Dual-channel Supply Chain Game Considering the Retailer's Sales Effort

Lei Meng, Yan-ran Shi, Xiao-xu Zhang, and Mei-xiang Hong

Abstract—**This study explores the effect of the manufacturer's dual-channel structure on the pricing strategies and sales effort in a two-echelon supply chain. The retailer makes sales effort to achieve its economic goal. Two decision-making models—centralized and decentralized—were considered, and two game structures, manufacturer-Stackelberg (MS) and retailer-Stackelberg (RS), were developed and their optimal solutions derived. Finally, the results of the proposed game models were analysed via a numerical example. The results showed that in the centralized decision-making model, the prices in the two channels are consistent. Meanwhile, in the decentralized decision-making model, although the retailer makes sales effort, online sales prices are always higher than those of the retailer in the manufacturer-led supply chain. However, when the supply chain is dominated by retailers, the opposite result is observed.**

Index Terms—**Dual-channel supply chain, Sales effort, Stackelberg game**

I. Introduction

WITH the development of e-commerce and network technology, manufacturing enterprises have increasingly employed the e-market to sell products to consumers directly while simultaneously distributing them through traditional retail channels, thus reconstructing their distribution channels. Thus, in the era of e-commerce, the e-channel has become an important retail channel. In recent years, a combination of the traditional channel and e-channel has become the main retail mode of many brand manufacturers. For example, many famous enterprises, such as HP and IBM, have achieved success by adopting a multi-channel strategy. Kodak, Nike, and Apple, which earlier used traditional channels, have also established e-channels [1], while enterprises mainly using e-channels have begun selling through traditional channels [2]. However, some manufacturers, such as Levi's, closed their online stores to reduce conflicts with traditional retailers [3]. Correspondingly, customers are increasingly willing to shop in dual-channel supply chains [4]. Many industry and government statistical reports show that e-commerce is growing rapidly because the direct marketing channel reduces intermediate links in the supply chain, which attracts more consumers. The supply chain uses both the traditional retail and direct marketing channels to sell products, which can increase market share, help understand demand information further, and improve the brand competitiveness of products. However, after the addition of network channels, upstream enterprises that previously served as only suppliers become competitors of retailers as well. The new direct marketing channels will compete with the original retail channels, which may lead to channel conflicts and even the breakdown of traditional retail channels, as was the case for Revlon [5]. Therefore, the game between direct marketing channels and retail channels has an important impact on suppliers and retailers as well as on the whole supply chain. For a manufacturer, whether to add a direct marketing channel is a problem worthy of study.

In recent years, several scholars have examined the decision making in dual-channel supply chains. Based on the consumer choice model, Chiang et al. [6] studied the pricing problem of dual channels under fixed demand and pointed out that manufacturers can influence retailers' pricing decisions through direct channels, reduce the dual marginal utility, and increase revenue. Yao & Liu [7] discussed static and dynamic equilibrium pricing strategies of a product in different channels, considering that price and service affect demand simultaneously. Cattani et al. [8] investigated the pricing problem of manufacturers in dual channels based on the consumer utility theory. Guo & Zhao [9] examined the Stackelberg and Bertrand pricing strategies dominated by manufacturers in dual channels and proposed the optimal strategy under different conditions. Liu & Wu [10] showed that competition between suppliers' online direct sales channels and retailers' offline retail channels can promote the latter's sales effort, alleviate the competition, and generate profits for both the parties under certain conditions.

The pricing decision of dual-channel supply chains is a hot topic of research. Under different dominance structures of enterprises in the supply chain, if the price strategy is formulated to obtain the maximum profit and the conflict between channels coordinated, the optimal situation can be achieved. Sang [11] examined pricing and service decisions of supply chains in an uncertain environment. Yan & Pei [12] abstracted physical and online stores into uniformly distributed points and centres, respectively, established a ring market model, and analysed the pricing strategy and competitive behaviour of retailers in a dual-channel environment. They found that if the number of physical

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stores is higher, so is the cost of online shopping, and the dual-channel strategy may not necessarily bring a competitive advantage; furthermore, the price in onffline stores should be lower than that of physical stores, but whether it is lower than that of other retailers' physical stores varies based on other independent variables. Pu et al. [13] studied the pricing strategy of the supply chain under the three-channel power structure of the manufacturer Stackelberg, the physical store Stackelberg, and the vertical Nash equilibrium. Yan et al. [14] explored the centralized and master–slave strategies of traditional supply chain independent channels and e-market dual-source channels, respectively. Huang et al. [15] analysed the pricing and production problems of a two-stage dual-channel supply chain with demand interruption and calculated the profit difference between manufacturers and retailers in decentralized decision-making. Ma et al. [16] examined the effect of the leading manufacturer's dominant strategy in a two-level supply chain on other members and customers, pointing out that retailers and customers always benefit from channel advantages.

The existing research on the pricing strategy of dual-channel supply chains is relatively comprehensive. It has examined related situations based on not only the manufacturer's leadership, but also the retailer's. Wang et al., [17] considering the hierarchical structure of the supply chain, construct a two-level programming model under the guidance of the manufacturer. Hong [18] analysed the Stackelberg game in a retailer-led supply chain, and Hong & Men [19] examined it in a retailer-led supply chain considering the sales effort. This has ensured in-depth examination of supply chains. In the real economy, demand distribution or demand transfer of various channels is closely related to prices, especially in a competitive environment where alternative products exist. When the price of a product decreases in a channel, it will lead to demand transfer of the product between different channels and demand transfer of alternative products. In other words, with intensifying competition between direct and offline channels, retailers will face greater pressure and motivation to make sales effort. In reality, with their continuous development and growth, retailers' status in the supply chain channel structure is gradually improving. Considering different dominance structures, it is of great practical significance to combine retail sales effort with pricing decisions. Thus, this study explores the optimal dual-channel pricing strategy in a two-level supply chain when the retailer makes sales effort in centralized and decentralized decision-making models.

II. MODEL DESCRIPTION

To study our game in the dual-channel supply chain considering sales effort, this study constructs a two-level supply chain with one manufacturer and one retailer, in which the manufacturer's products can be sold through both an offline retail channel and online direct channel. Therefore, the dual channel in this study is mainly applicable to manufacturers, as shown in Fig. 1.

The product circulation process is as follows: in the offline channel, the manufacturer sells the product to the consumer through the retailer at the wholesale price, and in the online channel, the manufacturer sells the product to the consumer through the online direct channel established by the manufacturer at the wholesale price. The offline retailer buys products at the wholesale price and then sells them to consumers through offline channels at the retail price. In the sales process, the retailer makes sales effort to improve customers' consumption experience and thus increase sales. The basic notations are shown in Table I.

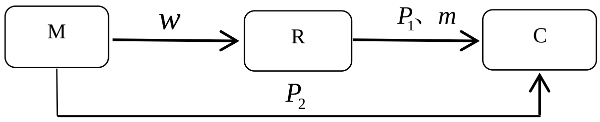

Fig. 1. Structure of Dual-channel Supply Chain

Table I

Parameter	Meaning
D_1	Demand of the offline retail channel
D_2	Demand of the online retail channel
P_1	Retail price in the offline retail channel
P_2	Retail price in the online direct channel
w	Wholesale price, $P_1 \ge P_2 \succ w$
α	Potential market demand
β	Elasticity of demand to price
θ	Market share of the offline retailer without sales effort
C	Cost of the offline retailer's sales effort behaviour
γ	The increase in market share through the offline retailer's sales effort is directly proportional to the cost of sales effort, $\gamma=kC$, $0 \prec k \prec 1/C$.
π_m^i	The profit of the i -th member in the m -decision model, $m = C, DE$; C stands for centralized decision-making, and DE stands for decentralized decision-making.

To formulate the problem, some assumptions are made:

Assumption 1. The members of the supply chain are independent entities with symmetrical information;

Assumption 2. The manufacturer has sufficient capacity to meet the market demand, that is, there will be no shortage of supply;

Assumption 3. To ensure the existence of an optimal solution, we assume that $\beta/(\beta-\lambda) \succ 2\theta(1-\theta)$.

Assumption 4. The customer's demand function is a linear function of the wholesale price, unit profit margin, other market's price, and sales effort cost. Thus, the customer's demand function is:

$$D_1 = \theta\alpha - \beta P_1 + \lambda P_2 \quad (1)$$

$$D_2 = (1\text{-}\theta)\alpha - \beta P_2 + \lambda P_1 \quad (2)$$

III. MODEL ANALYSIS

In this section, we discuss the structures of the supply chain members and how to set their optimal policies in different decision-making models with different power structures. In the decentralized decision-making model, we analyse the situation when the retailer makes sales effort in two conditions: the manufacturer dominates the supply chain, or the retailer dominates the supply chain. In the following discussion, we use superscripts *MS* and *RS* to denote that the corresponding quantities are for the manufacturer-Stackelberg (MS) and retailer-Stackelberg (RS) structures.

A. *Centralized Decision-making Analysis*

In the centralized decision-making model, the manufacturer and retailer form a strategic alliance, which has a common goal: to maximize the revenue of the whole supply chain. In this case, the competition between offline sales channels and online sales channels will be weakened, and the retailer does not need to make additional sales effort to improve product sales. In this model, the specific forms of the demand and profit functions are as follows:

$$\pi_C^{sc} = P_1 D_1 + P_2 D_2 \qquad (3)$$

Substituting equations (1) and (2) into (3), we get the following:

$$\pi_C^{sc} = P_1(\theta\alpha - \beta P_1 + \lambda P_2) + P_2[(1\text{-}\theta)\alpha - \beta P_2 + \lambda P_1] \qquad (4)$$

The game model is:

$$\max_{P_1,P_2} \pi_C^{sc} = P_1(\theta\alpha - \beta P_1 + \lambda P_2) + P_2[(1\text{-}\theta)\alpha - \beta P_2 + \lambda P_1]$$

Theorem 1. In the centralized decision-making model, the optimal price strategy for manufacturers and retailers is:

$$P_1 = \frac{(1-\theta)\alpha\lambda + \theta\alpha\beta}{2(\beta^2 - \lambda^2)} \qquad (5)$$

$$P_2 = \frac{\theta\alpha\lambda + (1-\theta)\alpha\beta}{2(\beta^2 - \lambda^2)} \qquad (6)$$

Proof. First, the first partial derivative of equation (5) with respect to P_1 and P_2 is obtained.

$$\frac{\partial \pi_C^{SC}}{\partial P_1} = \theta\alpha - 2\beta P_1 + 2\lambda P_2 \; ; \; \frac{\partial \pi_C^{SC}}{\partial P_2} = (1-\theta)\alpha - 2\beta P_2 + 2\lambda P_1$$

Second, we obtain the second-order partial derivatives of equation (5) with respect to P_1 and P_2, respectively.

$$\frac{\partial^2 \pi_C^{SC}}{\partial P_1^2} = -2\beta \; ; \; \frac{\partial^2 \pi_C^{SC}}{\partial P_2^2} = -2\beta$$

Then, the Hessian matrix of π_C^{sc} is

$$H = \begin{bmatrix} \frac{\partial^2 \pi_C^{SC}}{\partial P_1^2} & \frac{\partial^2 \pi_C^{SC}}{\partial P_1 \partial P_2} \\ \frac{\partial^2 \pi_C^{SC}}{\partial P_2 \partial P_1} & \frac{\partial^2 \pi_C^{SC}}{\partial P_2^2} \end{bmatrix} = \begin{bmatrix} -2\beta & 2\lambda \\ 2\lambda & -2\beta \end{bmatrix}$$

Since $\beta \succ \lambda \succ 0$, the Hessian matrix of π_C^{sc} is negative definite. It can be seen that π_C^{sc} is the concave function of P_1 and P_2 . Therefore, the optimal pricing of the manufacturer and retailer can be obtained according to the following first-order conditions on P_1 and P_2:

$$\theta\alpha - 2\beta P_1 + 2\lambda P_2 = 0 \qquad (7)$$

$$(1-\theta)\alpha - 2\beta P_2 + 2\lambda P_1 = 0 \qquad (8)$$

Solving the simultaneous equations (7) and (8), we get the manufacturer's optimal online sales price and the retailer's optimal offline sales price, respectively:

$$P_1 = \frac{(1-\theta)\alpha\lambda + \theta\alpha\beta}{2(\beta^2 - \lambda^2)} \; ; \; P_2 = \frac{\theta\alpha\lambda + (1-\theta)\alpha\beta}{2(\beta^2 - \lambda^2)}$$

Thus, the proof of Theorem 1 is complete.

According to equations (1), (2), (3), (5), and (6), we can conclude that the maximum profit of the supply chain is:

$$\pi_C^{sc} = \frac{\alpha^2\beta}{4(\beta^2 - \lambda^2)} - \frac{\theta(1-\theta)\alpha^2}{2(\beta + \lambda)}$$

Proposition 1. In the centralized decision-making model, as the potential demand increases, the price difference between offline retail channels and online direct channels also increases.

Proof.

$$\left|P_2^* - P_1^*\right| = \left|\frac{\alpha}{2(\beta - \lambda)}\right| \qquad (9)$$

$\left|P_2^* - P_1^*\right|$ indicates the price difference between online direct channels and offline retail channels, and α represents the potential demand in the market. As shown in equation (9), with the increase in α , the price difference between the two channels also increases.

The proof of Proposition 1 is hence complete.

Proposition 2. In the centralized decision-making model, the optimal price in the online direct channel and the of the offline retail channel are both proportional to the potential demand.

Proof. Taking the derivative of P_1 and P_2 with respect to α , we get

$$\frac{dP_1^*}{d\alpha} = \frac{(1-\theta)\lambda + \theta\beta}{2(\beta^2 - \lambda^2)} \; ; \; \frac{dP_2^*}{d\alpha} = \frac{\theta\lambda + (1-\theta)\beta}{2(\beta^2 - \lambda^2)}$$

Since $\beta \succ \lambda \succ 0$ and $\theta \succ 0$, we can get $\frac{dP_1^*}{d\alpha} \succ 0$ and $\frac{dP_2^*}{d\alpha} \succ 0$ easily; that is, both P_1^* and P_2^* increase with the increase in α .

This shows that in the centralized decision-making model, as the potential market demand increases, supply chain members can increase their profits by increasing prices.

B. Decentralized Decision-making Analysis

In the decentralized decision-making model, the manufacturer and retailer play a Stackelberg game, which is applicable to an oligopoly market structure with unequal status of firms. We assume there are two firms in the market, one the dominant (i.e. leader) and the other the follower. The leader first sets the price to maximize its own interests, and the follower makes its optimal response according to the price strategy of the leader. To solve the Stackelberg game, the reverse induction method is usually employed. First, the follower's optimal response function is solved, which is then substituted into the leader's profit function, and the leader's optimal price strategy is obtained by solving its profit maximization problem. Finally, the optimal price strategy is substituted into the follower's optimal response function to obtain its optimal price strategy, and the equilibrium solution of the Stackelberg game is thus obtained.

In the centralized decision-making model, the manufacturer sells its products through both online and offline channels. On the one hand, the manufacturer sells its products to the retailer at the wholesale price of w; on the other hand, the manufacturer sells its products through online direct channels to customers at the price of P_2. When the offline retailer buys goods at the wholesale price of w, they sell their products to consumers through offline channels at the price of P_1 with unit profit m; that is, $P_1 = w + m$. To increase sales, the offline retailer makes sales effort, such as advertising and improving the service level. C is the cost of such sales effort made by the offline retailer, and γ is the market share gained through this effort. Therefore, in the decentralized decision-making model, the demand and revenue functions can be set as follows:

$$D_1=(\theta+\gamma)\alpha-\beta P_1+\lambda P_2=(\theta+\gamma)\alpha-\beta(w+m)+\lambda P_2 \quad (10)$$

$$D_2=(1\text{-}\theta\text{-}\gamma)\alpha-\beta P_2+\lambda(w+m) \quad (11)$$

$$\pi_{DE}^R=(P_1-w)D_1-C=mD_1-C \quad (12)$$

$$\pi_{DE}^M=wD_1+P_2D_2 \quad (13)$$

$$\pi_{DE}^{SC}=\pi_{DE}^M+\pi_{DE}^R \quad (14)$$

Accordingly, in the decentralized decision-making model, the optimal pricing strategy of the supply chain members should satisfy the following equations:

$$\begin{cases} \max_w \pi_{DE}^R=\max_w\{(P_1-w)[(\theta+\gamma)\alpha-\beta P_1+\lambda P_2]-C\} \\ \max_w \pi_{DE}^R=\max_w\{w[(\theta+\gamma)\alpha-\beta P_1+\lambda P_2]+P_2[(1-\theta-\gamma)\alpha-\beta P_2+\lambda P_1]\} \end{cases}$$

That is,

$$\begin{cases} \max_w \pi_{DE}^R=\max_w m[(\theta+\gamma)\alpha-\beta(w+m)+\lambda P_2]-C \\ \max_w \pi_{DE}^R=\max_w w[(\theta+\gamma)\alpha-\beta(w+m)+\lambda P_2]+P_2[(1-\theta-\gamma)\alpha-\beta P_2+\lambda(w+m)] \end{cases}$$

In the Stackelberg game of the supply chain, we consider two situations: the manufacturer as the leader and the retailer as the follower, and the retailer as the leader and the manufacturer as the follower. We call the former the MS power structure, and the latter, the RS power structure. Concurrently, we assume that the retailer make sales effort in any circumstance. The pricing strategies of the manufacturer and the retailer in MS and RS are discussed below.

1) MS game model

In the MS supply chain structure, the scale and market share of the retailer are relatively small; thus, it is in a subordinate position to the manufacturer, who is the leader in the market. The order of the game is as follows: first, the manufacturer sets the wholesale price w and the online direct selling price P_2 according to its profit maximization goal. When the retailer observes the manufacturer's pricing strategy, it responds; in other words, according to its own profit maximization goal, it sets the price of the product sold through the offline channel as the sum of unit profit m and price P_2. The reverse induction method is used in the solution, which is mainly reflected in the fact that the manufacturer accounts for the response function of the follower (the retailer) in its profit function when deciding the optimal pricing strategy. Therefore, the description of the MS game model is:

$$\begin{aligned} \max_{w,P_2} \pi_{DE}^M &= wD_1+P_2D_2 \\ &= w[(\theta+\gamma)\alpha-\beta P_1+\lambda P_2]+P_2[(1-\theta-\gamma)\alpha-\beta P_2+\lambda P_1] \\ &= w[(\theta+\gamma)\alpha-\beta(w+m)+\lambda P_2]+P_2[(1-\theta-\gamma)\alpha-\beta P_2+\lambda(w+m)] \end{aligned}$$

$$s.t.\begin{cases} m=\arg\max \pi_{DE}^R \\ \max_m \pi_{DE}^R=\max_m[(P-w)D_1-C]=\max_m(mD_1-C) \\ \qquad =\max_m\{m[(\theta+\gamma)\alpha-\beta(w+m)+\lambda P_2]-C\} \end{cases}$$

Theorem 2. In the MS game model, the optimal pricing strategies for the manufacturer and retailer are:

$$w^{MS}=\frac{(\theta+\gamma)\alpha(\beta+2\lambda)}{2\beta(\beta+\lambda)}+\frac{\alpha\lambda}{2(\beta^2-\lambda^2)} \quad (15)$$

$$P_2^{MS}=\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)}+\frac{\alpha\beta}{2(\beta^2-\lambda^2)} \quad (16)$$

$$m^{MS}=\frac{(\theta+\lambda)\alpha}{4\beta} \quad (17)$$

Proof. According to the reverse induction method, first, we need to solve the retailer's response function. Assuming that the manufacturer has decided the optimal wholesale price and online sales price, the retailer's profit function is maximized. The solution is derived as follows:

$$\begin{aligned} \max_m \pi_{DE}^R &= \max_m(mD_1-C) \\ &= \max_m\{m[(\theta+\gamma)\alpha-\beta(w+m)+\lambda P_2]-C\} \end{aligned} \quad (18)$$

By solving the first derivative of the above formula with respect to m, we get

$$\frac{\partial \pi_{DE}^R}{\partial m}=(\theta+\gamma)\alpha-\beta(w+2m)+\lambda P_2$$

By solving the second derivative with respect to m, we get

$$\frac{\partial^2 \pi_{DE}^R}{\partial m^2}=-2\beta$$

As $\beta\succ 0$, it is evident that the second derivative of π_{DE}^R is negative. Therefore, π_{DE}^R is a concave function with respect to m. Then, the solution for the retailer's optimal response function can be based on the first-order condition of the function π_{DE}^R

$$(\theta+\gamma)\alpha-\beta(w+2m)+\lambda P_2=0 \quad (19)$$

Solving the above formula, the optimal response function of the retailer can be obtained as:

$$m^{ms}(w,P_2)=\frac{(\theta+\gamma)\alpha+\lambda P_2}{2\beta}-\frac{w}{2} \quad (20)$$

Next, consider the manufacturer's profit function as

$$\begin{aligned} \pi_{DE}^M &= w[(\theta+\gamma)\alpha-\beta(w+m)+\lambda P_2] \\ &\quad +P_2[(1-\theta-\gamma)\alpha-\beta P_2+\lambda(w+m)] \end{aligned} \quad (21)$$

By substituting $m^{ms}(w,P_2)$ into the profit function of the manufacturer, the profit maximization problem can be expressed as:

$$\begin{aligned} \max_{w,P_2} \pi_{DE}^M &= \frac{w}{2}[(\theta+\gamma)\alpha-\beta w+\lambda P_2] \\ &\quad +P_2[(1-\theta-\gamma)\alpha-\beta P_2+\frac{\lambda w}{2}+\frac{\lambda(\theta+\gamma)\alpha+\lambda^2P_2}{2\beta}] \end{aligned} \quad (22)$$

Its first-order condition is

$$\frac{\partial \pi_{DE}^M}{\partial w}=\frac{(\theta+\gamma)\alpha}{2}-\beta w+\lambda\beta_2$$

$$\frac{\partial \pi_{DE}^M}{\partial P_2}=(1-\theta-\gamma)\alpha+\lambda w+(\frac{\lambda^2}{\beta}-2\beta)P_2+\frac{\lambda(\theta+\gamma)\alpha}{2\beta}$$

Therefore, the available Hessian matrix of the manufacturer's profit function is

$$H=\begin{bmatrix} \frac{\partial^2\pi_{DE}^M}{\partial w^2} & \frac{\partial^2\pi_{DE}^M}{\partial w\partial P_2} \\ \frac{\partial^2\pi_{DE}^M}{\partial P_2\partial w} & \frac{\partial^2\pi_{DE}^M}{\partial P_2^2} \end{bmatrix}=\begin{bmatrix} \text{-}\beta & \lambda \\ \lambda & \frac{\lambda^2}{\beta}\text{-}2\beta \end{bmatrix}$$

As $\beta\succ 0$ and $\beta\succ\lambda$, the Hessian matrix of the manufacturer's profit function is negative definite. Therefore, the manufacturer's profit function is a concave function of w and P_2. Therefore, the solution to the manufacturer's profit maximization can be achieved through the two first-order conditions of the function.

$$\frac{(\theta+\gamma)\alpha}{2}-\beta w+\lambda\beta_2=0 \quad (23)$$

$$(1-\theta-\gamma)\alpha+\lambda w+(\frac{\lambda^2}{\beta}-2\beta)P_2+\frac{\lambda(\theta+\gamma)\alpha}{2\beta}=0 \quad (24)$$

By solving the equation set consisting of these two first-order conditions, we obtain the optimal wholesale price w^{ms} and the optimal online sales price P_2^{MS} set by the manufacturer:

$$w^{MS}=\frac{(\theta+\gamma)\alpha(\beta+2\lambda)}{2\beta(\beta+\lambda)}+\frac{\lambda\alpha}{2(\beta^2-\lambda^2)}$$

$$P_2^{MS}=\frac{(\theta+\gamma)\alpha}{2\beta(\beta+\lambda)}+\frac{\alpha\beta}{2(\beta^2-\lambda^2)}$$

By substituting w^{MS} and P_2^{MS} into equation (16), the retailer's optimal marginal profit m^{MS} is obtained.

$$m^{MS}=\frac{(\theta+\gamma)\alpha}{4\beta}$$

Thus, the proof of Theorem 2 is complete.

Based on the above results, the offline sales price P_1^{MS} and the online and offline market demand of the retailers are:

$$P_1^{MS}=\frac{(\theta+\gamma)\alpha(3\beta+5\lambda)}{4\beta(\beta+\lambda)}+\frac{\lambda\alpha}{2(\beta^2-\lambda^2)} \tag{25}$$

$$D_1=(\theta+\gamma)\alpha-\beta P_1+\lambda P_2=\frac{(\theta+\gamma)\alpha}{4} \tag{26}$$

$$D_2=\frac{\alpha}{2}-\frac{(\theta+\gamma)\alpha(6\beta-5\lambda)}{4\beta} \tag{27}$$

According to equations (15), (16), and (17) and (12), (13), and (14), we can deduce the maximum profits of the retailer, the manufacturer, and the whole supply chain.

$$\pi_{DE}^{R}=\frac{(\theta+\gamma)^2\alpha^2}{16\beta}-C \tag{28}$$

$$\pi_{DE}^{M}=\frac{(\theta+\gamma)^2\alpha^2(7\lambda-5\beta)}{8\beta(\beta+\lambda)}-\frac{(\theta+\gamma)\alpha^2}{2(\beta+\lambda)}+\frac{\alpha^2\beta}{4(\beta^2-\lambda^2)} \tag{29}$$

$$\pi_{DE}^{SC}=\frac{(\theta+\gamma)^2\alpha^2(7\lambda-5\beta)}{8\beta(\beta+\lambda)}-\frac{(\theta+\gamma)\alpha^2}{2(\beta+\lambda)}+\frac{\alpha^2\beta}{4(\beta^2-\lambda^2)}+\frac{(\theta+\gamma)^2\alpha^2}{16\beta}-C \tag{30}$$

Proposition 3. For both the offline retail channel and the online direct channel, the price is directly proportional to the potential market demand.

Proof. The derivatives of equations (25) and (16) with respect to α, respectively, can be obtained.

$$\frac{\partial P_1^{MS}}{\partial\alpha}=\frac{(\theta+\gamma)(3\beta+5\lambda)}{4\beta(\beta+\lambda)}+\frac{\lambda}{2(\beta^2-\lambda^2)} \tag{31}$$

$$\frac{\partial P_2^{MS}}{\partial\alpha}=\frac{(\theta+\gamma)}{2\beta(\beta+\lambda)}+\frac{\beta}{2(\beta^2-\lambda^2)} \tag{32}$$

Since $\beta\succ 0$ and $\beta\succ\lambda$, we get $\frac{\partial P_2^{MS}}{\partial\alpha}\succ 0$ and $\frac{\partial P_1^{MS}}{\partial\alpha}\succ 0$ easily. Therefore, for both the offline retail channel and the online direct channel, the sales price is proportional to the potential market demand α.

Thus, the proof of Proposition 3 is complete.

Proposition 4. The optimal price in offline retail channels is proportional to the sales effort made by the retailer; the optimal price in online direct channels is also proportional to this sales effort.

Proof. Since $\gamma=kC$, the derivative of equations (25) and (16) with respect to C, respectively, can be obtained.

$$\frac{\partial P_1^{MS}}{\partial C}=\frac{k\alpha(3\beta+5\lambda)}{4\beta(\beta+\lambda)} \tag{33}$$

$$\frac{\partial P_2^{MS}}{\partial C}=\frac{k\alpha}{2\beta(\beta+\lambda)} \tag{34}$$

Since $\beta\succ 0$, $\lambda\succ 0$, $k\succ 0$, and $\alpha\succ 0$, we get $\frac{\partial P_1^{MS}}{\partial C}\succ 0$ and $\frac{\partial P_2^{MS}}{\partial C}\succ 0$. Therefore, the optimal prices in the offline retail channel and the online direct channel are both in direct proportion to the retailer's sales effort. The reason is as follows. Although the offline retailer's sales effort to achieve more market share and greater profit and the online direct channels form a competitive relationship, it will also attract more potential, high-quality customers and expand the market space of the products; thus, the price in offline retail channels and online direct channels will increase simultaneously. Concomitantly, the benefits in the two channels also increases.

On the contrary, with an increase in the offline retailer's sales effort, the cost of this effort also increases, which may subsequently reduce its profit.

The derivative of equation (28) with respect to the sales effort cost C is

$$\frac{\partial\pi_{DE}^{R}}{\partial C}=\frac{(\theta+\gamma)k\alpha^2}{8\beta}-1 \tag{35}$$

Using the first-order condition $\frac{\partial\pi_{DE}^{R}}{\partial C}=0$, the optimal sales effort cost can be obtained as follows:

$$C^*=\frac{8\beta}{k^2\alpha^2}-\frac{\theta}{k}$$

Through the above analysis, we find that increasing the sales effort of the offline retailer will only reduce its own profit. Only when the offline retailer makes an appropriate level of sales effort can it obtain the greatest benefits while occupying a certain market position.

2) RS game model

In the RS supply chain structure, the manufacturer's scale and market share are relatively small, so it is in a subordinate position to the retailer. In this case, the retailer is the leader of the market, and the manufacturer is the follower. The game sequence is as follows: First, the retailer sets the unit profit m and price of products P_1 sold through offline channels based on its profit maximization goal. After observing the retailer's pricing strategy, the manufacturer responds; that is, according to the retailer's profit maximization goal, it sets the wholesale price w of the product and the online direct price P_2. Again, reverse induction is used in the solution, which is mainly reflected in the fact that the retailer incorporates the manufacturer's (follower's) response function into its profit function when deciding the optimal pricing strategy. Therefore, the expression of the RS game model is:

$$\begin{aligned}\max_{m}\pi_{DE}^{R}&=(P_1-w)D_1-C\\&=m[(\theta+\gamma)\alpha-\beta(w+m)+\lambda P_2]-C\end{aligned}$$

$$s.t.\begin{cases} w,P_2=\arg\max\pi_{DE}^{M}\\ \max\limits_{w,P_2}\pi_{DE}^{M}=\max\limits_{w,P_2}(wD_1+P_2D_2)\\ =\max\limits_{w,P_2}\{w[(\theta+\gamma)\alpha-\beta P_1+\lambda P_2]+P_2[(1-\theta-\gamma)\alpha-\beta P_2+\lambda P_1]\}\\ =\max\limits_{w,P_2}\{w[(\theta+\gamma)\alpha-\beta(w+m)+\lambda P_2]+P_2[(1-\theta-\gamma)\alpha-\beta P_2+\lambda(w+m)]\}\end{cases}$$

Theorem 3. In the RS game model, the optimal pricing strategies of the retailer and the manufacturer are:

$$m^{RS}=\frac{(\theta+\gamma)\alpha}{2\beta} \tag{36}$$

$$w^{RS}=\frac{(\theta+\gamma)\alpha(\beta-\lambda)}{4\beta(\beta+\lambda)}+\frac{\lambda\alpha}{2(\beta^2-\lambda^2)} \tag{37}$$

$$P_1^{RS}=\frac{(\theta+\gamma)\alpha(3\beta+\lambda)}{4\beta(\beta+\lambda)}+\frac{\lambda\alpha}{2(\beta^2-\lambda^2)} \quad (38)$$

$$P_2^{RS}=\frac{\alpha\beta}{2(\beta^2-\lambda^2)}-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)} \quad (39)$$

Proof. Following the reverse induction method, we first solve the manufacturer's response function. Assuming that the retailer has made the optimal unit profit and offline sales price decisions, the manufacturer's profit maximization problem is solved as follows:

$$\begin{aligned}&\max_{w,P_2}\pi_{DE}^M\\&=\max_{w,P_2}\{w[(\theta+\gamma)\alpha-\beta(w+m)+\lambda P_2]+P_2[(1-\theta-\gamma)\alpha-\beta P_2+\lambda(w+m)]\}\end{aligned} \quad (40)$$

The first-order partial derivatives of equation (38) with respect to w and P_2, respectively, are:

$$\frac{\partial\pi_{DE}^M}{\partial w}=(\theta+\gamma)\alpha-2\beta w-\beta m+2\lambda P_2$$

$$\frac{\partial\pi_{DE}^M}{\partial P_2}=(1-\theta-\gamma)\alpha+2\lambda w+\lambda m-2\beta P_2$$

The second-order partial derivatives of equation (40) with respect to w and P_2, respectively, are:

$$\frac{\partial^2\pi_{DE}^M}{\partial w^2}=-2\beta\text{；}\ \frac{\partial^2\pi_{DE}^M}{\partial P_2^2}=-2\beta$$

Therefore, the Hessian matrix of π_{DE}^M is

$$H=\begin{bmatrix}\frac{\partial^2\pi_{DE}^M}{\partial w_1^2} & \frac{\partial^2\pi_{DE}^M}{\partial w\partial P_2}\\ \frac{\partial^2\pi_{DE}^M}{\partial P_2\partial w} & \frac{\partial^2\pi_{DE}^M}{\partial P_2^2}\end{bmatrix}=\begin{bmatrix}-2\beta & 2\lambda\\ 2\lambda & -2\beta\end{bmatrix}$$

Since $\beta\succ\lambda\succ0$, the Hessian matrix of π_{DE}^M is negative definite. It is evident that π_{DE}^M is the concave function of w and P_2. Therefore, according to the following first-order conditions on w and P_2, the manufacturer's optimal pricing strategy can be obtained.

$$P_2^{RS}=\frac{\alpha\beta}{2(\beta^2-\lambda^2)}-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)}$$

$$w^{RS}(m)=\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)}+\frac{\lambda\alpha}{2(\beta^2-\lambda^2)}-\frac{m}{2}$$

By substituting $w^{RS}(m)$ and P_2^{RS} into the retailer's profit function, we get

$$\begin{aligned}&\max_m\pi_{DE}^R\\&=m[(\theta+\gamma)\alpha-\frac{(\theta+\gamma)\alpha\beta}{2(\beta+\lambda)}-\frac{\lambda\alpha\beta}{2(\beta^2-\lambda^2)}-\frac{\beta}{2}m+\frac{\alpha\beta\lambda}{2(\beta^2-\lambda^2)}-\frac{(\theta+\gamma)\alpha\lambda}{2(\beta+\lambda)}{}_2]-C\end{aligned} \quad (41)$$

The first-order derivative of (43) with respect to m is:

$$\frac{\partial\pi_{DE}^R}{\partial m}=(\theta+\gamma)\alpha-\beta m-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)}(\beta+\lambda)$$

The second-order derivative of $\partial\pi_{DE}^R$ with respect to m is:

$$\frac{\partial\pi_{DE}^R}{\partial m}=-\beta$$

Since $\beta\succ0$, the second-order derivative of π_{DE}^R is negative. Therefore, π_{DE}^R is a concave function with respect to m. Then, the solution to the retailer's profit maximization problem can be based on the first-order condition of the function.

$$(\theta+\gamma)\alpha-\beta m-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)}(\beta+\lambda)=0 \quad (42)$$

Solving the above formula, we can get the unit profit when the retailer's profit is maximized as follows.

$$m^{RS}=\frac{(\theta+\gamma)\alpha}{2\beta}$$

The proof of Theorem 3 is hence complete.

According to the above results, the online and offline market demands of available products are:

$$D_1=\frac{(\theta+\gamma)\alpha}{4} \quad (43)$$

$$D_2=\frac{\alpha}{2}-\frac{(\theta+\gamma)\alpha(2\beta-\lambda)}{4\beta} \quad (44)$$

According to equations (38), (39), and (41) and equations (12), (13), and (14), the maximum profit of the retailer, the manufacturer, and the whole supply chain can be derived.

$$\pi_{DE}^R=\frac{(\theta+\gamma)^2\alpha^2}{8\beta}-C \quad (45)$$

$$\pi_{DE}^M=\frac{(\theta+\gamma)^2\alpha^2(\beta-3\lambda)}{16\beta(\beta+\lambda)}+\frac{\alpha^2\beta}{4(\beta^2-\lambda^2)}-\frac{(\theta+\gamma)\alpha^2}{4(\beta+\lambda)} \quad (46)$$

$$\pi_{DE}^{SC}=\frac{(\theta+\gamma)^2\alpha^2(3\beta-\lambda)}{16\beta(\beta+\lambda)}+\frac{\alpha^2\beta}{4(\beta^2-\lambda^2)}-\frac{(\theta+\gamma)\alpha^2}{4(\beta+\lambda)}-C \quad (47)$$

Proposition 5. For both offline retail channels or online direct channels, the price is proportional to the potential market demand.

Proof. The derivatives of equations (43) and (44) with respect to α are:

$$\frac{\partial P_1^{RS}}{\partial\alpha}=\frac{(\theta+\gamma)(3\beta+\lambda)}{4\beta(\beta+\lambda)}+\frac{\lambda}{2(\beta^2-\lambda^2)} \quad (48)$$

$$\frac{\partial P_2^{RS}}{\partial\alpha}=\frac{\beta}{2(\beta^2-\lambda^2)}-\frac{\theta+\gamma}{2(\beta+\lambda)} \quad (49)$$

Since $\beta\succ0$, $\beta\succ\lambda$, and $\frac{\beta}{\beta-\lambda}\succ\theta+\gamma$, it is easy to get $\frac{\partial P_2^{RS}}{\partial\alpha}\succ0$ and $\frac{\partial P_1^{RS}}{\partial\alpha}\succ0$. Therefore, for both offline retail channels and online direct channels, the sales price is proportional to the potential market demand α.

Proposition 6. The optimal price in offline retail channels is directly proportional to the retailer's sales effort, while the optimal price in online direct channels is inversely proportional to this effort.

Proof. Because $\gamma=kC$, the derivative of equations (38) and (39) with respect to C are as follows:

$$\frac{\partial P_1^{RS}}{\partial C}=\frac{k\alpha(3\beta+\lambda)}{4\beta(\beta+\lambda)} \quad (50)$$

$$\frac{\partial P_2^{RS}}{\partial C}=-\frac{k\alpha}{2(\beta+\lambda)} \quad (51)$$

Since $\beta\succ0$, $\lambda\succ0$, $k\succ0$, and $\alpha\succ0$, it is easy to get $\frac{\partial P_1^{RS}}{\partial C}\succ0$ and $\frac{\partial P_2^{RS}}{\partial C}\prec0$. Therefore, the optimal price P_1^{RS} of offline retail channels increases with an increase in the offline retailer's sales effort, while the optimal price P_2^{RS} of online direct channels decreases with an increase in the offline retailer's sales effort. The reason is as follows: when the retailer occupies the dominant market position, the offline retailer makes more sales effort to achieve greater market share and competes with the manufacturer, and thus, it may lower the price in online direct channels in response to this competition.

However, simultaneously, as the sales effort of the offline retailer increases, so does the cost C of this effort, which may reduce the profit of the offline retailer.

The first-order derivative of equation (45) with respect to the sales effort C is:

$$\frac{\partial \pi_{DE}^{R}}{\partial C}=\frac{(\theta+kC)k\alpha^{2}}{4\beta}-1$$

The second-order derivative of equation (45) with respect to the sales effort C is:

$$\frac{\partial \pi_{DE}^{R}}{\partial C}=\frac{k^{2}\alpha^{2}}{4\beta}$$

Since $\beta \succ 0$, $k \succ 0$, and $\alpha \succ 0$, it is easy to get $\frac{\partial \pi_{DE}^{R}}{\partial C} \succ 0$. Therefore, π_{DE}^{R} has a minimum value.

The optimal sales effort cost can be obtained by using the first-order condition $\frac{\partial \pi_{DE}^{R}}{\partial C}=0$,

$$C^{*}=\frac{4\beta}{k^{2}\alpha^{2}}-\frac{\theta}{k}$$

Through the above analysis, we find that excessive sales effort by the offline retailer reduces its profit. Only when the offline retailer makes an appropriate level of sales effort can it gain the maximum benefits while occupying a certain market position.

C. Model Comparison

So far, we have examined the optimal pricing strategies of the manufacturer and retailer in a dual-channel supply chain in centralized and decentralized decision-making models. In this section, we conduct a comparative analysis of the two decision models from two aspects: the influence of cross-price elasticity coefficient λ on P_1 and P_2 and the influence of the market share θ of the offline sales channel on P_1 and P_2.

Proposition 7. In the centralized decision-making model, the offline sales price is proportional to the online sales price and the cross elasticity of demand. In the decentralized decision-making model, when the manufacturer dominates the supply chain, the offline sales price is proportional to the cross elasticity of demand, and the online sales price is the concave function of this cross elasticity. When the retailer dominates the supply chain, the online sales price is proportional to the cross elasticity of demand, and the offline sales price is the concave function of this cross elasticity. That is,

$$\frac{\partial P_1^{C}}{\partial \lambda}\succ 0,\quad \frac{\partial P_2^{C}}{\partial \lambda}\succ 0;\quad \frac{\partial P_1^{MS}}{\partial \lambda}\succ 0,$$

If $(\theta+\gamma)\prec\frac{2\beta\lambda}{(\beta-\lambda)^{2}}$, then $\frac{\partial P_2^{MS}}{\partial \lambda}\prec 0$;

If $(\theta+\gamma)=\frac{2\beta\lambda}{(\beta-\lambda)^{2}}$, then $\frac{\partial P_2^{MS}}{\partial \lambda}=0$;

If $(\theta+\gamma)\succ\frac{2\beta\lambda}{(\beta-\lambda)^{2}}$, then $\frac{\partial P_2^{MS}}{\partial \lambda}\succ 0$.

If $(\theta+\gamma)\prec\frac{\beta^{2}+\lambda^{2}}{(\beta-\lambda)^{2}}$, then $\frac{\partial P_1^{RS}}{\partial \lambda}\prec 0$;

If $(\theta+\gamma)=\frac{\beta^{2}+\lambda^{2}}{(\beta-\lambda)^{2}}$, then $\frac{\partial P_1^{RS}}{\partial \lambda}=0$;

If $(\theta+\gamma)\succ\frac{\beta^{2}+\lambda^{2}}{(\beta-\lambda)^{2}}$, then $\frac{\partial P_1^{RS}}{\partial \lambda}\succ 0$.

$$\frac{\partial P_2^{RS}}{\partial \lambda}\prec 0.$$

Proof. Since $\alpha \succ 0$, $\beta \succ 0$, $\lambda \succ 0$, and $\beta \succ \lambda$, it is easy to verify that

$$\frac{\partial P_1^{C}}{\partial \lambda}=\frac{(1-\theta)\alpha}{2(\beta^{2}-\lambda^{2})}+\frac{\theta\alpha\beta\lambda}{(\beta^{2}-\lambda^{2})^{2}}\succ 0$$

$$\frac{\partial P_2}{\partial \lambda}=\frac{\theta\alpha(\beta^{2}+\lambda^{2})}{2(\beta^{2}-\lambda^{2})^{2}}+\frac{(1-\theta)\alpha\beta\lambda}{(\beta^{2}-\lambda^{2})^{2}}\succ 0$$

$$\frac{\partial P_1^{MS}}{\partial \lambda}=\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)^{2}}+\frac{\alpha(\beta^{2}+\lambda^{2})}{2(\beta^{2}-\lambda^{2})^{2}}\succ 0$$

$$\frac{\partial P_2^{MS}}{\partial \lambda}=-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)^{2}}+\frac{\alpha\beta\lambda}{(\beta^{2}-\lambda^{2})^{2}}$$

When $(\theta+\gamma)\prec\frac{2\beta\lambda}{(\beta-\lambda)^{2}}$, we obtain

$$\frac{\partial P_2^{MS}}{\partial \lambda}=-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)^{2}}+\frac{\alpha\beta\lambda}{(\beta^{2}-\lambda^{2})^{2}}\prec 0;$$

When $(\theta+\gamma)=\frac{2\beta\lambda}{(\beta-\lambda)^{2}}$, we obtain

$$\frac{\partial P_2^{MS}}{\partial \lambda}=-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)^{2}}+\frac{\alpha\beta\lambda}{(\beta^{2}-\lambda^{2})^{2}}=0;$$

When $(\theta+\gamma)\succ\frac{2\beta\lambda}{(\beta-\lambda)^{2}}$, we obtain

$$\frac{\partial P_2^{MS}}{\partial \lambda}=-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)^{2}}+\frac{\alpha\beta\lambda}{(\beta^{2}-\lambda^{2})^{2}}\succ 0.$$

$$\frac{\partial P_1^{RS}}{\partial \lambda}=-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)^{2}}+\frac{\alpha(\beta^{2}+\lambda^{2})}{2(\beta^{2}-\lambda^{2})^{2}}$$

When $(\theta+\gamma)\prec\frac{\beta^{2}+\lambda^{2}}{(\beta-\lambda)^{2}}$, we obtain

$$\frac{\partial P_1^{RS}}{\partial \lambda}=-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)^{2}}+\frac{\alpha(\beta^{2}+\lambda^{2})}{2(\beta^{2}-\lambda^{2})^{2}}\prec 0;$$

When $(\theta+\gamma)=\frac{\beta^{2}+\lambda^{2}}{(\beta-\lambda)^{2}}$, we obtain

$$\frac{\partial P_1^{RS}}{\partial \lambda}=-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)^{2}}+\frac{\alpha(\beta^{2}+\lambda^{2})}{2(\beta^{2}-\lambda^{2})^{2}}=0;$$

When $(\theta+\gamma)\succ\frac{\beta^{2}+\lambda^{2}}{(\beta-\lambda)^{2}}$, we obtain

$$\frac{\partial P_1^{RS}}{\partial \lambda}=-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)^{2}}+\frac{\alpha(\beta^{2}+\lambda^{2})}{2(\beta^{2}-\lambda^{2})^{2}}\succ 0.$$

$$\frac{\partial P_2^{RS}}{\partial \lambda}=-\frac{(\theta+\gamma)\alpha}{2(\beta+\lambda)^{2}}-\frac{\alpha\beta\lambda}{(\beta^{2}-\lambda^{2})^{2}}\prec 0.$$

Thus, the proof of Proposition 7 is complete.

Proposition 8. In the centralized decision-making model, the offline sales price is directly proportional to the offline sales market share, while the online sales price is inversely proportional to this share. Conversely, in the decentralized decision-making model, when the manufacturer is in the dominant position, the offline sales price and online sales channel are both directly proportional to the market share of the offline sales. Meanwhile, when the retailer is in the dominant position, the offline sales price is directly proportional to the offline sales market share, while the online sales price is inversely proportional to this share. That is,

$$\frac{\partial P_1^C}{\partial\theta} \succ 0, \quad \frac{\partial P_2^C}{\partial\theta} \prec 0;$$

$$\frac{\partial P_1^{MS}}{\partial\theta} \succ 0, \frac{\partial P_2^{MS}}{\partial\theta} \succ 0;$$

$$\frac{\partial P_1^{RS}}{\partial\theta} \succ 0, \frac{\partial P_2^{RS}}{\partial\theta} \succ 0.$$

Proof. Since $\alpha \succ 0$, $\beta \succ 0$ and $\lambda \succ 0$, it is easy to verify that

$$\frac{\partial P_1^C}{\partial\theta} = \frac{\alpha}{2(\beta+\lambda)} \succ 0;$$

$$\frac{\partial P_2^C}{\partial\theta} = -\frac{\alpha}{2(\beta+\lambda)} \prec 0;$$

$$\frac{\partial P_1^{MS}}{\partial\theta} = \frac{\alpha(3\beta+5\lambda)}{4\beta(\beta+\lambda)} \succ 0;$$

$$\frac{\partial P_2^{MS}}{\partial\theta} = \frac{\alpha}{2(\beta+\lambda)} \succ 0;$$

$$\frac{\partial P_1^{RS}}{\partial\theta} = \frac{\alpha(3\beta+\lambda)}{4\beta(\beta+\lambda)} \succ 0;$$

$$\frac{\partial P_2^{RS}}{\partial\theta} = -\frac{\alpha}{2(\beta+\lambda)} \prec 0.$$

Therefore, the proof of Proposition 8 is complete.

IV. A NUMERICAL EXAMPLE

In this section, we use a numerical example to explain the three game models above in the two decision-making models. We analyse the impact of dual-channel sales behaviour on corporate pricing decisions. We assume that the values of each parameter are: α=200, β=10, γ=6.

According to the calculation results in section III, the dual-channel optimal pricing strategy will vary with the cross elasticity of demand and the offline market share. The optimal pricing level in the centralized model and the decentralized model is shown in Table II.

Table II

The relationship between the optimal pricing strategy and λ and θ in the centralised and decentralised decision-making models

		λ	Centralized decision-making model		Decentralized decision-making model: Manufacturer-led		Decentralized decision-making model: Retailer-led	
			P_1	P_2	P_1	P_2	P_1	P_2
γ=0.1	θ=0.1	1	1.92	9.19	4.19	11.92	3.83	8.28
		2	2.92	9.58	5.42	12.08	4.75	8.75
		5	7.33	12.67	10.33	14.67	9.00	12.00
		6	10.00	15.00	13.13	16.88	11.63	14.38
		9	47.89	52.11	51.32	53.68	49.42	51.58
	θ=0.5	1	5.56	5.56	10.56	15.56	9.46	4.65
		2	6.25	6.25	12.08	15.42	10.08	5.42
		5	10.00	10.00	17.67	17.33	13.67	9.33
		6	12.50	12.50	20.63	19.38	16.13	11.88
		9	50.00	50.00	59.21	55.79	53.53	49.47
	θ=0.6	1	6.46	4.65	12.15	16.46	10.87	3.74
		2	7.08	5.42	13.75	16.25	11.42	4.58
		5	10.67	9.33	19.50	18.00	14.83	8.67
		6	13.13	11.88	22.50	20.00	17.25	11.25
		9	50.53	49.47	61.18	56.32	54.55	48.95
γ=0.3	θ=0.1	1	1.92	9.19	7.37	13.74	6.65	6.46
		2	2.92	9.58	8.75	13.75	7.42	7.08
		5	7.33	12.67	14.00	16.00	11.33	10.67
		6	10.00	15.00	16.88	18.13	13.88	13.13
		9	47.89	52.11	55.26	54.74	51.47	50.53
	θ=0.5	1	5.56	5.56	13.74	17.37	12.28	2.83
		2	6.25	6.25	15.42	17.08	12.75	3.75
		5	10.00	10.00	21.33	18.67	16.00	8.00
		6	12.50	12.50	24.38	20.63	18.38	10.63
		9	50.00	50.00	63.16	56.84	55.58	48.42
	θ=0.6	1	6.46	4.65	15.33	18.28	13.69	1.92
		2	7.08	5.42	17.08	17.92	14.08	2.92
		5	10.67	9.33	23.17	19.33	17.17	7.33
		6	13.13	11.88	26.25	21.25	19.50	10.00
		9	50.53	49.47	65.13	57.37	56.61	47.89
γ=0.5	θ=0.1	1	1.92	9.19	10.56	15.56	9.46	4.65
		2	2.92	9.58	12.08	15.42	10.08	5.42
		5	7.33	12.67	17.67	17.33	13.67	9.33
		6	10.00	15.00	20.63	19.38	16.13	11.88
		9	47.89	52.11	59.21	55.79	53.53	49.47
	θ=0.5	1	5.56	5.56	16.92	19.19	15.10	1.01
		2	6.25	6.25	18.75	18.75	15.42	2.08
		5	10.00	10.00	25.00	20.00	18.33	6.67
		6	12.50	12.50	28.13	21.88	20.63	9.38
		9	50.00	50.00	67.11	57.89	57.63	47.37
	θ=0.6	1	6.46	4.65	18.51	20.10	16.51	0.10
		2	7.08	5.42	20.42	19.58	16.75	1.25
		5	10.67	9.33	26.83	20.67	19.50	6.00
		6	13.13	11.88	30.00	22.50	21.75	8.75
		9	50.53	49.47	69.08	58.42	58.66	46.84

Table II shows that the results of the numerical example prove the reliability of the foregoing conclusions. The numerical example revealed the following:

1) When γ and θ remain unchanged, as the cross elasticity of demand between the two channels increases, the prices in online sales channels in the centralized and the decentralized decision-making models both increase. The online sales price is always higher than the offline sales price, but the gap between these two will gradually narrow. Regarding decentralized decision-making, the results reveal that regardless of whether the supply chain is dominated by the manufacturer or the retailer, the online sales price of the manufacturer is always higher than the offline sales price of the retailer, considering the sales effort of the latter. As the cross elasticity of demand increases, the gap between the two becomes smaller. Compared with the retail-dominated supply chain, the equilibrium price in both offline retail channels and online direct channels in the manufacturer-dominated situation is higher.

2) When γ and λ remain unchanged, because the retailer's market share increases when it does not make sales effort, the offline sales price increases, while the online sales price decreases. When the retailer's market share is greater, online sales prices are higher than offline sales prices; however, as its market share increases, offline sales prices gradually become higher than online sales prices, and the gap between the two increases. Regarding decentralized decision-making, the results reveal that in a retailer-dominated supply chain, offline sales prices gradually increase, online sales prices gradually decrease, and the gap between the two gradually increases. In a manufacturer-dominated supply chain, both offline sales prices and online sales prices increase proportionately, online sales prices are higher than offline sales prices, and the gap between the two gradually decreases.

3) When λ and θ remain unchanged, with an increase in market share due to sales effort, the retailer's offline sales price and the manufacturer's online sales price in the centralized decision-making model are not subject to changes due to the increased effort. Regarding decentralized decision-making, the results reveal that with an increase in market share brought about by sales effort, the offline sales price gradually increases in the retailer-led supply chain, the price in online sales channels gradually decreases, and the gap between the two increases. In the manufacturer-led supply chain, the online sales price is higher than the offline sales price, and both increase with the increase in market share due to sales effort.

V. Conclusion

This study identifies a dual-channel optimal pricing strategy based on the retailer's sales effort, considering two scenarios: centralized and decentralized decision-making. For the latter, the different supply chain structures led by manufacturers and retailers are analysed. We also examine the manufacturer's dual-channel optimal pricing strategy and market capacity, demand elasticity, demand cross-elasticity between offline and online markets, the retailer's market share, sales effort to increase market share, as well as the relationship between other variables.

The results reveal that in the centralized decision-making model, the prices in the dual channels are consistent. In the decentralized decision-making model, although retailers make sales effort, the online sales price is always higher than the offline sales price of the retailer in a manufacturer-led supply chain, indicating that the retailer's sales effort increases the market demand. Although the manufacturer-led supply chain leads to an increase in market share and sales price, it also has a positive impact on the manufacturer: its online sales price and profit increase. However, when the supply chain is dominated by the retailer, the manufacturer's online direct sales prices continue to decline as the retailer increases its market share due to sales effort. Consequently, the retailer's offline sales channels continue to increase in price, and its profits expand.

This study has some limitations. First, we only consider the manufacturer's dual-channel pricing strategy in the case of retailers making sales effort. In the future, researchers can consider both the retailer's and manufacturer's sales effort. Second, we only consider a secondary supply chain consisting of one manufacturer and one retailer, which can be expanded to a multi-level supply chain of multiple manufacturers and retailers in future research to identify pricing strategies. Third, the demand function and sales effort function set in this study are both linear, and other types of functions can be used to expand the related research.

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1) Date of modification: June 23, 2023
2) Brief description of the changes:

Indicate the corresponding author to Mei-xiang Hong.