# A Hybrid Metaheuristic Algorithm for the School Bus Routing Problem with Multi-School Planning Scenarios

Yane Hou, Ning Zhao, Lanxue Dang, and Bingbing Liu

***Abstract*—In the practice of multi-school bus route planning, the characteristics of the bus fleet and the operation modes of buses have produced a variety of bus planning scenarios. In recent years, many methods have been used to arrange the routes of school buses for the school bus routing problem (SBRP) with a specific planning scenario. However, it is still a challenging task to develop a general-purpose algorithm that can effectively apply to the bus route planning for a variety of bus planning scenarios. This paper tries to develop a hybrid iterated local search (ILS) metaheuristic algorithm for SBRP with multiple planning scenarios, which including homogenous or heterogeneous fleets, single load or mixed load operation modes. Within the framework of ILS, a variety of neighborhood structures are used to improve the initial solution as well as the routes generated in the process of local search are recorded. For heterogeneous SBRP, the fleet adjustment strategy based on route segments is also applied. In addition, the perturbation mechanism and the acceptance of worse solutions within a certain deviation range are adopted to enhance the diversity of solutions. Finally, the local best solution is further promoted by the set partitioning procedure (SP), which is modeled by the history routes in the local search process. The experiment results prove that the proposed algorithm is effective and it also outperforms the existing algorithms for multi-school SBRP.**

***Index Terms*—school bus routing problem, general-purpose, iterated local search, set partitioning, hybrid metaheuristic.**

## I. Introduction

WITH the rapid development of China's the compulsory education and the continuous improvement of the national quality of life, the demand for the school bus service has become very urgent. Providing school bus services for the students in the stage of compulsory education not only can reduce the burden of the family, but also alleviate the pressure of traffic, especially for the relatively densely populated areas. Planning school bus routes is an important part of the school bus operation management. It is generally agreed that reasonable arrangement of school bus routes can not only provide high quality service, but also reduce the operation costs of the buses. School bus routing problem (SBRP) studies how to arrange a fleet of school buses to pick up some students from bus stations and deliver them to their school under conditions of constraints, such as bus capacity, maximum riding time or school time windows and so on [1]. SBRP belongs to vehicle routing problem (VRP), and it can be considered a variant of VRP. Like VRP, SBRP is a kind of NP-hard problem with extremely high computational complexity [1],[2]. Because of its complexity, it is quit inappropriate to arrange the routes of school bus manually. Since firstly introduced by Newton and Thomas [3], SBRP have been continuously researched for many years. A recent review of literature can be found in [1],[2].

From the classification of SBRP, the most common classification method is according to the number of schools served by the school bus. The SBRP can be a single-school or multi-school based bus service system. The single-school SBRP is similar with capacitated vehicle routing problem (CVRP) while just only considering capacity constraint, which have been studies in many existing SBRP literatures [4],[5]. For multi-school SBRP, there is one or many schools in a school bus route. It has single load and mixed load two operation modes. For single load SBRP, the schools are served by school bus in a certain order. While for mixed load SBRP, it allows the students from different schools to stay in the same school bus at the same time. The multi-school problem is more harder than single-school problem, because it has more complex constraints, such as visiting sequence of stops and schools, multiple schools time windows and so on. Thus the multi-school problem is always solved by heuristic algorithms [6],[7]. In recent ten years, metaheuristic algorithms are gradually applied for this problem [8],[9],[10]. In additional, the school bus may has different vehicle characteristics such as bus capacity, fixed cost, and per unit distance variable cost [1]. When planning bus routes with heterogeneous fleet, the SBRP problem is called heterogeneous school bus routing problem (HSBRP).

In the practice of school bus route planning for multiple schools, different planning scenarios will be generated, because there exist many factors such as the operation mode and the type of school bus, etc. Each of these factors can be considered as the characteristics of a particular multi-school SBRP, including school bus characteristics (homogeneous or heterogeneous) and the operation mode (single load or mixed load). In existing literatures, some methods are usually specially designed to solve the multi-school SBRP with one or more characteristics [8],[9],[10]. Although these methods could effectively solve the specific multi-school SBRP, but they have poor portability. To our best knowledge, some

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good unified algorithms have emerged for solving a class of VRP problems in recent years, such as tabu [11], iterated local search [12],[13], genetic algorithm [14], etc. While for SBRP, the literatures about such unified algorithms are still relatively few and they have been received very limited attention.

The goal of this paper is to develop a general-purpose hybrid metaheuristic algorithm solving several multi-school SBRP problems with different problem characteristics. In this paper, there are four multi-school SBRP problems to be solved, which include multi-school SBRP with homogeneous or heterogeneous school bus fleets, and each one also has single load and mixed load two kinds of operation modes. We propose a hybrid metaheuristic algorithm combining iterated local search (ILS) with the set partitioning procedure (SP). It has proved that SP can promote the quality of algorithm effectively [12],[15],[16]. In our former studies [17],[18], we find that the application of SP in iterated local search algorithm has an advantage of improving performance and parameters insensitivity. It has the application potential to be a unified algorithm solving for several SBRP problems.

In the framework of ILS metaheuristic algorithm, we use three neighborhood structures, which were original designed for pickup and delivery vehicle routing problem with time window (PDPTW), to explore solution space. For the obtained neighborhood solution, we also use perturbation method and allow accepting worse solution to add the diversity of the algorithm. To avoid short-sighted of local search, the routes found in the local search process are also recorded in the route pool. Then a SP model is made by these routes and then solved by CPLEX optimization software. The SP can be used as a kind of post optimization technology for our proposed algorithm. The results on benchmark instances verify the validity of the proposed algorithm.

The remaining of the paper is organized in the following. Section II gives the problem description of multi-school SBRP. Section III describes the design of our algorithm in detail. Computational results and findings are described in Section IV. Finally, some concluding remarks of this work are presented in Section V.

## II. Problem Description

Now we describe the multi-school SBRP problems. In this paper, we consider four multi-school SBRP problems, including homogeneous or heterogeneous school bus fleets, single load or mixed load operation mode. In order to describe these research issues, an unified problem model of multi-school heterogeneous SBRP with mixed load (MLHSBRP) is built for them. When the fleets are the same, we can consider the problem as a specific case of MLHSBRP. In additional, if we require the students from different schools not to stay simultaneously in the same bus, it is the single load SBRP. From this point of view, the single load SBRP can be also considered as a specific case of mixed load by adding the constraints to mixed load SBRP. Therefore, the MLHSBRP could be changed to other research issues by adding constraints or ignoring the difference of capacity and cost of different bus fleets.

The MLHSBRP could be modeled as a m-1 PDPTW problem like the mixed load SBRP in [10] or a set partitioning procedure model. In this article, we model the MLHSBRP as the set partitioning problem, and its formulation model based on SP is defined in the following.

Let $C$ be the set of stops. Assume $R$ be the set of all possible routes for multi-school SBRP and $S$ be a subset of $R$ ($S \subseteq R$). The set of bus types is denoted as $M = \{1, 2, 3..., m\}$. When all the bus types are the same, $M = \{1\}$. For the bus type $k$ ,its fixed cost and variable cost per mile are $f_k$ and $v_k$ . The $R_k$ is denoted as the subset of routes using bus type of $k$ and $S$= $\bigcup_{k\in M} R_k$. Each route $r$ ($r \in R_k$) has an associated cost $c_r$ and a binary variable $x_r$. When $x_r$ =1, it means that the route $r$ is one route of the final solution. We assume that $d_r$ is the total travel distance of route $r$. For homogeneous school bus fleets, we introduce a relatively large positive integer $M_0$ to ensure the number of routes as the first optimization objective. The total cost $c_r$ of route $r$ is defined as $M_0$ + $d_r$. While for heterogeneous SBRP, the total cost of route $r$ is related with the bus type, which including bus fixed cost and variable cost. When the bus type of route $r$ is $k$, $c_r$ is defined as $f_k$ + $d_r * v_k$. Let $I$ be the subset of the routes covering stop $i$ ($i \in C$,$I \subseteq S$). A set partitioning formulation for the four multi-school SBRP problems is given as follows:

Minimize

$$\sum_{r\in S} c_r x_r \tag{1}$$

s.t.

$$\sum_{r\in I} x_r = 1 \tag{2}$$

$$x_r \in \{0, 1\}, \forall r \in S \tag{3}$$

This SP model tries to select an optimal SBRP solution from the possible routes. The objective function is given in (1) which minimizes the total cost of all routes. Constraint in equation (2) guarantees that each stop must be covered exactly once. Constraint (3) defines the binary decision variables.

## III. Proposed Algorithm

### A. Description of the Proposed Algorithm

The proposed hybrid algorithm (ILS-SP) consists of an iterated local search algorithm and a set partitioning procedure. ILS is a metaheuristic algorithm, which has been successfully applied to various VRP variants [13]. In this paper, the components of the ILS are well designed to solve several SBRP problems, including neighborhood operators, perturbation methods and acceptance rules. At the same time, the intermediate routes in the locally optimal solution identified by ILS are recorded. After the execution of ILS, the optimal solution obtained by ILS is also recorded. Then, a SP model will be built based on the routes recorded in route pool and then is solved by CPLEX software. Finally, the solution solved by CPLEX software will compare with the solution obtained by ILS, and the solution with smaller objective value is the final solution.

### B. Initial Solution Construction

For the multi-school SBRP, obtaining a feasible initial solution is a very difficult job due to its complexity, especially for mixed load SBRP. Therefore, we use a two-stage method to get the initial solution of this problem. The first stage is to construct the initial routes of every school, and the second is to combine these routes to an initial solution of multi-school SBRP.

The initial construction method for multi-school SBRP are described in the following.

(1) First, divide a complex multi-school SBRP into several sub problems with only having one school according by the school node.

(2) Then, obtain an initial solution for every single-school SBRP. In this step, the initial solution of single-school SBRP is obtained by the cheapest insertion method [18].

(3) Finally, the initial feasible solution for multi-school SBRP is composed of the routes of every single-school SBRP.

## C. Neighborhood Structures

In the local search process of ILS metaheuristic algorithm, several neighborhood structures are used iteratively to explore the solution space to find the local optima solution. For multi-school SBRP, a student station and its paired school station must be in the same route. It is impossible to directly apply the neighborhood operators such as 2-opt and or-opt designed for general SBRP to solve the multi-school SBRP, especially for mixed load SBRP. Therefore, we use three point pair neighborhood operators, such as single paired insertion (SPI), swapping pairs between routes (SBR), and within route insertion (WRI), which were designed and applied successfully to solve PDPTW [19]. These three operations can be found in [10], and they also are described as follows.

(1) SPI tries to shift one student station from one route to another route. When a student station is moved from one route to another one, it and its destination school node must be in the same route. If the target school has already been in the target route, it just only need move the student station. Otherwise, the destination school node also must be inserted to the target route. In additional, we also determine whether necessary to remove the destination school node from the original route. If there exists some students from other student stops on original route belongs to the destination school of shifted student stop, the destination school must be stayed on the route. Otherwise, we remove it from the original route.

(2) SBR occurs in the two different routes, which exchanges a pair of student and school stations in one route with another pair of student and school stations in the other route. For SBR, the swap operation of the destination school nodes also need to consider the insertion of them to the new route as well as the removal of them from the original route.

(3) WRI relocates a pair of student and school stations to the best position in the same route. It is usually used to reduce the total travel distance.

These point pair neighborhood operators are executed in a fixed sequence, which SPI are followed by WRI and SBR. The moves of neighborhood operators will be accepted or rejected by the rules. First, a feasible neighborhood solution must be obtained by every move of these operators. And then, the move will bring about the costs saving for the current solution or the new neighborhood solution meets the demand for the rules of acceptance. While for multi-school SBRP with heterogeneous bus fleet, the fleet adjustment will be performed after SPI and SBR operators to decrease the total cost of objective.

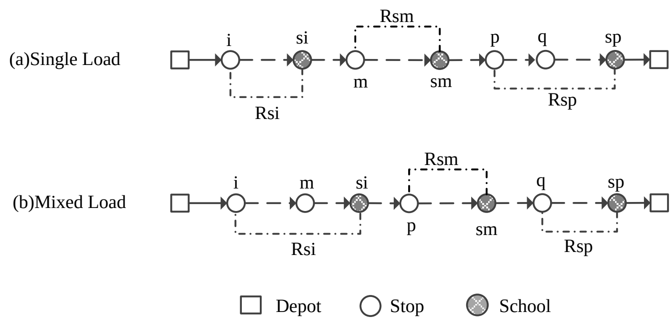



Fig. 1. An example for the partition of route segment

## D. Fleet Type Adjustment Strategy

For HSBRP, adjusting the bus type for a route not only can reduce its cost, but also can enhance the diversity of solutions in the local search process. We have successfully applied the fleet type adjustment strategy to single-school HSBRP in [18]. In order to get better solution, it is necessary to extend and apply the fleet type adjustment strategy to multi-school HSBRP. Therefore, in this section, we are going to focus on the fleet type adjustment strategy, which is used for multi-school HSBRP.

For multi-school HSBRP, there are some student stops and their destination school nodes in a route, while the route may be include several different school nodes. When the students destined to different schools could be permitted staying on the same bus, this is mixed load, the students getting on the bus and getting off the bus may be exist simultaneously. Because of possible empty load in the bus, it is unfeasible to adjust the bus fleet type directly on the whole route.

For this reason, we design a fleet adjustment strategy based on route segment. The core of this strategy is to divide the route into several path segments by the school node, and then find the available bus to meet the capacity requirements of all path segments. For the maximum actual load of the school bus found for each path segment, the fleet type adjustment is carried out by using the relationship between the maximum value and the current bus capacity. This fleet adjustment strategy can be also used for single-school HSBRP, when the route has only one route segment. From this point, the single-school HSBRP can be regarded as a special case of the multi-school HSBRP.

The fleet adjustment strategy based on route segment consists of partition of route segment and adjusting bus fleets. The first step is to find all route segments of a route. A route segment is defined as the set of a serial student stops and their target school nodes. The route can be divided into several route segments by the school nodes in it. Every route segment starts a student stops and ends with a school node. This partition method will generate a group of independent route segments without stops or school node crossing. If there exist two or more continuous school nodes on the route, only the valid route segments are considered. Fig 1 shows the partition of route segment for multi-school SBRP.

In Fig 1, $i$, $m$, $p$ and $q$ are student stops, $si$ and $sm$ are stops $i$ and $m$ destined school nodes respectively. The stop $p$ and stop $q$ will go to the same school $sq$. When the bus operation mode is single load (Fig 1(a)), the schools are visited in order, and the route can be divided into three route segments, such as $Rsi$, $Rsm$ and $Rsp$. While for mixed load SBRP, each route segment may be include student stops with different targeted school. As shown in Fig 1(b), route segment $Rsi$ includes two student stops $i$ and $m$ with different targeted school. There are also three route segments in the route. But if there are no student stops between each two adjacent school nodes, the number of route segment will decrease. If student stop $q$ is not in the middle of $sm$ and $sp$, the route segment $Rsp$ is invalid. That is to say, the actual route segments of the route are $Rsi$ and $Rsm$.

The second step of fleet adjustment strategy tries to find a lower-cost bus in a feasible route or a little bigger bus in a route just only violating capacity constraints. Under these condition, it needs to consider the relationship between the actual load of every route segment and the capacity of bus fleet type. Meanwhile, the fleet adjustment may fail because there may be not any school bus to serve the route. If it happens, the fleet adjustment operation will do nothing.

The fleet type adjustment strategy is described in the following. Let the route $r$ using bus type of $k$ and the bus capacity is $Q_k$. The total cost of route $r$ is ${C_r}^k$. The set of route segments in route $r$ is denoted as $R = \{R_1, R_2, ..., R_n\}$. The actual load of route segment $R_1$ is $Q_{R1}$, and the maximal actual load of school bus in all route segments is $Q_{max} = max\{Q_{R1}, Q_{R2}, ..., Q_{Rn}\}$. The set of bus types is denoted as $M = \{1, 2, ..., m\}$, which is ordered by capacity of bus type ascending. If $Q_k = Q_{max}$ , then leave it alone. If $Q_{max} < Q_k$, then it tries to find a low-cost bus to server this route. For each bus type in the set of bus types $\{1, 2, ..., k-1\}$ , seek the bus type $j$ that is satisfy with $Q_{max} \leq Q_j$ and makes ${C_r}^k - {C_r}^j$ with the minimum value. When we cannot find the bus type $j$ or $k$ is the smallest bus type in $M$, it will do nothing. If $Q_{max} > Q_k$, then it represents the number of students exceed the bus capacity constraints, it needs to find a new bus to server this route. For each bus type in the set of bus types $\{k+1, k+2, ..., m\}$, search the bus type $j$ that is satisfy with $Q_{max} \leq Q_j$ and makes ${C_r}^j - {C_r}^k$ with the minimum value. When the bus type that meets the above constraints cannot be found, or $k$ is the biggest bus type in $M$, we will not adjust the fleet type.

### *E. Perturbation Mechanism*

ILS uses perturbation methods to skip out the local optima, and it repeats the local search from another starting point to improve the performance of ILS. Usually, the multi-point shift or multi-point swap can be utilized to perturb the current solution [17],[18]. While these perturbation methods are not suitable for multi-school SBRP. It is a time-consuming work to find a feasible neighborhood solution after perturbing the current local solution because of difficultly induced by large scale multi-school problem.

In order to decrease the complexity of disturbance method, we randomly select a route with more than two school nodes, and then truncate it into multiple feasible routes by school nodes. At the same time, we also design another perturbation method based on SPI neighborhood operator. We randomly select 2∼5 student stops, and then use SPI operator move them to another routes. These two perturbation methods are randomly executed when the objective value of solution has not been decreased in five consecutive iterations.

### *F. Acceptance Rules*

The optimization objective is decided by the problem type of solved SBRP. For homogenous SBRP, the number of routes or total travel distance is the mainly the optimization objectives. If two solutions obtained by ILS with the same number of school buses, we think the solution with shorter total travel distance is better. So, we adopt a lexicographic neighborhood solution function defined in [10] to determine whether accept the solution or not. If the number of routes is reduced, the solution must be accepted. If the number of routes does not change, it tends to accept the shorter total travel distance. Otherwise, the solution is accepted or not by the record-to-record travel (RRT) [20] acceptance criterion. The little worse solution could be accepted, if the product of its total travel distance and deviation coefficient is in a certain range of values.

For heterogeneous SBRP, the optimization objective is to minimize the total cost, which includes fixed and variable costs of routes. The total cost of solution is closely related to the number of buses required and the combination of the fleet types. The total cost could be reduced by decreasing the number of bus routes, because the fixed cost is usually more higher than the travel cost per distance unit for each school bus. Therefore, we first evaluate the number of routes and then the total cost. Similarly, some worsening neighborhood solutions could be accepted according to the RRT acceptance criterion [20].

### *G. Set Partitioning Procedure*

The final step of our proposed algorithm is set partitioning procedure, which may find a better SBRP solution from a global point view. ILS algorithm has been successfully used to solving various routing problems, but this kind of local search-based algorithm has the shortcoming of short-sighted owing to the relative small neighborhood search space. We use the set partition procedure as a post improvement technology to enhance the performance of the algorithm on the basis of existing successful experience in [12],[15],[16],[17],[18].

In the local search process of our ILS algorithm, the routes explored by the neighborhood operators are recorded in a route pool. These historical routes are collected to build a SP model according to the definition in Section II. We utilize CPLEX optimization software to solve the SP model. Because of its weakly NP-hard, SP model can be solved efficiently by a MIP solver in reasonable time. The combining ILS with SP can make full use of the advantages of ILS and exact algorithm, and it also will improve the effective of the algorithm.

## IV. Computation Results

The proposed ILS-SP algorithm described in Section III was implemented by C # programming. All the experiments have been executed on a personal computer with an Intel i7-6700 3.40GHz CPU, and with 8GB of memory. The number of maximum iteration is set to 50, and the number of moved nodes in perturbation methods is a random integer number within the range of [2, 5] to balance the quality and computation time. The value of deviation coefficient that is used in acceptance rules is $10^{-5}$. The integer $M_0$ in SP model is set to $10^6$. The SP model are solved by CPLEX 12.6 software. For CPLEX, the maximum computation time was set to 60 seconds and the *MIPGap* parameter was set to $10^{-10}$. Meanwhile, each instance was executed 10 times by the ILS algorithm.

### A. Test Instances

We use the mixed load SBRP benchmark instances in this research, which were proposed by [21]. The instances including two groups: random spatial distribution of schools and bus stops (RSRB) and clustered distribution (CSCB). We select 8 benchmark instances, which are RSRB01∼RSRB04 and CSCB01∼CSCB04. The number of schools is 6∼25, and the number of stops is 250∼500. The settings of benchmark instances are the same as that defined in [10],[21],[22]. Just to be clear, the homogeneous school bus has the capacity of 66. The maximum riding time (MRT) of students are set to 2700 and 5400 seconds respectively. The distance between any two nodes is calculated by Manhattan distance. While for multi-school SBRP with heterogeneous bus fleet, we assume that all the instances used in this paper has three bus types donated as A, B and C. For each bus type of them, their capacity is 50, 60 and 70 respectively, and their fixed cost is 2500, 2800 and 3000. The average speed of each bus type is 20 mile per hour, and their variable cost per mile is set to 1.2, 1.3 and 1.5.

### B. Homogeneous multi-school SBRP

For homogeneous multi-school SBRP, the optimization objective is to minimize the number of school buses. We first use the ILS without SP (donated as ILS) and ILS-SP to solve two multi-school SBRP problems respectively, including single load and mixed load SBRP problems. In additional, we also compare our algorithm with other existing algorithms.

TABLE I shows the solutions obtained by ILS and ILS-SP for multi-school homogeneous SBRP instances with two operation modes. Columns *Name*, *Stops*, *Schools* are the basic description of instances, which include instance name, the number of student stops and the number of schools. The columns $N_s$ and $N_m$ denote the best route number respectively. The average computation time in seconds are shown in columns $T_s$ and $T_m$ respectively. Columns $G_s$ and $G_m$ represent the percentage deviation of the number of routes between the best solution obtained by ILS and that was found by ILS-SP for each instance respectively.

Compared with ILS algorithm, the ILS-SP algorithm is effective, which can find the best solution on average. For homogeneous SBRP with single load, the ILS-SP algorithm can reduces the number of buses by 0.57% on average. While for mixed load SBRP, the ILS-SP algorithm decrease the number of buses by 1% on average. The maximum percentage of improvement on these two SBRP problems are 3.57% and 4.17% separately. The ILS-SP algorithm decreases 4 and 6 buses in total respectively. The results show that combining ILS with the set partitioning procedure can improve the performance of ILS algorithm. Because of adding the set partitioning procedure, the ILS-SP algorithm needs more computation time to some extent.

To further evaluate the effective of ILS-SP algorithm, we compare it with the existing multi-school SBRP algorithms, such as post optimization heuristic algorithm (PH) [21], simulate annealing(SA) [22] and record-to-record travel (RRT) [10]. TABLE II shows the results found by ILS-SP algorithms and other algorithms for each instance. For single load SBRP, the columns *PH* and *SA* denote the results from [21] and [22]. While for mixed load SBRP, columns *PH* and *RRT* denote the results from [21] and [10] respectively. Columns $G_p$ , $G_s$ and $G_r$ represent the improvement percentage of the best route number obtained by ILS-SP algorithm for each instance when compared with PH, SA and RRT respectively.

As shown in TABLE II, ILS-SP algorithm is more competitive than existing algorithms. Compared with post optimization heuristic algorithm, the ILS-SP algorithm decreases the number of school buses by 25.10% and 10.62% on average respectively for single load and mixed load SBRP. For all the instances, ILS-SP algorithm obtain the better solutions than PH. The maximum percentages of improvement on two SBRP problems are 34.29% and 16.67% separately. When compared with SA designed for single load SBRP, the ILS-SP algorithm also outperforms it, which decreases the number of buses by 6.81%. The maximum percentage of improvement is 20.60%. The ILS-SP algorithm is also effective for mixed load SBRP, it uses less buses than RRT algorithm and the number of routes is reduced by 1.95% on average.

### C. Heterogeneous multi-school SBRP

For heterogeneous multi-school SBRP, we try to get the best fleet composition and the lowest total cost of solutions. Thus, the objective is the sum of fixed purchasing cost and variable cost. We also adopt the fleet adjustment strategy after every move made by neighborhood operators between different routes in local search of the proposed algorithm.

First of all, we use ILS algorithm and ILS-SP algorithm to solve all the instances respectively. The results of them are shown in TABLE III and TABLE IV. The columns, *TC* and *STD*, indicate the best and percentage deviation of costs among the 10 solutions respectively. The column *Fleet* denotes the best fleet composition. The average computation time in seconds is given in column *T*. The description of the remain columns are the same as TABLE I.

Seen from TABLE III and TABLE IV, the ILS-SP algorithm use less total cost than ILS algorithm. For single load SBRP, the ILS-SP algorithm decreases the total cost by 1.64% on average, when compared with ILS algorithm. For some instances, such as RSRB02, CSCB02, and CSCB04, the improvement percentage are all more than 3%. When the maximum ridding time of students are set to 2700 and 5400 seconds, on average, the total cost are decrease by 1.38% and 1.91% respectively. For mixed load SBRP, the results in TABLE IV show that the ILS-SP algorithm decreases the total cost by 1.44% compared with ILS algorithm. The maximal improvement percentage is 3.74% of all the instances. The total cost of all instances are reduced by 2.02% and 0.86%

TABLE I
COMPUTATIONAL RESULTS ON HOMOGENEOUS SBRP WITH SINGLE LOAD AND MIXED LOAD

| Instance | | | MRT | ILS | | | | ILS-SP | | | | Gap(%) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | Stops | Schools | | Ns | Ts | Nm | Tm | Ns | Ts | Nm | Tm | Gs | Gm |
| CSCB01 | 250 | 6 | 2700 | 28 | 37.54 | 26 | 74.09 | 27 | 95.72 | 26 | 136.98 | 3.57% | 0.00% |
| CSCB02 | 250 | 12 | 2700 | 25 | 31.07 | 25 | 77.32 | 25 | 85.55 | 25 | 140.12 | 0.00% | 0.00% |
| CSCB03 | 500 | 12 | 2700 | 53 | 108.5 | 48 | 386.92 | 52 | 178.27 | 48 | 452.53 | 1.89% | 0.00% |
| CSCB04 | 500 | 25 | 2700 | 58 | 81.44 | 56 | 350.68 | 57 | 149.33 | 54 | 421.25 | 1.72% | 3.57% |
| RSRB01 | 250 | 6 | 2700 | 26 | 22.98 | 26 | 53.89 | 26 | 84.89 | 25 | 86.18 | 0.00% | 3.85% |
| RSRB02 | 250 | 12 | 2700 | 26 | 20.36 | 26 | 57.69 | 26 | 81.44 | 26 | 79.88 | 0.00% | 0.00% |
| RSRB03 | 500 | 12 | 2700 | 51 | 85.05 | 51 | 287.61 | 50 | 126.25 | 51 | 352.63 | 1.96% | 0.00% |
| RSRB04 | 500 | 25 | 2700 | 54 | 75.03 | 53 | 327.48 | 54 | 108.89 | 52 | 395.38 | 0.00% | 1.89% |
| CSCB01 | 250 | 6 | 5400 | 23 | 36.93 | 23 | 40.39 | 23 | 99.84 | 23 | 85.03 | 0.00% | 0.00% |
| CSCB02 | 250 | 12 | 5400 | 20 | 32.53 | 19 | 55.22 | 20 | 95.94 | 19 | 97.26 | 0.00% | 0.00% |
| CSCB03 | 500 | 12 | 5400 | 40 | 110.65 | 39 | 254.26 | 40 | 165.88 | 38 | 317.33 | 0.00% | 2.56% |
| CSCB04 | 500 | 25 | 5400 | 41 | 89.52 | 40 | 260.31 | 41 | 135.87 | 40 | 330.83 | 0.00% | 0.00% |
| RSRB01 | 250 | 6 | 5400 | 23 | 28.88 | 24 | 33.74 | 23 | 90.20 | 23 | 83.52 | 0.00% | 4.17% |
| RSRB02 | 250 | 12 | 5400 | 22 | 25.71 | 22 | 41.01 | 22 | 89.51 | 22 | 98.87 | 0.00% | 0.00% |
| RSRB03 | 500 | 12 | 5400 | 46 | 98.35 | 46 | 193.6 | 46 | 171.75 | 46 | 258.6 | 0.00% | 0.00% |
| RSRB04 | 500 | 25 | 5400 | 41 | 86.09 | 40 | 233.9 | 41 | 155.68 | 40 | 308.11 | 0.00% | 0.00% |
| Avg | 375 | 13.75 | 4050 | 36.06 | 60.66 | 35.25 | 170.51 | 35.81 | 119.69 | 34.88 | 227.78 | 0.57% | 1.00% |

TABLE II
COMPARISON WITH OTHER ALGORITHMS FOR HOMOGENEOUS SBRP WITH TWO OPERATION MODES

| Instance | | | MRT | Single Load | | | | | Mixed Load | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Name | Stops | Schools | | PH | SA | ILS-SP | Gp | Gs | PH | RRT | ILS-SP | Gp | Gr |
| CSCB01 | 250 | 6 | 2700 | 39 | 31 | 27 | 30.77% | 12.90% | 30 | 27 | 26 | 13.33% | 3.70% |
| CSCB02 | 250 | 12 | 2700 | 33 | 26 | 25 | 24.24% | 3.85% | 30 | 26 | 25 | 16.67% | 3.85% |
| CSCB03 | 500 | 12 | 2700 | 66 | 59 | 52 | 21.21% | 11.86% | 55 | 49 | 48 | 12.73% | 2.04% |
| CSCB04 | 500 | 25 | 2700 | 72 | 61 | 57 | 20.83% | 6.56% | 62 | 57 | 54 | 12.90% | 5.26% |
| RSRB01 | 250 | 6 | 2700 | 35 | 26 | 26 | 25.71% | 0.00% | 30 | 26 | 25 | 16.67% | 3.85% |
| RSRB02 | 250 | 12 | 2700 | 32 | 27 | 26 | 18.75% | 3.70% | 29 | 27 | 26 | 10.34% | 3.70% |
| RSRB03 | 500 | 12 | 2700 | 66 | 47 | 50 | 24.24% | -6.38% | 56 | 53 | 51 | 8.93% | 3.77% |
| RSRB04 | 500 | 25 | 2700 | 68 | 58 | 54 | 20.59% | 6.90% | 59 | 52 | 52 | 11.86% | 0.00% |
| CSCB01 | 250 | 6 | 5400 | 35 | 29 | 23 | 34.29% | 20.69% | 24 | 23 | 23 | 4.17% | 0.00% |
| CSCB02 | 250 | 12 | 5400 | 27 | 23 | 20 | 25.93% | 13.04% | 22 | 19 | 19 | 13.64% | 0.00% |
| CSCB03 | 500 | 12 | 5400 | 52 | 42 | 40 | 23.08% | 4.76% | 41 | 39 | 38 | 7.32% | 2.56% |
| CSCB04 | 500 | 25 | 5400 | 57 | 45 | 41 | 28.07% | 8.89% | 43 | 37 | 40 | 6.98% | -8.11% |
| RSRB01 | 250 | 6 | 5400 | 31 | 28 | 23 | 25.81% | 17.86% | 27 | 24 | 23 | 14.81% | 4.17% |
| RSRB02 | 250 | 12 | 5400 | 30 | 23 | 22 | 26.67% | 4.35% | 23 | 23 | 22 | 4.35% | 4.35% |
| RSRB03 | 500 | 12 | 5400 | 61 | 46 | 46 | 24.59% | 0.00% | 47 | 47 | 46 | 2.13% | 2.13% |
| RSRB04 | 500 | 25 | 5400 | 56 | 41 | 41 | 26.79% | 0.00% | 46 | 40 | 40 | 13.04% | 0.00% |
| Avg | 375 | 13.75 | 4050 | 47.50 | 38.25 | 35.81 | 25.10% | 6.81% | 39 | 35.56 | 34.88 | 10.62% | 1.95% |

on average, when the maximum ridding time of students are set to 2700 and 5400 seconds respectively. Moreover, the percentage deviation is controlled within 2%. It shows that our ILS-SP algorithm is relatively stable. In general, the results in these two tables show that ILS-SP algorithm is very effective.

Further, we compare the ILS-SP algorithm with the existing algorithms solving for multi-school heterogeneous SBRP. These algorithms include adaptive location based heuristic(ALBH) [7],random location based heuristic(RLBH) [6] and RRT algorithm proposed in [8] and so on. We have implemented these three algorithms according to the description of them in [6], [7] and [8] to compare them with our proposed algorithm. The parameter settings of these three algorithms are the same as our proposed algorithm. TABLE V shows the results of these algorithms on the benchmark

TABLE III
COMPUTATIONAL RESULTS ON HETEROGENEOUS SBRP WITH SINGLE LOAD

| Instance | | | MRT | ILS | | ILS-SP | | | | Gap(%) |
|---|---|---|---|---|---|---|---|---|---|---|
| Name | Stops | Schools | | TC | T | TC | Std(%) | Fleet | T | |
| CSCB01 | 250 | 6 | 2700 | 80864.15 | 47.96 | 80603.48 | 2.07% | 16A4B9C | 98.55 | 0.32% |
| CSCB02 | 250 | 12 | 2700 | 72057.53 | 42.29 | 71940.05 | 2.59% | 23A4C | 83.76 | 0.16% |
| CSCB03 | 500 | 12 | 2700 | 152623.22 | 148.89 | 152296.48 | 1.28% | 34A6B15C | 216.34 | 0.21% |
| CSCB04 | 500 | 25 | 2700 | 164024.32 | 166.67 | 162587.27 | 1.41% | 27A6B24C | 217.67 | 0.88% |
| RSRB01 | 250 | 6 | 2700 | 81758.28 | 40.39 | 80458.02 | 1.52% | 29A2B | 96.57 | 1.59% |
| RSRB02 | 250 | 12 | 2700 | 79006.90 | 49.82 | 75945.93 | 1.42% | 15A2B10C | 68.49 | 3.87% |
| RSRB03 | 500 | 12 | 2700 | 154727.50 | 119.39 | 151287.36 | 1.41% | 5A1B35C | 200.47 | 2.22% |
| RSRB04 | 500 | 25 | 2700 | 158562.35 | 137.59 | 155765.00 | 1.72% | 35A21C | 199.8 | 1.76% |
| CSCB01 | 250 | 6 | 5400 | 70056.64 | 49.15 | 68023.96 | 2.07% | 1A21C | 78.63 | 2.90% |
| CSCB02 | 250 | 12 | 5400 | 60674.66 | 46.84 | 58823.07 | 1.91% | 13A8C | 68.85 | 3.05% |
| CSCB03 | 500 | 12 | 5400 | 124919.86 | 126.91 | 122153.89 | 1.41% | 1A1B37C | 211.28 | 2.21% |
| CSCB04 | 500 | 25 | 5400 | 127549.64 | 147.41 | 123057.63 | 2.49% | 5A35C | 199.61 | 3.52% |
| RSRB01 | 250 | 6 | 5400 | 73739.09 | 33.48 | 72954.14 | 1.69% | 3A1B20C | 84.72 | 1.06% |
| RSRB02 | 250 | 12 | 5400 | 63062.71 | 45.56 | 62548.89 | 2.45% | 1A19C | 60.31 | 0.81% |
| RSRB03 | 500 | 12 | 5400 | 138608.09 | 146.42 | 138607.49 | 1.25% | 9A1B36C | 185.13 | 0.00% |
| RSRB04 | 500 | 25 | 5400 | 129252.28 | 124.14 | 127056.54 | 2.36% | 13A3B27C | 192.33 | 1.70% |
| Avg | 375 | 13.75 | 4050 | 108217.95 | 92.06 | 106506.83 | 1.82% | - | 141.41 | 1.64% |

TABLE IV
COMPUTATIONAL RESULTS ON HETEROGENEOUS SBRP WITH MIXED LOAD

| Instance | | | MRT | ILS | | ILS-SP | | | | Gap(%) |
|---|---|---|---|---|---|---|---|---|---|---|
| Name | Stops | Schools | | TC | T | TC | Std(%) | Fleet | T | |
| CSCB01 | 250 | 6 | 2700 | 78588.32 | 65.82 | 75646.03 | 1.22% | 8A3B15C | 101.25 | 3.74% |
| CSCB02 | 250 | 12 | 2700 | 71249.23 | 65.53 | 68824.43 | 1.67% | 16A3B6C | 100.75 | 3.40% |
| CSCB03 | 500 | 12 | 2700 | 140998.88 | 231.28 | 137948.06 | 1.58% | A2B19C28 | 286.48 | 2.16% |
| CSCB04 | 500 | 25 | 2700 | 156409.01 | 174.63 | 152360.77 | 1.06% | 24A5B24C | 236.18 | 2.59% |
| RSRB01 | 250 | 6 | 2700 | 81480.43 | 44.45 | 80875.94 | 1.39% | 28A3B | 89.71 | 0.74% |
| RSRB02 | 250 | 12 | 2700 | 76118.90 | 45.89 | 75590.44 | 1.97% | 16A3B8C | 73.85 | 0.69% |
| RSRB03 | 500 | 12 | 2700 | 154135.70 | 134.23 | 151767.05 | 1.13% | 30A3B21C | 228.68 | 1.54% |
| RSRB04 | 500 | 25 | 2700 | 153061.90 | 159.87 | 151071.97 | 1.29% | 38A1B16C | 211.59 | 1.30% |
| CSCB01 | 250 | 6 | 5400 | 67754.86 | 67.89 | 67481.62 | 1.72% | 2A20C | 87.23 | 0.40% |
| CSCB02 | 250 | 12 | 5400 | 57597.21 | 66.84 | 57041.52 | 2.24% | 4A1B14C | 71.68 | 0.96% |
| CSCB03 | 500 | 12 | 5400 | 116509.42 | 224.54 | 115080.36 | 2.38% | 2A35C | 284.76 | 1.23% |
| CSCB04 | 500 | 25 | 5400 | 118611.09 | 213.25 | 115737.65 | 2.49% | 2A1B34C | 268.18 | 2.42% |
| RSRB01 | 250 | 6 | 5400 | 73786.99 | 43.91 | 73113.65 | 2.13% | 8A2B15C | 78.43 | 0.91% |
| RSRB02 | 250 | 12 | 5400 | 62622.53 | 45.98 | 62527.04 | 2.16% | 1A19C | 62.86 | 0.15% |
| RSRB03 | 500 | 12 | 5400 | 139659.56 | 135.98 | 138945.23 | 0.93% | 14A1B32C | 188.25 | 0.51% |
| RSRB04 | 500 | 25 | 5400 | 123892.87 | 174.37 | 123494.71 | 2.12% | 8A4B29C | 203.42 | 0.32% |
| Avg | 375 | 13.75 | 4050 | 104529.81 | 118.40 | 102969.15 | 1.72% | - | 160.83 | 1.44% |

instances.

As shown in TABLE V, ILS-SP algorithm is more effective than existing algorithms [6],[7],[8] for heterogeneous multi-school SBRP. Compared with ALBH [7], RLBH [6] and RRT [8] algorithms, ILS-SP algorithm can reduces the total cost on average by 38.13%, 28.32% and 6.09% respectively. When the operation mode is mixed load, the ILS-SP algorithm can decrease the total cost on average by 35.74%, 31.99% and 6.2% respectively. Because the ALBH and RLBH algorithms are constructive heuristics, ILS-SP algorithm is remarkable better than them. For the RRT meta-heuristic algorithm proposed in [8], the ILS-SP algorithm is also very competitive. The ILS-SP algorithm overcomes the short-sighted shortcoming of local-search based algorithms because of combining the SP procedure. The results in TABLE V show the ILS-SP algorithm is effective again.

TABLE V
COMPARISON WITH OTHER ALGORITHMS FOR HETEROGENEOUS SBRP WITH TWO OPERATION MODES

| Instance | MRT | Single Load | | | | Mixed Load | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | ALBH | RLBH | RRT | ILS-SP | ALBH | RLBH | RRT | ILS-SP |
| CSCB01 | 2700 | 128591.49 | 110593.02 | 83416.94 | 80603.48 | 115940.25 | 113275.18 | 77326.51 | 75646.03 |
| CSCB02 | 2700 | 127199.59 | 112026.34 | 77546.25 | 71940.05 | 123439.74 | 111280.21 | 74484.74 | 68824.43 |
| CSCB03 | 2700 | 265827.83 | 207966.90 | 161914.23 | 152296.48 | 218367.30 | 205769.17 | 143410.50 | 137948.06 |
| CSCB04 | 2700 | 295366.78 | 255514.99 | 180064.96 | 162587.27 | 279958.64 | 253832.53 | 174158.57 | 152360.77 |
| RSRB01 | 2700 | 129591.58 | 111351.18 | 80528.02 | 80458.02 | 112524.91 | 109166.74 | 80465.92 | 80875.94 |
| RSRB02 | 2700 | 152984.43 | 110567.49 | 80585.55 | 75945.93 | 129453.80 | 117878.37 | 83734.15 | 75590.44 |
| RSRB03 | 2700 | 264493.47 | 230727.13 | 152029.08 | 151287.36 | 214386.55 | 214092.19 | 152174.87 | 151767.05 |
| RSRB04 | 2700 | 267452.78 | 240011.31 | 167449.28 | 155765.00 | 240874.22 | 234870.24 | 164533.32 | 151071.97 |
| CSCB01 | 5400 | 105246.91 | 88480.98 | 71576.71 | 68023.96 | 105246.91 | 92983.28 | 71485.11 | 67481.62 |
| CSCB02 | 5400 | 89917.36 | 78530.22 | 62403.68 | 58823.07 | 89917.36 | 84570.64 | 62156.32 | 57041.52 |
| CSCB03 | 5400 | 173060.06 | 153844.00 | 131565.85 | 122153.89 | 170112.54 | 152476.59 | 128402.38 | 115080.36 |
| CSCB04 | 5400 | 207265.28 | 162250.28 | 144392.80 | 123057.63 | 218984.09 | 186439.28 | 132036.07 | 115737.65 |
| RSRB01 | 5400 | 101760.70 | 86791.02 | 74603.33 | 72954.14 | 101760.70 | 94525.81 | 71522.76 | 73113.65 |
| RSRB02 | 5400 | 104173.48 | 88992.89 | 72114.50 | 62548.89 | 97772.75 | 93685.18 | 68905.21 | 62527.04 |
| RSRB03 | 5400 | 160589.67 | 166384.03 | 140170.72 | 138607.49 | 165532.43 | 175558.17 | 140167.93 | 138945.23 |
| RSRB04 | 5400 | 180839.73 | 173248.69 | 134263.37 | 127056.54 | 179697.08 | 182042.90 | 131455.92 | 123494.71 |
| Avg | 4050 | 172147.57 | 148580.03 | 113414.08 | 106506.83 | 160248.08 | 151402.91 | 109776.27 | 102969.15 |

### *D. Performance Analysis of ILS-SP Algorithm*

According to the results from TABLE I to TABLE V, the ILS-SP algorithm can effectively solve the multi-school homogeneous or heterogeneous SBRP with single load and mixed load. The comparison result of ILS-SP algorithm with other algorithms is shown in Fig 2.

Seen from the Fig 2, we can find that ILS-SP algorithm is more effective than existing algorithm for multi-school SBRP. For single load and mixed load homogeneous SBRP, the ILS-SP algorithm uses least the number of routes. Compared with post optimization heuristic [21], ILS-SP algorithm reduces the number of routes by more than 10%. While for SA [22] and RRT [10], ILS-SP algorithm can also decrease the routes on average by 6.8% and 1.95% respectively. While for single load and mixed load heterogeneous SBRP, ILS-SP algorithm is significantly better than ALBH [7] and RLBH [6] heuristic algorithms. When compared with ILS algorithm, it reduces the total cost on average by 1.64% and 1.44% respectively. The ILS-SP algorithm also outperforms the RRT metaheuristic algorithm proposed by [8], and it decreases the average total cost by 6.09% and 6.2% respectively. All in all, we can draw the conclusion that the ILS-SP algorithm is very effective, and it can outperform existing algorithms for homogeneous or heterogeneous SBRP with single load and mixed load.

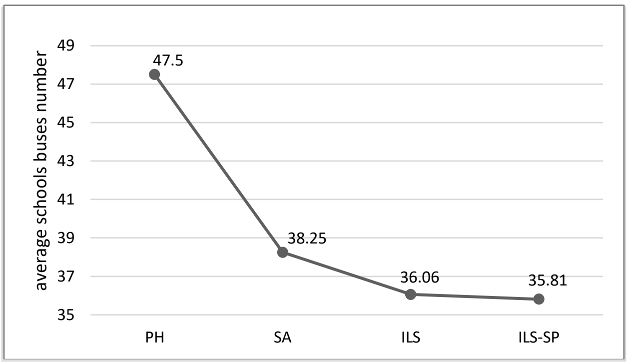
Among of these algorithms, post optimization heuristic [21] and simulated annealing proposed by [22] are both two-stage algorithms, which firstly get routes of every single school and then combine them with some certain strategies. The former, that is post optimization heuristic, uses simple heuristic to combine, while the latter combines the routes in the framework of simulated annealing metaheuristic. Although they can solve the multi-school SBRP quickly, they lack global considerations in the optimization process. The ALBH [6] and RLBH [7] are both constructive heuristic algorithms, which both have the limited ability to find the better solution. While for the two record-to-record travel algorithms proposed by [8] and [10], they just only use the acceptance rule based on deviation factor to enhance the diversity of solution. Compared with these two algorithms, our ILS-SP algorithm adopts several heuristic strategies, including perturbation methods and allowing accepting worse solution, to explore the diversity of neighborhood solutions and avoid trapping local optima.

In additional, the ILS-SP algorithm uses the set partitioning procedure to optimize the solution from a global point of view. The ILS-SP algorithm takes advantages of ILS algorithm and exact algorithm, which can use SP to improve further ILS algorithm. Although the computation time of the ILS-SP algorithm increases to some extent, the increase in computation time may be negligible for large-scale multi-school SBRP, because planning school bus routes for schools usually has one time before each semester begins.

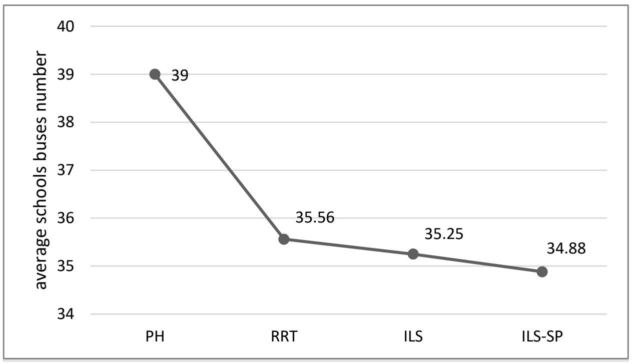
### *E. Analysis of ILS-SP algorithm on Different Instances Groups*

In this section, we tries to analysis the performance of ILS-SP algorithm on two different instances groups. The average school buses number of all the instance for two homogeneous SBRP problems are calculated. At the same time, the average total cost of all the instance for two heterogeneous SBRP problems are also computed. The results of them are shown in Fig 3 and Fig 4 respectively.

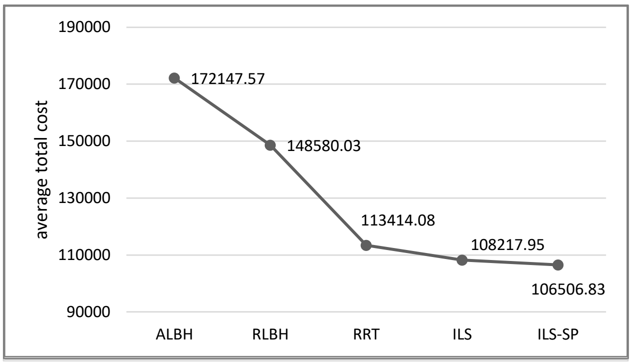
There are some findings from Fig 3 and Fig 4. First, the ILS-SP algorithm can find better solutions on CSCB instances. Because of stops and schools clustered distribution in CSCB instances, the ILS-SP algorithm with the limited iterations could be easy to find the better solutions than it solves the RSRB instances. Second, when the maximum ridding time of students are set to 5400, it causes difficult to find



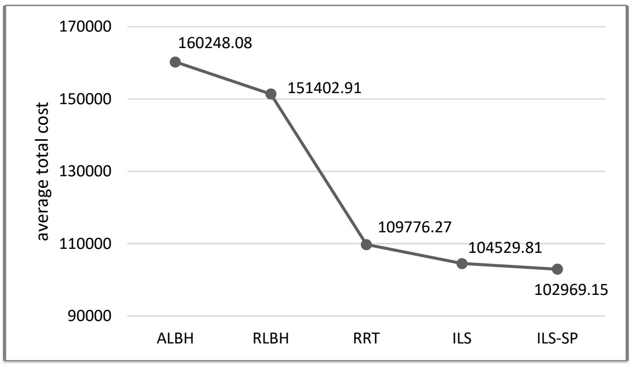

(a) homogeneous SBRP with single load




(b) homogeneous SBRP with mixed load

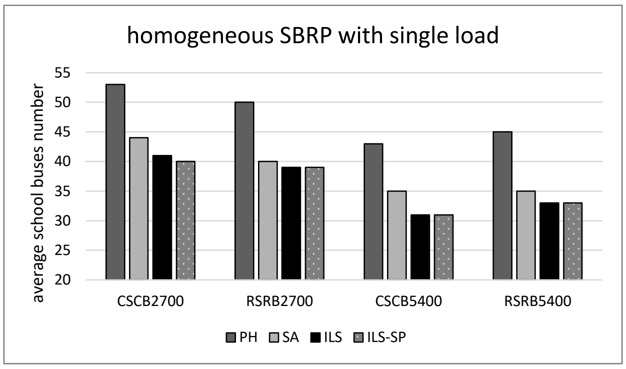



(c) heterogeneous SBRP with single load

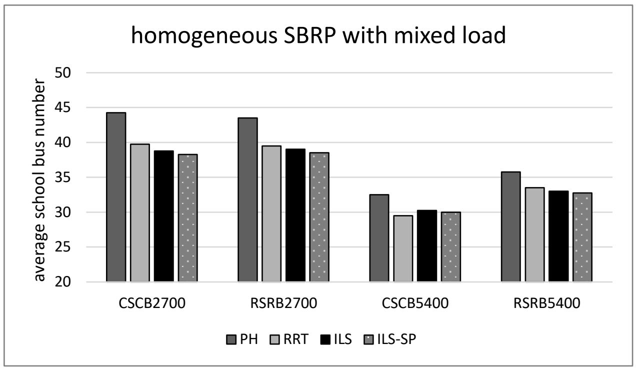



(d) heterogeneous SBRP with mixed load

Fig. 2. Comparison of different algorithms for homogeneous and heterogeneous multi-school SBRP

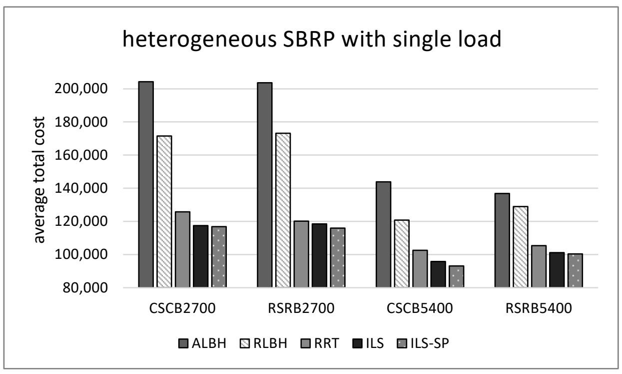



(a) single load

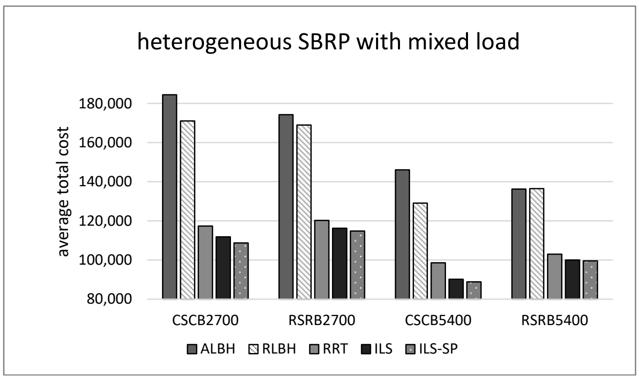



(b) mixed load

Fig. 3. Average buses number of different algorithms for homogeneous multi-school SBRP




(a) single load




(b) mixed load

Fig. 4. Average total cost of different algorithms for heterogeneous multi-school SBRP

best local solution. The maximum ridding time constraint is relaxed, it means that it is easy to obtain the neighborhood solution. But because of limited iterations and relative big neighborhood solution space, the best local solution is difficult to obtain. Therefore, relaxing the maximum ridding time constraint cannot reduce the complexity of multi-school SBRP.

## V. Conclusion

This paper proposes a hybrid metaheuristic algorithm(ILS-SP) algorithm to solve the school bus routing problem with multiple schools. We consider four multi-school SBRP problems, which including homogeneous fleets and heterogeneous fleets, single load and mixed load operation mode. The problems are firstly modeled as a kind of heterogeneous mixed load SBRP, and the problem model can be converted to other problems by decreasing the constraints. Then the ILS-SP algorithm is implemented, which combines an ILS metaheuristic with SP. The addition of SP enhances the ability of the ILS algorithm to find better solutions, because it can effectively overcome the short-term behavior of the local search-based algorithm. Finally, the algorithm is evaluated using the benchmark instances and compared with other existing algorithms for SBRP. The results prove that our proposed algorithm is very competitive. The ILS-SP algorithm can effectively solve multi-school SBRP problems and it is also an effective general-purpose algorithm for large-scale multi-school SBRP.

In the future, we intent to improve and extend the ILS-SP algorithm for solving other SBRP variants, which have additional attributes such as split delivery and multiple depots.

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