method simply adopts interval uncertainty linguistic variables, which is not complicated, while simultaneously considering evaluations of actual situations. The grey method is insensitive to the volume of data; hence, it tends to address insufficient or limited information. Therefore, the method can be applied even when there are only a few experts. Furthermore, the grey target TOPSIS decision method adopted in this paper is recommended to aid companies in achieving more objective assessments. Companies tend to delay using a complex method. Hence, with the use of computer algorithms, it is easier to implement the proposed method.

VII. CONCLUSION

FMEA has been an essential and effective method of risk analysis. It is applied extensively in practice due to its simplicity and efficiency. Nevertheless, the traditional FMEA method has limitations. Therefore, improvement of the traditional FMEA has attracted the attention of many scholars, and it is a hot topic in the FMEA research filed. Many researchers have attempted to enhance the shortcomings of the traditional FMEA through various methods. However, little research has adopted the grey method, which can work adequately with limited data. This paper uses interval grey uncertain linguistic variables to deal with expert evaluation and simultaneously considers grey and fuzzy attributes. To be more objective, the paper utilizes the optimization method to determine the weights of the three factors and the weights of the experts. The case study reveals the validity of the proposed method. With the rapid development of big data analytics and machine learning, several researchers are paying special attention to these emerging methods and attempting to deploy them to tackle problems. Therefore, improvements in FMEA with big data analytics and machine learning may be a promising direction for future research.

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