# Adaptive Fuzzy Prescribed Performance Control for Strict-Feedback Stochastic Nonlinear System with Input Constraint

Yilin Ma, Nannan Zhao, Xinyu Ouyang, Haibo Xu and Yiqin Zhou

*Abstract*—**This paper studies the tracking control and constraint control problem of strict-feedback stochastic nonlinear system with input saturation constraint. First, a novel error transformation function is designed and applied to the design process of the prescribed performance controller, so that the steady-state and transient performance can be guaranteed and the error of the system can be effectively constrained. Then, a anti-trigonometric function is used to approach the system input function, in this way, the controller can be designed directly by backstepping technology. Next, the stability analysis proves that all the signals in the closed-loop system are uniform ultimately bounded (UUB) in probability. Lastly, the results of simulation further verify the effectiveness of the proposed method.**

*Index Terms*—**backstepping, prescribed performance control (PPC), input saturation constraint, strict-feedback stochastic nonlinear system.**

## I. Introduction

IN the past few decades, great achievements have been made in the study of stochastic nonlinear systems by back-stepping technique [1]–[5] and approximation technology [6]–[12]. For example, for a class of strict-feedback stochastic nonlinear systems with respect to an unknown virtual control gain function, the author in [13] proposed an adaptive fuzzy control method and ensured that the tracking error converges to a small neighborhood and all the closed-loop signals are bounded in probability. In [14], Chen presented an adaptive asymptotic tracking controller for stochastic strict-feedback systems by introducing a novel gain suppressing inequality approach. Furthermore, adaptive fuzzy control method for stochastic nonlinear system has been applied to many practical problems [15]–[17]. For example, in [15], adaptive fuzzy actuator fault compensation control has been studied for stochastic nonlinear system with unknown functions, unmeasurable states and actuator faults. In [17], the Takagi-Sugeno fuzzy system was used to establish the system model for the general electric XTE46 turbine engine, and the tracking performance of the system is guaranteed by the stable fault-tolerant adaptive fuzzy control method.

In many practical control processes, the transient and steady-state performance of the system is as important as the stability of the stochastic nonlinear system, so prescribed performance control (PPC) has become a research hotspot in recent years [18]–[23]. In [18]–[20], PPC was used to ensure the tracking performance, the steady-state and transient performance of the system content the requirement. Compared with [18]–[20], in [21] and [22], the Nussbaum function was applied to eliminate the influence of uncertain control direction, then adaptive fuzzy PPC was applied to uncertain strict-feedback system with unknown control direction. Inspire by [18]–[22], in [23], the Nussbeum-type gain was introduced to eliminate the influence of unknown control direction, and the adaptive rate was constructed by prediction error to solve the approximate accuracy problem of generalized fuzzy hyperbolic model (GFHM), the prescribed performance tracking control of MIMO system with uncertain control direction was realized.

In practical system, due to the performance limitations of the actuator, nonlinear system with input constraint [24]–[29] has been widely mentioned recently. Typically, in [25], to solve the non-differentiable problem in saturated nonlinearity, a smooth nonlinear function was introduced to approximate it, and an adaptive tracking controller has been designed for a class of pure-feedback stochastic systems. Afterward, many papers further investigated other kinds of nonlinear systems with different input constraints, such as stochastic nonlinear system with dead-zone [27], and strict-feedback nonlinear system with time-delay [29].

Inspired by [18] and [25], this paper designs an adaptive prescribe performance controller for the strict-feedback stochastic nonlinear system with input constraints. The mainly contributions of this work are including:

(1) The adaptive fuzzy prescribed performance control method is applied to solve the tracking control and constrain control problems of strict-feedback stochastic nonlinear system with input saturation constraints.

(2) A new error transfer function in the form of logarithmic function is introduced into the controller design process, because of the existence of stochastic disturbance, the original error transfer function has to be reconstructed.

(3) The simulation results show the effectiveness of the designed controller.

Manuscript received Feburary 24, 2021; revised Marh 19, 2021. The work is supported in part by the Scientific Research Fundation of Liaoning Provincial Education Department of China (Grant Nos. 2019LNJC13 and 2019LNJC11), and the National Natural Science Found of China (Grant No. U173110085).

Yi-lin Ma is Postgraduate of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO114051, China. (e-mail: 1747678359@qq.com).

Nan-nan Zhao is Associate Professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (Corresponding author: 723306003@qq.com).

Xin-yu Ouyang is Professor of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (Corresponding author: e-mail: 13392862@qq.com).

Hai-bo Xu is Postgraduate of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (e-mail: 502679740@qq.com).

Yi-qin Zhou is Postgraduate of School of Electronic and Information Engineering, University of Science and Technology Liaoning, Anshan, Liaoning, CO 114051, China. (e-mail: 1179820057@qq.com).

## II. System descriptions and basic knowledge

### A. System descriptions

Let's give some important definitions and assumptions first. A stochastic system is considered as follows:

$$dx = f(x)dt + \varphi(x)d\omega, \forall x \in \mathbb{R}^n \tag{1}$$

where $x \in \mathbb{R}^n$ is the state of the system, $\omega$ is an $r$-dimensional independent standard brownian motion, which defined on the complete probability space $(\Omega, F, p)$, $f_i(\cdot)$ and $\varphi_i(\cdot) : \mathbb{R}^i \to \mathbb{R}$ are all the unknown functions.

**Definition 1.** *For any given $V(x) \in \mathbb{C}^2$, associated with the stochastic differential (1), define the differential operator L as follows:*

$$LV = (\frac{\partial V}{\partial x})f + \frac{1}{2}Tr\left[h^T(\frac{\partial^2 V}{\partial x^2})h\right] \tag{2}$$

*where Tr(A) is the trace of matrix A.*

**Remark 1.** *In the Itô correction term $(1/2)Tr\{h^T(\partial^2 V/\partial x^2)h\}$, the second-order differential $\partial^2 V/\partial x^2$ makes the design of controller very hard.*

**Assumption 1.** *The direction of function $g_i(\bar{x}_i), 1 \leq i \leq n-1$ is certain, and positive constants $b_{m1}$ and $b_M$ are uncertain such that*

$$0 < b_{m1} \leqslant |g_i(\bar{x}_i)| \leqslant b_M < \infty \tag{3}$$

**Assumption 2.** *For the function $G_\delta$, there exist an unknown positive constant $G_m$ such that*

$$0 < G_m \leqslant G_\sigma < 1 \tag{4}$$

**Remark 2.** *Without affecting the analysis, we can further suppose that $g_i > 0$. Denote $b_m = \min\{b_{m1}, b_{m1}G_m\}$, one has*

$$b_m \leq g_i \leq b_M,\ 1 \leq i \leq n-1$$

$$b_m \leq g_n G_\sigma \leq b_M$$

Consider a strict-feedback stochastic system with input saturation nonlinearity as follows:

$$\begin{cases} dx_i = (f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1}(t))dt + \varphi_i^T(x)d\omega \\ dx_n = (f_n(\bar{x}_n) + g_n(\bar{x}_n)u(t))dt + \varphi_n^T(x)d\omega \\ y = x_1 \end{cases} \tag{5}$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T \in \mathbb{R}^i$ $(i = 1, 2, \cdots, n-1)$ and $y \in \mathbb{R}$ are the state vector and the output of the system, respectively, $g_i(\cdot)$ is an unknown smooth function, and $u$ is the input of the system with asymmetric saturated nonlinear characteristics, it can be described by

$$u(t) = S(v) = \begin{cases} u_{\max}, v \geq u_{\max} \\ v, u_{\min} < v < u_{\max} \\ u_{\min}, v \leq u_{\min} \end{cases} \tag{6}$$

where $v$ is the actual signal of input, $u_{\max} > 0$ and $u_{\min} < 0$ indicate the upper and lower bounds of input signal which are unknown constants. From (6),we know that when $v = u_{max}$ and $v = u_{min}$, there will be two non-differentiable points, which make the backstepping method unable to be applied to the construction of input signal. So we use a smooth curve to replace the original input function defined as follows:

$$\begin{aligned} G(v) &= \begin{cases} u_{\max} * \tanh(\frac{v}{u_{\max}}), v \geq 0 \\ u_{\min} * \tanh(\frac{v}{u_{\min}}), v < 0 \end{cases} \\ &= \begin{cases} u_{\max} * \frac{e^{\frac{v}{u_{\max}}} - e^{\frac{-v}{u_{\max}}}}{e^{\frac{v}{u_{\max}}} + e^{\frac{-v}{u_{\max}}}}, v \geq 0 \\ u_{\min} * \frac{e^{\frac{v}{u_{\min}}} - e^{\frac{-v}{u_{\min}}}}{e^{\frac{v}{u_{\min}}} + e^{\frac{-v}{u_{\min}}}}, v < 0 \end{cases} \end{aligned} \tag{7}$$

Then, $S(v)$ in (6) will become:

$$u = S(v) = G(v) + h(v) \tag{8}$$

where $G(v) = G_\sigma(v)v$ and $G_\sigma$ is expressed as follows:

$$G_\sigma = \left.\frac{\partial G(v)}{\partial v}\right|_{v=v_\sigma} v_\sigma = \sigma v, 0 < \sigma < 1 \tag{9}$$

we have the function as follows

$$S(v) = G_\sigma(v)v + h(v) \tag{10}$$

where $h(v) = S(v) - G(v)$ is a bounded function expressed by

$$\begin{aligned} |h(v)| &= |S(v) - G(v)| \\ &\leq \max\{u_{\max}(1 - \tanh(1)), u_{\min}(1 - \tanh(1))\} \\ &= B \end{aligned} \tag{11}$$

Substituting (10) into (5) result in

$$\begin{cases} dx_i = (f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1}(t))dt + \varphi_i^T(\bar{x}_i)d\omega \\ dx_n = (f_n(\bar{x}_n) + g_n G_\sigma v + g_n h(v))dt + \varphi_n^T(\bar{x}_n)d\omega \end{cases} \tag{12}$$

### B. Prescribed performance control

In this section, backstepping technology will be applied to the design of the controller. For the $i$th subsystem, let's define the virtual control signal $\alpha_i$ and control input $v$ as

$$\alpha_1 = -\frac{\kappa_1}{\Gamma_1}(a_1 + \frac{3}{4} + \frac{\kappa_1^2}{2c_1^2}\hat{\theta}\psi_1^T\psi_1) \tag{13}$$

where $\psi_1$ is the function of $[x_1, y_d, \dot{y}_d, \rho_1, \dot{\rho}_1]$ and $\alpha_1$ is the function of $[x_1, \hat{\theta}, y_d, \dot{y}_d, \rho_1, \dot{\rho}_1]$. $a_1$ and $c_1$ are designed positive constants.

$$\alpha_i = -(a_i + \frac{3}{4})z_i - \frac{1}{2c_i^2}z_i^3\hat{\theta}\psi_i^T\psi_i \tag{14}$$

$$\alpha_n = v = -(a_n + \frac{3}{4\Theta^2})z_n - \frac{1}{2c_n^2}z_n^3\hat{\theta}\psi_n^T\psi_n \tag{15}$$

$\psi_i$ is the function of $[\bar{x}_i, \hat{\theta}, y_d, \dot{y}_d \cdots y_d^{(i)}, \rho_1, \dot{\rho}_1 \cdots \rho_1^{(i)}]$ and $\alpha_i$ is the function of $[\bar{x}_i, \hat{\theta}, y_d, \dot{y}_d \cdots y_d^{(i)}, \rho_1, \dot{\rho}_1 \cdots \rho_1^{(i)}]$, $\Theta, a_i, c_i, (i = 2, 3, ..., n)$ are designed positive constants. $\hat{\theta}$ is an unknown constant representing the estimated value of $\theta$, which is defined as follows:

$$\theta = \max_{1 \leq i \leq n}\{\frac{1}{b_m}\|W_i^*\|^2\} \tag{16}$$

The adaptive laws is defined as follows:

$$\dot{\hat{\theta}} = \frac{\gamma}{2c_1^2}\kappa_1^6\psi_1^T\psi_1 + \sum_{i=2}^{n}\frac{\gamma}{2c_i^2}z_i^6\psi_i^T\psi_i - \delta\hat{\theta} \tag{17}$$

The state error are defined as follows:

$$\begin{aligned} z_1 &= x_1 - y_d, \\ z_i &= x_i - \alpha_{i-1}, \quad i = 2, 3, \cdots, n \end{aligned} \tag{18}$$

where $y_d$ is the control objective. The boundary function of state error $z_1$ is defined as follows:

$$\rho_1 = (\rho_0 - \rho_\infty)e^{-\varrho t} + \rho_\infty, \tag{19}$$

with $\rho_0 > \rho_\infty$, where $\rho_0$ is the initial value of the boundary and the maximum value allowed by state error $z_1$, $\rho_\infty$, is the final value of the boundary and the maximum value allowed by the preset steady-state error, $\varrho$ is the preset boundary convergence rate, which can determine the adjustment time of the system, the above three parameters are preset postive constants. The initial state error $z_i(0)$ of the system should satisfy the condition $|z_i(0)| < |\rho_0|$. For the sake of making the system (5) stable and making the state error $z_i$ satisfy the preset, we design a new error transformation functoin as follows:

$$\kappa_1 = \ln(\frac{\rho_1 + z_1}{\rho_1 - z_1}) \tag{20}$$

and its time-derivative is

$$d\kappa_1 = \Gamma_1(f_1 + g_1 x_2 - \dot{y}_d - \frac{z_1\dot{\rho}_1}{\rho_1})dt + \Gamma_1\varphi_1^T d\omega \tag{21}$$

where $\Gamma_1 = \frac{2\rho_1}{\rho_1^2 - z_1^2}$.

**Lemma 1.** *For $\forall(x, y) \in R^2$, we can have:*

$$xy \le \frac{\eta^p}{p}|x|^p + \frac{1}{q\eta^q}|y|^q \tag{22}$$

*where $\eta > 0,\ p > 1,\ q > 1$, and $(p-1)(q-1) = 1$.*

## III. Controller Design

### A. Controller design

in the following, we will design the controller in $n$ steps.

**Step** 1: A positive definite Lyapunov function is defined as follows:

$$V_1 = \frac{1}{4}\kappa_1^4 + \frac{b_m}{2\gamma}\tilde{\theta}^2 \tag{23}$$

By combining (2) and (21), the derivative of $V_1(t)$ can be obtained

$$\begin{aligned} LV_1 &= \kappa_1^3\Gamma_1(f_1 + g_1x_2 - \dot{y}_d - \frac{\dot{\rho}_1 z_1}{\rho_1}) \\ &\quad + \frac{3}{2}\kappa_1^2\|\Gamma_1\varphi_1\|^2 - \frac{b_m}{\gamma}\tilde{\theta}\dot{\hat{\theta}} \end{aligned} \tag{24}$$

Based on Lemma 1, it is not hard to obtain that $\frac{3}{2}\kappa_1^2\|\Gamma_1\varphi_1\|^2 \le \frac{3}{4l_1^2}\kappa_1^4\|\Gamma_1\varphi_1\|^4 + \frac{3l_1^2}{4}$ where $l_i$ $(i = 1, 2, ..., n)$ are positive design paraments. Using this inequality into (24) gives

$$\begin{aligned} LV_1 &\le \kappa_1^3\Gamma_1(f_1 + g_1x_2 - \dot{y}_d - \frac{\dot{\rho}_1 z_1}{\rho_1}) \\ &\quad + \frac{3}{4l_1^2}\kappa_1^4\|\Gamma_1\varphi_1\|^4 + \frac{3l_1^2}{4} - \frac{b_m}{\gamma}\tilde{\theta}\dot{\hat{\theta}} \\ &\le \kappa_1^3\Gamma_1 g_1 x_2 + \kappa_1^3 F_1 - \frac{3}{4}\kappa_1^4 + \frac{3}{4}l_1^2 - \frac{b_m}{\gamma}\tilde{\theta}\dot{\hat{\theta}} \end{aligned} \tag{25}$$

where $F_1 = \Gamma_1(f_1 - \dot{y}_d - \frac{\dot{\rho}_1 z_1}{\rho_1}) + \frac{3}{4l_1^2}\kappa_1\|\Gamma_1\varphi_1\|^4 + \frac{3}{4}\kappa_1$. Since $F_1$ include the unknown functions $f_1(\bar{x}_1)$, $F_1$ cannot be used to design virtual control signal $\alpha_1$. By employing a fuzzy logic system $W_1^T\psi_1(Z_1)$ to approximate $F_1$, where $Z_1 = [x_1, y_r, \dot{y}_r, \rho_1, \dot{\rho}_1]^T \in \Omega_{Z_1} \subset R^5$, $F_1$ can be expressed as

$$F_1 = W_1^{*T}\psi_1(Z_1) + \tau_1(Z_1), |\tau_1(Z_1)| \le \bar{\tau}_1 \tag{26}$$

where $\tau_1(Z_1)$ and $\bar{\tau}_1$ are the approximation error and a given positive constant respectively. By Young's inequality, one has

$$\begin{aligned} \kappa_1^3 F_1 &\le \frac{1}{2c_1^2}\kappa_1^6\left\|W_1^{*T}\right\|^2\psi_1^T\psi_1 + \frac{c_1^2}{2} + \frac{3\kappa_1^4}{4} + \frac{\bar{\tau}_1^4}{4} \\ &\le \frac{b_m}{2c_1^2}\kappa_1^6\theta\psi_1^T\psi_1 + \frac{c_1^2}{2} + \frac{3\kappa_1^4}{4} + \frac{\bar{\tau}_1^4}{4} \end{aligned} \tag{27}$$

Based on (13)

$$\kappa_1^3\Gamma_1 g_1\alpha_1 \le -a_1 b_m\kappa_1^4 - \frac{3}{4}g_1\kappa_1^4 - \frac{b_m}{2c_1^2}\kappa_1^6\hat{\theta}\psi_1^T\psi_1 \tag{28}$$

Base on Lemma 1

$$\kappa_1^3\Gamma_1 g_1 z_2 \le \frac{3}{4}g_1\kappa_1^4 + \frac{1}{4}b_M(\Gamma_1 z_2)^4 \tag{29}$$

so we have

$$\begin{aligned} LV_1 &\le \frac{b_m}{2c_1^2}\kappa_1^6\theta\psi_1^T\psi_1 + \frac{c_1^2}{2} - a_1 b_m\kappa_1^4 + \frac{\bar{\tau}_1^4}{4} \\ &\quad + \frac{1}{4}b_M(\Gamma_1 z_2)^4 - \frac{b_m}{2c_1^2}\kappa_1^6\hat{\theta}\psi_1^T\psi_1 + \frac{3l_1^2}{4} \\ &\quad - \frac{b_m}{\gamma}\tilde{\theta}\dot{\hat{\theta}} \\ &\le -a_1 b_m\kappa_1^4 + \frac{1}{4}b_M(\Gamma_1 z_2)^4 + \varpi_1 \\ &\quad + \frac{b_m}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\kappa_1^6\psi_1^T\psi_1 - \dot{\hat{\theta}}) \end{aligned} \tag{30}$$

where $\varpi_1 = \frac{c_1^2}{2} + \frac{3l_1^2}{4} + \frac{\bar{\tau}_1^4}{4}$.

**Step** 2: According to the $z_2 = x_2 - \alpha_1$, we have

$$dz_2 = (g_2x_3 + f_2 - L\alpha_1)dt + (\varphi_2 - \frac{\partial\alpha_1}{\partial x_1}\varphi_1)^T d\omega \tag{31}$$

where

$$\begin{aligned} L\alpha_1 &= \frac{\partial\alpha_1}{\partial x_1}(g_1x_2 + f_1) + \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(i)}}y_d^{(i+1)} + \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial\rho_1^{(i)}}\rho_1^{(i+1)} \\ &\quad + \frac{\partial\alpha_1}{\partial\hat{\theta}}\dot{\hat{\theta}} + \frac{1}{2}\frac{\partial^2\alpha_1}{\partial x_1^2}\varphi_1^T\varphi_1 \end{aligned}$$

We can define Lyapunov function as follows:

$$V_2 = V_1 + \frac{1}{2}z_2^4 \tag{32}$$

By combining (30), (31) and $It\hat{o}$ formula, the time derivative of $V_2$ is

$$\begin{aligned} LV_2 &\le -a_1 b_m\kappa_1^4 + \varpi_1 + \frac{b_m}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\kappa_1^6\psi_1^T\psi_1 - \dot{\hat{\theta}}) \\ &\quad + z_2^3(g_2x_3 + f_2 - L\alpha_1 + \frac{1}{4}b_M\Gamma_1^4 z_2) \\ &\quad + \frac{3}{2}z_2^2(\varphi_2 - \frac{\partial\alpha_1}{\partial x_1}\varphi_1)^T(\varphi_2 - \frac{\partial\alpha_1}{\partial x_1}\varphi_1) \end{aligned} \tag{33}$$

Similar to $step$ 1, we can have

$$\begin{aligned} LV_2 &\le -a_1 b_m \kappa_1^4 + \varpi_1 + \frac{3}{4} l_2^2 + z_2^3 g_2 x_3 + z_2^3 F_2 \\ &+ \frac{b_m}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\kappa_1^6 \psi_1^T \psi_1 - \dot{\hat{\theta}}) - \frac{3}{4} z_2^4 \\ &- \frac{\partial \alpha_1}{\partial \hat{\theta}} z_2^3 \sum_{i=3}^{n} \frac{\gamma}{2c_i^2} z_i^6 \psi_i^T \psi_i \end{aligned} \tag{34}$$

where $F_2 = f_2 + \frac{3}{4l_2^2} z_2 \left\| \varphi_2 - \frac{\partial \alpha_1}{\partial x_1}\varphi_1 \right\|^4 + \frac{3}{4} z_2 + \frac{1}{4} b_M \Gamma_1^4 z_2 - \frac{\partial \alpha_1}{\partial x_1}(f_1 + g_1 x_2) - \sum_{k=0}^{1} \frac{\partial \alpha_1}{\partial \rho_1^{(k)}} \rho_1^{(k+1)} - \sum_{k=0}^{1} \frac{\partial \alpha_1}{\partial y_d^{(k)}} y_d^{(k+1)} - \frac{1}{2}\frac{\partial^2 \alpha_1}{\partial x_1^2}\varphi_1^T \varphi_1 - \frac{\partial \alpha_1}{\partial \hat{\theta}}(\frac{\gamma}{2c_1^2}\kappa_1^6 \psi_1^T \psi_1 + \frac{\gamma}{2c_2^2} z_2^6 \psi_2^T \psi_2) + \frac{\partial \alpha_1}{\partial \hat{\theta}}\delta\hat{\theta}$

Apply the same estimation method as (26). We can get the following inequality

$$z_2^3 F_2 \le \frac{b_m}{2c_2^2} z_2^6 \theta \psi_2^T \psi_2 + \frac{c_2^2}{2} + \frac{3z_2^4}{4} + \frac{\bar{\tau}_2^4}{4} \tag{35}$$

By substituting (35) and $x_3 = z_3 + \alpha_2$ into (34), we can have

$$\begin{aligned} LV_2 &\le -a_1 b_m \kappa_1^4 + \varpi_1 - \frac{\partial \alpha_1}{\partial \hat{\theta}} z_2^3 \sum_{i=3}^{n} \frac{\gamma}{2c_i^2} z_i^6 \psi_i^T \psi_i \\ &+ \frac{b_m}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\kappa_1^6 \psi_1^T \psi_1 - \dot{\hat{\theta}}) + \frac{b_m}{2c_2^2} z_2^6 \theta \psi_2^T \psi_2 \\ &+ z_2^3 g_2 z_3 + z_2^3 g_2 \alpha_2 + \frac{1}{2} c_2^2 + \frac{3}{4} l_2^2 + \frac{1}{4}\bar{\tau}_2^4 \end{aligned} \tag{36}$$

Bring $i = 2$ into virtual control law, the following inequality can be obtained:

$$z_2^3 g_2 \alpha_2 \le -a_2 b_m z_2^4 - \frac{3}{4} g_2 z_2^4 - \frac{b_m}{2c_2^2} z_2^6 \hat{\theta} \psi_2^T \psi_2 \tag{37}$$

Base on Lemma 1, we can have

$$z_2^3 g_2 z_3 \le \frac{3}{4} g_2 z_2^4 + \frac{1}{4} b_M z_3^4 \tag{38}$$

By substituting (37) and (38) into (36), we obtain $LV_2$ as

$$\begin{aligned} LV_2 &\le -a_1 b_m \kappa_1^4 - a_2 b_m z_2^4 + \frac{1}{4} b_M z_3^4 + \sum_{j=1}^{2} \varpi_j \\ &+ \frac{b_m}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\kappa_1^6 \psi_1^T \psi_1 + \frac{\gamma}{2c_2^2} z_2^6 \psi_2^T \psi_2 - \dot{\hat{\theta}}) \\ &- \frac{\partial \alpha_1}{\partial \hat{\theta}} z_2^3 \sum_{i=3}^{n} \frac{\gamma}{2c_i^2} z_i^6 \psi_i^T \psi_i \end{aligned} \tag{39}$$

where $\varpi_j = \frac{1}{2} c_j^2 + \frac{3}{4} l_j^2 + \frac{1}{4}\bar{\tau}_j^4$.

**Step** $i$ $(i = 3, 4, \cdots, n-1)$: By using $z_i = x_i - \alpha_{i-1}$, we have

$$\begin{aligned} dz_i &= (g_i x_{i+1} + f_i - L\alpha_{i-1})dt \\ &+ (\varphi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\varphi_j)^T d\omega \end{aligned} \tag{40}$$

where

$$\begin{aligned} L\alpha_{i-1} &= \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(g_j x_{j+1} + f_j) + \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)} \\ &+ \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \rho_1^{(j)}} \rho_1^{(j+1)} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} \\ &+ \frac{1}{2} \sum_{p=1,q=1}^{i-1} \frac{\partial^2 \alpha_{i-1}}{\partial x_p \partial x_p}\varphi_p^T \varphi_q \end{aligned}$$

We can define Lyapunov function as follows:

$$V_i = V_{i-1} + \frac{1}{2} z_i^4 \tag{41}$$

Base on $Step$ 2, by iteration, we can obtain

$$\begin{aligned} LV_{i-1} &\le -a_1 b_m \kappa_1^4 - \sum_{j=2}^{i-1} a_j b_m z_j^4 + \sum_{j=1}^{i-1} \varpi_j + \frac{1}{4} b_M {z_i}^4 \\ &+ \frac{b_m}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\kappa_1^6 \psi_1^T \psi_1 + \sum_{j=2}^{i-1} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j - \dot{\hat{\theta}}) \\ &- \sum_{m=1}^{i-2} \frac{\partial \alpha_m}{\partial \hat{\theta}} z_{m+1}^3 \sum_{j=i}^{n} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j \end{aligned}$$

Substitute $LV_{i-1}$ into (41) and use the $It\hat{o}$ formula, we have

$$\begin{aligned} LV_i &\le -a_1 b_m \kappa_1^4 - \sum_{j=2}^{i-1} a_j b_m z_j^4 + \sum_{j=1}^{i-1} \varpi_j \\ &+ \frac{b_m}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\kappa_1^6 \psi_1^T \psi_1 + \sum_{j=2}^{i-1} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j - \dot{\hat{\theta}}) \\ &- \sum_{m=1}^{i-2} \frac{\partial \alpha_m}{\partial \hat{\theta}} z_{m+1}^3 \sum_{j=i}^{n} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j \\ &+ z_i^3 (g_i x_{i+1} + f_i - L\alpha_{i-1} + \frac{1}{4} b_M z_i) \\ &+ \frac{3}{2} z_i^2 (\varphi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\varphi_j)^T (\varphi_i - \\ &\sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\varphi_j) \end{aligned} \tag{42}$$

By repeating the previous two steps, we can have

$$\begin{aligned} LV_i &\le -a_1 b_m \kappa_1^4 - \sum_{j=2}^{i-1} a_j b_m z_j^4 + \sum_{j=1}^{i-1} \varpi_j \\ &- \sum_{m=1}^{i-1} \frac{\partial \alpha_m}{\partial \hat{\theta}} z_{m+1}^3 \sum_{j=i+1}^{n} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j \\ &+ \frac{b_m}{\gamma}\tilde{\theta}(\frac{\gamma}{2c_1^2}\kappa_1^6 \psi_1^T \psi_1 + \sum_{j=2}^{i-1} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j - \dot{\hat{\theta}}) \\ &+ z_i^3 (g_i x_{i+1} + F_i) - \frac{3}{4} z_i^4 + \frac{3l_i^2}{4} \end{aligned} \tag{43}$$

where $F_i = f_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(g_j x_{j+1} + f_j) - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)} - \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \rho_1^{(j)}} \rho_1^{(j+1)} - \frac{1}{2} \sum_{p=1,q=1}^{i-1} \frac{\partial^2 \alpha_{i-1}}{\partial x_p \partial x_q}\varphi_p^T \varphi_q + \frac{1}{4} b_M z_i + \frac{3}{4} z_i + \frac{3}{4l_i^2} z_i \left\| \varphi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\varphi_j \right\|^4 - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}(\frac{\gamma}{2c_1^2}\kappa_1^6 \psi_1^T \psi_1 + \sum_{j=2}^{i} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j - \delta\hat{\theta}) - \frac{\gamma z_i^3 \psi_i^T \psi_i}{2c_i^2} \sum_{j=1}^{i-2} \frac{\partial \alpha_j}{\partial \hat{\theta}} z_{j+1}^3$

Apply the same estimation method as (26). We can get the following inequality

$$z_i^3 F_i \le \frac{b_m}{2c_i^2} z_i^6 \theta \psi_i^T \psi_i + \frac{c_i^2}{2} + \frac{3z_i^4}{4} + \frac{\bar{\tau}_i^4}{4} \tag{44}$$

By substituting (44) and $x_i = z_i + \alpha_{i-1}$ into (43), we can have

$$\begin{aligned} LV_i \ &\leq -a_1 b_m \kappa_1^4 - \sum_{j=2}^{i-1} a_j b_m z_j^4 + \sum_{j=1}^{i-1} \varpi_j \\ &- \sum_{m=1}^{i-1} \frac{\partial \alpha_m}{\partial \hat{\theta}} z_{m+1}^3 \sum_{j=i+1}^{n} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j \\ &+ \frac{b_m}{\gamma} \tilde{\theta} (\frac{\gamma}{2c_1^2} \kappa_1^6 \psi_1^T \psi_1 + \sum_{j=2}^{i-1} \frac{\gamma}{2c_j^2} z_j^6 \theta \psi_j^T \psi_j \\ &- \dot{\hat{\theta}}) + \frac{b_m}{2c_i^2} z_i^6 \theta \psi_i^T \psi_i + z_i^3 g_i z_{i+1} + z_i^3 g_i \alpha_i \\ &+ \frac{1}{2} c_i^2 + \frac{3}{4} l_i^2 + \frac{1}{4} \bar{\tau}_i^4 \end{aligned} \tag{45}$$

Using the virtual control law, the following inequality can be obtained:

$$z_i^3 g_i \alpha_i \leq -a_i b_m z_i^4 - \frac{3}{4} g_i z_i^4 - \frac{b_m}{2c_i^2} z_i^6 \hat{\theta} \psi_i^T \psi_i \tag{46}$$

Base on Lemma 1

$$z_i^3 g_i z_{i+1} \leq \frac{3}{4} g_i z_i^4 + \frac{1}{4} b_M z_{i+1}^4 \tag{47}$$

By substituting (46) and (47) into (45), we can have

$$\begin{aligned} LV_i \ &\leq -a_1 b_m \kappa_1^4 - \sum_{j=2}^{i} a_j b_m z_j^4 + \frac{1}{4} b_M z_{i+1}^4 \\ &- \sum_{m=1}^{i-1} \frac{\partial \alpha_m}{\partial \hat{\theta}} z_{m+1}^3 \sum_{j=i+1}^{n} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j \\ &+ \frac{b_m}{\gamma} \tilde{\theta} (\frac{\gamma}{2c_1^2} \kappa_1^6 \psi_1^T \psi_1 + \sum_{j=2}^{i} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j \\ &- \dot{\hat{\theta}}) + \sum_{j=1}^{i} \varpi_j \end{aligned} \tag{48}$$

where $\varpi_j = \frac{1}{4}\bar{\tau}_j^4 + \frac{1}{2}c_j^2 + \frac{3}{4}l_j^2$.

**Step** $n$ : According to the $z_n = x_n - \alpha_{n-1}$, we have

$$\begin{aligned} dz_n \ &= (g_n G_\sigma v + g_n h + f_n - L\alpha_{n-1}) dt \\ &+ (\varphi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \varphi_j)^T d\omega \end{aligned} \tag{49}$$

where

$$\begin{aligned} L\alpha_{n-1} \ &= \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (g_j x_{j+1} + f_j) + \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(j)}} y_d^{(j+1)} \\ &+ \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \rho_1^{(j)}} \rho_1^{(j+1)} + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \\ &+ \frac{1}{2} \sum_{p=1,q=1}^{n-1} \frac{\partial^2 \alpha_{n-1}}{\partial x_p \partial x_q} \varphi_p^T \varphi_q \end{aligned}$$

We can define Lyapunov function as follows:

$$V_n = V_{n-1} + \frac{1}{2} z_n^4 \tag{50}$$

By substituting $i = n-1$ into (48) and using $It\hat{o}$ formula for (49), we have

$$\begin{aligned} LV_n \ &\leq -a_1 b_m \kappa_1^4 - \sum_{j=2}^{n-1} a_j b_m z_j^4 + \sum_{j=1}^{n-1} \varpi_j + \\ &\frac{b_m}{\gamma} \tilde{\theta} (\frac{\gamma}{2c_1^2} \kappa_1^6 \psi_1^T \psi_1 + \sum_{j=2}^{n-1} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j - \dot{\hat{\theta}}) \\ &+ z_n^3 (g_n G_\sigma v + g_n h + f_n - L\alpha_{i-1} \\ &+ \frac{1}{4} b_M z_n - \frac{\gamma}{2c_n^2} z_n^3 \psi_n^T \psi_n \sum_{m=1}^{n-2} \frac{\partial \alpha_m}{\partial \hat{\theta}} z_{m+1}^3) \\ &+ \frac{3}{2} z_n^2 (\varphi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \varphi_j)^T (\varphi_n \\ &- \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \varphi_j) \end{aligned} \tag{51}$$

By repeating the previous steps, we can have

$$\begin{aligned} LV_n \ &\leq -a_1 b_m \kappa_1^4 - \sum_{j=2}^{n-1} a_j b_m z_j^4 + \sum_{j=1}^{n-1} \varpi_j \\ &+ \frac{b_m}{\gamma} \tilde{\theta} (\frac{\gamma}{2c_1^2} \kappa_1^6 \psi_1^T \psi_1 + \sum_{j=2}^{n-1} \frac{\gamma}{2c_j^2} z_j^6 \psi_j^T \psi_j \\ &- \dot{\hat{\theta}}) + z_n^3 (g_n G_\sigma v + g_n h + F_n) + \frac{3l_n^2}{4} \\ &- \frac{3}{4} z_n^4 \end{aligned} \tag{52}$$

where $F_n = f_n + \frac{1}{4} b_M z_n + \frac{3}{4} z_n + \frac{3}{4l_n^2} z_n \left\| \varphi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \varphi_j \right\|^4 - \frac{\gamma}{2c_n^2} z_n^3 \psi_n^T \psi_n \sum_{m=1}^{n-2} \frac{\partial \alpha_m}{\partial \hat{\theta}} z_{m+1}^3 - L\alpha_{n-1}$

Apply the same estimation method as (26). The following inequality can be obtainedp

$$z_n^3 F_n \leq \frac{b_m}{2c_n^2} z_n^6 \theta \psi_n^T \psi_n + \frac{c_n^2}{2} + \frac{3z_n^4}{4} + \frac{\bar{\tau}_n^4}{4} \tag{53}$$

By substituting (53) into (52), we can have

$$\begin{aligned} LV_n \ &\leq -a_1 b_m \kappa_1^4 - \sum_{j=2}^{n-1} a_j b_m z_j^4 + \sum_{j=1}^{n-1} \varpi_j \\ &+ \frac{b_m}{\gamma} \tilde{\theta} (\frac{\gamma}{2c_1^2} \kappa_1^6 \psi_1^T \psi_1 + \sum_{j=2}^{n-1} \frac{b_m}{2c_j^2} z_j^6 \theta \psi_j^T \psi_j \\ &- \dot{\hat{\theta}}) + \frac{b_m}{2c_n^2} z_n^6 \theta \psi_n^T \psi_n + z_n^3 (g_n G_\sigma v + g_n h) \\ &+ \frac{1}{2} c_n^2 + \frac{3}{4} l_n^2 + \frac{1}{4} \bar{\tau}_n^4 \end{aligned} \tag{54}$$

Now by using the virtual control law in (15), the following inequality can be obtained:

$$z_n^3 g_n G_\sigma v \leq -a_n b_m z_n^4 - \frac{3}{4\Theta^2} g_n G_\sigma z_n^4 - \frac{b_m}{2c_n^2} z_n^6 \hat{\theta} \psi_n^T \psi_n \tag{55}$$

Base on Lemma 1

$$z_n^3 g_n h \leq \frac{3}{4\Theta^2} g_n G_\sigma z_n^4 + \frac{b_M}{4b_m} \Theta^2 B^4 \tag{56}$$

where $\Theta$ is a positive constant.

By substituting (55) and (56) into (54), we can have

$$\begin{aligned} LV_n &\leq -a_1 b_m \kappa_1^4 - \sum_{j=2}^{n} a_j b_m z_j^4 + \sum_{j=1}^{n} \varpi_j \\ &+ \frac{b_m}{\gamma}\tilde{\theta}(\frac{\gamma}{2C_1^2}\kappa_1^6 \psi_1^T \psi_1 + \sum_{j=2}^{n} \frac{\gamma}{2C_j^2} z_j^6 \psi_j^T \psi_j \\ &- \dot{\hat{\theta}}) + \frac{b_M}{4b_m}\Theta^2 B^4 \end{aligned} \tag{57}$$

where $\varpi_j = \frac{1}{4}\bar{\tau}_j^4 + \frac{1}{2}c_j^2 + \frac{3}{4}l_j^2$

Substituting adaptive laws (17) into (57)

$$\begin{aligned} LV_n &\leq -a_1 b_m \kappa_1^4 - \sum_{j=2}^{n} a_j b_m z_j^4 + \sum_{j=1}^{n} \varpi_j \\ &+ \frac{b_m}{\gamma}\delta\tilde{\theta}\hat{\theta} + \frac{b_M}{4b_m}\Theta^2 B^4 \end{aligned} \tag{58}$$

By using the perfect square formula

$$\tilde{\theta}\hat{\theta} \leq \frac{\theta^2}{2} - \frac{\tilde{\theta}^2}{2} \tag{59}$$

Substituting (59) into (58), we obtain $LV_n$ as

$$\begin{aligned} LV_n &\leq -a_1 b_m \kappa_1^4 - \sum_{j=2}^{n} a_j b_m z_j^4 + \frac{b_M}{4b_m}\Theta^2 B^4 \\ &+ \frac{b_m}{\gamma}\delta(\frac{\theta^2}{2} - \frac{\tilde{\theta}^2}{2}) + \sum_{j=1}^{n} \varpi_j \end{aligned} \tag{60}$$

We define $\Psi_0$ as $\Psi_0 = \frac{b_m \delta \theta^2}{2\gamma} + \frac{b_M \Theta^2 B^4}{4b_m} + \sum_{j=1}^{n} \varpi_j$. Define $\Upsilon_0$ as $\Upsilon_0 = min\{4b_m a_i, \delta, i = 1, 2, ..., n\}$, so (60) can be rewritten as

$$LV_n \leq \Upsilon_0 V + \Psi_0 \tag{61}$$

### B. Stability analysis

In this section, we will use Lyapunov theory to analyze the stability of strict feedback stochastic nonlinear systems with the above controllers.

**Theorem 1.** *For closed-loop systems of (5) with unknown input saturation (6), and its controller (15) and adaptive law (17), if Assumptions 1 and 2 are satisfied, and support that $F_i(x_i)$ defined in a compact set $\Omega_0$ can be approached by fuzzy logic system with a desired level of accuracy $\bar{\tau}_i$ for i = 1, 2, ... , n. Then the following properties hold:*

*(1) All the signals of the closed-loop system are bounded in probability. Specially, the constraint function $\kappa_1$, the error signal $z_i, i = 2, 3, ..., n$ and $\hat{\theta}$ ultimately converge to a compact set $\Omega_z$ given by*

$$\begin{aligned} \Omega_z = \{\kappa_1, z_i, \hat{\theta} | E(\kappa_1^4) \leq \frac{4\Psi_0}{\Upsilon_0}, E(z_i^4) \leq \frac{4\Psi_0}{\Upsilon_0}, \\ E(\tilde{\theta}^2) \leq \frac{2\gamma\Psi_0}{b_m \Upsilon_0}, i = 2, 3, ..., n\} \end{aligned} \tag{62}$$

*(2) The tracking error $z_1$ is bounded by the boundary function $\rho_1$, and the transient and steady-state performance will be guaranteed with desired trajectory $y_d(t)$.*

**Proof 1.** *The inequality (61) satisfies*

$$E(V(t)) \leq (V(0) - \frac{\Psi_0}{\Upsilon_0})e^{-\Upsilon_0 t} + \frac{\Psi_0}{\Upsilon_0} \tag{63}$$

*From the definition of $V(t)$ and (63), it implies that $V(t)$, $\kappa_1, Z_i, \hat{\theta}$ and $x_i$ are bounded in probability. In addition, as $t \to \infty$, we have $e^{-\kappa_0 t} \to 0$, we can obtain*

$$E(V(t)) \leq \frac{\Psi}{\Upsilon} \tag{64}$$

*So, the constraint function $\kappa_1$, the error signal $z_i$, $i = 2, 3, ..., n$ and $\hat{\theta}$ ultimately converge to a compact set $\Omega_z$ given by (62), which means that all the signals of the system are uniformly ultimately bounded. In addition, in order to prove $z_1$ is bounded by the prescribed performance function $\rho_1$, recalling $V_1$ and (64) give*

$$E(\frac{z_1^8}{64(\rho_1^2 - z_1^2)}) \leq \frac{\Psi_0}{\Upsilon_0} \tag{65}$$

*If let $\sqrt{\Psi_0/\Upsilon_0} \leq 1/8$, then we have $E(z_1^4/(\rho_1^2 - z_1^2)^2) \leq 1$ which implies*

$$E(2z_1^2) \leq \rho_1 \tag{66}$$

*Further, we have*

$$E(|z_1|) \leq |\rho_1| \tag{67}$$

*which implies that the tracking error $z_1$ is bounded by the boundary function $\rho_1$ all the time in probability.*

**Remark 3.** *Based on the above proof, the designed adaptive controllers (13)-(15) with parameter update law (17) not only ensure that the tracking error do not violate the preset performance function, but also ensure that all signals of the closed-loop system are bounded.*

## IV. Simulation

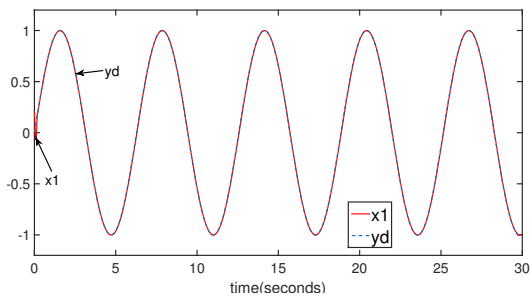


Fig. 1: System output $y(t)$ and refererence signal $y_d(t)$

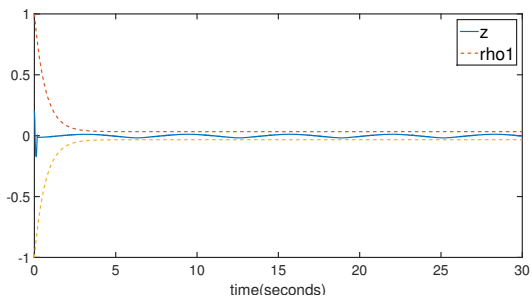


Fig. 2: Tracking error $z_1(t)$ under $\rho_1(t)$

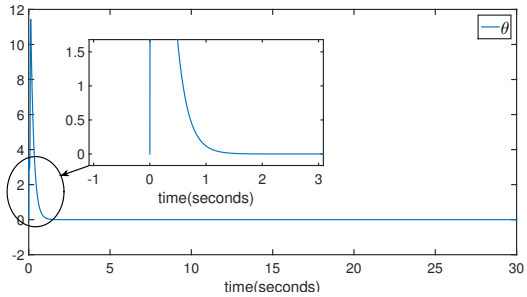


Fig. 3: Adaptive law $\dot{\hat{\theta}}(t)$

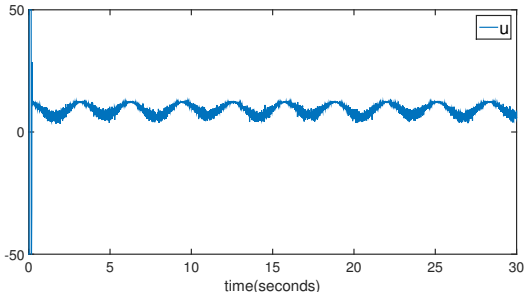


Fig. 4: Actual control signals u(t) with saturation constraints

In order to prove the effectiveness of above results, consider a strict-feedback stochastic nonlinear system with input saturation as follows:

$$\begin{cases} \dot{x}_1 = x_2 + 0.1sin(x_1^2)d\omega, \\ \dot{x}_2 = -\dfrac{mgl}{J}cos(x_1) + \dfrac{1}{J}u + 0.1x_2cos(x_1)\omega, \\ y = x_1 \end{cases} \tag{68}$$

where $x_1$, $x_2$ and $l$ denote the angular position, relative angular velocity and the length of manipulator respectively, $m$ is the load mass, $g$ denotes the gravity, and $J = \frac{4}{3}ml^2$ represents the inertia coefficient. $y$ is the system output, $u$ is the system input defined in (6), and we choose $u_{max} = 50$ and $u_{min} = -50$ as the limits of symmetric input saturation.

We select -7;-5;-3;-1;0;1;3;5;7 as nine fuzzy sets. The parameters of the boundary function are set as follows: the maximum allowable error is $\rho_0 = 0.8$, the maximunm allowable steady-state error is $\rho_\infty = 0.06$, and the minimum convergence speed is $\varrho = 1$. In this way, we can get the boundary function $\rho_1 = (1 - 0.033)e^{-1.5} + 0.033$. The coordinate transformation $z_1 = x_1 - \alpha_0$ with $\alpha_0 = y_d$ and error transform function $\kappa = ln((\rho_1 + z_1)/(\rho_1 - z_1))$.

According to (13) and (15),we can define the virtual control signal and the actual control input as follows:

$$\begin{aligned} \alpha_1 &= -\frac{\kappa_1}{\Gamma_1}(a_1 + \frac{3}{4} + \frac{\kappa_1^2}{2c_1^2}\hat{\theta}\psi_1^T\psi_1) \\ v &= -(a_2 + \frac{3}{4\Theta^2})z_2 - \frac{1}{2c_2^2}z_2^3\hat{\theta}\psi_2^T\psi_2 \end{aligned} \tag{69}$$

And the adaptive law is

$$\dot{\hat{\theta}} = \frac{\gamma}{2c_1^2}\kappa_1^6\psi_1^T\psi_1 + \frac{\gamma}{2c_2^2}z_2^6\psi_2^T\psi_2 - \delta\hat{\theta} \tag{70}$$

where $a_1, a_2, c_1, c_2, \Theta, \gamma$ $and$ $\delta$ appropriately selected positive parameters.

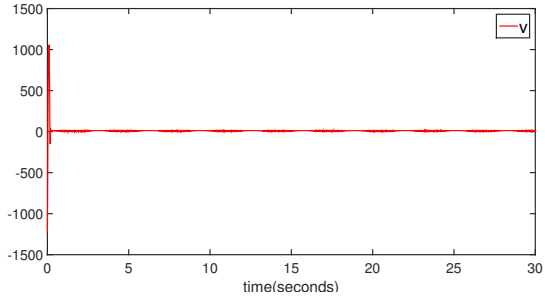


Fig. 5: Input signal v(t)

The model parameters are chosen as $m = 5, g = 9.8, l = 0.25$. The control parameters are selected as $a_1 = 80, a_2 = 80, c_1 = 100, c_2 = 80, \Theta = 1, \gamma = 1$ $and$ $\delta = 5.25$. The initial conditions are $[x_1(0), x_2(0)]^T = [0.2, -0.5]^T$ $and$ $\theta(0) = 0$. The objective function is $y_d(t) = 0.5sin(t)$. The tracking performance is shown in Fig.1. The tracking error and boundary function are shown in Fig.2. The adaptive law $\dot{\hat{\theta}}$ is shown in Fig.3. The actual control signals u(t) with saturation constraints are shown in Fig.4 and Fig.5 respectively. The state variable $x_2$ is shown in Fig.6.

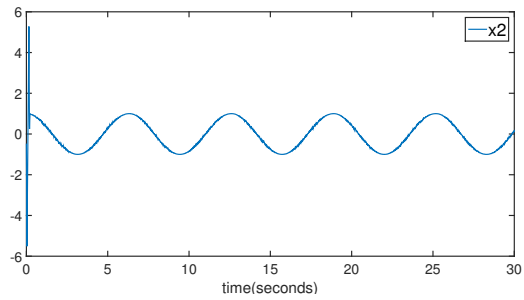


Fig. 6: State variables $x_2(t)$

## V. Conclusion

In this paper, a controller for strict-feedback stochastic nonlinear systems with input constraints has been designed. A modified prescribed performance constraint method has been proposed to achieve that the system error is constrained within the set boundary. The controller designed can keep all signals in the closed-loop system with bounded in probability, and the tracking error can be converged to any small neighborhood near the origin in the sense of quartic mean.

## References

[1] S.-C. Tong, Y.-M. Li, G. Feng, and T.-S. Li, "Observer-based adaptive fuzzy backstepping dynamic surface control for a class of mimo nonlinear systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 41, no. 4, pp. 1124–1135, 2011.

[2] S. Tong, Y. Li, Y. Li, and Y. Liu, "Observer-based adaptive fuzzy backstepping control for a class of stochastic nonlinear strict-feedback systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 41, no. 6, pp. 1693–1704, 2011.

[3] H. Li, L. Wang, H. Du, and A. Boulkroune, "Adaptive fuzzy backstepping tracking control for strict-feedback systems with input delay," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 3, pp. 642–652, 2016.

[4] Y. Li, S. Tong, and T. Li, "Adaptive fuzzy backstepping control design for a class of pure-feedback switched nonlinear systems," *Nonlinear Analysis: Hybrid Systems*, vol. 16, pp. 72–80, 2015.

[5] S. Tong, Y. Li, and P. Shi, "Observer-based adaptive fuzzy backstepping output feedback control of uncertain mimo pure-feedback nonlinear systems," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 4, pp. 771–785, 2012.

[6] H. Wang, P. X. Liu, S. Li, and D. Wang, "Adaptive neural output-feedback control for a class of nonlower triangular nonlinear systems with unmodeled dynamics," *IEEE transactions on neural networks and learning systems*, vol. 29, no. 8, pp. 3658–3668, 2017.

[7] C. P. Chen, Y.-J. Liu, and G.-X. Wen, "Fuzzy neural network-based adaptive control for a class of uncertain nonlinear stochastic systems," *IEEE Transactions on Cybernetics*, vol. 44, no. 5, pp. 583–593, 2013.

[8] J. Dong and G.-H. Yang, "Reliable state feedback control of t–s fuzzy systems with sensor faults," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 2, pp. 421–433, 2014.

[9] H. Wang, W. Liu, J. Qiu, and P. X. Liu, "Adaptive fuzzy decentralized control for a class of strong interconnected nonlinear systems with unmodeled dynamics," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 2, pp. 836–846, 2017.

[10] Z. Zhang, S. Xu, and B. Zhang, "Asymptotic tracking control of uncertain nonlinear systems with unknown actuator nonlinearity," *IEEE Transactions on Automatic Control*, vol. 59, no. 5, pp. 1336–1341, 2013.

[11] L. Wang and L. Wang, "Global exponential stabilization for some impulsive ts fuzzy systems with uncertainties," *IAENG International Journal of Applied Mathematics*, vol. 47, no. 4, pp. 425–430, 2017.

[12] G. Wang and Q. Zhang, "Dynamical behavior of first-order nonlinear fuzzy difference equation," *IAENG International Journal of Computer Science*, vol. 45, no. 4, pp. 552–559, 2018.

[13] Y.-c. Wang, H. Hang, and Y.-z. Wang, "Fuzzy adaptive control of stochastic nonlinear systems with unknown virtual control gain function," *Acta Automatica Sinica*, vol. 32, no. 2, p. 170, 2006.

[14] C. Chen, Z. Liu, Y. Zhang, C. L. P. Chen, and S. Xie, "Asymptotic fuzzy tracking control for a class of stochastic strict-feedback systems," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 3, pp. 556–568, 2017.

[15] S. Tong, T. Wang, and Y. Li, "Fuzzy adaptive actuator failure compensation control of uncertain stochastic nonlinear systems with unmodeled dynamics," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 3, pp. 563–574, 2013.

[16] S. Tong, Y. Xu, and Y. Li, "Adaptive fuzzy decentralised control for stochastic nonlinear large-scale systems in pure-feedback form," *International Journal of Systems Science*, vol. 46, no. 8, pp. 1510–1524, 2015.

[17] Yixin Diao and K. M. Passino, "Stable fault-tolerant adaptive fuzzy/neural control for a turbine engine," *IEEE Transactions on Control Systems Technology*, vol. 9, no. 3, pp. 494–509, 2001.

[18] X. Liu, H. Wang, C. Gao, and M. Chen, "Adaptive fuzzy funnel control for a class of strict feedback nonlinear systems," *Neurocomputing*, vol. 241, pp. 71–80, 2017.

[19] C. P. Bechlioulis and G. A. Rovithakis, "A low-complexity global approximation-free control scheme with prescribed performance for unknown pure feedback systems," *Automatica*, vol. 50, no. 4, pp. 1217–1226, 2014.

[20] A. Theodorakopoulos and G. A. Rovithakis, "Guaranteeing preselected tracking quality for uncertain strict-feedback systems with deadzone input nonlinearity and disturbances via low-complexity control," *Automatica*, vol. 54, pp. 135–145, 2015.

[21] W. Wang, D. Wang, Z. Peng, and T. Li, "Prescribed performance consensus of uncertain nonlinear strict-feedback systems with unknown control directions," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 46, no. 9, pp. 1279–1286, 2015.

[22] W. Xiang and H. Liu, "Fuzzy adaptive prescribed performance tracking control for uncertain nonlinear systems with unknown control gain signs," *IEEE Access*, vol. 7, pp. 149 867–149 877, 2019.

[23] L. Liu, Z. Wang, Z. Huang, and H. Zhang, "Adaptive predefined performance control for mimo systems with unknown direction via generalized fuzzy hyperbolic model," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 3, pp. 527–542, 2016.

[24] X. Jin, J. Qin, Y. Shi, and W. X. Zheng, "Auxiliary fault tolerant control with actuator amplitude saturation and limited rate," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 10, pp. 1816–1825, 2017.

[25] H. Wang, B. Chen, X. Liu, K. Liu, and C. Lin, "Robust adaptive fuzzy tracking control for pure-feedback stochastic nonlinear systems with input constraints," *IEEE Transactions on Cybernetics*, vol. 43, no. 6, pp. 2093–2104, 2013.

[26] K. Yan, M. Chen, Q. Wu, Y. Wang, and R. Zhu, "Prescribed performance fault tolerant control for uncertain nonlinear systems with input saturation," *International Journal of Systems Science*, vol. 51, no. 2, pp. 258–274, 2020.

[27] L.-B. Wu and G.-H. Yang, "Adaptive output neural network control for a class of stochastic nonlinear systems with dead-zone nonlinearities," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 3, pp. 726–739, 2015.

[28] X. Tian, Z. Yang, and Z. Yang, "Adaptive stabilization of fractional-order energy supply-demand system with dead-zone nonlinear inputs," *IAENG International Journal of Applied Mathematics*, vol. 49, no. 4, pp. 500–504, 2019.

[29] S. Yin, P. Shi, and H. Yang, "Adaptive fuzzy control of strict-feedback nonlinear time-delay systems with unmodeled dynamics," *IEEE Transactions on Cybernetics*, vol. 46, no. 8, pp. 1926–1938, 2016.