A Node Ranking Method Based on Local Structure Information in Complex Networks

Jieming Yang, Jinghan Lu, Yun Wu, Tianyang Li, and Yuehua Yang

***Abstract*—It exerts significant impact on the research of complex networks to evaluate the importance of nodes. A new method of the node ranks based on neighbor lines and local network structure was proposed in this paper. Firstly, the local network structure is considered as a sub-network structure which is consisted of a number of layers of neighbor nodes. Then, the attributes of neighbor nodes are used to compute the importance of the nodes within local network structure. Finally, the contributions of neighbor lines are merged to estimate the importance value of nodes. Both the attribute information of the neighbor lines and local structure information are taken into consideration in this paper. In order to verify the performance of the method presented in this paper, the experiments have been carried out on European grid, dolphin social network, contiguous USA network and the US grid. The results indicate the method proposed in this paper is feasible and the nodes of the complex networks can be ranked effectively by importance.**

***Index Terms*—complex network, node importance, neighbor node, network efficiency**

I. Introduction

WITH the deepening of researches on the characteristics of information transmission of the social network, transportation network, communication network and disease transmission network, complex network theory has attracted widespread attention [1]–[2]. Complex networks can be regarded as an abstract representation of a variety of real networks [3]. The transfer of information between network nodes is the principal characteristics of complex network. An influential node can result in more rapid transmission of information [4]–[5]. Therefore, identifying the key nodes of the complex network is of great significance.

Nowadays, there exists some problems which remain to be solved by complex networks, such as the transmission of the virus, traffic jams on the transportation network, flight delays, large area blackout [6], etc. In order to predict and control the complex network system effectively, lots of theories and methods of complex network are applied to analyze the functional characteristics of complex network system [1].

A network is composed of nodes and lines, and each node and its associated edge in the network contains plenty of local and global information of the network [7]. For example, the large area blackout is mainly caused by the vulnerability of power systems [2]. However, it is possible to reduce or even avoid economic losses and disasters if the key nodes of the power grid can be found in advance for prevention and control [2]. Compared with other nodes, the key nodes have great influence on the structure and function of the network [8]. Therefore, it is important to identify the key nodes of networks. Generally speaking, the number of the critical nodes in complex network is usually small, however, the propagation speed of the critical nodes is extremely rapid and other nodes in the network system will be affected instantaneously [9]–[10].

A large number of studies used to evaluate the node importance in the complex network have been presented [11]–[13], such as Degree Centrality (DC) [14], Betweenness Centrality (BC) [15], Closeness Centrality (CC) and K-kernel decomposition method, etc [16]–[17]. DC evaluates the importance of a node by computing the total number of whose neighbors, the more the number of neighbors node increases, the more important it is. DC is the most intuitive and simple index in the network, and it is also an index of low computational complexity. However, it lacks the description of the location of the neighbor nodes and other surrounding information. so how to achieve the more accurate evaluation results of nodes still needs to be studied. BC relates to the shortest path through the node, because the information flow is generally propagated through the shortest path, it is hard to avoid the network congestion. Because of its high computational complexity, BC doesn't work well in the large complex networks [18]. CC measures the importance by calculating the average distance from one node to other nodes in a network. In a word, CC can be understood as the following: the closer the distance from one node to other nodes in the network is, the greater the node contribution for the network is, which means the node should be more important.

The edges between neighbor nodes have not received enough attention from the researches mentioned upon. In fact, the importance of a node in a network depends on not only its own location and neighbor nodes, but also its adjacent side. Chen Yong [19] analyzed the importance of nodes by deleting nodes and edges in real networks, and analyzed the importance of nodes based on the number of spanning trees of graphs when some nodes and edges has been removed. Supposed that a node has been removed, the fewer spanning

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trees exist in the new graph, the more important the node will be, which is the node spanning tree theory of the centrality method. The principle of node contraction method is that a node can merges with its neighbors in the network with the aid of network contraction to form a new node. Tan [20] believed that the contraction of node v_i would be more important if its makes the whole network more cohesive in a network.

Liu [21] believed that the importance of lines should be described with their connecting ability and irreplaceability. Thus, both the importance of lines and the degree of nodes are introduced at the same time by the DIL method . When the importance of nodes is evaluated using the DIL method, the weights of lines will be redefined using the importance of lines. Thus, it is different from some traditional methods which only consider the attributes of nodes themselves. However, the DIL method didn’t pay attention to the attribute of nodes and the local topology property of the node [22].

In this paper, a new effective method named LSI which ranks nodes based on neighbor lines and local network structure was proposed. Both the attribute information of the neighbor lines and local structure information are put into account simultaneously by LSI. The local network structure of nodes is considered as a sub-network structure formed by several layers of neighbor nodes. In this way, the attributes of local network can be considered with low computational complexity comprehensively, and the accuracy of node ranking can be greatly increased. To evaluate the performace of the proposed method, the comparison with the classical node importance methods has been made in term of network efficiency, propagation model and correlation coefficient.

II. NODE IMPORTANCE RANKING METHOD BASED ON LOCAL STRUCTURE INFORMATION (LSI)

A. Enlightenment

DIL centrality is a novel method which ranks nodes based on the importance of the node that measured with the degree of nodes and the importance of lines. Accordingly, it answered the question whether the nodes are important or not. However, DIL centrality only considered the degree of the node, the influence of global topology attribute of the node was not put into account [21].

Fig. 1 and 2 are two simple networks which has been used to explore the problems in the existing studies on node ranking method.

It can be concluded that all nodes had a degree of one when DIL centrality method is applied to calculate the importance of nodes in the network shown in Fig. 1, which means that they are of equal importance. The reason for this is that the lines attached to the nodes with degree of one can not form a triangle with the other lines, which means the importance of edge is zero, DIL only focuses on the degree of nodes and draw a conclusion that the degree of most nodes is equal to one. As shown in Fig. 1, the node v_4 connects the left and right sub-graphs, and it can be concluded based on the principle of the spatial autocorrelation that the closer the two nodes are, the greater the interdependence on each other. Based on the spatial autocorrelation theory, it can be argued that the closer the node to the current node, the greater the contribution to the importance of the node. So node v_4's neighbor node v_{25} is important. However, DIL centrality considers that node v_{25} is as important as other nodes with degree of one, which is obviously unreasonable.

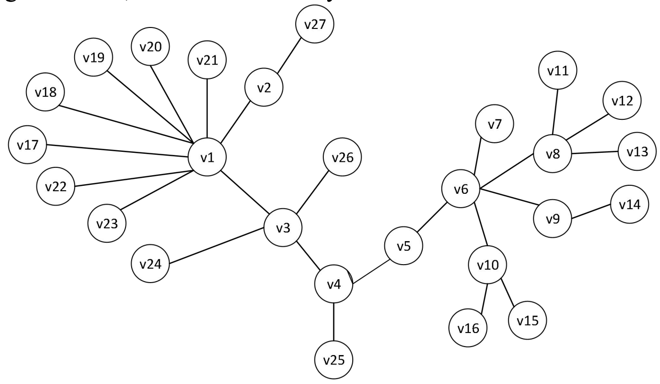

Fig. 1. The topology of a simple network

When the importance of nodes in Fig.2 is calculated using the DIL centrality method, $Cdil(v_1)$=6.3, $Cdil(v_5)$=6.7. It turns out that the importance of node v_5 is much greater than that of node v_1. However, based on the deletion of node v_1, it can be seen that Fig.2 will be partitioned into three sub-graphs, such as the sub-graph connected to node v_3, the sub-graph connected to node v_4, and node v_2. Based on the deletion of node v_5, Fig.2 will be partitioned into two sub-graphs, namely the node v_7 and the remaining nodes respectively. Compared with the node v_5, node v_1 exerts a greater influence on the destruction of graph connectivity, which indicates that the importance of the node v_1 is much greater than that of the node v_5. This is in inconsistent with the conclusion of DIL algorithm. In summary, there exists two underlying issues in the centrality method of DIL, namely the ranking results of nodes whose degree with value of 1 in the network are identical, and the attribute of the network topology has been ignored. As a result, when the associated edges have the same effect on nodes, the ranking result of nodes completely depends on theirs degrees.

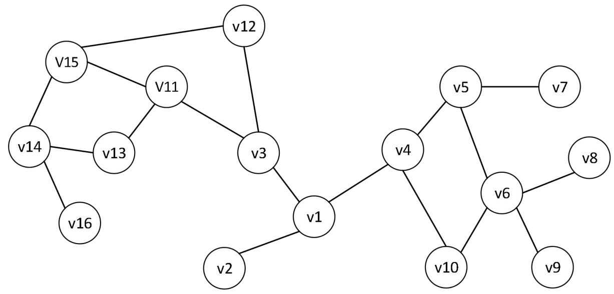

Fig. 2. The topology of a network which contains sixteen nodes

B. Node sorting method based on node importance calculated by neighbor lines and local information(*LSI*)

To solve problems mentioned above, the LSI node ranking method based on neighbor lines and local network structure was proposed in this paper. The local network structure of nodes is a sub-network structure formed by a number of layers of neighbor nodes, and the contribution values, which arise from its neighbor lines, are incorporated into the calculation of node importance. As mentioned above, the attribute information of the neighbor lines of the node has been introduced. Thus, the corresponding network structure information will be more accurate. Meanwhile, the computational complexity will be reduced as much as possible.

The specific definition of the LSI node ranking method is as follows：

It assumes there is an undirected and unweighted network $G=(V,E)$, which has N nodes and M edges. Where $V=\{v_1,...v_i,...v_n\}$ is the set of all nodes in the network, N is the number of nodes in the set, v_i is the *ith* node in the network, $E=\{e_{11},...e_{ij},...e_{mn}\}$ is the set of all edges in the network, M is the number of edges in the set, e_{ij} is an edge from node v_i to node v_j and the contribution of e_{ij} in the network can be calculated by $Ie_{ij}=U/\lambda$. where U is the edge connectivity of e_{ij}, which can be calculated by $U=(k(v_i)-p-1)\cdot(k(v_j)-p-1)$, $k(v_i)$ and $k(v_j)$ are the degree of node v_i and v_j, respectively, p is the number of triangles which one edge is e_{ij}, $\lambda=p/2+1$ is the alternative index for edge e_{ij}. The calculation formula of the node importance is shown as follow:

$$C(v_i)=\sum_{j\in\Gamma(i)}N(v_j)+\sum_{j\in\Gamma(i)}Ie_{ij}\cdot\frac{k(v_i)-1}{k(v_i)+k(v_j)-2} \quad (1)$$

Where $\Gamma(i)$ is a set, which includes all neighbors of node v_i, v_j is the *jth* neighbor of node v_i.

When the importance of node v_1 and node v_5 in Fig.2 was calculated based on DIL centrality method, only the degree of the node was introduced, therefore, node v_5 is more important than node v_1 in the case of that the importance degree of lines is same. Although node v_1 and node v_5 have the same degree, from the perspective of the damage to the connectivity of the graph by deleting nodes, node v_1 is more important than node v_5. The results obtained by formula (1) indicate that the importance of node v_1 and v_5 is 18.4 and 17.4, respectively, which means node v_1 is more important than node v_5 in terms of the node importance. When the node importance is calculated by the LSI method, the local structure of nodes has been considered, therefore, the importance of nodes can be further divided in detail.

Then Fig. 1 will be taken as an example to calculate the importance degree of node v_{26} and node v_{27}. Based on DIL centrality method, the results are $Cdil(v_{26})=Cdil(v_{27})=1$. However, when the LSI method is applied, the importance degree of node v_{26} is 18 and that of node v_{27} is 10. It is not difficult to concluded that the LSI method can make an more reasonable estimation for the importance of nodes than DIL centrality method.

III. Experiments and discussions

A. Data sources

The LSI method is evaluated on the European grid [23], dolphin social network [24], contiguous USA network, and the US grid [25], respectively. The European grid contains 1514 nodes, which represent power stations or substations, and connections between them represent transmission from the grid. The dolphin social network has 62 nodes, one dolphin is represented by one node in the network and there is an edge between two nodes if two dolphins communicate with each other. Contiguous USA network includes 49 nodes. The nodes represent the boundary positions of contiguous states and the edges is the connections between contiguous states. The US grid contains 4941 nodes, which represent power stations or substations, and connections between them represent transmission from the grid.

B. Evaluation

In this paper, three different types of measures were used to verify the performance of the LSI method. These measures are based on two different mechanisms, one is transmission dynamics, the other is that the importance of a node is same to the extent of network damage after the node is deleted [25].

a. Network efficiency

In the complex network research, the network efficiency [29]–[30] is a common method which describes the connectivity of the network. The network efficiency increases with the improvement of the connectivity of the network. The formula of network efficiency η is shown as following:

$$\eta=\frac{1}{N(N-1)}\sum_{V_i\neq V_j\in V}\eta_{ij} \quad (2)$$

η_{ij} is the network efficiency, which is related to the node v_i and v_j and can be calculated with $\eta_{ij}=1/d_{ij}$, d_{ij} is the length of the shortest path from node v_i to node v_j. The amount of nodes in the network is denoted by N. Usually, μ represents the decline rate of network efficiency, which is defined as $\mu=1-\eta/\eta_0$.

η is the network efficiency when it is attacked by removing node. η_0 is the initial efficiency of the network. The larger μ is, the more damage the deleted node would done to the network, which indicates that the deleted node is more critical.

b. SI epidemic model

The transmission dynamics model, such as SI epidemic model, is usually applied to measure the advantages and disadvantages of each ranking algorithm [26]–[28]. SI epidemic model was usually used to simulate the dynamic transmission of the disease.

As far as SI epidemic model is concerned, nodes of network are in two discrete states : (1) S state is the state of susceptible to infection; (2) I state is the infection state. It is usually assumed that the stronger the propagation ability of a node is and the greater its influence on the structure and function of the network is, then the node is more important.

In our experiments, the top 10% of the important nodes extracted from the calculation results of various methods were fed into the SI epidemic model, and the results has been further analyzed. Then, it can be concluded that the stronger the propagation ability of a node is, the greater the effect of the node is. The experimental results show that LSI method perform better under the SI epidemic model.

c. Correlation coefficient

Generally speaking, the stronger the communication ability is, the more important the node is. Therefore, the effectiveness of the algorithm can be investigated by analyzing the linear relation between the results of the ranking method and the propagation capability of the nodes. The higher the correlation of two methods is, the higher the performance of the method is. The correlation coefficient is a classical and intuitive representation method. There are many correlation coefficients. Spearman rank correlation coefficient has been applied to explore the effectiveness of the method.

In our researches, the top 10% nodes selected by each method were evaluated using correlation coefficient method. It is assumed that the infection probability of SI epidemic model is in the 0.0-0.1 range. Then, according to different infection probability, the Spearman correlation coefficient between the results of different algorithms and the result simulated by SI epidemic model was calculated.The effectiveness of the method can be investigated by the Spearman correlation coefficient between the ranked results calculated by this method and the simulation results by SI epidemic model.

C. Experimental analysis

a. Network efficiency assessment method

In our researches, four classic methods such as DC, CC, BC and DIL are selected to compare with the LSI method proposed in this paper on four real networks, such as the European grid, dolphin social network, contiguous USA network, and the US grid.

In each network, the top 10% of nodes are deleted in turn according to the ranking results generated by every method, and then the decline rate of efficiency is evaluated. It can be obtained that the decline rate of network efficiency is proportional to node importance. In other words, the decline rate of network efficiency after a node was deleted from the network can indicate the importance of this node.

Fig. 3-6 demonstrate the curves of five ranking methods on four real networks. The magnitude of the decline in network efficiency indicates the importance of deleted node. The height of the curve in the figure indicates the importance of the nodes sorted by each method. The higher the curve is, the more important the deleted node is and the more accurate the corresponding method is.

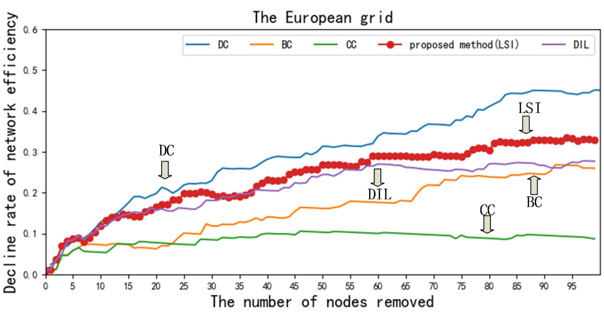

Fig. 3. The tendency of the decline rate of network efficiency when the node was removed in turn from the European grid.

As shown in Fig. 3, when the European grid is used in our experiment, the red curve indicates the decline rate of network efficiency calculated by the LSI method, the blue line indicates DC. It can be concluded that DC method and the LSI method have the best performance, while CC method has the worst performance. It can be seen from Fig. 4 that CC method is better when dolphin social network is applied. Similarly, as far as the contiguous USA network is concerned, LSI method and DC method perform better than others, which has been shown in Fig. 5. LSI method perform best on the US grid, as shown in Fig. 6. To sum up, the LSI node ranking method perform fairly well in term of network efficiency.

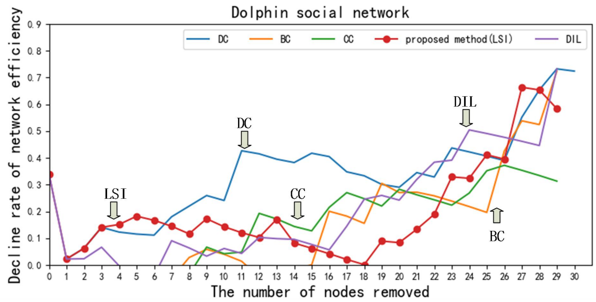

Fig. 4. The tendency of the decline rate of network efficiency when the node was removed in turn from dolphin social network.

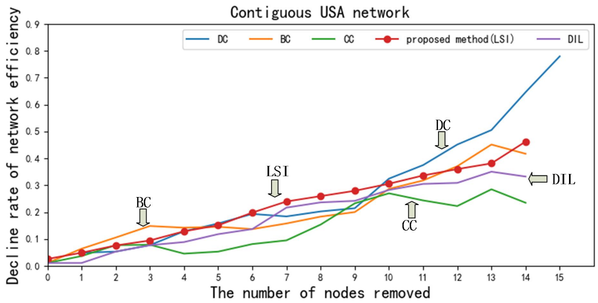

Fig. 5. The tendency of the decline rate of network efficiency when the node was removed in turn from contiguous USA network.

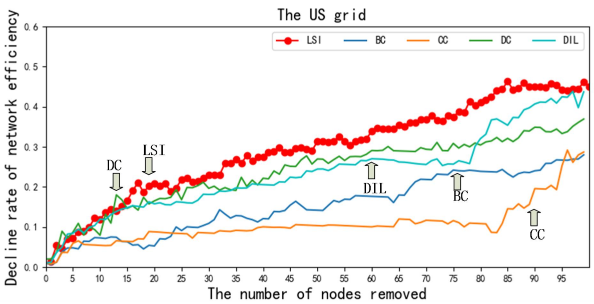

Fig. 6. The tendency of the decline rate of network efficiency when the node was removed in turn from the US grid.

TABLE I
THE RELATION BETWEEN INFECTION CAPACITY AND SPREAD NODES FROM THE EUROPEAN GRID

Dataset	The European grid				
Spread nodes	LSI	DC	BC	CC	DIL
1	**62**	53	62	50	62
2	60	47	62	39	50
3	55	58	23	16	22
4	45	41	47	46	41
5	55	35	11	27	61
6	**52**	24	28	41	30
7	**45**	36	10	36	45
8	**51**	33	34	23	32
9	**37**	26	17	13	22
10	28	31	15	35	47

b. The average infectivity of the top 10 percent nodes

In our researches, the top 10% nodes selected by each method were evaluated using SI epidemic model, and experiments were carried out in accordance with the principle that the more nodes one node can infect with, the more powerful its propagation ability is, therefore the more important the node is. Table I-IV show the propagation capability of the top-10 nodes selected by each method on four real networks.

TABLE II
THE RELATION BETWEEN INFECTION CAPACITY AND SPREAD NODES FROM DOLPHIN SOCIAL NETWORK

Dataset	Dolphin social network				
Spread nodes	LSI	DC	BC	CC	DIL
1	**52**	42	50	51	39
2	**53**	46	24	45	43
3	45	43	38	37	46
4	39	35	39	42	45
5	38	32	35	38	44
6	**45**	25	16	36	39
7	39	38	42	28	30
8	**38**	38	32	31	34
9	37	23	37	24	38
10	**35**	17	30	18	22

TABLE III
THE RELATION BETWEEN INFECTION CAPACITY AND SPREAD NODES FROM CONTIGUOUS USA NETWORK

Dataset	Contiguous USA network				
Spread nodes	LSI	DC	BC	CC	DIL
1	**33**	30	30	26	30
2	**35**	21	26	21	26
3	29	25	20	20	32
4	27	33	16	23	26
5	**23**	10	18	13	23
6	20	21	21	21	18
7	**30**	18	22	17	18
8	**24**	18	19	10	25
9	20	21	20	21	19
10	18	21	19	16	16

TABLE IV
THE RELATION BETWEEN INFECTION CAPACITY AND SPREAD NODES FROM THE US GRID

Dataset	The US grid				
Spread nodes	LSI	DC	BC	CC	DIL
1	62	53	63	50	72
2	60	50	62	39	60
3	**75**	48	43	26	52
4	**65**	41	47	46	41
5	**72**	35	20	37	61
6	**65**	24	28	41	50
7	**59**	26	20	36	45
8	**44**	29	24	23	42
9	39	26	20	13	40
10	38	27	16	35	45

The declining of the ability of transmission indicates that the importance of nodes decreases in turn. The infection capacity of one node is proportional to its importance in the network. The bigger the amount of infected nodes is, the more accurate the ranking method is.

The items in bold in the Table I-IV represent the infection capacity of the LSI method. It can be seen from Table IV that the infection capacity of nodes extracted by the LSI node ranking method is stronger than the nodes by other ranking methods when the US grid is used, which means that the nodes ranking by the LSI method are more important. Meanwhile, it can be concluded from Table I-III that the LSI ranking method performs better on the European grid, dolphin social network and contiguous USA network.

c. The spearman correlation coefficient

The correlation of the ranking results generated by five method with that of the SI model method has been shown in Fig. 11-14.

Fig. 11 indicates that the results of LSI , DIL have the maximum matching degree with that of SI epidemic model in term of the spearman correlation coefficient on the European grid. As shown in Fig. 12, The spearman correlation coefficient of LSI method is slightly higher than that of other methods on the dolphin social network and the result of CC centrality algorithm is unstable with the increase of infection probability. Fig. 13 indicates that there exists maximum matching degree between LSI ranking method and SI epidemic model on the contiguous USA network.

It is not difficult to draw a conclusion that the LSI slightly performs better than other node ranking methods. As far as the US grid is concerned, the LSI method is slightly higher than other node ranking methods, while CC method has the worst performance.

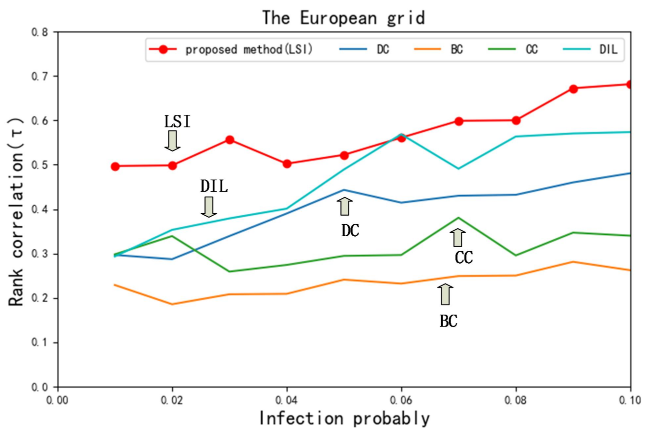

Fig.11. The correlation coefficient of five algorithms and SI epidemic model on the European grid.

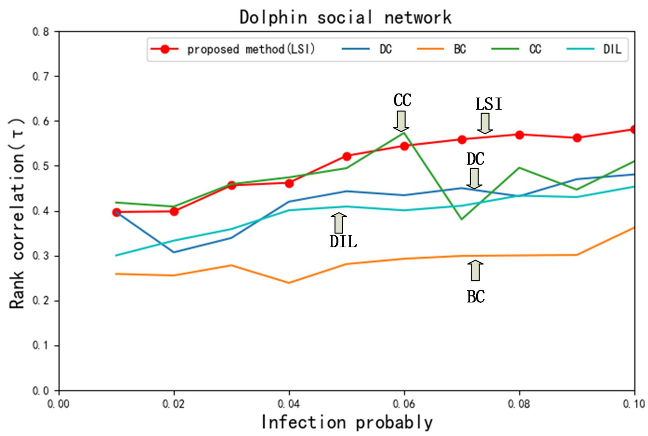

Fig.12. The correlation coefficient of five algorithms and SI epidemic model

in dolphin social network.

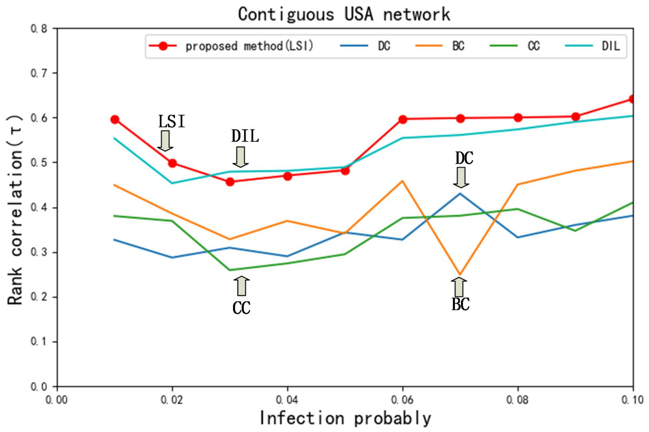

Fig.13. The correlation coefficient between different algorithms and SI epidemic model in contiguous USA network.

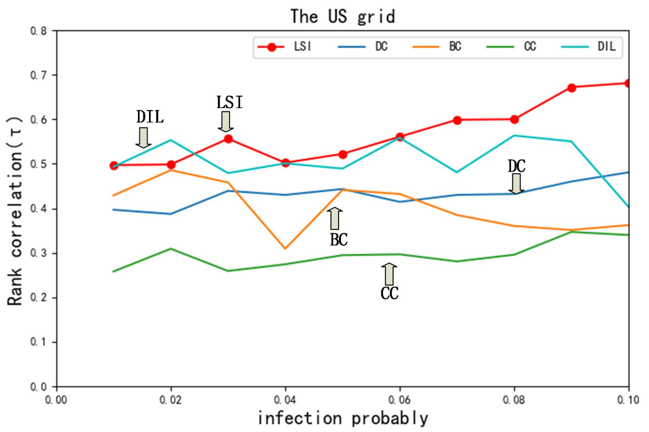

Fig.14. The correlation coefficient between different algorithms and SI epidemic model in the US grid.

IV. Conclusion

In this paper, a new ranking method of node importance is proposed, which incorporates the neighbor lines and local information and comprehensively evaluates the importance of nodes with lower time complexity. SI epidemic model method, network efficiency and correlation coefficient were used to evaluate the ranking results. In the end, the experimental results have been analyzed on four real networks. It can be argued that the LSI method perform fairly well, and can better identify the important degree of bridge nodes as well.

The LSI method is only applicable to static networks. Whether it can be utilized to address dynamic networks or not remains to explore further. Therefore, future researches will focus on the ranking and verification methods of the importance of nodes with the proposed solution based on dynamic networks.

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