A Mathematical Model of Salinity Control in a River with an Effect of Internal Waves using Two Explicit Finite Difference Methods

Pornpon Othata, and Nopparat Pochai

***Abstract*—Salinity is related to the quantity of salt in rivers, and that salt can take several different forms. Salinity is determined by water evaporation to dryness, and form residual measuring. The total salt content in water can in influence the taste of water. Drinking water at levels of salinity greater than around 1.0 g/L is drastically and rapidly unpalatable. In this research, two models, the internal wave hydrodynamic model and the salinity dispersion model, are proposed. The internal waves hydrodynamic model provides salinity internal wave and salinity flow velocity. The salinity dispersion model provides the salinity level. A modified salinity control model is also used in a river with a barrage dam. The suggested model provides salinity control releasing fresh water from a barrage dam where the salinity level is not higher than the standard and does not waste too much fresh water. An explicit finite difference method which is unconditionally stable is used for approximating the degree of salinity from the proposed model under many conditions.**

***Index Terms*—Salinity Control, Internal Waves, Explicit Finite Difference Methods, Hydrodynamic model, Salinity dispersion model**

I. Introduction

Water creation implies the evacuation of surface water or crude water from characteristic water sources, for example, streams, channels, stores, and the ocean, in the generation procedure, meeting the quality and amount prerequisites for tap water and unadulterated water for buyer use or use in horticulture and industry. Each sort of creation water can utilize diverse generation advances.

Surface water or crude water is available in the water which will be utilized for utilization, horticulture, and certain enterprises that don't require top notch water. There are numerous elements that influence the nature of the water delivered, for example, saltiness of the water. It is a significant factor under way, on the grounds that it can't be dealt with customary ways. Thus, when carrying water into the treatment procedure, it is important to have a saltiness standard.

Thailand's Waterworks Authority has nine water quality control stations all over a river. Each station is within reaching distance of the estuary as seen in Table 1.

At present, the pumping station has an excess of salinity, which affects the quality of tap water in Bangkok. This impacts the quality of water produced.

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TABLE I
DISTANCE FROM THE ESTUARY OF EACH OBSERVATION STATIONS

Stations	Distance from the estuary(km)
S_1	12
S_2	27
S_3	35
S_4	50
S_5	64
S_6	91
S_7	96
S_8	108

In [1], the finite difference method was utilized to explain water contamination models. In [17] and [18], the three dimension advection diffusion model was introduced. In [13], the implicit finite difference method was to solve the water pollution model. In [5], [15] and [16], the fluid flow model was proposed. In [14], the numerical method was used to solve the fluid flow problem. In [2], a hydrodynamic model with constant coefficients in a closed uniform reservoir was solved by using the finite difference method. In [3], an theoretical solution for a hydrodynamic model in a closed uniform reservoir was suggested. In [4], The Lax-Wendroff scheme was also proposed for approximating salinity elevation and salinity flow rate. Research reports on the effect of salt drinking water on standards have been undertaken, such as [6]. The water was excessively salty, up to levels that influence the body. In this way, the expansion of salinity water in Chao Phraya River has been introduced in [7]. In [8], the one-dimensional salinity water model was proposed

$$A(x)\frac{\partial S}{\partial t} + Q(x,t)\frac{\partial S}{\partial x} = \frac{\partial}{\partial x}\left[A(x)D_x\frac{\partial S}{\partial x}\right], \qquad (1)$$

where $A(x)$ is the river cross-sectional area (m^2), $Q(x,t)$ is flow rates (m^3/s), The coefficient of water diffusion is D_x (m^2/s), S is water salinity level (ppt), x is the length of river (m) and t is times (s)

To solve this problem, a one-dimensional salinity management model with non-uniform internal waves in a river is proposed. A one-dimensional hydrodynamic model of the internal saltiness flow in a river is introduced. A saltiness control in a river with a barrage dam model is also proposed. A modified Lax-diffusive method is used to approximate the solution of the internal wave hydrodynamic model. the Sualyev scheme is used to estimate the saltiness level. The suggested evaluation method for water supply forms can be used in reasonable circumstances.

II. Salinity measurement model in a River with the effect of internal wave

2.1 One-dimensional salinity internal wave hydrodynamic model

Under the assumptions, by combining the Navier-Stokes equations over the flow direction as a hydrostatic pressure distribution and a heavy downward slope, one-dimensional shallow water equations are obtained. The hydrodynamic flows are swift and can be regarded as shallow salinity flows driven by advection. Hence the concept of eddy viscosity may be overlooked. The system of partial differential equations of governing equation and vector form equations [10] can be written as

$$\partial_x \begin{pmatrix} h \\ hu \end{pmatrix} + \partial_t \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix} = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix}, \quad (2)$$

where x is the horizontal difference across a river(m), t is time(s), $h(x,t)$ is the salinity wave lifting above the bottom(m/s), $z(x)$ is the feature that characterizes the underneath geography(m), and $u(x,t)$ is component of velocity(m/s), for all $(x,t) \in [0,L] \times [0,T]$. We can defined the initial conditions by

$$u\,(x,0) \;=\; u_1(x) \;\; for\ all\ \ x \in [0,L] \quad (3)$$

and

$$h\,(x,0) \;=\; h_1(x) \;\; for\ all\ \ x \in [0,L] \quad (4)$$

The boundary conditions shall also be determined by

$$\frac{\partial u(0,t)}{\partial x} = f_1(t), t > 0, \quad (5)$$

$$\frac{\partial u(L,t)}{\partial x} = f_2(t), t > 0, \quad (6)$$

$$h\,(0,t) = g_1(t), t > 0, \quad (7)$$

$$\frac{\partial h(L,t)}{\partial x} = g_2(t), t > 0. \quad (8)$$

So as to be predictable with the physical marvel of a hydrodynamic from the left to one side at $t = 0$.

2.2 One-dimensional salinity water measurement model

The fluid one-dimensional advection-dispersion equation is the governing equation in the salinity dispersion model. Simpler representation, The equation is measured over the waters, as appeared in [11]

$$\frac{\partial c}{\partial t} + u(x,t)\frac{\partial c}{\partial x} = D\frac{\partial^2 c}{\partial x^2}, \quad (9)$$

where $(x,t) \in [0,L] \times [0,T]$, u is the speed of the salinity flow and D is coefficient of diffusion.

Expecting that the saltiness is weakened by the fresh water. These are then the saltiness shift in weather conditions level is decreased by the fresh water speed. The rate of fresh water efficiency to weaken saltiness is accepted by $k \in [0,1]$. In [11], the model for salinity problem was proposed

$$\frac{\partial c}{\partial t} + (u_s - ku_w)\frac{\partial c}{\partial x} = D_s\frac{\partial^2 c}{\partial x^2}, \quad (10)$$

where $c(x,t)$ is the saltiness level of water(kg/m^3), k is the efficiency rate of water salinity removal, u_s is the salinity water advective velocity (m/s) and u_w is the velocity of fresh water flow

2.2.1 Initial conditions: A Lagrange interpolating polynomials of the salinity data of salinity data is defined as the initial condition, along the river form the estuary is taken into consideration using salinity data, defined by

$$c(x,0) = f(x), \quad (11)$$

where $x \in [0,L]$ and $f(x)$ is a measured salinity data interpolation function.

2.2.2 Left boundary condition: The left boundary condition is a Lagrange interpolating polynomials of estimated crude information, it depends on the saltiness of a stream at the main station that shut to the estuary, defined by

$$c(0,t) = g(t), \quad (12)$$

where $t \in [0,T]$ and $g(t)$ is a given Lagrange interpolating polynomials at the first observation station via calculated salinity data.

2.2.3 Right boundary condition: The right boundary condition were determined by the change in salinity at the last station, defined by

$$\frac{\partial c}{\partial x} = C_R, \quad (13)$$

where $t \in [0,T]$ and C_R is an estimated the rate of change of the saltiness level at the last observation station.

III. Explicit finite difference scheme for one-dimensional salinity internal wave hydrodynamic model

We can discriminate against the domain by splitting the interval $[0,L]$ into M sub intervals such that $M\Delta x = L$ and the time interval $[0,T]$ into N subintervals such that $N\Delta t = T$. The grid points (x_m, t_n) are defined by $x_m = m\Delta x$ for all $m = 1,2,3,...,M$ and $t_n = n\Delta t$ for all $n = 1,2,3,...,N$ in which M and N are positive integers. We can then estimate $c\,(x_m, t_n)$ by C_m^n, value of the difference approximation of $c\,(x,t)$ at point $x = m\Delta x$ and $t = n\Delta t$, where $m \in [0,M]$ and $n \in [0,N]$.

3.1 A modified Lax-diffusive method for the hydrodynamic model

The hydrodynamic model gives the speed field and the height of the water at that point the model's calculated effect would be the contribution to the dispersion model that provides the target area of pollution. This section suggests the alteration approach of a standard Lax-diffusive method for the hydrodynamic model of [12].

We'll adjust f^* from the standard formula of [12] to be the sum of three points. The semi-discrete scheme is applied to Eq.(2) and usage of an existing spatial grid $(x_m, t_n) = (m\Delta x, n\Delta t)$, we can define

$$f_x = \frac{f_{m+1}^n - f_{m-1}^n}{2\Delta x}, \quad (14)$$

$$f_t = \frac{f_m^{n+1} - f_m^*}{\Delta t} \quad (15)$$

where

$$f^* = \frac{f_{m+1}^n + f_m^n + f_{m-1}^n}{3} \quad (16)$$

The partial derivative of h and u is estimated by using Eqs.(14-16), respectively, with respect to x and t. We will see that Eq.(2) is in a form of a matrix as

$$A_t + B_x + C = 0, \tag{17}$$

where

$$A = \begin{pmatrix} h \\ hu \end{pmatrix}, B = \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}gh^2 \end{pmatrix},$$

$$C = \begin{pmatrix} 0 \\ -gh\partial_x z \end{pmatrix} \tag{18}$$

It implies that the standardized spatial grids will write the Eq.(18) as

$$A_m^n = \begin{pmatrix} h_m^n \\ h_m^n u_m^n \end{pmatrix}, B = \begin{pmatrix} h_m^n u_m^n \\ h_m^n (u_m^n)^2 + \frac{1}{2}g(h_m^n)^2 \end{pmatrix},$$

$$C = \begin{pmatrix} 0 \\ -gh_m^n \partial_x z \end{pmatrix}, \tag{19}$$

Substituting Eqs.(14-15) and Eq.(16) into Eq.(17), we obtain that

$$A_m^{n+1} = \frac{\Delta t}{2\Delta x}\left(B_{m-1}^n - B_{m+1}^n\right) + A^*, \tag{20}$$

where $A^* = \begin{pmatrix} h^* \\ (hu)^* \end{pmatrix}$. Substituting Eq.(19) into Eq.(20), we can see that

$$\begin{pmatrix} h_m^{n+1} \\ h_m^{n+1}u_m^{n+1} \end{pmatrix} = \begin{pmatrix} h_{m-1}^n + h_m^n + h_{m+1}^n \\ h_{m-1}^n u_{m-1}^n + h_m^n u_m^n + h_{m+1}^n u_{m+1}^n \end{pmatrix}$$
$$+\frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{m-1}^n u_{m-1}^n - h_{m+1}^n u_{m+1}^n \\ h_{m-1}^n \left(u_{m-1}^n\right)^2 - h_{m+1}^n \left(u_{m+1}^n\right)^2 \\ +\frac{1}{2}g\left(\left(h_{m-1}^n\right)^2 - \left(h_{m+1}^n\right)^2\right) \end{pmatrix}, \tag{21}$$

for all $m \in [1, M]$ and $n \in [0, N-1]$. For upper boundary, where $m = 0$, replaced by $u_{-1}^n = u_0^n$ and $h_{-1}^n = h_0^n$ into Eq.(21), we get

$$\begin{pmatrix} h_1^{n+1} \\ h_1^{n+1}u_1^{n+1} \end{pmatrix} = \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_0^n u_0^n - h_1^n u_1^n \\ h_0^n (u_0^n)^2 - h_1^n (u_1^n)^2 \\ +\frac{1}{2}g\left((h_0^n)^2 - (h_1^n)^2\right) \end{pmatrix}$$
$$+ \begin{pmatrix} 2h_0^n + h_1^n \\ 2h_0^n u_0^n + h_1^n u_1^n \end{pmatrix}, \tag{22}$$

For lower boundary, where $m = M$, substituted by boundary conditions for the estimated unknown value of the right boundary, we can let $u_{M+1}^n = u_M^n$ and $h_{M+1}^n = h_M^n$ by rearranging, we get

$$\begin{pmatrix} h_M^{n+1} \\ h_M^{n+1}u_M^{n+1} \end{pmatrix}$$
$$= \frac{\Delta t}{2\Delta x}\begin{pmatrix} h_{M-1}^n u_{M-1}^n - h_M^n u_M^n \\ h_{M-1}^n \left(u_{M-1}^n\right)^2 - h_M^n (u_M^n)^2 \\ +\frac{1}{2}g\left(\left(h_{M-1}^n\right)^2 - (h_M^n)^2\right) \end{pmatrix}$$
$$+ \begin{pmatrix} h_{M-1}^n + 2h_M^n \\ h_{M-1}^n u_{M-1}^n + 2h_M^n u_M^n \end{pmatrix}, \tag{23}$$

The scheme's stability state required CFL number as the [12],

$$C_n = u_{\max}\left(\frac{\Delta t}{\Delta x}\right) \leq 1. \tag{24}$$

3.2 Saulyev method for the salinity dispersion model

The Saulyev scheme is unconditionally stable [2]. Obviously the non strictly dependability prerequisite of Saulyev scheme is the principle of preferred position and conservative to utilize. Taking the Saulyev scheme into Eq.(10), it appears to have the following discretization:

$$c(x_m, t_n) \cong C_m^n, \tag{25}$$
$$\left.\frac{\partial c}{\partial t}\right|_{(x_m,t_n)} \cong \frac{C_m^{n+1} - C_m^n}{\Delta t}, \tag{26}$$
$$\left.\frac{\partial c}{\partial x}\right|_{(x_m,t_n)} \cong \frac{C_{m+1}^n - C_{m-1}^{n+1}}{2\Delta x}, \tag{27}$$
$$\left.\frac{\partial^2 c}{\partial x^2}\right|_{(x_m,t_n)} \cong \frac{C_{m+1}^n - C_m^n - C_m^{n+1} + C_{m-1}^{n+1}}{(\Delta x)^2}, \tag{28}$$
$$u_{s_m}^n \cong u_m^n, \tag{29}$$
$$u_{w_m}^n = u_w(x_m, t_n). \tag{30}$$

Substituting Eqs.(25-30) into Eq.(10), We obtain the equation of finite difference,

$$\frac{C_m^{n+1} - C_m^n}{\Delta t} + \left(u_{s_m}^n - ku_{w_m}^n\right)\left(\frac{C_{m+1}^n - C_{m-1}^{n+1}}{2\Delta x}\right)$$
$$= D_s\left(\frac{C_{m+1}^n - C_m^n - C_m^{n+1} + C_{m-1}^{n+1}}{(\Delta x)^2}\right). \tag{31}$$

The explicit equation of finite difference then becomes

$$C_{m+1}^{n+1} = \left(\frac{1}{1+\lambda}\right)\left[\left(\lambda + \frac{1}{2}r_m^n\right)C_{m-1}^{n+1} + (1-\lambda)C_m^n \right.$$
$$\left. + \left(\lambda - \frac{1}{2}r_m^n\right)C_{m+1}^n\right]. \tag{32}$$

where $i = 1, 2, 3, ..., M-1$, $\lambda = \frac{D_s\Delta t}{(\Delta x)^2}$ and $r_m^n = \frac{\left(u_{s_m}^n - ku_{w_m}^n\right)\Delta t}{\Delta x}$. For $i = M$, replaced the unknown value in Eq.(5), we obtain

$$C_{M+1}^n = \left(\frac{C_{M_2}^n - C_{M_1}^n}{L_2 - L_1}\right)\Delta x + C_{M-1}^n. \tag{33}$$

The truncation error of Sualyev scheme is $O\left\{(\Delta x)^2 + (\Delta t)^2 + (\Delta t/\Delta x)^2\right\}$.

IV. Numerical simulations

4.1 Simulation 1 : the salinity flow velocity and the salinity elevation in a river with effect of internal wave.

The observation stations with 90 km along the river are considered, as appeared in Table 1. We find the approximate solution of a one-dimensional salinity internal wave hydrodynamic model Eq.(2). the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation is 10 days. The boundary and initial condition functions are seen in Tables 2 and 3, respectively.

TABLE II
Parameters of physical of simulation 1.

D_s (m^2/s)	u_w (m/s)	K	L(km)	T(days)
0.1	0.25	0.3	108	10

The salinity diffusion coefficient is 0.1 m^2/s. By using a modified Lax-diffusive method for the hydrodynamic model,

TABLE III
THE BOUNDARY AND INITIAL CONDITION FUNCTION OF SIMULATION 1.

Parameters	Given functions
$u_1(x)$	0.06
$h_1(x)$	$0.05 + 0.1sin(m\Delta t)$
$f_1(t)$	0
$f_2(t)$	0
$g_1(t)$	$0.05 + 0.1sin(m\Delta t)$
$g_2(t)$	0

Eqs.(21-23), we obtain a graph of elevation of the water above the bottom and, by using the hydrodynamic model, we obtain a graph of the salinity flow, seen in Fig 1 and 2, respectively. We can see that this simulation is to approximate the salinity flow velocity from the estuary.

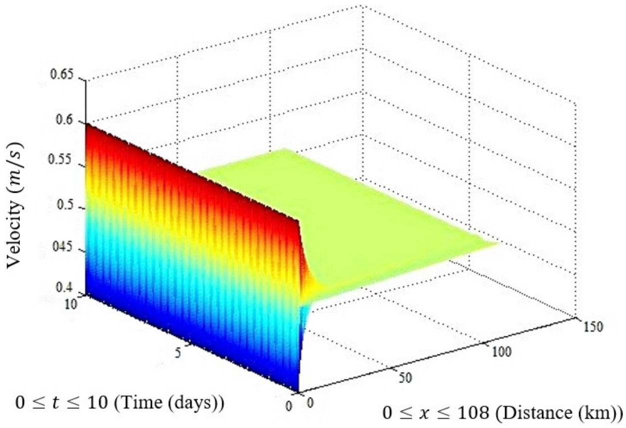

Fig. 1. The salinity flow velocity for $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

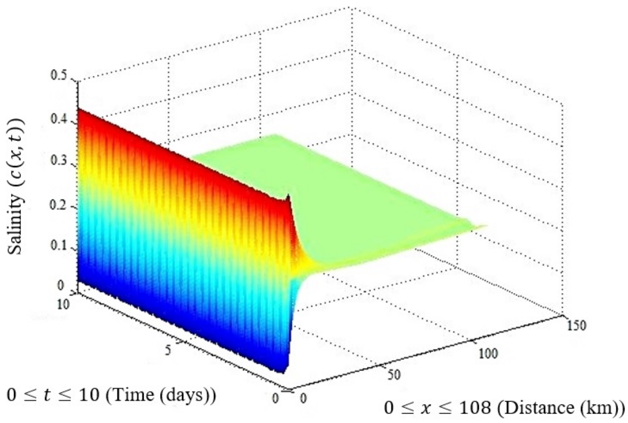

Fig. 2. The salinity flow velocity for $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

4.2 Simulation 2 : release fresh water from the barrage dam to dilute the salinity without the salinity internal wave factor.

We will to find the approximate solution of Eq.(10) for all observation stations with 108 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow $u_s = 0.065\ m/s$, the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation 10 days. The parameters of physical are appeared in Table 4. We get the approximated solution $c(x, t)$ by using Sualyev

TABLE IV
PARAMETERS OF PHYSICAL OF SIMULATION 2.

D_s (m^2/s)	u_w	u_w (m/s)	K	L (km)	T (days)
0.1	0.065	0.25	0.3	108	10

scheme are appeared in Table 5. The level of salinity at the operated S_7 observation station will be standardized, as seen in Figs 3 and 4, respectively. We can see that this simulation is to reduce the saltiness level which releases fresh water from the dam.

TABLE V
THE ESTIMATED SALINITY LEVEL OF SIMULATION 2 FOR ALL OBSERVATION STATIONS.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	11.7025	3.7114	2.0435	1.0136
10000	11.2247	3.5663	2.0230	1.0088
15000	10.8874	3.4530	1.9999	0.9991
20000	10.6373	3.3562	1.9699	0.9854
t	S_5	S_6	S_7	S_8
1	0.8025	0.5325	0.5053	0.1618
5000	0.7730	0.4865	0.3955	0.0148
10000	0.7458	0.4081	0.2975	0.0061
15000	0.7221	0.3370	0.2304	0.0029
20000	0.6975	0.2798	0.1836	0.0015

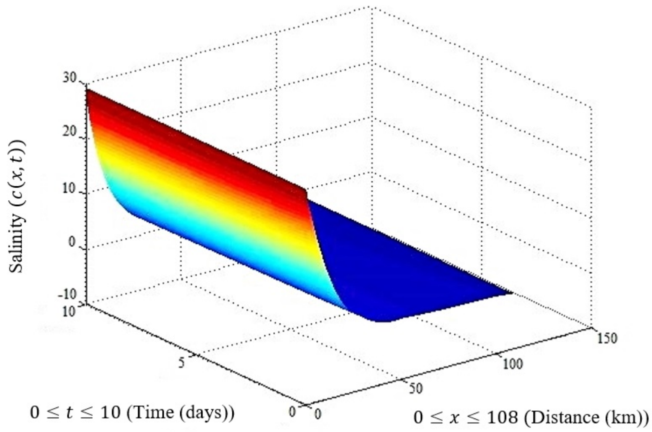

Fig. 3. The estimated salinity level of simulation 2 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

4.3 Simulation 3 : release fresh water from the barrage dam to dilute the salinity with the salinity internal wave factor.

We will to find the approximate solution of Eq.(10) for all observation stations with 108 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow from simulation 1, the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation 10 days. The parameters of physical are appeared in Table 6.

We get the approximated solution $c(x, t)$ by using Sualyev scheme are appeared in Table 7. The saltiness concentration level at the station S_7 becomes a standard level. We can see that this simulation is to reduce the salinity level which

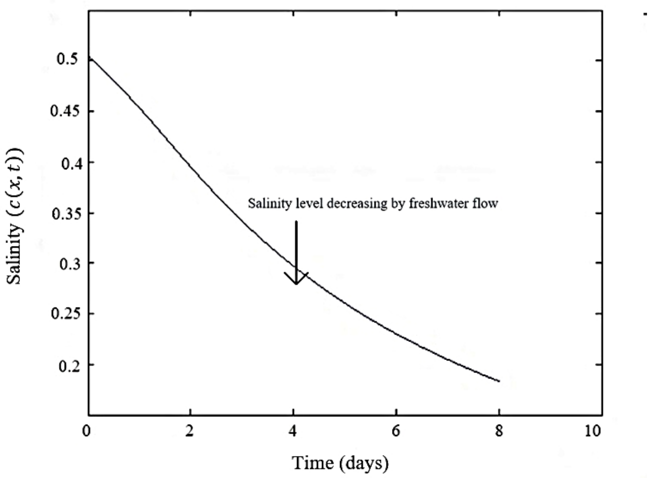

Fig. 4. The estimated salinity level of simulation 2 at station S_7 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $t \in [0, 10]$.

TABLE VI
PARAMETERS OF PHYSICAL OF SIMULATION 3.

D_s (m^2/s)	u_w (m/s)	K	L (km)	T (days)
0.1	0.2	0.3	108	10
0.1	0.25	0.3	108	10
0.1	0.3	0.3	108	10

releases fresh water from the dam. A comparison of the simulated salinity levels at the controlled observation station as seen in Figures 5-6.

TABLE VII
THE ESTIMATED SALINITY LEVEL OF SIMULATION 3 FOR ALL OBSERVATION STATIONS.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	8.6881	2.7478	1.5514	0.9373
10000	6.0982	1.9581	1.2625	0.8286
15000	4.4861	1.5021	1.0642	0.7351
20000	3.4869	1.2107	0.9131	0.6494
t	S_5	S_6	S_7	S_8
1	0.8025	0.5325	0.5053	0.1618
5000	0.7122	0.4204	0.2960	0.0040
10000	0.6393	0.2369	0.1302	0.0022
15000	0.5627	0.1206	0.0574	0.0017
20000	0.4604	0.0604	0.0262	0.0010

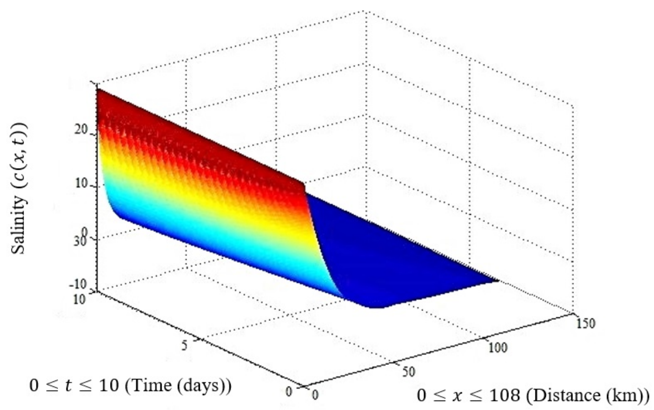

Fig. 5. The estimated salinity level of simulation 3 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

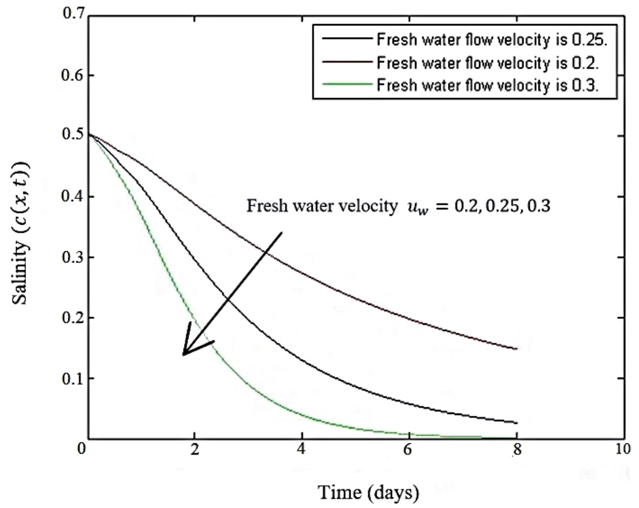

Fig. 6. The estimated salinity level of simulation 3 at station S_7 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $t \in [0, 10]$.

Since, the water fresh water flow velocity is 0.25 m/s can diluted the salinity water in time and not waste too much water from the barrage dam and the saltiness flow velocity that flows into the river in nature is not constant at all times, the comparison of The salinity concentration level at the controlled observation station S_7 of the simulation 2 and 3 is seen in Fig 7. Therefore, in the following simulations, it will

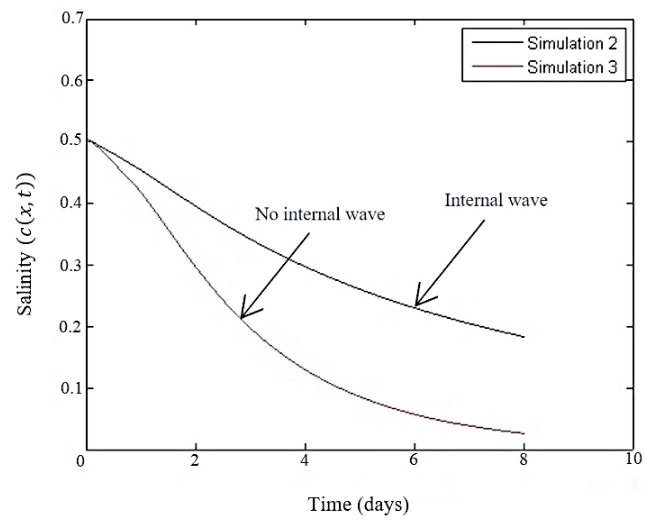

Fig. 7. The comparison of The saltiness concentration level at station S_7 of simulation 2 and 3 by $\Delta x = 0.5$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

be using the saltiness flow velocity obtained from simulation 1.

4.4 Simulation 4 : maintaining a constant level of salinity to the standard by reducing the speed of water discharge from the barrage dam.

We will to find the approximate solution of Eq.(10) for all observation stations with 108 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow from the simulation 1, the efficiency of eliminating saltiness of fresh water discharge is $k = 30\%$, and time of simulation is 10 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.3$ kg/m^3 by the controlled release of water from barrage dams with the following process:

1) Release at high speed when the salinity level $c(96, t) > C_{ST}$ at the station S_6.

2) Release at low speed when the salinity level $c(96, t) < C_{ST}$ at the station S_6.

Their parameters of physical are appeared in Table 8.

The approximate salinity level for all observation station can be obtained, as appeared in Table 9 and Fig 8. The saltiness level at the various observation stations S_7, as appeared in Fig 9. We can see that the technique is to reduce the saltiness level which releases fresh water from the dam and use the amount of fresh water not too much.

TABLE VIII
PARAMETERS OF PHYSICAL OF SIMULATION 4.

$c(x,t)$ at S_7	D (m^2/s)	u_w (m/s)	K
$> C_{ST}$	0.1	0.25	0.3
$< C_{ST}$	0.1	0.15	0.3
	T (days)	L (km)	c(0,t)
	10	108	g(t)
	10	108	g(t)

TABLE IX
THE ESTIMATED SALINITY LEVEL OF SIMULATION 4 FOR ALL OBSERVATION STATIONS.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	8.7906	2.7751	1.5642	0.9396
10000	11.5783	3.3642	1.9124	0.9836
15000	13.9586	4.1634	2.3118	1.0693
20000	15.6397	5.1400	2.8139	1.1899
t	S_5	S_6	S_7	S_8
1	0.8025	0.5325	0.5053	0.1618
5000	0.7140	0.4230	0.2997	0.0112
10000	0.7322	0.3990	0.2946	0.0126
15000	0.7514	0.3897	0.2931	0.0149
20000	0.7776	0.3852	0.2910	0.0153

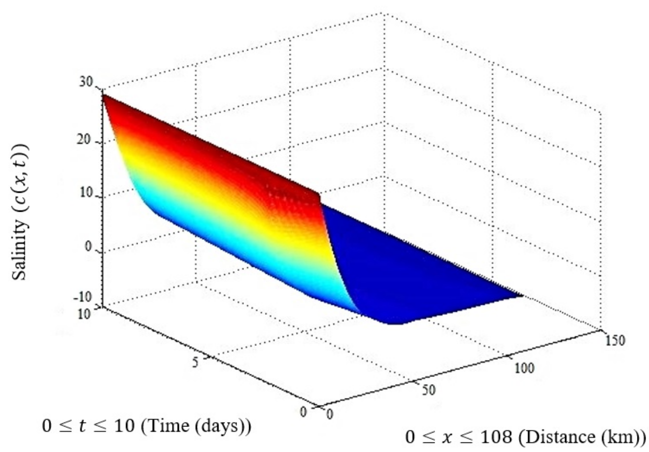

Fig. 8. The estimated salinity level of simulation 4 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

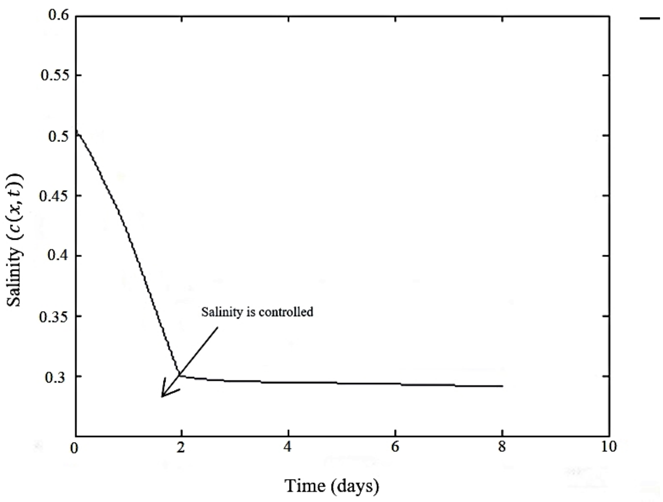

Fig. 9. The estimated salinity level of simulation 4 at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 10]$.

4.5 Simulation 5 : reduce the level of salinity before the salinity exceeds the standard.

We will to find the approximate solution of Eq.(10) for all observation stations with 108 km along the river, as appeared in Table 1. Provided that saltiness water coefficient of diffusion is 0.1 m^2/s, the speed of saltiness water flow from the simulation 1, the efficiency of eliminating salinity of fresh water discharge is $k = 30\%$, and time of simulation is 10 days. We want to monitor the salinity of the water at the station S_6 to be less than the specified salinity $C_{ST} = 0.3$ kg/m^3 about 3 days by the controlled release of water from barrage dams with the following process:

1) Release at normal speed when the salinity level $c(79, t) < C_{ST}$ at the station S_5.

2) Release at high speed when the salinity level $c(79, t) > C_{ST}$ at the station S_5

Their parameters of physical are appeared in Table 10.

The approximate salinity level for all observation station can be obtained, as appeared in Table 11 and Fig 10. The saltiness level at the various observation stations S_7, as appeared in Fig 11. We can see that the technique is to reduce the saltiness level which releases fresh water from the dam before the salinity is higher than standard.

TABLE X
PARAMETERS OF PHYSICAL OF SIMULATION 5.

$c(x,t)$ at S_5	D (m^2/s)	u_s (m/s)	u_w (m/s)
$< C_{ST}$	0.1	0.065	0
$> C_{ST}$	0.1	0.065	0.25
K	T (days)	L (km)	c(0,t)
0.3	10	108	g(t)
0.3	10	108	g(t)

V. DISCUSSION

In simulation 1, the approximate solutions of salinity flow velocity are obtained by using the modified Lax-diffusive system for the hydrodynamic model, as seen in Figs 1-2. In simulation 2, we can obtain salinity of water at the pumping station without the salinity flow velocity form simulation 1 by using the Sualyev technique. We can see that this

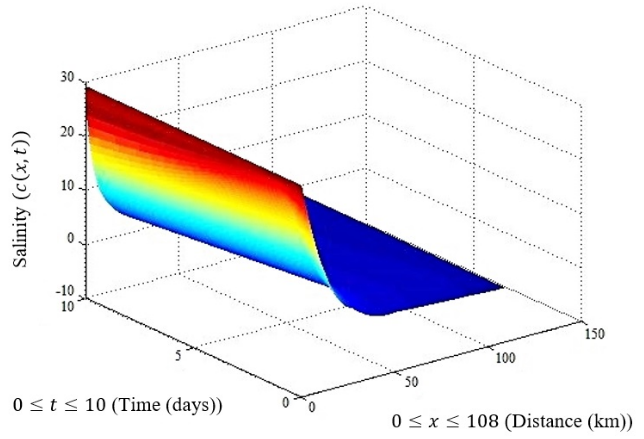

Fig. 10. The estimated salinity level of simulation 5 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $x \in [0, 108]$ and $t \in [0, 10]$.

TABLE XI
THE ESTIMATED SALINITY LEVEL OF SIMULATION 5 FOR ALL OBSERVATION STATIONS.

t	S_1	S_2	S_3	S_4
1	12.5019	4.0615	2.0976	1.0746
5000	19.5112	6.9051	3.7499	1.3210
10000	13.4092	4.6944	2.6005	1.1293
15000	9.3605	3.2451	1.9022	0.9671
20000	6.6951	2.3258	1.4632	0.8235
t	S_5	S_6	S_7	S_8
1	0.8033	0.4052	0.2641	0.1378
5000	0.9057	0.4607	0.3732	0.0097
10000	0.7853	0.3005	0.1826	0.0041
15000	0.6692	0.1595	0.0806	0.0032
20000	0.5456	0.0808	0.0365	0.0019

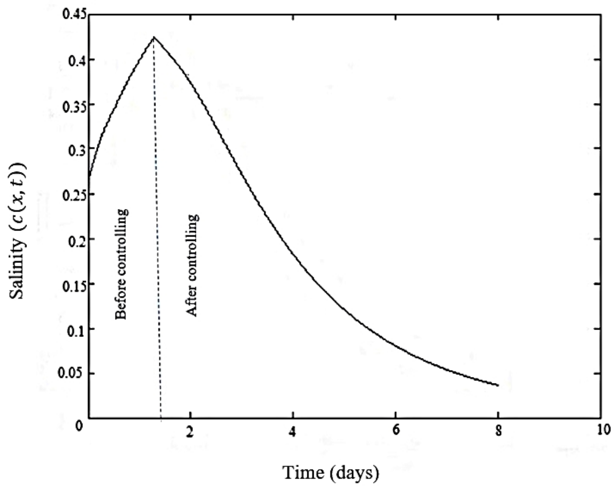

Fig. 11. The estimated salinity level of simulation 5 at station S_7 by $\Delta x = 0.1$ and $\Delta t = 0.05$ for all $t \in [0, 10]$.

simulation can decrease the salinity level with salinity flow velocity is constant by releasing fresh water from the barrage dam, as seen in Figs 3-4. In simulation 3, we can obtain salinity of water at the pumping station with the salinity flow velocity from simulation 1 by using the Sualyev technique. We can see that this simulation can decrease the salinity level with the affect of salinity internal wave by releasing fresh water from the barrage dam, as seen in Figs 5-6 and the comparison of the degree of salinity at a S_7 monitored observation station, as seen in Figure 7. In simulation 4, a salinity control process is simulated with the salinity flow velocity from Simulation 1, as seen in Figs 3-4. The salinity is reduced to the standard level after this, as seen in Figs 8-9, we'll lower the fresh water flow rate to preserve the normal salinity level. In simulation 5, a process of with the salinity flow velocity from simulation 1 was proposed. The salinity is that until the normal salinity amount is reached. The suggested process may reduce the salinity level at least, as seen in Figs 10-11, when the barrage dam releases fresh water.

VI. CONCLUSION

We also suggested a mathematical model for saltiness water measurement in one-dimension. The proposed model concerns the saltiness advection to the river and the effect of salinity flow velocity from an estuary into the river and the dam's release of fresh water. There are several scenarios in which the effects of internal salinity waves are simulated. The suggested simulations can be use in practical salinity measurement in similar topographical rivers. In the salinity control aspect, the suggested process may decrease the salinity level until the level reaches the normal line and the quantity of fresh water may not be used too much. Suggested simulations can be used in realistic salinity management situations in rivers with barrage dam systems along with savings in the volume of fresh water used.

ACKNOWLEDGEMENT

This paper is being sponsored by the Centre of Excellence in Mathematics, the Commission on Higher Education, Thailand. We are very grateful to experts for their appropriate and constructive suggestions to improve this paper.

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